Soft margin Classifier

problem 1: $\min\limits_{w} \frac{1}{2}\|w\|^2$

s.t. $y_i(w^T x_i) \geq 1$

problem 2: $\min\limits_{w,\xi} \frac{\lambda}{2}\|w\|^2 + \|\xi\|^2$

s.t. $y_i(w^T x_i) \geq 1 - \xi_i$ $\forall x_i$

called slack variables ← $\xi_i \geq 0$

Q: is problem 2 still a QP? yes!

I $\min \frac{\lambda}{2} w^T w + \frac{1}{2}\xi^T\xi$

s.t. $\phi w \leq 1 - \xi$

$\xi \geq 0$

$x, w \in \mathbb{R}^m$

$\xi \in \mathbb{R}^n$

$\phi \in \mathbb{R}^{n \times m}$

II

$$\min_{\xi, w} \ \frac{1}{2} \begin{bmatrix} w \\ \xi \end{bmatrix}^T \begin{bmatrix} \lambda I_m & 0_{m \times n} \\ 0_{n \times m} & I_n \end{bmatrix} \begin{bmatrix} w \\ \xi \end{bmatrix}$$

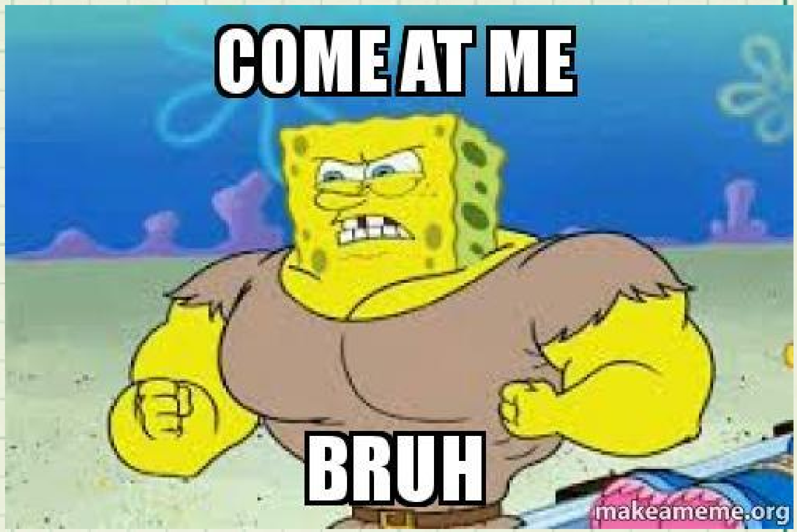
s.t.

$$\begin{bmatrix} -\infty_n \\ 0_n \end{bmatrix} \leqslant \underbrace{\begin{bmatrix} \phi_{n \times m} & I_n \\ 0_{n \times m} & I_n \end{bmatrix}}_{2n \times (m+n)} \begin{bmatrix} w \\ \xi \end{bmatrix} \leqslant \begin{bmatrix} 1_n \\ \infty_n \end{bmatrix}$$

$$1_n = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{n \times 1}, \quad 0_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n \times 1}$$

$$\phi = \begin{bmatrix} -y_1 x_1^T \\ \vdots \\ -y_n x_n^T \end{bmatrix}_{n \times m}$$

Nonlinear Classifier



$$y = \sum_i w_i x_i = w^T x$$

A feature map : $x \longmapsto \varphi(x)$

$$\varphi_c(x) = \exp\left(\frac{-\|x - x_c\|^2}{2s^2}\right)$$

$$y = \sum_j \alpha_j \varphi_j = \alpha^T \varphi \quad , \quad \varphi_j = \exp\left(\frac{-\|x - x_j\|^2}{2s^2}\right)$$

n data points $\longrightarrow$ $\alpha \in \mathbb{R}^{n+1}$

$m = n+1$