Rob 101 - Computational Linear Algebra
Recitation #4

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1 Vectors in $\mathbb{R}^n$

We identify $\mathbb{R}^n$ with the set of column vectors on length n

So for us, saying $x \in \mathbb{R}^n$ is the same as saying $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, where, $x_i \in \mathbb{R}$

Thus any matrix A of n rows and m columns is a set of m vectors in $\mathbb{R}^n$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} = \begin{bmatrix} a_1^{\text{col}} & a_2^{\text{col}} & \cdots & a_m^{\text{col}} \end{bmatrix} \iff a_j^{\text{col}} := \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{nj} \end{bmatrix} \in \mathbb{R}^n, 1 \leq j \leq n \tag{1}$$

Example:

Given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 5 & 6 & 8 \end{bmatrix}$ is a set of 3 vectors in $\mathbb{R}^3$

Write down this set of vectors:

2 Linear Combination

A vector, $v \in \mathbb{R}^n$ is said to be a Linear Combination of vectors $v_1, v_2 \cdots v_m \in \mathbb{R}^n$ if there exits real numbers $\alpha_1, \alpha_2 \cdots \alpha_m$ such that

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_m v_m$$

Using this formulation, find if the given vector v, is a linear combination of the vectors $v_1, v_2 \cdots v_m$ in the question, if true, also find the vector of coefficients, α

1. $v = \begin{bmatrix} 8 \\ 1 \\ -8 \end{bmatrix}$

   $v_1 = \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ 1 \\ 6 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

2. $v = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$

   $v_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

3. $v = \begin{bmatrix} 7 \\ 5 \\ 4 \end{bmatrix}$

$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

3 Linear Independence

the vectors $\{v_1, v_2, ..., v_m\}$ are **linearly independent** if the **only** real numbers $\alpha_1, \alpha_2, \ldots, \alpha_m$ yielding a linear combination of vectors that adds up to the zero vector,

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_m v_m = 0, \tag{2}$$

are $\boldsymbol{\alpha}_1 = 0, \boldsymbol{\alpha}_2 = 0, \ldots, \boldsymbol{\alpha}_m = 0$.
Concise definition of Linear Independence:

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_m v_m = 0 \iff \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Using this Definition, determine if the following vectors are linearly independent.

1. $v_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

2. $v_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $v_4 = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$