ROB 101 - Computational Linear Algebra
Recitation #1

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1 Linear systems

A system of equations with no Non-linearity ($\cos, \sin, \log, x^2$ etc.)

1.1 Solution of Linear System of Equations

Lets work out some examples:
Find a solution by for the following set of equations by substitution, if it exists and also sketch a rough solution to corroborate:

1.1.1 Part A

$$x + 2y = 6 \tag{1}$$
$$2x - y = 4 \tag{2}$$

1.1.2 Part B

$$x + 2y = 6 \tag{3}$$
$$3x + 6y = 9 \tag{4}$$

1.1.3 Part C

$$x + 2y = 6 \tag{5}$$

$$3x + 6y = 18 \tag{6}$$

Lets try and express these in the matrix format ($Ax = b$) and determine if the solution is unique $det(A) \neq 0$

Review: Determinant Facts:

- det(A) is a real number
- $Ax = b$, a system of equations with n equations and n unknowns has a unique solution for any b if an only if $det(A) \neq 0$
- When $det(A) = 0$, the system may have either infinite or no solution
- $det(A) = ad - bc$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

1. $x + 2y = 6$
   $2x - y = 4$

   $A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ $x = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ $b = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

   $det(A) =$

2. $x + 2y = 6$
   $3x + 6y = 10$

   $A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ $x = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ $b = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

   $det(A) =$

3. $x + 2y = 6$
   $3x + 6y = 18$

   $A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ $x = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ $b = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

   $det(A) =$

4. $4x = 10$
   $x + 6y = 2$

   $A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ $x = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ $b = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

   $det(A) =$

5. $y - 2x = 4$
   $y = 2$

   $A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ $x = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ $b = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

   $det(A) =$

6. $2x - y = 3$
   $6x - 3y = 1$

   $A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$ $x = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ $b = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

   $det(A) =$

2 Quadratic equation

$$ax^2 + bx + c = 0$$

So on the x-y axis we want to plot:

$$y = ax^2 + bx + c$$

Finding the roots at $y = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac \geq 0$, Roots are real
If $b^2 - 4ac < 0$, Roots are complex

Lets see some examples and how to plot these standard quadratic forms:

$$x^2 - 10x + 21 = 0 \tag{7}$$

$$x^2 - 10x + 25 = 0 \tag{8}$$

$$x^2 - 10x + 26 = 0 \tag{9}$$