Rob 101 - Computational Linear Algebra
Recitation #7

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1 HW 4 Supplement Questions

1.1 LU factorization with permutation

Why does the P matrix show up?
Solve the following system of equations:

$$\begin{aligned} x_2 &= 3 \\ x_1 + 2x_2 &= 7 \end{aligned}$$

We know the solution from simple substitution $x_1 = 1, x_2 = 3$
This was easy because the we have 2 equations and 2 unknowns. What if we have a much larger system of equations, and we try to solve $Ax = b$ for a A of size 100x100.
We need to use LU factorization. Let see how we would do that!

How to solve for system of equations with permutations?
You are given that

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{P} \cdot \underbrace{\begin{bmatrix} 2 & -1 & 2 \\ 6 & -3 & 9 \\ 2 & -3 & 6 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix}}_{L} \cdot \underbrace{\begin{bmatrix} 6 & -3 & 9 \\ 0 & -2 & 3 \\ 0 & 0 & -1 \end{bmatrix}}_{U}$$

Using the LU Factorization, find the solution to $Ax = b$, for

$$b = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}.$$

Linear Independence

Ways to check for Linear Independence of the columns of a matrix A of dimension mxn:

- By Definition: If the only solution to $A\alpha = 0_{mx1}$ is the trivial solution, i.e. $\alpha = 0_{nx1}$. Then columns of A are Linearly Independent.
- Using Pro-tip: If the $det(A^T A) \neq 0$ or there are no zero terms in the diagonal of U, where $PA^T A = LU$

Ways to find the number of Linear Independent columns of a matrix A of dimension mxn:

- By Definition: If the only solution to $A\alpha = 0_{mx1}$ is the trivial solution, i.e. $\alpha = 0_{nx1}$. Then columns of A are Linearly Independent. So we find the subset columns that satisfy this condition.
- Using Semi Pro-tip: Count the number of zeros in the LU decomposition of $A^T * A$ and use the semi Pro-tip (Appendix!)

We will go over a coding example of doing the same.

Application of Linear Independence to $\mathbf{Ax = b}$

We can comment on two properties of the solution of Ax=b using the concept of Linear Independence:

- Existence: If b is a linear combination of columns of A, a solution exists. Why?

- Uniqueness: If a solution exists, and all columns of A are linearly independent, then the solution is also unique.