COMPUTER SCIENCE & ENGINEERING

EECS498-008
Formal Verification of Systems Software

Material and slides created by

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Some new Dafny syntax

Datatype member functions

```
datatype Pet = Dog | Cat | Ant | Spider {
  function CountLegs() : int {
    match this
      case Dog => 4
      case Cat => 4
      case Ant => 6
      case Spider => 8
  }
}

function ShoesForTwo(pet: Pet) : int {
  2 * pet.CountLegs()
}
```

Some new Dafny syntax

Calc statements

```
assert a == b;
assert b == c;
assert c == d;

calc {
  a;
  b;
  c;
  d;
}
```

Some new Dafny syntax

Calc statements

```
assert a == b;
assert b == c;
assert c == d;

calc {
  a;
  { MyUsefulLemma(a,b); }
  b;
  c;
  d;
}
```

Some new Dafny syntax

Calc statements

```
assert a == b;
assert b == c;
assert c == d;

calc ==> {
  a;
  { MyUsefulLemma(a,b); }
  b;
  c;
  d;
}
```

Some new Dafny syntax

Choose operator

```
assert 1 % 7 == 1;
assert exists x :: x % 7 == 1;
var x :| x % 7 == 1;
```

Choose x such that…

Administrivia

Remember that Problem Set 1 is due this Friday

I’m still missing some of your pictures. Please send me your picture (manosk@umich.edu) with the Subject “EECS498-008 picture”

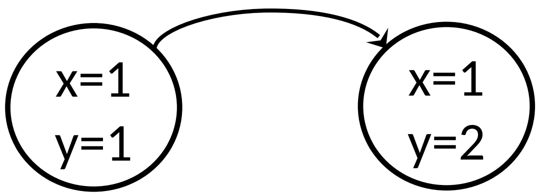
Chapter 3: Building state machines

A state is an assignment of values to variables

An action is a transition from one state to another

An execution is a sequence of states

We will capture executions with state machines



Building state machines

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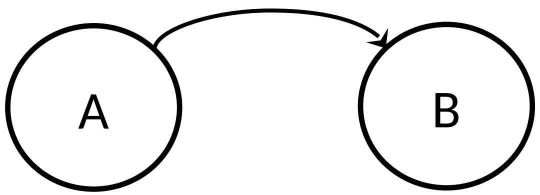
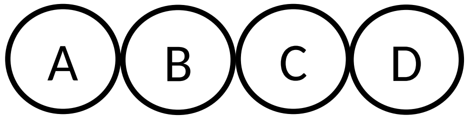
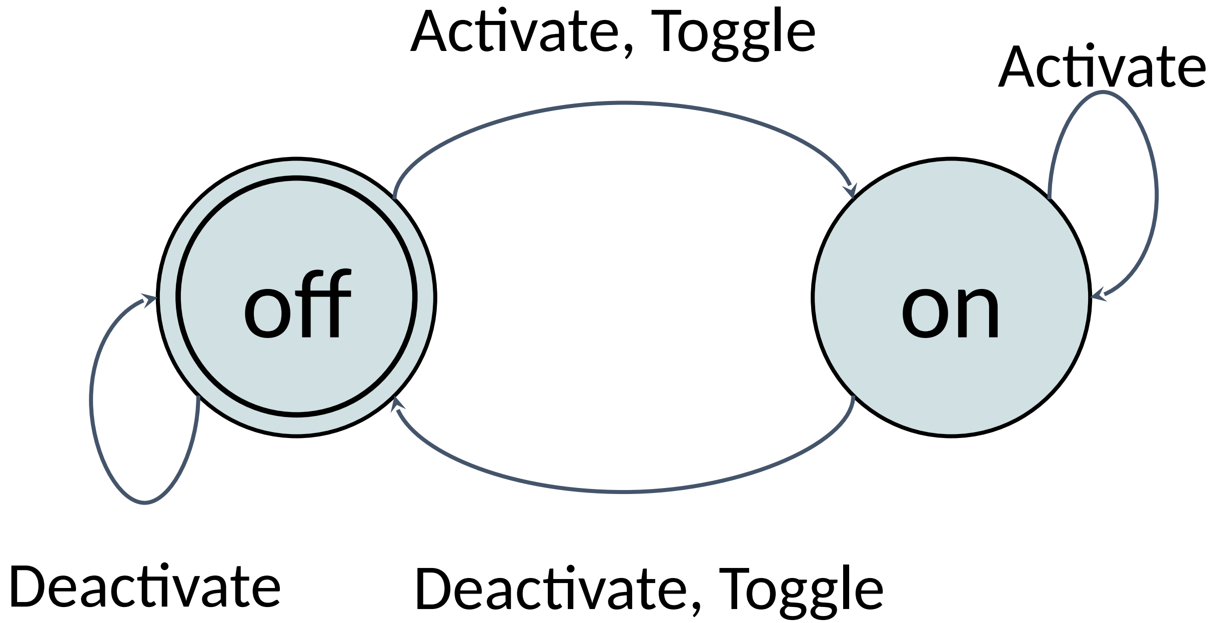
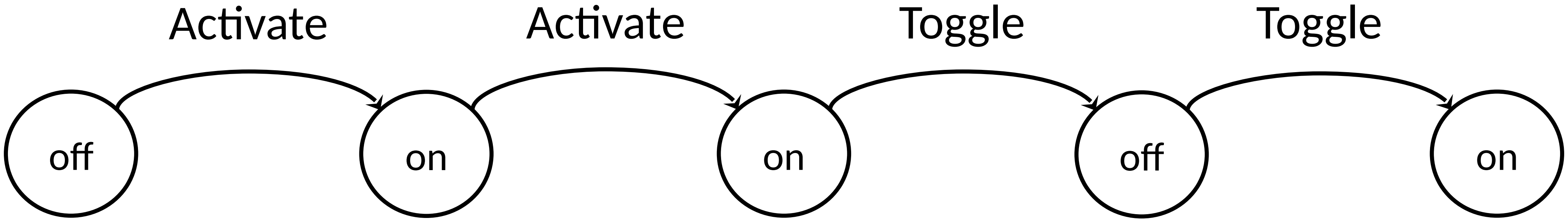


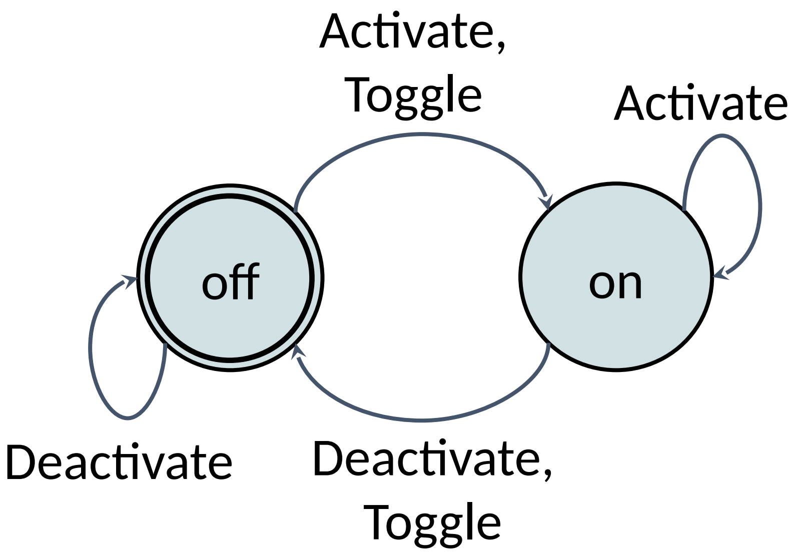


The Switch state machine





The Switch state machine




```
datatype SwitchState = On | Off
datatype Variables =
        Variables(switch:SwitchState)
predicate Init(v:Variables) {
    v.switch == Off
}
```

```
predicate Activate(v:Variables, v':Variables) {
    v'.switch == On
}
predicate Deactivate(v:Variables, v':Variables)
{
    v'.switch == Off
}
predicate Toggle(v:Variables, v':Variables) {
    v'.switch == if v.switch.On? then Off else
On
}
predicate Next(v:Variables, v':Variables) {
    || Activate(v, v')
    || Deactivate(v, v')
    || Toggle(v, v')
}
```

The Game of Nim

$$11 \xrightarrow{\text{Play(1)}} 10 \xrightarrow{\text{Play(3)}} 7 \xrightarrow{\text{Play(2)}} 5 \xrightarrow{\text{Play(4)}} 1$$

The Nim state machine

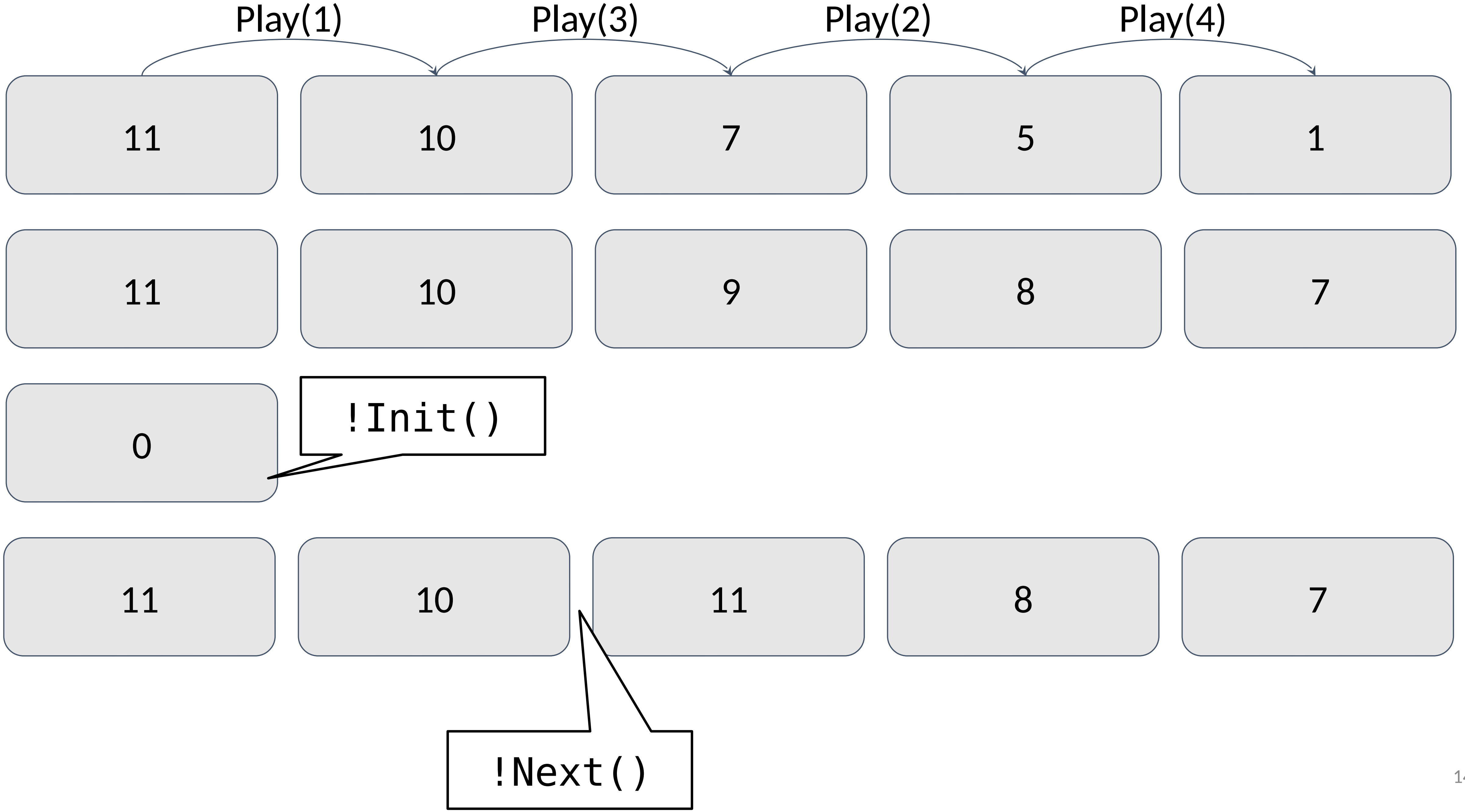
```
datatype Variables = Variables(tokens:nat)
predicate Init(v:Variables) {
    v.tokens > 0
}

predicate Play(v:Variables, v':Variables, take:nat) {
    && 1 <= take <= 4
    && take <= v.tokens
    && v'.tokens == v.tokens - take
}

predicate Next(v:Variables, v':Variables)
{
    exists take :: Play(v, v', take)
}
```

enabling condition

“update”


Play(1)
Play(3)
Play(2)
Play(4)
11
10
7
5
1
11
10
9
8
7
0
!Init()
11
10
11
8
7
!Next()

A **state** is an assignment of values to va

```
datatype Card = Shelf | Patron(name: string)
datatype Book = Book(title: string)
type Library = map<Book, Card>
```

The state space is the set of possible assignments.

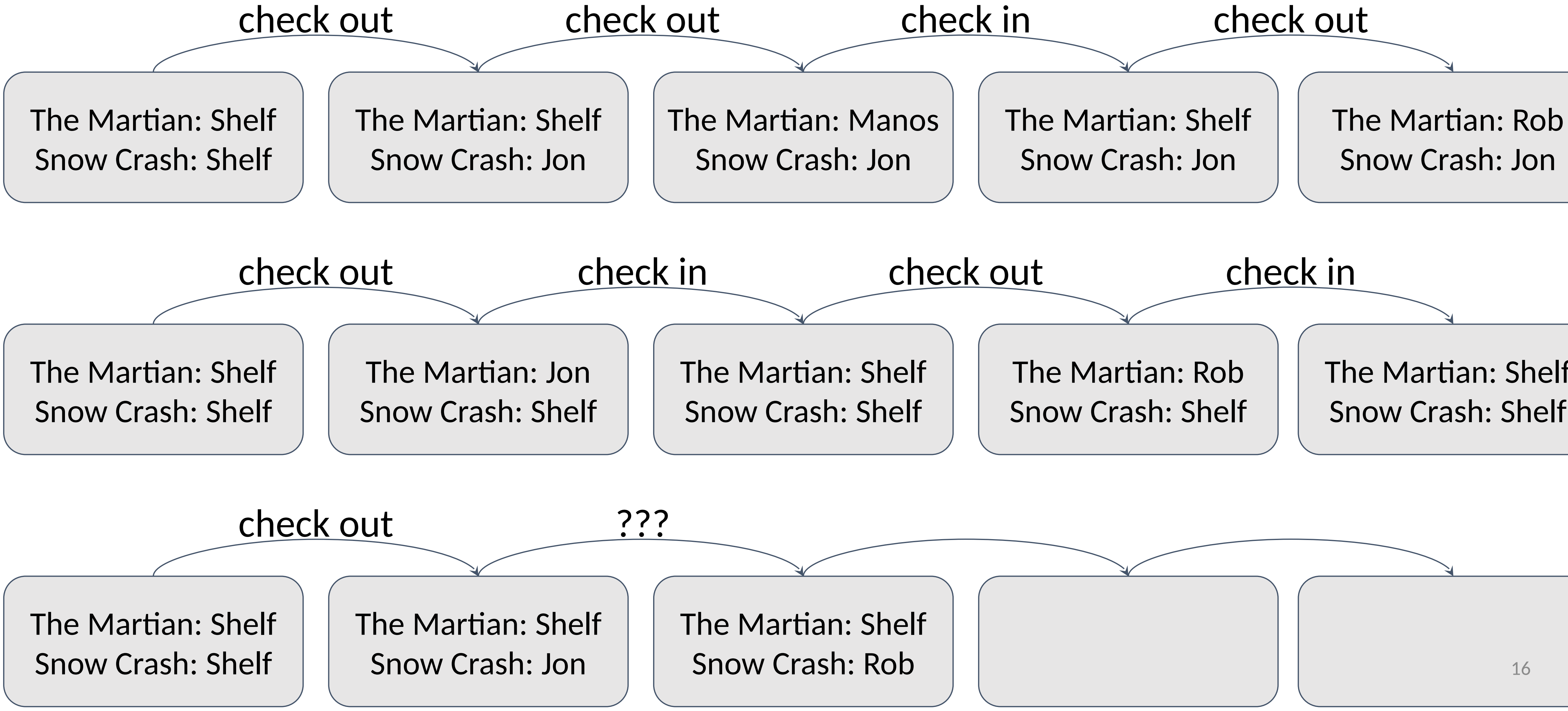
The Martian: Shelf
Snow Crash: Shelf

The Martian: Shelf
Snow Crash: Jon

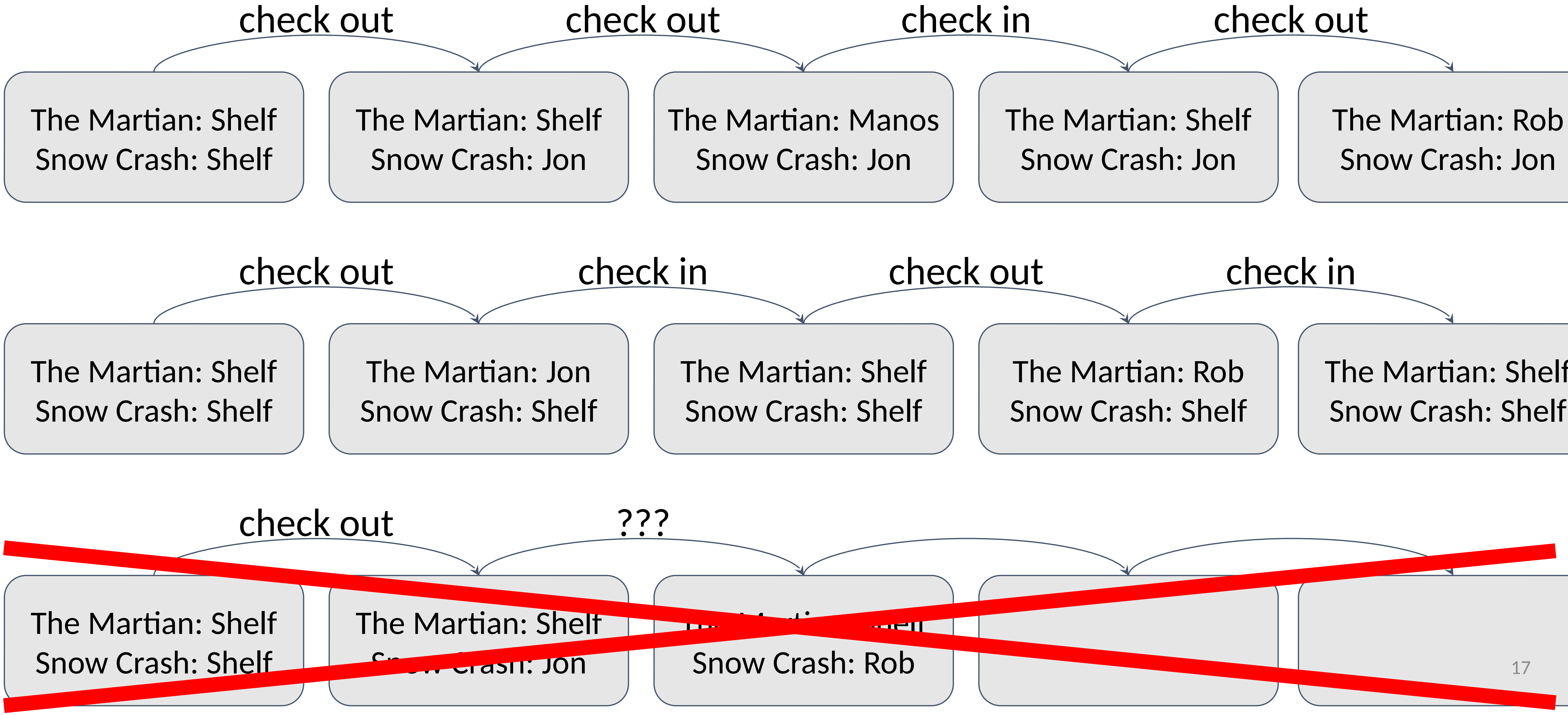
~~The Martian: Jon~~
~~Snow Crash: Jon~~

The Martian: Manos
Snow Crash: Jon

An execution is an infinite sequence of states



A **behavior** is the set of **all possible** executions



How should we define a behavior?

With a **program**?

Its variables define its state space
Its executions define its behavior

Weaknesses:

- concreteness
- nondeterminism
- asynchrony
- environment

How should we define a behavior?

With a **state machine**

Its type defines its state space
Its initial states and transitions define its behavior

A state machine definition

```
datatype Card = Shelf | Patron(name: string)
datatype Book = Book(title: string)
type Library = map<Book, Card>
```

```
predicate Init(v: Library) {
  forall book | book in v :: v[book] == Shelf
}
predicate CheckOut(v : Library, v' : Library, book: Book, name: string) {
  && book in v
  && v[book] == Shelf
  && (forall book | book in v :: v[book] != Patron(name))
  && v' == v[book := Patron(name)]
}
predicate CheckIn(v : Library, v' : Library, book: Book, name: string) {
  && book in v
  && v[book] == Patron(name)
  && v' == v[book := Shelf]
}
predicate Next(v: Library, v': Library) {
  || (exists book, name :: CheckOut(v, v', book, name))
  || (exists book, name :: CheckIn(v, v', book, name))
}
```

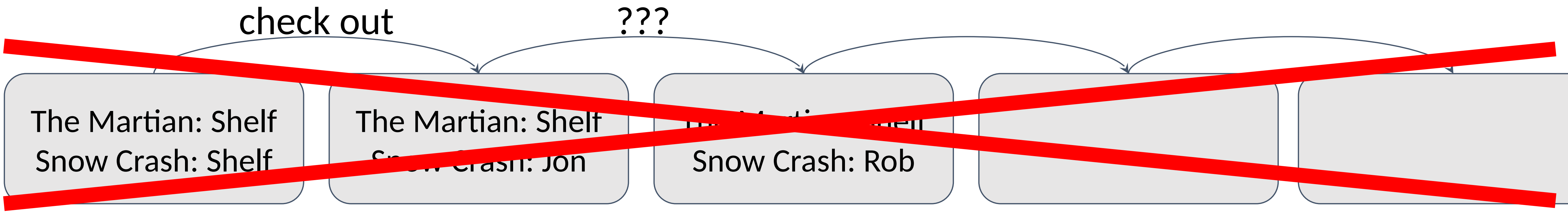
enabling condition

“update”

Nondeterministic definition

A behavior is the set of all possible executions

```
predicate CheckOut(v, v’, book, name) {
  && book in v
  && v[book] == Shelf
  && (forall book | book in v :: v[book] !=
Patron(name))
  && v’ == v[book := Patron(name)]
}
predicate CheckIn(v, v’, book, name) {
  && book in v
  && v[book] == Patron(name)
  && v’ == v[book := Shelf]
}
```



State machine strengths

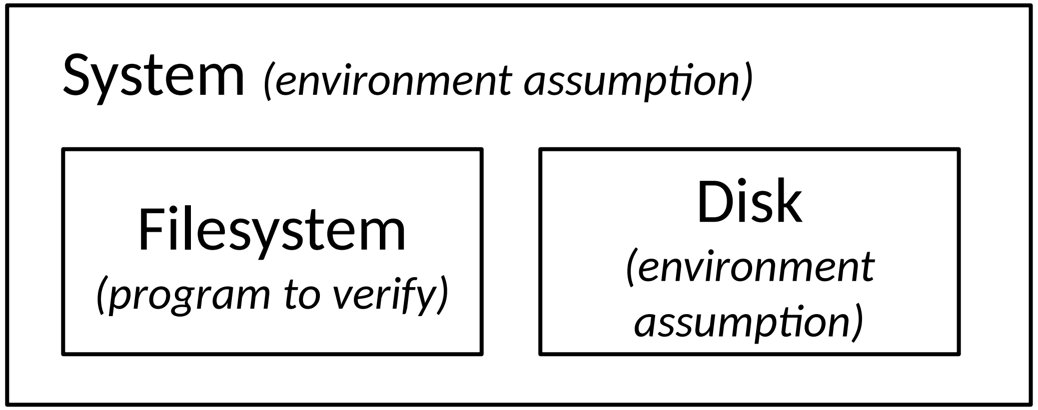
- Abstraction
  - States can be abstract
    - Model an infinite map instead of an efficient pivot table
  - Next predicate is nondeterministic:
    - Implementation may only select some of the choices
    - Can model Murphy’s law (e.g. crash tolerance) or an adversary

State machine strengths

- Abstraction
- Asynchrony
  - Each step of a state machine is conceptually atomic
  - Interleaved steps capture asynchrony: threads, host processes, adversaries
  - Designer decides how precisely to model interleaving; can refine/reduce

State machine strengths

- Abstraction
- Asynchrony
- Environment
  - Model a proposed program with one state machine (verified)
  - Model adversarial environment with another (trusted)
  - Compound state machine models their interactions (trusted)



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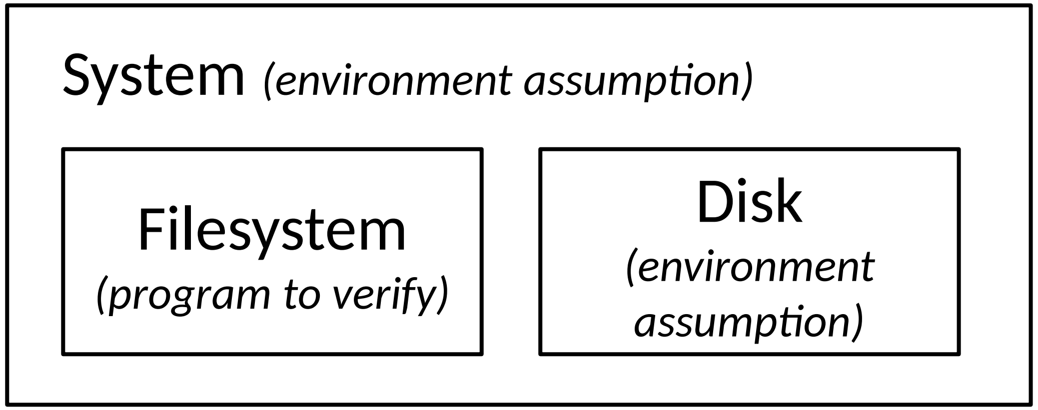
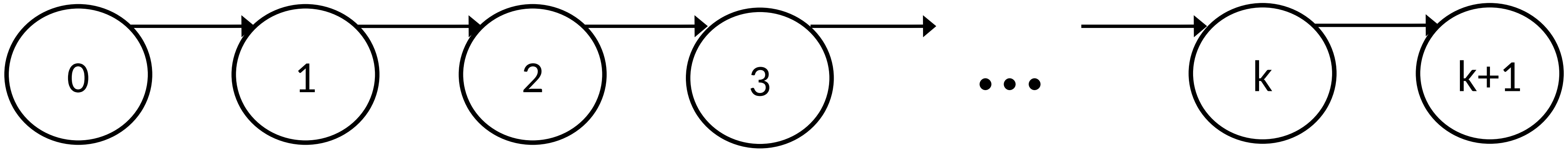


Chapter 4: Proving properties

Expressing a system as a state machine allows us to prove that it has certain properties

- We will focus on safety properties

Basic tool: induction



- Show that the property holds on state 0
- Show that if the property holds on state k, it must hold on state k+1

Let’s prove a safety invariant!

```
predicate Safety(v:Library) {
  true // TBD
}

lemma SafetyProof()
  ensures forall v :: Init(v) ==> Safety(v)
  ensures forall v, v' :: Safety(v) && Next(v, v') ==> Safety(v')
{
}
```

Base case

Inductive Step

Let’s prove a safety invariant!

Interactive proof development in editor

- *Bisection debugging,*
- *case analysis,*
- *existential instantiation*