## Drone failure management

This automaton is a model of a 6 propeller drone. The automaton simulates the behaviour of the hovering drone as it attempts to remain stable after some of its motors shutdown. Thus, each location corresponds to a failure scenario and the associated flow comprises the equation of the system's dynamic. Failure scenarios and equations of dynamic originate from previous work on the control of hovering hexacopter during failure [1]. The drone is located by the position and orientation of its center of mass. To describe the system's dynamic, both position and velocities have to be known. The state of the system is described in the vector:

$$S = (x \ y \ z \ \phi \ \theta \ \psi \ u \ v \ w \ p \ q \ s)^T$$

Where  $(\phi, \theta, \psi)$  are the Euler angle, (u, v, w) the acceleration and (p, q, r) the angular acceleration. In addition, the variables of the system are the rotation speed of each propeller  $\Omega_i$ . In the original paper, the speed of each controller is supposed to be positive, therefore the change of variable  $\Omega_i = \Omega_{r,i}^2$  has been made. Each rotor speed can be individually controlled. Thus, each flow will be comprised of the PFD to update S and the equations of control to update S.

$$\Omega = (\Omega_{r,1}, \Omega_{r,2}, \Omega_{r,3}, \Omega_{r,i4}, \Omega_{r,5}, \Omega_{r,6})^T$$

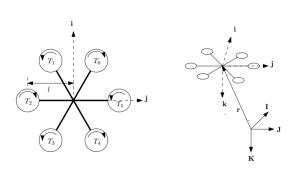


Figure 1: Geometry of studied system

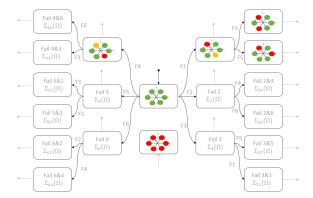


Figure 2: simplified view of the automaton

**Hypothesis.** The forces exerted on the system are the thrust, the rotor drag, the air resistance and the weight. The moments exerted on the system are the torques due to rotational rate difference between rotors, gyroscopic effect and yaw counter torque. We will use the same notation as in the original paper:  $J_r$  is the inertia of one propeller, l is the distance between a propeller and the center of mass, d is the drag factor, b is the thrust factor,  $K_T$  and  $K_D$  are constants

**Model.** The system of equations satisfied by velocities and acceleration can be derived from the fundamental principle of dynamic as follow.

$$\begin{cases} \dot{x} = \cos\theta\cos\psi u + (\cos\psi\sin\phi\sin\theta - \cos\psi\cos\phi)v + (\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta)w \\ \dot{y} = \cos\theta\sin\psi u + (\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi)v + (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)w \\ \dot{z} = -\sin\theta u + \cos\theta\sin\phi v + \cos\theta\cos\phi w \\ \dot{\phi} = p + \sin\phi\tan\theta q + \cos\phi\tan\theta r \\ \dot{\theta} = \cos\phi q - \sin\phi r \\ \dot{\psi} = \frac{\sin\phi}{\cos\theta}q + \frac{\cos\phi}{\cos\theta}r \end{cases}$$

$$\begin{cases} &\dot{u}=vr-qw-g\sin\theta-\frac{K_T+K_DV}{T}u\\ &\dot{v}=pw-ru+g\sin\phi\cos\theta-\frac{K_T+K_DV}{T}v\\ &\dot{w}=qu-pv+g\cos\phi\cos\theta-\frac{T_t}{m}-\frac{m_{K_DV}}{m}w\\ &\dot{p}=\frac{I_{yy}-I_{zz}}{I_{xx}}qr+\frac{M_{\phi}-J_r\eta\Omega_R}{I_{xx}}\\ &\dot{q}=\frac{I_{zz}-I_{xx}}{I_{yy}}pr+\frac{M_{\phi}+J_r\Omega_R}{I_{yy}}\\ &\dot{r}=\frac{I_{xx}-I_{yy}}{I_{zz}}pq+\frac{M_{\psi}-J_r\Omega_R}{I_{zz}} \end{cases}$$

The key difference between each state of the system is the way in which the action of gravity is compensated by the thrust force. This action has to be distributed across motors so that the drone can remain in the air while not experiencing torque along its pitch and roll axis. Let  $F_t = b \sum \Omega_i^2$  the thrust force.

• case 1 : 
$$\Omega_i = \sqrt{\frac{mg}{6b}} \quad \forall i$$

• case 2:

$$\Omega_6 = \sqrt{\frac{2mg}{5b}}$$

$$\Omega_i = \sqrt{\frac{mg}{5b}} \quad \forall i \in \{2, 3, 4\}$$

Where the following abbreviations have been made:

$$V = \sqrt{u^2 + v^2 + w^2}$$

$$T_t = b \sum_{i=1}^6 \Omega_{r,i}^4$$

$$M_\phi = \frac{bl}{2} (2\Omega_{r,2}^4 - 2\Omega_{r,5}^4 + \Omega_{r,1}^4 + \Omega_{r,3}^4 - \Omega_{r,4}^4 - \Omega_{r,6}^4)$$

$$M_\theta = \frac{\sqrt{3}bl}{2} (\Omega_{r,1}^4 + \Omega_{r,6}^4 - \Omega_{r,3}^4 - \Omega_{r,4}^4)$$

$$M_\psi = d(\Omega_{r,1}^4 - \Omega_{r,2}^4 + \Omega_{r,3}^4 - \Omega_{r,4}^4 + \Omega_{r,5}^4 - \Omega_{r,6}^4)$$

$$\Omega_R = \sum_{i=1}^6 (-1)^i \Omega_{r,i}^2$$

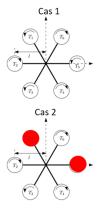


Figure 3: Two possible states of the system

Finally, this model allows for the use of different control strategy. For now, only a simple feedback loop with proportional correction is implemented and the coefficient have only been tuned via trial and error. In addition, control is limited to three DOF: pitch, roll and translation along K axis. Equation 1 presents the control equation for rotor i. The only difference between rotors is the sign of  $K_{\theta}$  and  $K_{\phi}$  which depends on the contribution of the propeller to pitch and roll.

$$\dot{\Omega}_i = K_w w + K_z (z_0 - z) + K_\theta^i \theta + K_\phi^i \phi \tag{1}$$

This control strategy does not succeed in managing successive failure scenarios if the two successive states involve an important variation relative to the stable state of pitch and roll. Further work is required to implement more complex control strategies.

**Automaton.** A simplified view of the automaton is presented figure 3. The locations correspond to the failure scenarios presented in the original paper in which the hovering of the drone remains manageable. In the case of too many motors failing, the drone ends up in a uncontrollable terminal state. The motors highlighted in red are considered no longer usable whereas the motors in yellow are only temporally shutdown for balancing the drone. This automaton is not autonomous. Thus it is able to observe the following set of discrete variables that play the role of events triggering the transitions.

$$F = \{F_1, F_2, F_3, F_4, F_5, F_6\} \quad \text{where } F_i = \begin{cases} 0 & \text{while motor i is operational} \\ 1 & \text{when motor i fails} \end{cases}$$
 (2)

## References

[1] Fu-Hsuan Wen, Fu-Yuen Hsiao and Jaw-Kuen Shiau, Analysis and Management of Motor Failures of Hexacopter in Hover