SoCraTes Kata Workshop #1 (?)
Time Integrators

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For SoCraTes Chile

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We will have little time and lots to do, hence:

1. Get your computers & IDE up and running!!!
2. Download the materials (follow QR-code)
3. Pick two persons to be on your “team” (best if strangers)

Disclaimer

Hi, I am **Gaël**. 😊

I will be your **<u>animator</u>** today…

…but I am not the ~~**expert**~~!

Today we **exchange** together!

Disclaimer

This is **Workshop #1!**

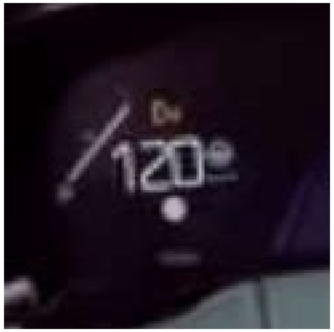
So I’ll do my best, but it could get hectic… 😅

Time Integrators

Time integrators
Explained by example

It is $t^n = 1$ O'clock

You are at
$x^n = 501km$



You drive at
$v(t^n) = 120km/h$

Where will you be
at $t^{n+1} = 6$ O'clock?

$$x^{n+1} = x^n + v(t^n)\,(t^{n+1} - t^n)$$

Time integrators
Explained by Example

There are many ways to answer the question:

"Euler Explicit"
aka **EE**

Use speed $v^n = 120km/h$ at $t^n = 1$

$$x^{n+1} \leftarrow x^n + v^n.(t^{n+1} - t^n) \qquad = 1101km$$

"Euler Implicit"
aka **EI**

Use speed $v^{n+1} = 90km/h$ at $t^{n+1} = 6$

$$x^{n+1} \leftarrow x^n + v(t^{n+1})\,(t^{n+1} - t^n) \;= 801km$$

"Runge-Kutta Midpoint"
aka **RK2**

Use speed $v^n = 120km/h$ to predict $x^{n+\frac{1}{2}}$ and $t^{n+\frac{1}{2}}$

$$t^{n+\frac{1}{2}} \leftarrow \frac{1}{2}\,(t^{n+1} - t^n)$$

$$x^{n+\frac{1}{2}} \leftarrow x^n + v(t^n)\,(t^{n+\frac{1}{2}} - t^n)$$

Then, assume you'll be traveling at $v(t^{n+\frac{1}{2}})$ all along

$$x^{n+1} \leftarrow x^n + v(t^{n+\frac{1}{2}}).\,(t^{n+1} - t^n) = 1051km$$

Those four ways may give different estimations!

$$v(t) := -0.8t^2 - 0.4t + 121.2$$

Time integrators
Definitions

	General	Example
Objective	Estimate q^{n+1} knowing q^n, t^n and t^{n+1}	Estimate x^{n+1} knowing x^n, t^n and t^{n+1}
Equation	$\frac{\partial q}{\partial t} = Rhs(q,t)$	$\frac{\partial x}{\partial t} = v(t)$ $v(t) := -0.8t^2 - 0.4t + 121.2$
Variable	q	x
Right Hand Side	$Rhs(q,t)$	$v(t)$
EE	$q^{n+1} \leftarrow q^n + Rhs(q^n, t^n)\,(t^{n+1} - t^n)$	$x^{n+1} \leftarrow x^n + v(t^n)\,(t^{n+1} - t^n)$
EI	$q^{n+1} \leftarrow q^n + Rhs(q^{n+1}, t^{n+1})\,(t^{n+1} - t^n)$	$x^{n+1} \leftarrow x^n + v(t^{n+1})\,(t^{n+1} - t^n)$
RK2	$t^{n+\frac{1}{2}} \leftarrow \frac{t^{n+1}+t^n}{2}$ $q^{n+\frac{1}{2}} \leftarrow q^n + Rhs(q^n, t^n)\,(t^{n+\frac{1}{2}} - t^n)$ $q^{n+1} \leftarrow q^n + Rhs(q^{n+\frac{1}{2}}, t^{n+\frac{1}{2}})\,(t^{n+1} - t^n)$	$t^{n+\frac{1}{2}} \leftarrow \frac{t^{n+1}+t^n}{2}$ $x^{n+1} \leftarrow x^n + v(t^n)\,(t^{n+\frac{1}{2}} - t^n)$ $x^{n+1} \leftarrow x^n + v(t^{n+\frac{1}{2}})\,(t^{n+1} - t^n)$

Practical Work #1
Getting Familiar with
Time Integrators

Practical Work #1
How we are Going to Work

1 doer, 1 coding-mate, 1 observer

Then, we'll rotate.

"Do" Phase: do ; Doer + Mate do, Observer watches
"Discussion" Phase: Observer gives feedback, Doer & Mate react

Observers: start thinking of your feedback and take notes during Phase 1!

Practical Work #1
Go, GO, GOOO!!!

Implement one or several user cases.
Do not try to implement elaborate architectures now (keep it for later)
The goal here is to become comfortable with the concept of time integrator.

Doer: Implement
Mate: Assist the doer (with the understanding)
Observer: observe

Questions for observers:
How did they decide to approach the problem? Why?
What did they do?   How did they work?   Anything interesting detail?

Practical Work #1
Case #1: Position of Robot

$$\frac{dx}{dt} = v(t)$$

The robot starts moving at time $t^0 = 0\,s$ and position $x^0 = 0\,cm$

Every second until time $t = 10\,s$, the recorded velocity (cm/s) happens to be of the form

$$\begin{array}{ll}
v(t) := 1 & (x(t) = t) \\
v(t) := t & (x(t) = t^2/2) \\
v(t) := t^2 & (x(t) = t^3/3) \\
v(t) := 1 + sin(t) & (x(t) = t - cos(t) + 1
\end{array}$$

$$t^{n+1} - t^n = 1\,s$$

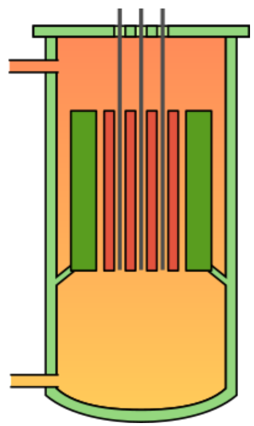
What is the distance travelled at each step? (i.e. every second)

Practical Work #1
Case #2: Nuclear Reactor

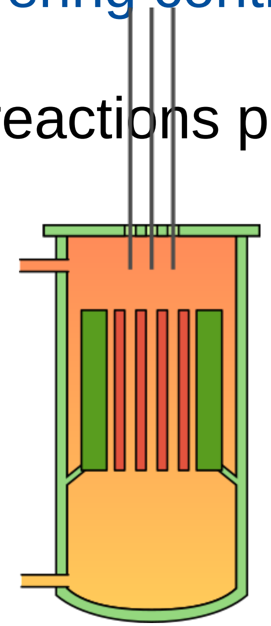
The number of (nuclear) reactions per second in a nuclear reactor is controlled by raising or lowering control rods

$$\frac{dN}{dt} = \alpha N$$

N is the number of reactions per hour



$\alpha = -0.01$



$\alpha = +0.01$

$$t^{n+1} - t^{n} = 1\,hour$$

$N_{ini} = 100\ reac./hr.$ $\quad N_{target} = 10^{9}$ $\quad N_{kaboum} = 10^{11}$

(Disclaimer: those are dummy numbers…)

Write a program that simulates the reaction rate + a code that lifts or raises the control rod to maintain a reaction rate of 10^9.
How much time to reach $N=10^9$?

Practical Work #1
Case #3: Cooling of Coffee

$$\frac{dT}{dt} = -k\,(T - T_{room})$$

T temperature of coffee
T_{room} room temperature
k heat dissipation factor

$T_{amb} = 25°C$
$k = 0.2\,min^{-1}$
$t^{n+1} - t^{n} = 1\,min$

How much time will it take for the coffee to become drinkable? I.e. that $T = 35°C$?

Suppose that if I add cold milk at a time t^n then
$$T^n \leftarrow 0.8\,T^n \; + \; 0.2\,T_{milk}$$
Will the coffee cool faster if I add the milk at t=0 or if I add it later?

Practical Work #1
Case #4: Prey-Predator

$$\frac{dx}{dt} = +\alpha\, x \;-\; \beta\, xy$$
$$\frac{dy}{dt} = -\gamma\, y \;+\; \zeta\, x\, y$$

(Lotka–Volterra equations)

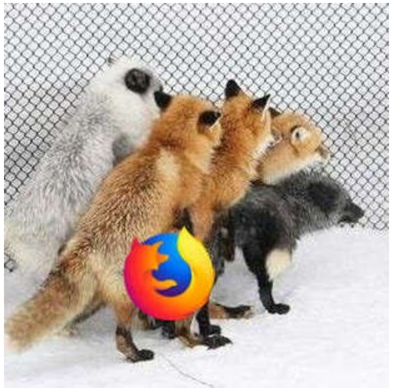
x is the number of rabbits
y is the number of foxes

α : Mating

β : Eaten

γ :Starvation (Overpopulation)

ζ : Feeding + Mating



$\alpha = 1.1$ $\quad \beta = 0.0004$ $\quad \gamma = 0.4$ $\quad \zeta = 0.0001$

$x^0 = 10\,000$ $\quad y^0 = 10\,000$

(Disclaimer: those are dummy numbers…)

How do the prey/predator ratio behave in time?

Practical Work #1
Go, GO, GOOO!!!

Implement one or several user cases.
Do not try to implement elaborate architectures now (keep it for later)
The goal here is to become comfortable with the concept of time integrator.

Doer: Implement
Mate: Assist the doer (with the understanding)
Observer: observe

Go, GO, GOOO!!!!

Questions for observers:
How did they decide to approach the problem? Why?
What did they do? How did they work? Anything interesting detail?

Practical Work #1
Go, GO, GOOO!!!

Discussion

Practical Work #2
Architecture

Practical Work #2
Go, GO, GOOO!!!

Time integrators are useful in many applications.
Hence you decide to implement it “once and for all” in a “library”
that you plan on using in the future.

Doer: Discuss the architecture + (if time) start implementation
Mate: Discuss the architecture + assist Doer in implementation
Observer: observe

Go, GO, GOOO!!!!

Questions for observers:
How did they decide to approach the problem? Why?
What did they do? How did they work? Anything interesting detail?

Practical Work #2
Go, GO, GOOO!!!

Discussion

Practical Work #3
Architecture

Practical Work #3
Go, GO, GOOO!!!

Follow-up of Practical Work #2:
Time integrators are useful in many applications.
Hence you decide to implement it “once and for all” in a “library”
that you plan on using in the future.

Doer: Decide what you want to work on
Mate: Assist the Doer
Observer: observe

Go, GO, GOOO!!!!

Questions for observers:
How did they decide to approach the problem? Why?
What did they do? How did they work? Anything interesting detail?

Practical Work #3
Go, GO, GOOO!!!

(Final) Discussion

Time integrators
Explained by example

There are many ways to answer the question:

“Euler Explicit”
aka **EE**

1. Use speed $v^n = 120km/h$ at $t^n = 1$

$$x^{n+1} = x^n + v^n.(t^{n+1} - t^n)$$

“Euler Implicit”
aka **EI**

2. Look at speed $v^{n+1} = 90km/h$ at $t^{n+1} = 6$

$$x^{n+1} = x^n + v^{n+1}\,(t^{n+1} - t^n)$$

“Crank-Nicolson”
aka **CN**

3. Do the average of previous methods

$$x^{n+1} = x^n + \frac{v^{n+1} + v^n}{2}\,(t^{n+1} - t^n)$$

“Runge-Kutta Midpoint”
aka **RK2**

4. Use speed $v^n = 120km/h$ to predict $x^{n+\frac{1}{2}}$ and $t^{n+\frac{1}{2}}$

$$t^{n+\frac{1}{2}} = \frac{1}{2}\,(t^{n+1} - t^n)$$

$$x^{n+\frac{1}{2}} = x^n + v^n\,(t^{n+\frac{1}{2}} - t^n)$$

Then, assume you’ll be traveling at $v(t^{n+\frac{1}{2}})$ all along

$$x^{n+1} = x^n + v(t^{n+\frac{1}{2}}).\,(t^{n+1} - t^n)$$

These four ways may give different answers!

$$v(t) := -0.8t^2 - 0.4t + 121.2$$