Rob 101 - Computational Linear Algebra
Recitation #3

Tribhi Kathuria

Sept 21, 2020

1 Matrix Multiplication

1.1 Partitioning Matrices

Let A be an $n \times m$ matrix.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

A partition of A into rows is

$$\begin{bmatrix} a_1^{\text{row}} \\ a_2^{\text{row}} \\ \vdots \\ a_n^{\text{row}} \end{bmatrix} = \begin{bmatrix} \boxed{a_{11} \quad a_{12} \quad \cdots \quad a_{1m}} \\ \boxed{a_{21} \quad a_{22} \quad \cdots \quad a_{2m}} \\ \vdots \\ \boxed{a_{n1} \quad a_{n2} \quad \cdots \quad a_{nm}} \end{bmatrix}.$$

A partition of A into columns is

$$\begin{bmatrix} a_1^{\text{col}} & a_2^{\text{col}} & \cdots & a_m^{\text{col}} \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{matrix}} & \boxed{\begin{matrix} a_{12} \\ a_{22} \\ \vdots \\ a_{n2} \end{matrix}} & \cdots & \boxed{\begin{matrix} a_{1m} \\ a_{2m} \\ \vdots \\ a_{nm} \end{matrix}} \end{bmatrix}.$$

Example

$$A = \begin{bmatrix} 5 & 3 & 8 \\ 4 & 2 & 7 \end{bmatrix}$$

1. $a_1^{row} =$
2. $a_2^{col} =$

1.2 Standard Matrix Multiplication

$C = A \cdot B$, Then,

$$C_{ij} := a_i^{\text{row}} \cdot b_j^{\text{col}}.$$

Example

1. $A = \begin{bmatrix} 2 & -4 & 2 \\ 3 & 4 & 8 \end{bmatrix} B = \begin{bmatrix} 2 & -5 \\ 6 & -3 \\ 3 & 2 \end{bmatrix}$

1.3 Multiplication by Summing over Columns and Rows

$$C = A \cdot B = \sum_{i=1}^{k} a_i^{\text{col}} \cdot b_i^{\text{row}},$$

Example

1. $A = \begin{bmatrix} 2 & -4 & 2 \\ 3 & 4 & 8 \end{bmatrix} B = \begin{bmatrix} 2 & -5 \\ 6 & -3 \\ 3 & 2 \end{bmatrix}$

2 LU Decomposition

Convert the following system of equations into $Ax = b$, and then Decompose the A matrix as $A = LU$. Finally, solve using,Forward Substitution for $Ly = U$ and Back substitution for $Ux = y$

$$
\begin{aligned}
x + 2y + z &= 13 \\
x - 3y + 4z &= -9 \\
3x + y - 2z &= 9
\end{aligned}
$$

3 Appendix

Review: The general form of a lower triangular system with a non-zero determinant is

$$\begin{aligned} a_{11}x_1 &= b_1 \quad (a_{11} \neq 0) \\ a_{21}x_1 + a_{22}x_2 &= b_2 \quad (a_{22} \neq 0) \\ \vdots &= \vdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n &= b_n \quad (a_{nn} \neq 0) \end{aligned} \tag{1}$$

and the solution proceeds from top to bottom, like this

$$\begin{aligned} x_1 &= \frac{b_1}{a_{11}} \quad (a_{11} \neq 0) \\ x_2 &= \frac{b_2 - a_{21}x_1}{a_{22}} \quad (a_{22} \neq 0) \\ \vdots &= \vdots \\ x_n &= \frac{b_n - a_{n1}x_1 - a_{n2}x_2 - \cdots - a_{n,n-1}x_{n-1}}{a_{nn}} \quad (a_{nn} \neq 0). \end{aligned} \tag{2}$$

Similarly, The general form of an upper triangular system with a non-zero determinant is

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \quad (a_{11} \neq 0) \\ a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \quad (a_{22} \neq 0) \\ a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \quad (a_{33} \neq 0) \\ \vdots &= \vdots \\ a_{nn}x_n &= b_n \quad (a_{nn} \neq 0), \end{aligned} \tag{3}$$

and the solution proceeds from bottom to top, like this,

$$\begin{aligned} x_1 &= \frac{b_1 - a_{12}x_2 - \cdots - a_{1n}x_n}{a_{11}} && (a_{11} \neq 0) \\ x_2 &= \frac{b_2 - a_{23}x_3 - \cdots - a_{2n}x_n}{a_{22}} && (a_{22} \neq 0) \\ \vdots &= \vdots && \vdots \\ x_{n-1} &= \frac{b_{n-1} - a_{n-1,n}x_n}{a_{n-1,n-1}} && (a_{n-1,n-1} \neq 0) \\ x_n &= \frac{b_n}{a_{nn}} && (a_{nn} \neq 0), \end{aligned} \tag{4}$$