Rob 101 - Computational Linear Algebra
Recitation #11

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1 Calculating Gradients

Calculate the Gradient of the given function:

$$f(x) = 2x^5 + 4x^2 + 9x \tag{1}$$

at $x_0 = 1$

1.1 Analytically

Using Calculus.

1.2 Numerically

Defined as a rise over run in three ways:

$$\textbf{Forward Difference: } \frac{df(x_0)}{dx} \approx \frac{f(x_0 + h) - f(x_0)}{h}, \tag{2}$$

$$\textbf{Backward Difference: } \frac{df(x_0)}{dx} \approx \frac{f(x_0) - f(x_0 - h)}{h}, \tag{3}$$

$$\textbf{Symmetric Difference: } \frac{df(x_0)}{dx} \approx \frac{f(x_0 + h) - f(x_0 - h)}{2h}. \tag{4}$$

2 Gradient Descent

In optimising with gradient descent we simply move in the direction of decreasing gradient. Thus, the update rule is given as:

$$x_{k+1} = x_k + \Delta x_k = x_k - s\frac{df(x_k)}{dx}$$

When x is column vector, The update rule is given as:

$$x_{k+1} = x_k - s\left[\nabla f(x_k)\right]^\top , \tag{5}$$

Using this calculate the minima of the following function:

$$f(x) = (10x_1^2 + x_2^2)/2 + 5\log(1 + e^{-x_1 - x_2})$$

3 Newtons Method

The update rule for a scalar optimization variable:

$$x_{k+1} = x_k - \left(\frac{d^2 f(x_k)}{dx^2} \right)^{-1} \frac{df(x_k)}{dx}, \tag{6}$$

The update rule for a vector optimization variable:

$$\nabla^2 f(x_k)\ \Delta x_k = -\left[\nabla f(x_k)\right]^\top \quad \text{(solve for } \ \Delta x_k\text{)} \tag{7}$$

$$x_{k+1} = x_k + \Delta x_k \quad \text{(use } \ \Delta x_k \ \text{ to update our estimate of the optimal value)} \tag{8}$$

Using this calculate the minima of the following function:

$$f(x) = (10x_1^2 + x_2^2)/2 + 5\log(1 + e^{-x_1 - x_2})$$