Inverted pendulum

This system (see figure 1) comprises an inverted pendulum mounted on a chariot able to translate on one axe. The automaton simulates the control of the moving part so that the pendulum remains in an unstable state. From an energetic perspective, the pendulum has to follow three behaviour, depending on its need to add, subtract or maintain its mechanical energy []. Thus, the state of the system is described by its mechanical energy E, its angular position and angular speed θ and $\dot{\theta}$.

$$J\ddot{\theta} = mgl\sin\theta - mal\cos\theta \tag{1}$$
$$E = 0.5J\dot{\theta}^2 + mgl(\cos\theta - 1) \tag{2}$$
$$\dot{E} = -mal\dot{\theta}\cos\theta \tag{3}$$

Physical system. The full demonstration of the control method is available in the original paper, the following is a summary of the principles used. Equation 1 is the application of Newton's principle to the system and equation 2 is the expression of the system's mechanical energy. m is the pendulum mass, J its moment of inertia, l is half its length and a is the acceleration of the moving platform. Deriving equation 2 and substituting $\ddot{\theta}$ via equation 1 gives equation 3. This expression shows that it is possible to control the variation of the system's energy by changing the acceleration of the platform.

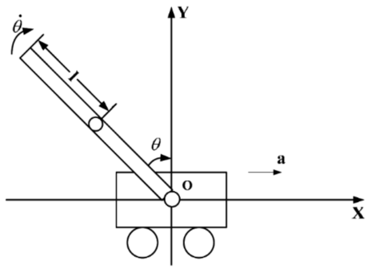

Figure 1: Physical system (source : original paper)

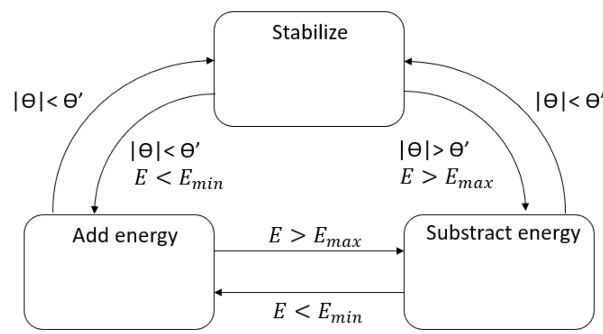

Figure 2: simplified view of the automaton

The automaton. The different behaviours of the system are dictated by the value of the parameter a (acceleration). Equation 3 provides the value of a to achieve the balance of the pendulum :

- when the system lacks energy, the choice $a = -\dot{\theta}\cos\theta$ allows the energy to increase.
- when the system has too much energy, the choice $a = \dot{\theta}\cos\theta$ allows the energy to reduce.
- when the system is close to the desired position ($\theta = 0$), the pendulum is stabilised with a proportional correction : $a = -k_1\theta - k_2\dot{\theta}$

Finally, let $S = [x_1,\ x_2,\ x_3]^T$ the state vector of the system. The following flow is verified in each location :

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = \frac{mgl\sin x_1 - mal\cos x_1}{J} \\ \dot{E} = -malx_2\cos x_1 \end{cases}$$