2020 Sept 02 ROB 101

Summary

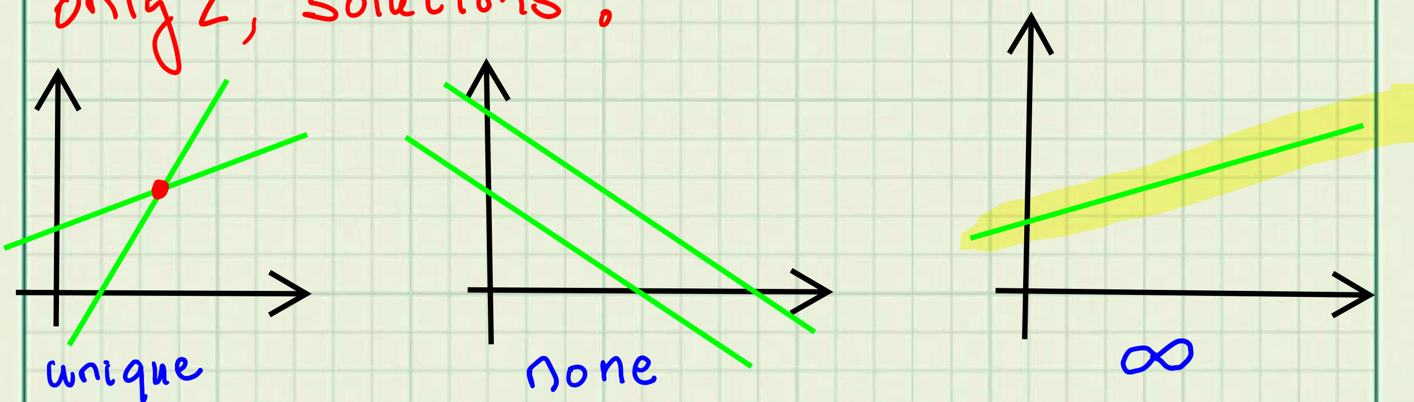
Summary So Far

Consider a set of two equations with two unknowns x and y

$$\begin{aligned} a_{11}x + a_{12}y &= b_1 \\ a_{21}x + a_{22}y &= b_2, \end{aligned} \tag{4}$$

constants $a_{11}, a_{12}, a_{21}, a_{22}$ and b_1, b_2. Depending on the values of the constants, the linear equations (4) can have a unique solution, no solution, or an infinity of solutions.

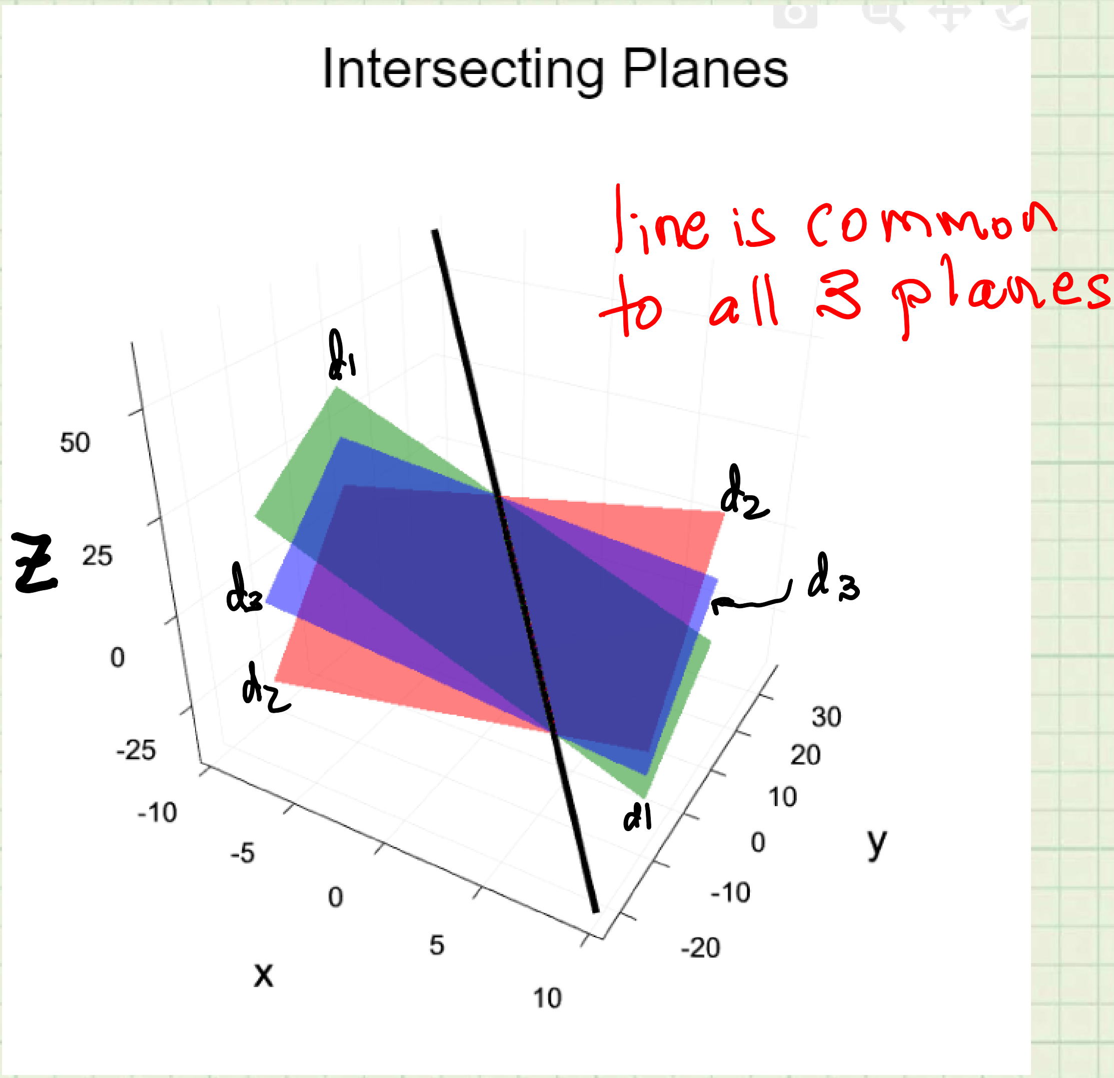
- It is NOT POSSIBLE to have 2, and only 2, solutions!



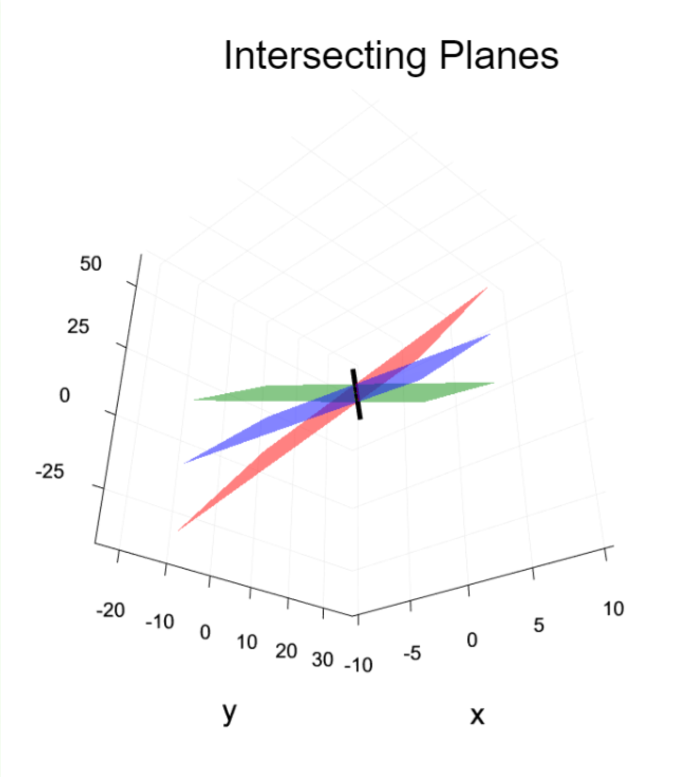
- Same is true for n linear equations in n unknowns, but with our currently clumsy notation, this is hard to show!

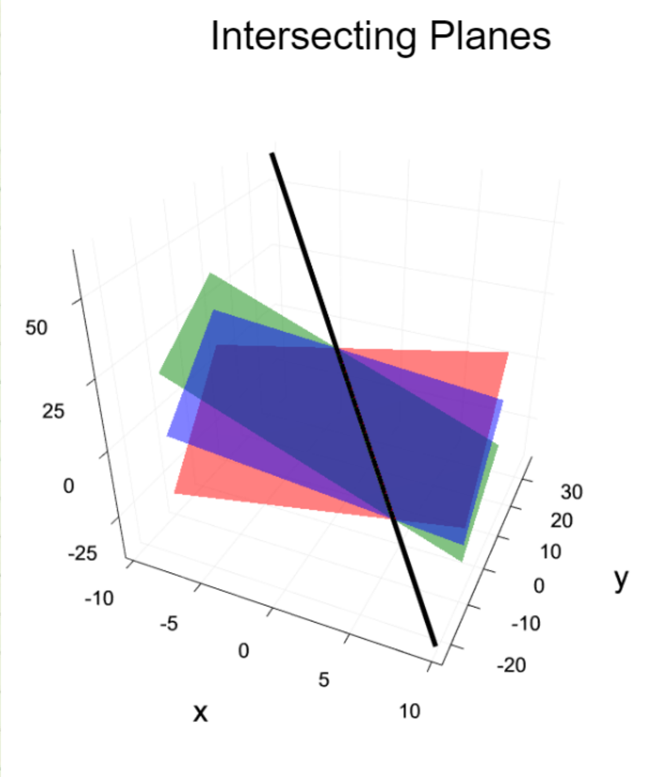
- "Pictures" (graphs) rapidly become less helpful as well for $n > 2$

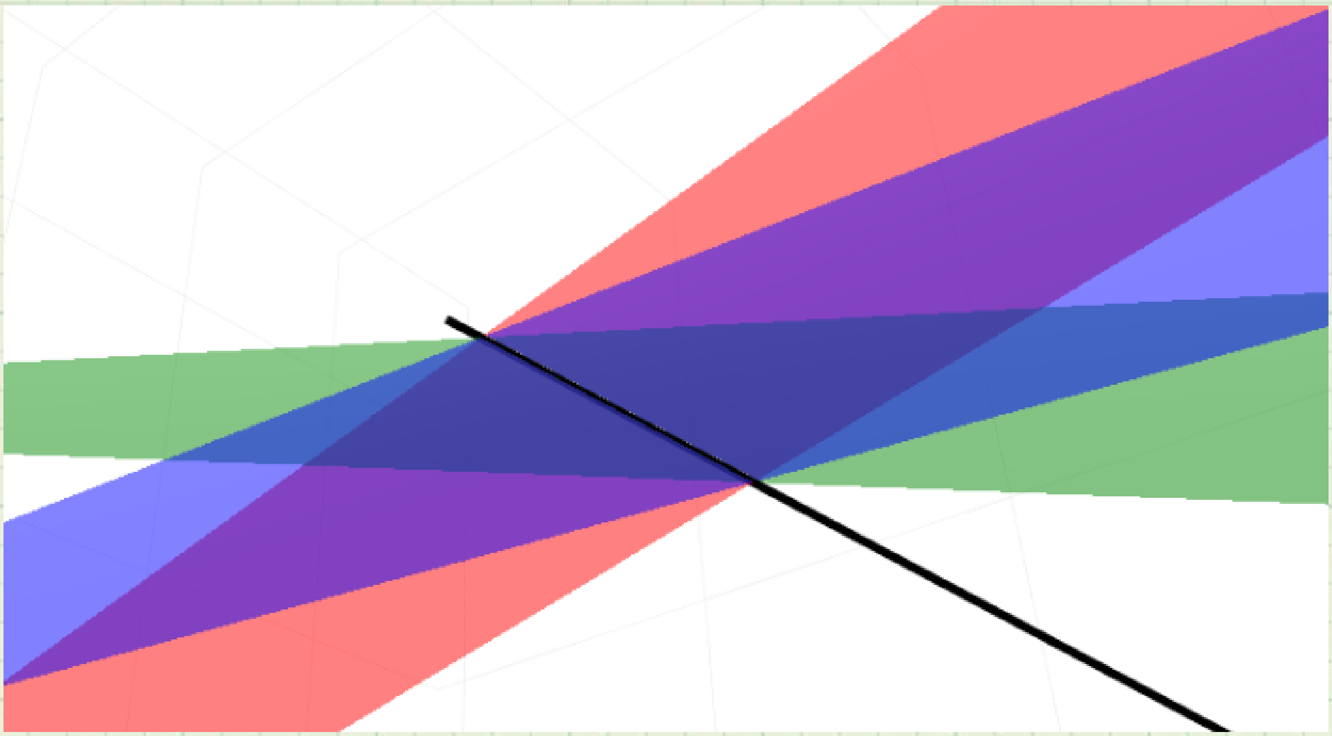
```julia
# 2*x -y + z = 4       (d1) ==> z = 4-2.0*x+y
# x + 2*y - z = 3      (d2) ==> z= -1.0*(3-x-2*y)
# -x + 3*y - 2*z = -1  (d3) ==> z= -(1/2.)*(-1.0-3*y+x)
d1(x,y)=4-2.0*x+y
d2(x,y)=-1.0*(3-x-2*y)
d3(x,y)=-(1/2.)*(-1.0-3*y+x)

plotly()
xmax=10.0
x=-xmax:0.05:xmax
y=x
titre="Intersecting Planes"
# camera=(-30,30),camera=(30,60),
plot(x,y,d2, st=:surface, color=:red, opacity = 0.5, showscale = false, label="d2")
xlabel!("x")
ylabel!("y")
#zlabel!("z")
plot!(x,y,d1, st=:surface,color=:green, opacity = 0.5, title=titre)
plot!(x,y,d3, st=:surface,color=:blue, opacity = 0.5)
line(x,y)=11-5*x  # z value on d1 intersects d2 intersects d3
yy(x)=7.0-3.0*x   # y value on d1 intersects d2 intersects d3
MyPlot=plot!(x,yy,line,linewidth=5,color=:black)
```



$$\text{line}(x) = \begin{bmatrix} x \\ 7-3x \\ 11-5x \end{bmatrix}$$


Intersecting Planes
50
25
0
-25
-20
-10
0
10
20
30
-10
-5
0
5
10
y
x


Intersecting Planes
50
25
0
-25
-10
-5
0
5
10
x
30
20
10
0
-10
-20
y



Today: More powerful notation

- Vectors
- Matrices
- $Ax = b$

Columns

Rows

	A	B	C	D	E	F	G
1	36	0.157822	114.2628	2536	10675	4808	14122
2	36	0.157822	114.2628	2536	10672	4808	14122
3	36	0.157822	114.2628	2536	10672	4808	14122
4	36	0.157822	114.2628	2536	10672	4808	14089
5	35	0.153377	111.0451	2535	10672	4808	14061
6	35	0.153377	111.0451	2535	10672	4808	14001
7	34	0.148936	107.8298	2534	10672	4808	13982
8	33	0.144499	104.617	2533	10672	4808	13911
9	32	0.140065	101.4068	2532	10672	4808	13823
10	30	0.131207	94.99374	2530	10672	4808	13756
11	30	0.131207	94.99374	2530	10672	4808	13683
12	29	0.126783	91.79099	2529	10672	4808	13609
13	24	0.104717	75.81477	2524	10672	4808	13494
14	24	0.104717	75.81477	2524	10672	4808	13470
15	20	0.087125	63.07885	2520	10672	4808	13290
16	6	0.025992	18.81852	2506	10673	4808	13209
17	-1	-0.00432	-3.12766	2499	10670	4807	13155
18	-18	-0.07723	-55.9149	2482	10666	4806	13119
19	-27	-0.11543	-83.5682	2473	10666	4806	13059
20	-56	-0.23659	-171.294	2444	10666	4806	12965

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Julia allows as many "dimensions" as you want (finite).

Naming variables

$x_1, x_2, x_3, x_4, \ldots$

$y_1, y_2, y_3, \ldots$

$z_1, z_2, \ldots$

$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \ldots, \alpha_{218}$

$V = \begin{bmatrix} \pi \\ -\sqrt{2} \\ 4 \end{bmatrix}$ is column vector.

It has 3 entries, $v_1 = \pi$, $v_2 = -\sqrt{2}$, $v_3 = 4$

The order matters.

$$\begin{bmatrix} -\sqrt{2} \\ 4 \\ \pi \end{bmatrix} \neq \begin{bmatrix} \pi \\ -\sqrt{2} \\ 4 \end{bmatrix}$$

$w = \begin{bmatrix} 1.6 & -11 & 2/3 \end{bmatrix}$ is a row vector. Just as with column vector, the order matters!

Heads up: In many textbooks

and research articles, one writes

$w = [\sqrt{2}, 1/8, 11]$ is a row vector.

But not in Julia!

$v = [v_1]$ is both a row vector and a column vector.

Heads up: $w = [9]$ is different than the number 9.

$w_1 = w[1] = 9$ is real number.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is a 2×3 matrix because it has 2 rows and 3 columns.

$A = \begin{bmatrix} \boxed{1 \quad 2 \quad 3} \\ \boxed{4 \quad 5 \quad 6} \end{bmatrix}$ are <u>rows</u> of A

and

$$A = \begin{bmatrix} \boxed{\begin{matrix}1\\4\end{matrix}} & \boxed{\begin{matrix}2\\5\end{matrix}} & \boxed{\begin{matrix}3\\6\end{matrix}} \end{bmatrix}$$ are columns of A.

$$B = \begin{bmatrix} \sqrt{2} & e \\ \pi & 6! \\ 47 & 19 \end{bmatrix}$$ is a 3×2 matrix.

$$C = \begin{bmatrix} 14 & 8 \\ 11 & \\ 12 & 13 \\ 2 & \sqrt{2} \end{bmatrix}$$ Wrong because the second row is not filled.

A general $n \times m$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2m} \\ \vdots & & & & \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nm} \end{bmatrix} \quad n, m \geq 1$$

$\boxed{\text{rectangular matrix}}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \ddots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$n = m$

Square matrix

a_{ij} = is an element on the i-th row and j-th column.

$a_{ij} = A[i,j]$

Suppose $A = \begin{bmatrix} 11 & 14 & -2 \\ 8 & \sqrt{2} & \pi \end{bmatrix}$

What are

$a_{12} = 14$

$a_{23} = \pi$

$a_{31} =$ Not defined

$a_{13} = -2$

JWG failed
Class passed

Expressing Linear Systems of Equations in the form "$Ax=b$"

e.1 $x_1 + x_2 - 4 = 0$
e.2 $2x_1 - x_2 + 1 = 0$

$$\xleftrightarrow{?} \quad Ax = b$$

<u>Step1</u> Order the equations as you wish and stack the unknowns into a vector.

For us $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Step2: Move all constants to the right hand side (RHS) and group (stack) them into a vector:

e.1 $x_1 + x_2 = +4$
e.2 $2x_1 - x_2 = -1$

$$\implies b = \begin{bmatrix} +4 \\ -1 \end{bmatrix}$$

<u>Step 3</u> Form A by a_{ij} = coefficient of x_j in the i-th equation

$x_1 + x_2 = 4$
$2x_1 - x_2 = -1$

$$\longleftrightarrow \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 4 \\ -1 \end{bmatrix}}_{b}$$