Université Paris Cité

INSTITUT DE RECHERCHE EN INFORMATIQUE FONDAMENTALE

Gaëtan LOPEZ

1st year PhD

supervised by

Giovanni Bernardi and Hugo Férée

Academic background

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- Studies in « Mathématiques Fondamentales » Université Paris Cité

⇒ Algebra, analysis, topology, probability, ...

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- Studies in « Mathématiques Fondamentales » Université Paris Cité

  ⇒ Algebra, analysis, topology, probability, …

- Studies in « Logique Mathématique et Fondements en Informatique » Université Paris Cité

  ⇒ Proof theory, calculability, formal proof, …

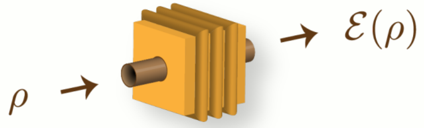
Thanks for listening !

Presention available at ：

GLMATHS GitHub

Main scientific interest <but not only>

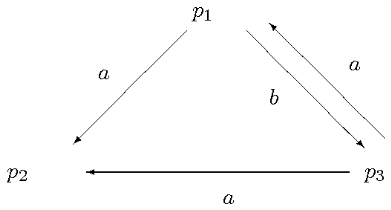
- Quantum Computing and Quantum Information



- Lambda Calculus

$$M := \lambda x.M \mid M_1\, M_2 \mid x$$

- Process Calculus



- COQ

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Inductive nat : Type :=
  | O : nat
  | S : nat -> nat.
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Main achievements

- Elaboration of a quantum lambda calculus with measurement, normalization and confluence
  <supervised by Claudia Faggian and Benoît Valiron>

- Formalization of CCS with Value Passing and Sequenciation in COQ
  + the Harmony Lemma in COQ
  <supervised by Giovanni Bernardi and Hugo Férée>

- Paper accepted at CSL 2025 :
  « A Rewriting Theory for Quantum λ-Calculus »
  <with Claudia Faggian and Benoît Valiron>

What's the plan ?

- Study linear quantum CCS

  <from Lorenzo Ceragioli, Fabio Gadducci, Giuseppe Lomurno and Gabriele Tedesch, 2023>

$$\frac{e \Downarrow tt}{\langle \rho, \mathbf{if}\ e\ \mathbf{then}\ P\ \mathbf{else}\ Q\rangle \xrightarrow{\tau} \overline{\langle \rho, P\rangle}}\ \textsc{IteT} \qquad \frac{e \Downarrow ff}{\langle \rho, \mathbf{if}\ e\ \mathbf{then}\ P\ \mathbf{else}\ Q\rangle \xrightarrow{\tau} \overline{\langle \rho, Q\rangle}}\ \textsc{IteF}$$

$$\frac{}{\langle \rho, \tau.P\rangle \xrightarrow{\tau} \overline{\langle \rho, P\rangle}}\ \textsc{Tau} \qquad \frac{e \Downarrow v}{\langle \rho, c!e.P\rangle \xrightarrow{c!v} \overline{\langle \rho, P\rangle}}\ \textsc{Send} \qquad \frac{v \in \Sigma_\rho}{\langle \rho, c?x.P\rangle \xrightarrow{c?v} \overline{\langle \rho, P[^v/_x]\rangle}}\ \textsc{Receive}$$

$$\frac{\langle \rho, P\rangle \xrightarrow{\mu} \Delta}{\langle \rho, P + Q\rangle \xrightarrow{\mu} \Delta}\ \textsc{SumL} \qquad \frac{\langle \rho, Q\rangle \xrightarrow{\mu} \Delta}{\langle \rho, P + Q\rangle \xrightarrow{\mu} \Delta}\ \textsc{SumR} \qquad \frac{\langle \rho, P\rangle \xrightarrow{\mu} \Delta \quad \mu \neq c!v, c?v}{\langle \rho, P \setminus c\rangle \xrightarrow{\mu} \Delta \setminus c}\ \textsc{Restrict}$$

$$\frac{}{\langle \rho, \mathcal{E}(\tilde q).P\rangle \xrightarrow{\tau} \overline{\langle \mathcal{E}_{\tilde q}(\rho), P\rangle}}\ \textsc{QOp} \qquad \frac{\rho_m = (M_m)_{\tilde q}(\rho) \quad p_m = \mathrm{tr}(\rho_m)}{\langle \rho, \mathcal{M}(\tilde q \rhd y).P\rangle \xrightarrow{\tau} \sum_{m=0}^{|M|-1} p_m \bullet \overline{\langle \frac{\rho_m}{p_m}, P[^m/_y]\rangle}}\ \textsc{QMeas}$$

$$\frac{\langle \rho, P\rangle \xrightarrow{\mu} \Delta \quad \mu = c?v \Rightarrow v \notin \Sigma_Q}{\langle \rho, P \parallel Q\rangle \xrightarrow{\mu} \Delta \parallel Q}\ \textsc{ParL} \qquad \frac{\langle \rho, P\rangle \xrightarrow{c!v} \overline{\langle \rho, P'\rangle} \quad \langle \rho, Q\rangle \xrightarrow{c?v} \overline{\langle \rho, Q'\rangle}}{\langle \rho, P \parallel Q\rangle \xrightarrow{\tau} \overline{\langle \rho, P' \parallel Q'\rangle}}\ \textsc{SynL}$$

$$\frac{\langle \rho, Q\rangle \xrightarrow{\mu} \Delta \quad \mu = c?v \Rightarrow v \notin \Sigma_P}{\langle \rho, P \parallel Q\rangle \xrightarrow{\mu} P \parallel \Delta}\ \textsc{ParR} \qquad \frac{\langle \rho, P\rangle \xrightarrow{c?v} \overline{\langle \rho, P'\rangle} \quad \langle \rho, Q\rangle \xrightarrow{c!v} \overline{\langle \rho, Q'\rangle}}{\langle \rho, P \parallel Q\rangle \xrightarrow{\tau} \overline{\langle \rho, P' \parallel Q'\rangle}}\ \textsc{SynR}$$

arXiv

What's the plan ?

- Study the MUST preorder with his characterization

  <from Giovanni Bernardi, Ilaria Castellani, Paul Laforgue, Léo Stefanesco, 2024>

Definition *We write*

- $p \preccurlyeq_{\mathsf{cnv}} q$ *whenever* $\forall s \in \mathsf{Act}^\star .\, p \Downarrow_A s$ *implies* $q \Downarrow_B s$,
- $p \preccurlyeq_{\mathsf{acc}} q$ *whenever* $\forall s \in \mathsf{Act}^\star .\, p \Downarrow_A s$ *implies* $\mathcal{A}(p, s, \longrightarrow_A) \ll \mathcal{A}(q, s, \longrightarrow_B)$,
- $p \preccurlyeq_{\mathsf{AS}} q$ *whenever* $p \preccurlyeq_{\mathsf{cnv}} q$ *and* $p \preccurlyeq_{\mathsf{acc}} q$.

Theorem *For every* $\mathcal{L}_A, \mathcal{L}_B \in OF$ *and* $p \in A, q \in B$, $p \sqsubseteq_{\scriptscriptstyle MUST} q$ *if and only if* $FW(p) \preccurlyeq_{\mathsf{AS}} FW(q)$.

What's the plan ?

Supervised by Giovanni Bernardi and Hugo Férée :

- Study linear quantum CCS with a testing theory
  <from Lorenzo Ceragioli, Fabio Gadducci, Giuseppe Lomurno and Gabriele Tedesch, 2023>

- Study the MUST preorder with his characterization
  <from Giovanni Bernardi, Ilaria Castellani, Paul Laforgue, Léo Stefanesco, 2024>

- Combine both to get wonderfull properties and/or new theories

- Formalize the results in COQ