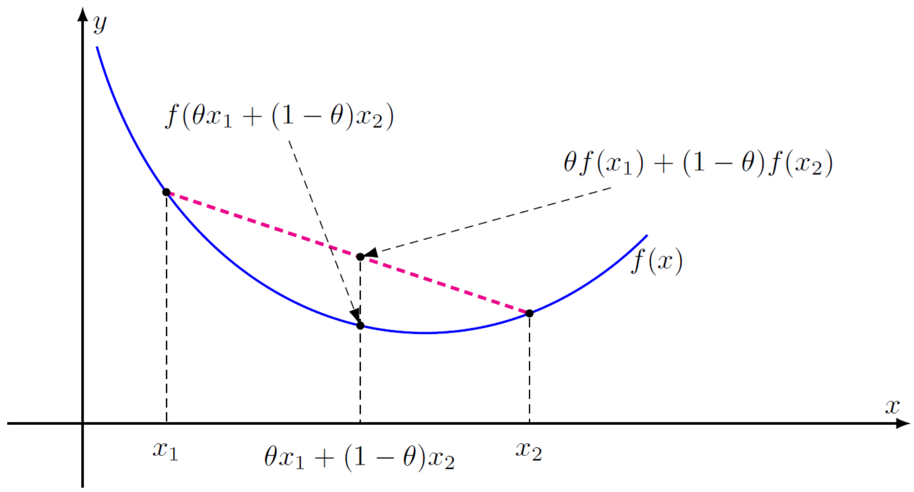
$f : \mathbb{R}^n \to \mathbb{R}$ ($\operatorname{dom} f = \mathbb{R}^n$) is convex iff:

1 For all $x_1, x_2 \in \mathbb{R}^n$ and all $\theta \in [0,1]$:

$$f(\theta x_1 + (1-\theta)x_2) \leq \theta f(x_1) + (1-\theta) f(x_2)$$

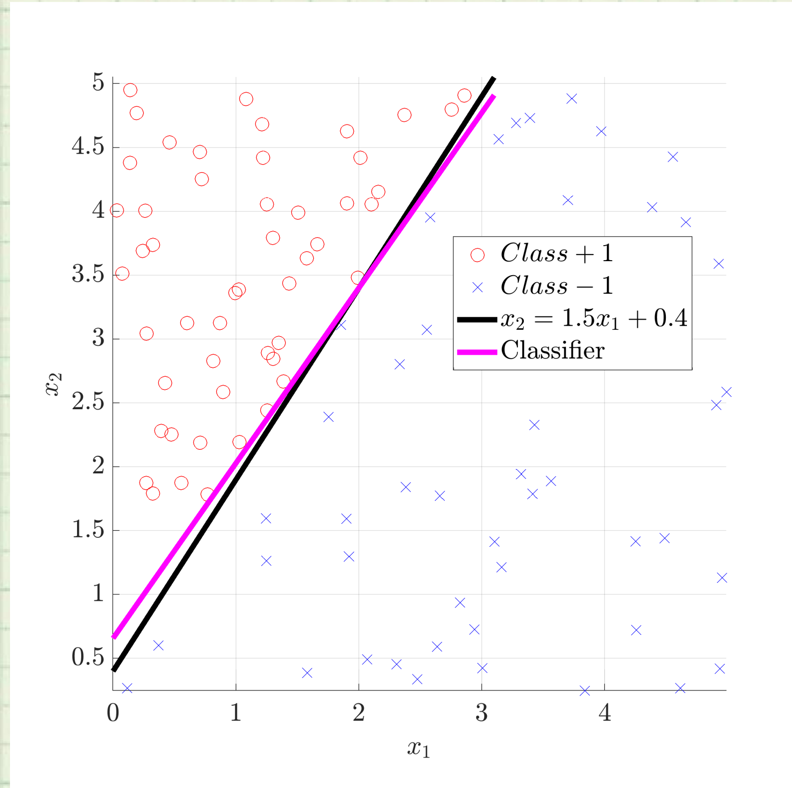


$\theta x_1 + (1-\theta) x_2$

$\theta = 0$, x_2

$\theta = 1$, x_1

- Hyperplanes

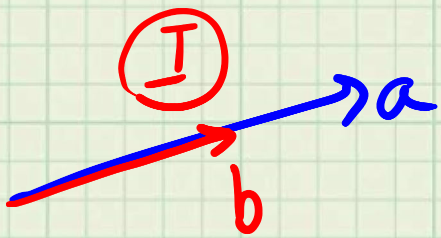


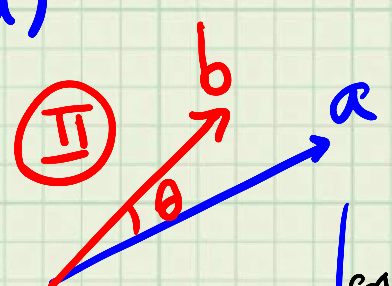
Reminder: Dot product

$$\langle a,b\rangle = a.b = a^T b = b^T a = a_1 b_1 + \cdots + a_n b_n$$

$$a, b \in \mathbb{R}^n$$

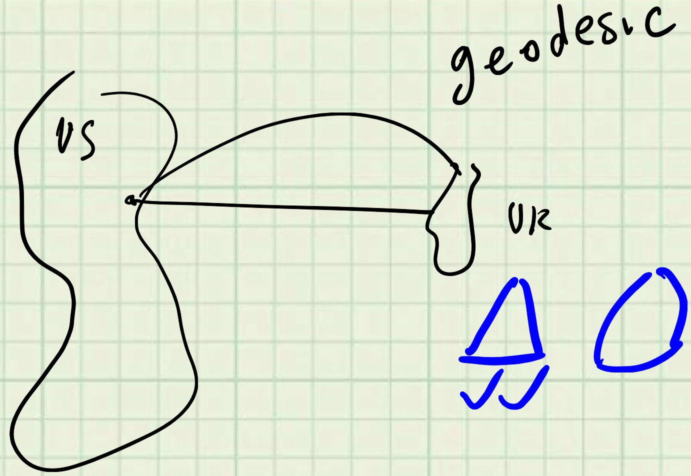
a.b is maximized when two vectors are parallel (aligned)

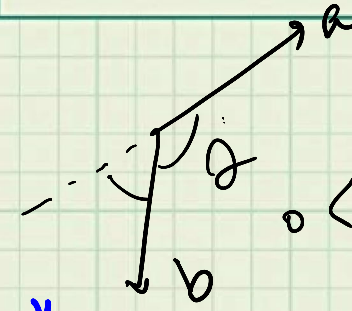




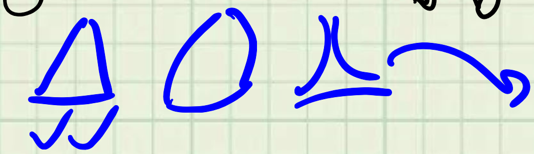
$$\cos\theta = \frac{a.b}{\|a\|.\|b\|}$$

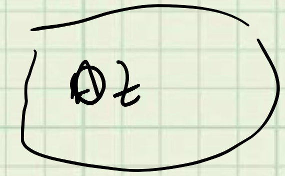
I $a \cdot b$ > II $a \cdot b$



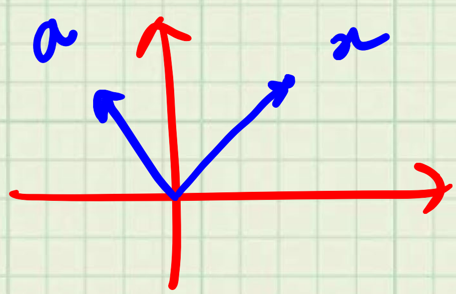


$0 < \theta < 180$



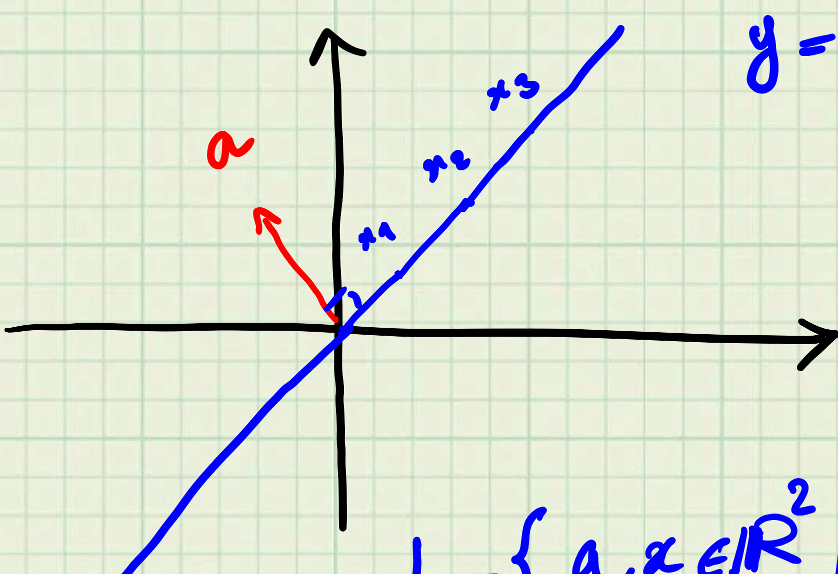


$a \cdot b$ is zero when $a \perp b$
(a is orthogonal to b)



$x - 0 = x$

Set of All points such that $a \cdot x = 0$



$y = mx + b$

$$L = \{ a, x \in \mathbb{R}^2 \mid a \cdot x = 0 \}$$

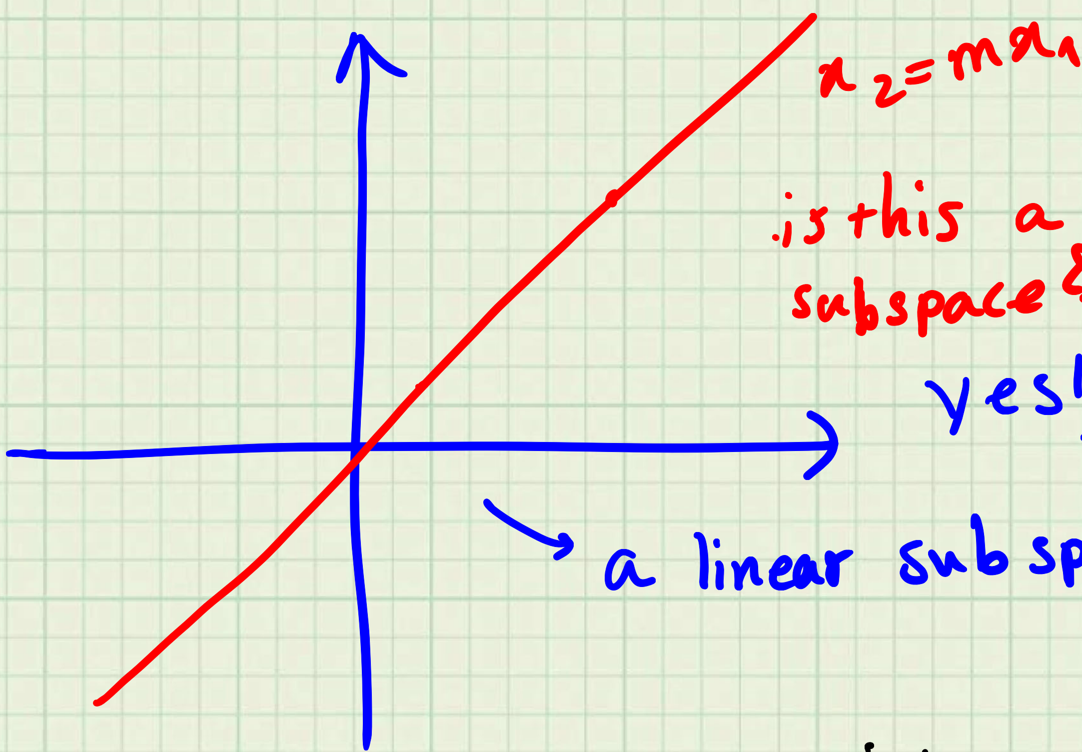
$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$a \cdot x = a_1 x_1 + a_2 x_2 = 0$$

$$\Rightarrow x_2 = -\frac{a_1}{a_2} x_1 , \quad a_2 \neq 0$$

↳ regressor form

$$m := -\frac{a_1}{a_2} \Rightarrow x_2 = m x_1$$

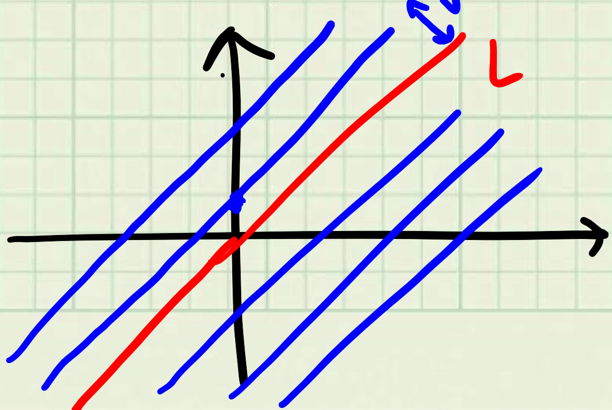


is this a subspace?

yes!

↳ a linear subspace

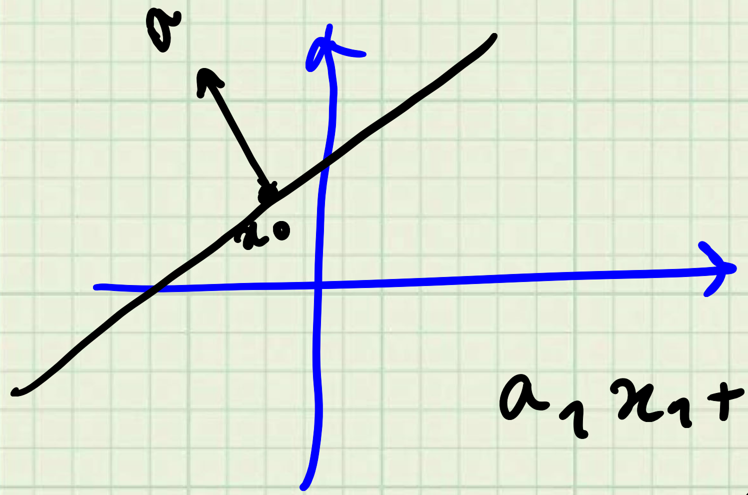
Affine subspace (linear variety, linear manifold)



$M = v + L$

$v \in \mathbb{R}^2$

pick any point x_0 and a



$$a \cdot (x - x_0) = 0$$

$$a \cdot x = a \cdot x_0 =: d$$

$$d \in \mathbb{R}$$

$$a_1 x_1 + a_2 x_2 = d$$

$$x_2 = -\frac{a_1}{a_2} x + \frac{d}{a_2}, \quad a_2 \neq 0$$

$$\left.\begin{aligned} m &:= -\frac{a_1}{a_2} \\ b &:= \frac{d}{a_2} \end{aligned}\right\} \Rightarrow \boxed{x_2 = m x_1 + b}$$