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FEDERATED DBSCAN

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CONTENTS

Abstract

In this thesis, federated versions of DBSCAN algorithm are introduced both for horizontally and vertically partitioned data.

DBSCAN algorithm is briefly overviewed in Chapter 1; then, in Chapter 2, an outline of federated learning is given. The proposed solutions for horizontal and vertical federated DBSCAN are described in Chapter 3, and a Python implementation of the algorithms is presented in Chapter 4. Finally, Chapter 5 reports an experimental analysis of both algorithms.

1. DBSCAN

1.1 Introduction

Clustering algorithms are used to solve identification problems in spatial databases. DBSCAN (Density Based Spatial Clustering of Applications with Noise) is a clustering algorithm which deals with good efficiency with the main problems that rise when applying clustering algorithms to large spatial databases: minimal requirements of domain knowledge to determine the input parameters and discovery of clusters with arbitrary shape.

For a gentle introduction to clustering algorithms refer to [1].
For a detailed description of DBSCAN refer to [2].

1.2 A Density Based Notion of Clusters

DBSCAN relies on a density-based notion of clusters. The algorithm is based on the key idea that for each point of a cluster the neighborhood of a given radius Eps has to contain at least a minimum number of points $MinPts$. Note that these parameters describes the density of the cluster: different clusters are described by different parameters. Ideally, then, we would have to know the appropriate parameters for each cluster in the dataset, but there is no easy way to retrieve this information. Thus, DBSCAN uses global values for Eps and $MinPts$ for all clusters. Good candidates for these parameter values are those specifying the density of the "thinnest" cluster of the database, i.e. the lowest density which is not considered to be noise.

Let D be a database of points of some n-dimensional space E, and let $dist(p, q)$ be a distance function between two points of D. The following definitions hold.

Definition 1: (Eps-neighborhood of a point) The *Eps-neighborhood* of a point p, denoted by $N_{Eps}(p)$, is defined by $N_{Eps}(p) = \{q \in D \mid dist(p, q) \leq Eps\}$.

Definition 2: (directly density-reachable) A point p is *directly density-reachable* from a point q wrt. Eps and $MinPts$ if:
- $p \in N_{Eps}(q)$, and
- $|N_{Eps}(q)| \geq MinPts$ (core point condition).

Definition 3: (density-reachable) A point p is *density-reachable* from a point q wrt. Eps and $MinPts$ if there is a chain of points p_1, …, p_n, such that $p_1 = q$, $p_n = p$ and p_{i+1} is directly reachable from p_i.

Definition 4: (density-connected) A point p is *density-connected* from a point q wrt. Eps and $MinPts$ if there is a point o such that both, p and q are density reachable from o wrt. Eps and $MinPts$.

Definition 5: (cluster) Let D be a database of points. A *cluster* C wrt. Eps and $MinPts$ is a non-empty subset of D satisfying the following conditions:

- $\forall\, p, q$: if $p \in C$ and q is density-reachable fro p wrt. Eps and $MinPts$, then $q \in C$. (Maximality)
- $\forall\, p, q \in C$: p is density-connected to q wrt. Eps and $MinPts$. (Connectivity)

Definition 6: (noise) Let C_1, …, C_k be the clusters of the database D wrt. Eps_i, $MinPts_i$, $i = 1, \ldots, k$. Then the *noise* is the set of points in the database D not belonging to any cluster C_i.

DBSCAN algorithm is designed to discover the clusters and the noise in a spatial database according to definitions 5 and 6, given Eps and $MinPts$.

1.3 The Algorithm

Initially, all points in the database D are marked as "**unvisited**". DBSCAN randomly selects an unvisited point p, marks it as "**visited**" and checks the core point condition. If not, p is marked as a "**noise**" point. Otherwise, a new cluster C is created for p, and all the points in its neighborhood are added to a candidate set, N. Then, the points in N that do not belong to any cluster are iteratively added to C. Furthermore, for a point p' in N that carries the label "**unvisited**", DBSCAN marks it as "**visited**" and check its core point condition. If the condition holds, all points in the Eps-neighborhood of p' are added to N. The loop continues adding points to C until N reaches emptiness. At this time, cluster C is completed. To compute the next cluster, DBSCAN randomly selects an "**unvisited**" point from the remaining ones, until all points are visited.

The following pseudocode describes DBSCAN algorithm.

```
DBSCAN

Input
$D$: database containing $n$ points
$Eps$: the radius parameter
$MinPts$: the neighborhood density threshold
```

```
**Output**
A set of density-based clusters

**Method**
• Mark each point as *unvisited*
• **do:**
  • Randomly select an *unvisited* point $p$
  • Mark $p$ as *visited*
  • **If** the Eps-neighborhood of $p$ has at least $MinPts$ points:
    • Create a new cluster $C$
    • Add $p$ to $C$
    • Let $N$ be the set of points in the Eps-neighborhood of $p$
    • **For** each point $p'$ in $N$:
      • **If** $p'$ is *unvisited*:
        • Mark $p'$ as *visited*
        • **If** the Eps-neighborhood of $p'$ has at least $MinPts$
          points: add those points to $N$
      • **If** $p'$ is not yet a member of any cluster: add $p'$ to $C$
    • Output $C$
  • **Else:** mark $p$ as *noise*
• **Until** no point is *unvisited*
```

2. Federated Learning

2.1 An Overview of Federated Learning

Federated learning deals with the possibility to train machine learning models using data from different end users, preserving owners privacy. This is crucial in real-world situations, where with the exception of a few industries, most resident data are limited or low quality. Still, in many situations, it is very difficult to break the barriers between data sources. This is due to industry competition, privacy security and complicated administrative procedures. In fact, as a result of new data regulations and privacy laws, it is generally forbidden to collect, fuse and process data from different places. Federated learning is a possible solution for this challenge.

To define what federated learning is, consider N data owners $\{P_1, \ldots, P_N\}$ with their respective data $\{D_1, \ldots, D_N\}$, and let M_{SUM} be the model trained by $D = D_1 \cup \ldots \cup D_N$. A federated-learning system is a learning process in which the data owners collaboratively train a model M_{FED} without exposing their own data to others. The accuracy of M_{FED}, V_{FED}, should be as close as possible to that of M_{SUM}, V_{SUM}. Formally, let δ be a non-negative real number; if $|V_{FED} - V_{SUM}| < \delta$, the federated learning algorithm is said to have δ-accuracy loss.

Federated learning systems can be coarsely categorized based on the data partitioning scheme, i.e. how data are distributed across the various data owners. To introduce this categorization, let F_i be the feature space and I_i the sample ID space of the data D_i. Then, we can distinguish *horizontal federated learning* from *vertical federated learning* as follows.

In horizontal federated learning the dataset is said to be *sample-partitioned*, and the following relations hold:

$$F_i = F_j, \qquad I_i \neq I_j, \qquad \forall\, D_i, D_j : i \neq j,$$

whilst in vertical federated learning the dataset is *feature-partitioned*:

$$F_i \neq F_j, \qquad I_i = I_j, \qquad \forall\, D_i, D_j : i \neq j.$$

Federated learning is widely overviewed in [3].

3. Federated DBSCAN

3.1 Horizontal Federated DBSCAN

In this section, we introduce a novel federated approach to DBSCAN clustering algorithm for horizontally portioned data. We refer to a set of clients willing to train a federated learning model interacting with a trustworthy server.

The key idea of the algorithm is the partitioning of each local feature space with a fixed granularity. The domain of definition of the features and the granularity of the grid are known and are the same for each client. This approach allows each client to share with the server only the number of points within the non-empty cells of the grid, preventing raw data sharing and thus preserving privacy.

Figure 1 describes the idea: the left picture represents a local dataset for a given client with the superimposed grid partitioning, supposing a two dimensional feature space, while the right one represents the information shared with the server, that is the number of points in each cell.

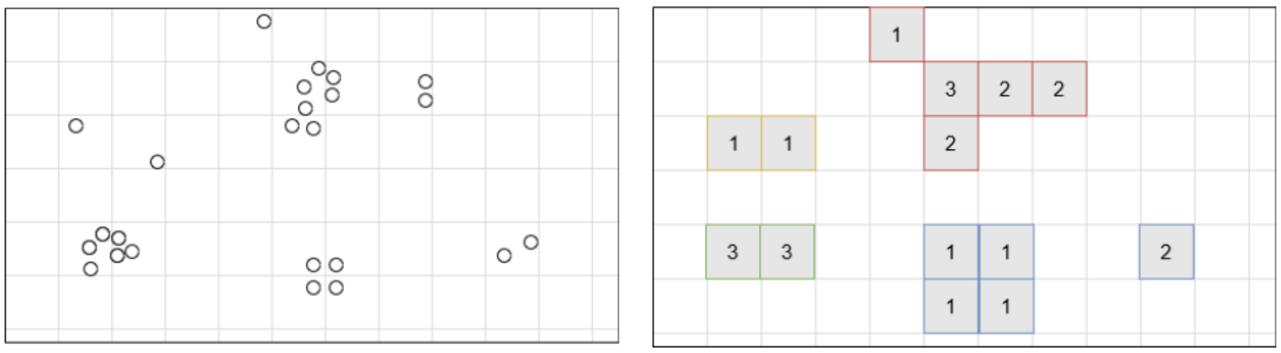

Fig. 1: the key idea behind horizontal federated DBSCAN. A partitioning on the input space prevents raw data sharing from clients to server. **Left:** local dataset for a given client (bidimensional attribute space), with superimposed grid partitioning. **Right:** Number of points in each cell (information shared with the server)

The server aggregates information received from the clients about the cells. Then, based on the $MinPts$ parameter, it decides whether a cell can be considered dense. Then, the server performs an adapted version of the classical DBSCAN algorithm, expanding the clusters of dense cells along adjacent ones. When all non-empty cells have been visited, the server returns to each client information about cluster membership of the dense cells. Finally, each client assigns local points to clusters according to the following criteria:

- If a cell is dense, then all points in the cell are assigned to the cluster the cell belongs to;

- If a cell is not dense but at least one of the adjacent cells is dense, then each point in the cell is assigned to the cluster of the nearest adjacent dense cell;
- If a cell is not dense and none of the adjacent cells is dense, then points are marked as outliers.

The following pseudocode describes horizontal federated DBSCAN.

```
Horizontal Federated DBSCAN

Input
$L$: the granularity of the cells
$MinPts$: the cell density threshold

Method
Each client $m$
• Compute grid for its local dataset, based on $L$
• Evaluate the number of points in each non-empty cell and
  transmits this information to the server
Server
• For each cell $c$:
    • Compute $N_c$, the overall number of points in the cell $c$,
      obtained as the sum of the contributions from all clients
    • If $N_C \geq MinPts$: Mark $c$ as a dense cell
• Evaluate clustering by expanding a cluster along adjacent dense
  cells
• Transmit to each client information about cluster membership of
  each dense cell
Each client $m$
• For each cell $c$:
    • If $c$ is dense: assign all the points in $c$ to the cluster the
      cell belongs to, as assigned by the server
    • Else:
        • If at least one of the cells adjacent to $c$ is dense: assign
          each point in $c$ to the cluster of the nearest adjacent
          dense cell
        • Else: Mark the points in the cell as outliers
```

3.2 Vertical Federated DBSCAN

As for the horizontal federated version, we still refer to a set of clients willing to train a federated learning model interacting with a trustworthy server.

The first step of vertical federated DBSCAN requires to locally compute a neighborhood matrix and share this matrix with the server. As an example of such neighborhood matrix computation, consider the dataset is given in Figure 2. Eps-neighborhood of points 2 and 4 are represented, so that it is possible to see that the only neighboring-couples of points are (1,2), (2,3), (4,5). The neighborhood matrix Sim related to a dataset of N points is

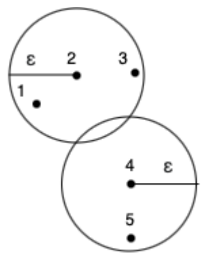

Fig. 2: a simple dataset

defined as a square matrix $N \times N$ whose (i, j)-th element is equal to 1 if and only if i and j are neighbors (otherwise it is equal to 0). Thus, the neighborhood matrix for the given dataset is:

$$Sim = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

(Notice that raw data are protected as the matrix only contains neighborhood information).

The server aggregates such local neighborhood matrices in a global neighborhood matrix, considering two points as neighbors if and only if they are neighbors for each client. Finally, the server executes an adapted version of the classical DBSCAN algorithm and shares the results with the clients.

The following pseudocode describes vertical federated DBSCAN.

```
Vertical Federated DBSCAN

Input
$Eps$: neighborhood radius
$MinPts$: the neighborhood density threshold

Method (Vertical DBSCAN)
Server
• Share with client $Eps$ parameter
Each client $m$
// Let $N$ be the number of points of the database
• For $i \in \{1,…,N\}$:
    • For $j \in \{1,…,N\}$:
• If distance($x_i$, $x_j$) $<$ $Eps$: $Sim_m[i,j] = Sim_m[j,i] = 1$
• Send $Sim_m$ matrix to server
Server
• For $i \in \{1,…,N\}$:
    • For $j \in \{1,…,N\}$:
        • If $Sim_m[i,j] = Sim_m[j,i] = 1$ for each client $m$: $Sim[i,j] = Sim[j,i] = 1$
• $Q$ = Server_DBSCAN($Sim$, $MinPts$)
• Send the $Q$ vector to the clients
```

```
Server_DBSCAN

Input
$Sim$: global neighborhood matrix
$MinPts$: the neighborhood density threshold
```

```
Method
• For each row $i$ in $Sim$:
    • If $i$ is visited: continue
    • Else:
        • Mark $i$ as visited
        • $NumPts = \text{sum}(i)$ // Sum of cells on the $i$-row equal to 1
        • If $NumPts < MinPts$: mark $i$ as noise
        • Else:
            • $c = newCluster$
            • $toVisit = \{j : Sim[i, j] = 1\}$
            • Expand_Cluster($Sim$, $i$, $toVisit$, $c$, $MinPts$)
• Let $Q = [q_1, \ldots, q_N]$ be the vector of cluster assignment of the
  points in the database
• Return $Q$
```

```
Expand_Cluster

Input
$Sim$: global neighborhood matrix
$i$: current row of the matrix, related to the $i$-th point
$c$: current cluster
$toVisit$: vector of points to visit
$MinPts$: the neighborhood density threshold

Method
• $q_i = c$ // Add $i$ to cluster $c$
• For $j$ in $toVisit$:
    • If $j$ is unvisited:
        • Mark $j$ as visited
        • $NumPts_j = \text{sum}(j)$ // Sum of cells on the $j$-row equal to 1
        • If $NumPts_j \geq MinPts$: $toVisit = toVisit \cup \{k : Sim[j, k] = 1\}$
    • If $j$ is not member of any cluster: $q_j = c$ // Add $j$ to cluster $c$
```

4. Python Implementation

4.1 Introduction

A Python implementation of the horizontal and vertical federated DBSCAN is here proposed. Flask and Flask-RESTful frameworks have been used to handle the clients-server network communication interface, based on HTTP post method.

Flask is a micro web framework: a web framework with a simple but extensible core. This means that Flask doesn't include by default many functionalities such as a database abstraction layer, form validation and so on; instead it supports extensions to add those functionalities to the application as they were implemented in Flask itself. Flask-RESTful is one of these extensions: it is a lightweight abstraction that allows to quickly build REST APIs. REST (REpresentational State Transfer) is a software architectural style that defines a set of constraints and guiding principles for the design and development of the architecture of Internet applications.

4.2 Utils

The file `utils.py` defines the functions that allow server and clients to communicate with each other.

```
# File: utils.py

import requests
import concurrent.futures
from typing import Dict, List

def send_post(url: str, data: Dict):
  r = requests.post(f'http://{url}', json = data)
  return r.json(), r.status_code

def process_http_posts(clients: List, data: Dict):
  with concurrent.futures.ThreadPoolExecutor() as executor:
    futures = [executor.submit(send_post, c, data) for c in clients]
    concurrent.futures.wait(futures)

  results = []
  failures = []

  for future in futures:
    failure = future.exception()
    if failure is not None:
      failures.append(failure)
    else:
      result = future.result()
      results.append(result[0])

  return results, failures
```

4.3 Horizontal Federated DBSCAN

The file `HF_DBSCAN/fd_dbscan.py` defines the methods of the federated server and client.

```
# File: HF_DBSCAN/fd_dbscan.py

import math
import numpy as np
from typing import List, Dict
from scipy.spatial import distance

def get_all_neighbors(cell: tuple):
  diag_coord = [(x - 1, x, x + 1) for x in cell]
  cartesian_product = [[]]
  for pool in diag_coord:
    cartesian_product = [(x + [y]) for x in cartesian_product for y in pool]

  neighbors = []
  for prod in cartesian_product:
    differential_coord = 0
    for i in range(len(prod)):
      if prod[i] != cell[i]:
        differential_coord += 1
    if differential_coord == 1:
      neighbors.append(tuple(prod))
  return neighbors

class FDBSCAN_Client():

  def initialize(self, params: Dict):
    self.__dataset = params['dataset']
    self.__L = params['L']
    self.__labels = []
    self.__true_labels = params['true_labels']
    self.__passive = True

  def get_dataset(self):
    return self.__dataset

  def get_labels(self):
    return self.__labels, self.__true_labels

  def is_passive(self):
    return self.__passive

  def __get_points(self, floor: bool = False):
    dimension = len(self.__dataset[0])
    points = []
    for row in self.__dataset:
      if floor:
        points.append(tuple(math.floor(row[i] / self.__L) for i in
range(dimension)))
```

```
      else:
        points.append(tuple(row[i] for i in range(dimension)))
    return points

  def compute_local_update(self):

    self.__passive = False

    cells = np.array(self.__get_points(floor = True))
    dimensions = len(cells[0])

    max_cell_coords = []
    min_cell_coords = []
    for i in range(dimensions):
      max_cell_coords.append(np.amax(cells[:, i]))
      min_cell_coords.append(np.amin(cells[:, i]))

    shifts = np.zeros(dimensions)
    for i in range(dimensions):
      if min_cell_coords[i] < 0:
        shifts[i] = -1 * min_cell_coords[i]

    shifted_dimensions = ()
    for i in range(dimensions):
      shifted_dimensions += (int(max_cell_coords[i] + 1 + shifts[i]), )

    count_matrix = np.zeros(shifted_dimensions)
    for cell in cells:
      shifted_cell_coords = ()
      for i in range(dimensions):
        shifted_cell_coords += (int(cell[i] + shifts[i]),)
      count_matrix[shifted_cell_coords] += 1

    non_zero = np.where(count_matrix > 0)
    non_zero_indexes = []
    for i in range(len(non_zero)):
      for j in range(len(non_zero[i])):
        if i == 0:
          non_zero_indexes.append((int(non_zero[i][j]), ))
        else:
          non_zero_indexes[j] += (int(non_zero[i][j]), )

    dict_to_return = {}
    for index in non_zero_indexes:
      shifted_index = ()
      for i in range(len(index)):
        shifted_index += (int(index[i] - shifts[i]), )
      dict_to_return[shifted_index] = count_matrix[index]

    return dict_to_return

  def assign_points_to_cluster(self, cells: List, labels: List):

    points = self.__get_points()

    dense_cells = []
    for row in cells:
      dense_cells.append(tuple(row))

    while len(points) > 0:
      actual_point = points.pop(0)
      actual_cell = tuple(math.floor(actual_point[i] / self.__L) for i in
range(len(actual_point)))
      outlier = True
      if actual_cell in dense_cells:
        self.__labels.append(labels[dense_cells.index(actual_cell)])
      else:
        min_dist = float('inf')
        cluster_to_assign = -1
        check_list = get_all_neighbors(actual_cell)
        for check_cell in check_list:
          if check_cell in dense_cells:
            cell_mid_point = tuple(cell_coord * self.__L + self.__L/2 for
cell_coord in check_cell)
            actual_dist = distance.euclidean(actual_point, cell_mid_point)
            if actual_dist < min_dist:
              min_dist = actual_dist
              cluster_to_assign = labels[dense_cells.index(check_cell)]
            outlier = False
        self.__labels.append(cluster_to_assign)

class FDBSCAN_Server():

  def initialize(self, params: Dict):
    self.__MIN_POINTS = params['MIN_POINTS']
    self.__running = False

  def get_running(self):
    return self.__running

  def run(self, value: bool = True):
    self.__running = value

  def compute_clusters(self, contribution_map: Dict):

    key_list = list(contribution_map.keys())
    value_list = list(contribution_map.values())

    n_cells = len(key_list)
    visited = np.zeros(n_cells)
    clustered = np.zeros(n_cells)
    cells = []
    labels = []
    cluster_ID = 0

    while 0 in visited:
      curr_index = np.random.choice(np.where(np.array(visited) == 0)[0])
      curr_cell = key_list[curr_index]
      visited[curr_index] = 1

      num_points = value_list[curr_index]
      if num_points >= self.__MIN_POINTS:
        cells.append(curr_cell)
        labels.append(cluster_ID)
        clustered[curr_index] = 1

        list_of_cells_to_check = get_all_neighbors(curr_cell)
        while len(list_of_cells_to_check) > 0:
          neighbor = list_of_cells_to_check.pop(0)
          neighbor_index = key_list.index(neighbor) if neighbor in key_list
else ""
          if neighbor in key_list and visited[neighbor_index] == 0:
            visited[neighbor_index] = 1
            if value_list[neighbor_index] >= self.__MIN_POINTS:
              list_of_cells_to_check += get_all_neighbors(neighbor)
            if clustered[neighbor_index] == 0:
              cells.append(neighbor)
              labels.append(cluster_ID)
              clustered[neighbor_index] = 1
        cluster_ID += 1

    return cells, labels
```

The server and client interfaces are defined in file `HF_DBSCAN/fd_server.py` and `HF_DBSCAN/fd_client.py`.

```
# File: HF_DBSCAN/fd_server.py

import random
from flask import Flask, request
from flask_restful import Resource, Api
from typing import Dict, List

from utils import process_http_posts
from fd_dbscan import FDBSCAN_Server

def training(clients: List, server: FDBSCAN_Server, clients_selection_seed:
int, missing_clients_percentage: int):
  import random
  random.seed(clients_selection_seed)
  n_clients = len(clients)
  n_clients_to_select = int(n_clients * (100 - missing_clients_percentage) /
100)
  selected_clients_idx = random.sample(range(n_clients), n_clients_to_select)
  selected_clients = [c for cli, c in enumerate(clients) if cli in
selected_clients_idx]
  print('Training started')
  print(f'Selected clients: {selected_clients_idx}')

  data = {'action': 'compute_local_update'}
  local_updates, failures = process_http_posts(selected_clients, data)
  contribution_map = {}
  for i in range(len(local_updates)):
    local_update = local_updates[i]
    for string_key, value in local_update.items():
      tuple_key = eval(string_key)
      if tuple_key in contribution_map:
        contribution_map[tuple_key] += value
      else:
        contribution_map[tuple_key] = value

  cells, labels = server.compute_clusters(contribution_map)

  data = {'action': 'assign_points_to_cluster', 'cells': cells, 'labels':
labels}
  process_http_posts(clients, data)

def connect(json_data: Dict, clients: List, client_ids: List, running: bool):
  if (not running):
    client_id = json_data.get('client_id')
    address = json_data.get('address')
    client_ids.append(client_id)
    clients.append(address)
    code = 200
    message = {'message': f'Client {client_id} Connected'}
  else:
    code = 500
    message = {'message': 'Unable to connect: a training process is runnning'}
  return message, code

def run_server(host: str, port: int, MIN_POINTS: int, clients_selection_seed:
int, missing_clients_percentage: int):

  clients = []
  client_ids = []

  params = {
    'MIN_POINTS': MIN_POINTS
  }
  server = FDBSCAN_Server()
  server.initialize(params)

class FederatedServer(Resource):

    def get(self):
      action = request.args.get('action')
      if (action == 'start'):
        server.run();
        training(clients, server, clients_selection_seed,
missing_clients_percentage)
        server.run(False);
        return {'message': 'Training completed'}
      else:
        return {'message': 'Hello world from server', 'clients': clients}

    def post(self):
      json_data = request.get_json()
      action = json_data.get('action')
      if (action == 'connect'):
        return connect(json_data, clients, client_ids, server.get_running())
      else:
        return {'message': 'Action not Supported'}, 201

  app = Flask(__name__)
  api = Api(app)
  api.add_resource(FederatedServer, '/')
  app.run(host = host, port = port)
```

```
# File: HF_DBSCAN/fd_client.py

from typing import List
from flask import Flask, request
from flask_restful import Resource, Api

from utils import send_post
from fd_dbscan import FDBSCAN_Client

def run_client(client_id: str, server_url: str, host: str, port: int, dataset:
List, L: float, true_labels: List):

  address = host + f':{port}'
  send_post(server_url, {'action': 'connect','client_id': client_id,
'address': address})

  params = {
    'dataset': dataset,
    'L': L,
    'true_labels': true_labels
  }
```

```
 client = FDBSCAN_Client()
  client.initialize(params)

  class FederatedClient(Resource):

    def get(self):
      action = request.args.get('action')
      if (action == 'results'):
        labels, true_labels = client.get_labels();
        return {'passive': client.is_passive(), 'dataset':
client.get_dataset().tolist(), 'labels': labels, 'true_labels':
true_labels.tolist()}
      else:
        return {'message': 'Hello World from Client', 'client_id': client_id}

    def post(self):
      json_data = request.json
      action = json_data.get('action')
      if (action == 'compute_local_update'):
        result = client.compute_local_update()
        to_return = {}
        for tuple_key in result:
          string_key = ','.join([str(coord) for coord in tuple_key])
          to_return[string_key] = result[tuple_key]
        return to_return
      elif (action == 'assign_points_to_cluster'):
        cells = json_data.get('cells')
        labels = json_data.get('labels')
        client.assign_points_to_cluster(cells, labels)
      else:
        return {'message': 'Action not Supported'}, 201

  app = Flask(client_id)
  api = Api(app)
  api.add_resource(FederatedClient, '/')
  app.run(host = host, port = port)
```

Finally, `HF_DBSCAN/main_server.py` runs the server and `HF_DBSCAN/main_client.py` runs the client.

```
# File: HF_DBSCAN/main_server.py

from fd_server import run_server

host = "127.0.0.1"
port = 8080
# MIN_POINTS = 4 # banana
MIN_POINTS = 15 # s-set1

clients_selection_seed = 1
missing_client_percentage = 10

run_server(host, port, MIN_POINTS, clients_selection_seed,
missing_client_percentage)
```

```
# File: HF_DBSCAN/main_client.py

import numpy as np
import pandas as pd
import uuid
```

```python
from typing import List
from threading import Thread
from sklearn.model_selection import StratifiedKFold
from sklearn.preprocessing import MinMaxScaler
from scipy.io import arff

from fd_client import run_client

def generate_dataset_chunks(X: np.array, Y: List, n_splits: int):
  if (n_splits == 1):
    return [X]
  skf = StratifiedKFold(n_splits = n_splits)
  dataset_chunks = []
  true_labels = []
  for train_index, test_index in skf.split(X, Y):
    dataset_chunks.append(X[test_index])
    true_labels.append(Y[test_index])
  return dataset_chunks, true_labels

def prepare_dataset(num_clients: int):
  dataset_dir = '../datasets'
  # dataset_file = 'banana.arff'
  dataset_file  = 's-set1.arff'
  dataset_path = f'{dataset_dir}/{dataset_file}'
  dataset = arff.loadarff(dataset_path)
  df = pd.DataFrame(dataset[0])

  Y = df['class'].tolist()
  Y = np.array([-1 if y == b'noise' else int(y) for y in Y])
  del df['class']
  X_original = np.array(df.values)
  min_max_scaler = MinMaxScaler()
  X = min_max_scaler.fit_transform(X_original)
  dataset_chunks, true_labels = generate_dataset_chunks(X, Y, num_clients)
  return dataset_chunks, true_labels

server_url = "127.0.0.1:8080"
host = "127.0.0.1"
start_port = 5000
N_clients = 10
dataset_chunks, true_labels = prepare_dataset(N_clients)
# L = 0.03 # banana
L = 0.03 # s-set1

threads = []
for cli in range(N_clients):
  client_id = uuid.uuid4().hex
  port = start_port + cli
  thread_obj = Thread(target = run_client, args = (client_id, server_url,
host, port, dataset_chunks[cli], L, true_labels[cli]))
  threads.append(thread_obj)
  thread_obj.start()

for t in threads:
  t.join()
```

4.3 Vertical Federated DBSCAN

As for the horizontal federated DBSCAN implementation, the file `F_DBSCAN/fd_dbscan.py` defines the methods of the federated server and client, their interfaces are defined in the files `VF_DBSCAN/fd_server.py` and `VF_DBSCAN/`

fd_client.py, and the VF_DBSCAN/main_server.py and VF_DBSCAN/main_client.py files run them.

```
# File: VF_DBSCAN/fd_dbscan.py

import numpy as np
from typing import List, Dict
from scipy.spatial import distance
from numba import jit

@jit(nopython=True)
def numba_euclidean_distance(u:np.ndarray, v:np.ndarray):
  return np.linalg.norm(u - v)

class FDBSCAN_Client():

  def initialize(self, params: Dict):
    self.__dataset = params['dataset']
    self.__labels = []

  def get_results(self):
    return self.__labels

  def __get_points(self):
    dimension = len(self.__dataset[0])
    points = []
    for row in self.__dataset:
      points.append(tuple(row[i] for i in range(dimension)))
    return points

  def compute_neighborhood_matrix(self, epsilon: float):
    points = self.__get_points()
    n_points = len(points)
    matrix = [[0] * n_points for i in range(n_points)]
    for i in range(n_points):
      for j in range(n_points):
        if (numba_euclidean_distance(points[i], points[j]) < epsilon):
          matrix[i][j] = 1
    return matrix

  def update_labels(self, labels: List):
    self.__labels = labels

class FDBSCAN_Server():

  def initialize(self, params: Dict):
    self.__MIN_POINTS = params['MIN_POINTS']
    self.__EPSILON = params['EPSILON']
    self.__running = False

  def get_running(self):
    return self.__running

  def run(self, value: bool = True):
    self.__running = value

  def get_epsilon(self):
    return self.__EPSILON
  def DBSCAN(self, global_matrix: np.ndarray):
    N = len(global_matrix)
    visited = np.zeros(N)
    labels = np.zeros(N)
    labels -= 1
    cluster_ID = 0
    for i in range(N):
      if visited[i]:
        continue
      else:
        visited[i] = 1
        num_points = np.sum(global_matrix[i])
        if num_points >= self.__MIN_POINTS:
          labels[i] = cluster_ID
          to_visit = np.where(global_matrix[i] == 1)[0].tolist()
          cluster_ID = self.__expand_cluster(to_visit, visited, global_matrix,
labels, cluster_ID)
    return labels

  def __expand_cluster(self, to_visit: List, visited: np.array,
global_matrix:np.ndarray, labels: np.array, cluster_ID: int):
    for j in to_visit:
      if visited[j] == 0:
        visited[j] = 1
        num_neighbors = np.sum(global_matrix[j])
        if num_neighbors >= self.__MIN_POINTS:
          to_visit += np.where(global_matrix[j] == 1)[0].tolist()
      if labels[j] == -1:
        labels[j] = cluster_ID
    cluster_ID += 1
    return cluster_ID
```

```
# File: VF_DBSCAN/fd_server.py

import numpy as np
from flask import Flask, request
from flask_restful import Resource, Api
from typing import Dict, List

from utils import process_http_posts
from fd_dbscan import FDBSCAN_Server

def training(clients: List, server: FDBSCAN_Server):
  data = {'action': 'compute_neighborhood_matrix', 'epsilon':
server.get_epsilon()}
  results, failures = process_http_posts(clients, data)

  N = len(results[0]['matrix'])
  global_matrix = np.zeros((N, N))
  for i in range(len(results)):
    matrix = np.array(results[i]['matrix'])
    global_matrix += matrix
  global_matrix = np.where(global_matrix < len(results), 0, 1)

  Q = server.DBSCAN(global_matrix)

  data = {'action': 'update_labels', 'labels': Q.tolist()}
  process_http_posts(clients, data)
```

```
def connect(json_data: Dict, clients: List, client_ids: List, running: bool):
  if (not running):
    client_id = json_data.get('client_id')
    address = json_data.get('address')
    client_ids.append(client_id)
    clients.append(address)
    code = 200
    message = {'message': f'Client {client_id} Connected'}
  else:
    code = 500
    message = {'message': 'Unable to connect: a training process is runnning'}
  return message, code

def run_server(host: str, port: int, MIN_POINTS: int, EPSILON: float):

  clients = []
  client_ids = []

  params = {
    'MIN_POINTS': MIN_POINTS,
    'EPSILON': EPSILON
  }
  server = FDBSCAN_Server()
  server.initialize(params)

  class FederatedServer(Resource):

    def get(self):
      action = request.args.get('action')
      if (action == 'start'):
        server.run();
        training(clients, server)
        server.run(False);
        return {'message': 'Training completed'}
      else:
        return {'message': 'Hello world from server', 'clients': clients}

    def post(self):
      json_data = request.get_json()
      action = json_data.get('action')
      if (action == 'connect'):
        return connect(json_data, clients, client_ids, server.get_running())
      else:
        return {'message': 'Action not Supported'}, 201

  app = Flask(__name__)
  api = Api(app)
  api.add_resource(FederatedServer, '/')
  app.run(host = host, port = port)
```

```
# File: VF_DBSCAN/fd_client.py

from typing import List
from flask import Flask, request
from flask_restful import Resource, Api

from utils import send_post
from fd_dbscan import FDBSCAN_Client
```

```
def run_client(client_id: str, server_url: str, host: str, port: int, dataset: 
List):

  address = host + f':{port}'
  send_post(server_url, {'action': 'connect','client_id': client_id, 
'address': address})

  params = {
    'dataset': dataset,
  }
  client = FDBSCAN_Client()
  client.initialize(params)

  class FederatedClient(Resource):

    def get(self):
      action = request.args.get('action')
      if (action == 'results'):
        labels = client.get_results();
        return {'labels': labels}
      else:
        return {'message': 'Hello World from Client', 'client_id': client_id}

    def post(self):
      json_data = request.json
      action = json_data.get('action')
      if (action == 'compute_neighborhood_matrix'):
        epsilon = json_data.get('epsilon')
        neighborhood_matrix = client.compute_neighborhood_matrix(epsilon)
        return {'matrix': neighborhood_matrix}
      elif (action == 'update_labels'):
        labels = json_data.get('labels')
        client.update_labels(labels)
      else:
        return {'message': 'Action not Supported'}, 201

  app = Flask(client_id)
  api = Api(app)
  api.add_resource(FederatedClient, '/')
  app.run(host = host, port = port)
```

```
# File: VF_DBSCAN/main_server.py

from fd_server import run_server

host = "127.0.0.1"
port = 8080

# MIN_POINTS = 4 # 3MC
MIN_POINTS = 6 # aggregation
# EPSILON = 0.1 # 3MC
EPSILON = 0.04 # aggregation

run_server(host, port, MIN_POINTS, EPSILON)
```

```
# File: VF_DBSCAN/main_client.py

import numpy as np
import pandas as pd
import uuid
from threading import Thread
from sklearn.preprocessing import MinMaxScaler
from scipy.io import arff

from fd_client import run_client

def generate_dataset_chunks(X: np.array, num_clients: int):
  dataset_chunks = []
  features_list = [i for i in range(len(X[0]))]
  for i in range(num_clients):
    dataset_chunks.append(X[:, features_list[i::num_clients]])
  return dataset_chunks

def prepare_dataset(num_clients: int):
  dataset_dir = '../datasets'
  # dataset_file = '3MC.arff'
  dataset_file = 'aggregation.arff'
  dataset_path = f'{dataset_dir}/{dataset_file}'
  dataset = arff.loadarff(dataset_path)
  df = pd.DataFrame(dataset[0])

  Y = df['class'].tolist()
  Y = np.array([-1 if y == b'noise' else int(y) for y in Y])
  del df['class']
  X_original = np.array(df.values)
  min_max_scaler = MinMaxScaler()
  X = min_max_scaler.fit_transform(X_original)
  dataset_chunks = generate_dataset_chunks(X, num_clients)
  return dataset_chunks

server_url = "127.0.0.1:8080"
host = "127.0.0.1"
start_port = 5000
N_clients = 2
dataset_chunks = prepare_dataset(N_clients)

threads = []
for cli in range(N_clients):
  client_id = uuid.uuid4().hex
  port = start_port + cli
  thread_obj = Thread(target = run_client, args = (client_id, server_url,
host, port, dataset_chunks[cli]))
  threads.append(thread_obj)
  thread_obj.start()

for t in threads:
  t.join()
```

5. Experimental Analysis

5.1 Experimental Setup

The presented horizontal and vertical federated versions of DBSCAN have been tested using the datasets described in Table 1 and represented in Figures 3 - 6.

	HORIZONTAL FEDERATED DBSCAN		**VERTICAL FEDERATED DBSCAN**	
Dataset	**banana.arff**	**s-set1.arff**	**aggregation.arff**	**3MC.arff**
Attributes	2	2	2	2
Points	4811	5000	788	400
Clusters	2	15	7	3
Outliers	0	0	0	0

Tab. 1: descriptive parameters of the datasets used to test horizontal federated DBSCAN (`banana.arff`, `s-set1.arff`) and vertical federated DBSCAN (`aggregation.arff`, `3MC.arff`)

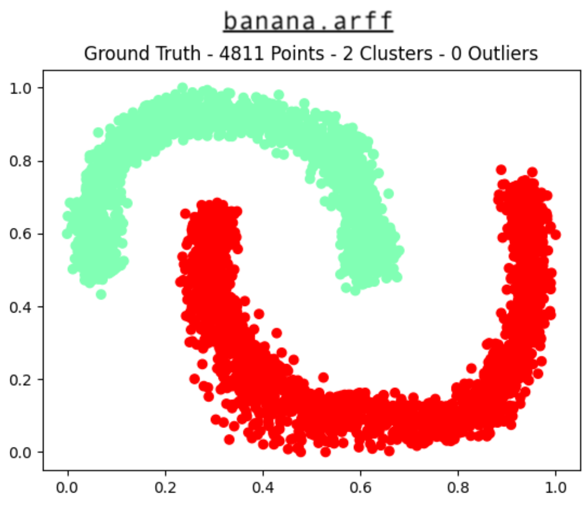

Fig. 3: `banana.arff` dataset, used to test horizontal federated DBSCAN

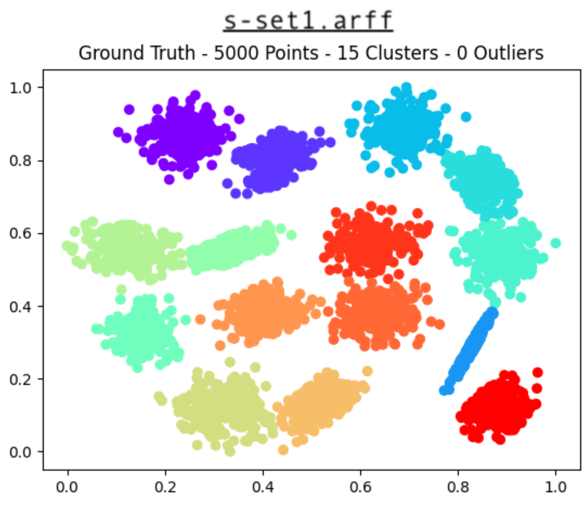

Fig. 4: `s-set1.arff` dataset, used to test horizontal federated DBSCAN

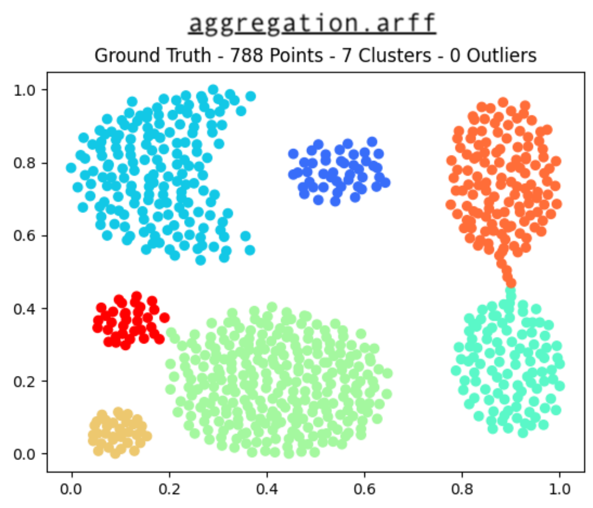

Fig. 5: aggregation.`arff` dataset, used to test vertical federated DBSCAN

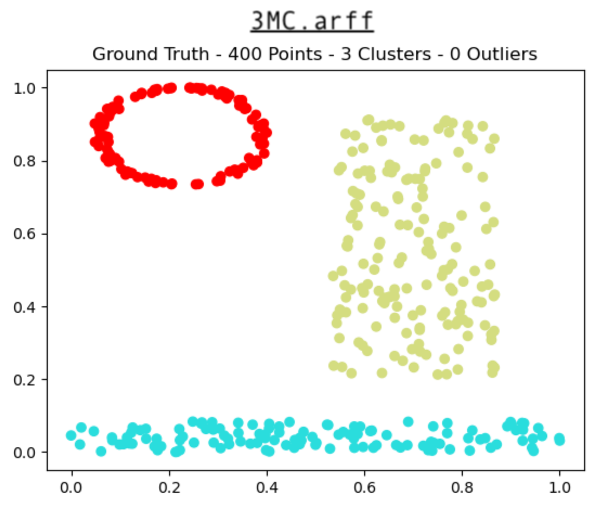

Fig. 6: aggregation.`arff` dataset, used to test vertical federated DBSCAN

Five different metrics, used to evaluate the presented algorithms, are here described: ARI and AMI scores, purity score, BCubed precision score and BCubed recall score. To introduce these metrics, consider a dataset $D = \{o_1, \ldots, o_N\}$ and two different class assignments: a ground truth class assignment (C) and a clustering algorithm class assignment (K).

ARI (Adjusted Rand Index) score is a function that measures the similarity of two class assignments. It is used to evaluate the similarity between K and C. It is defined as

$$\text{ARI} = \frac{\text{RI} - E[\text{RI}]}{\max(\text{RI}) - E[\text{RI}]}.$$

RI is the Rand Index score, defined as:

$$\text{RI} = \frac{a + b}{C_2^{n_{samples}}},$$

where a is the number of pairs of elements that are in the same set in C and in the same set in K, b is the number of pairs of elements that are in different sets in C and in different sets in K, and $C_2^{n_{samples}}$ is the total number of possible pairs in the dataset.

$E[\mathrm{RI}]$ is the expected RI of random labelings, and is discounted in the mathematical formulation of ARI to guarantee that random label assignments will get a value close to zero.

AMI (Adjusted Mutual Information) score is a function that measures the agreement of two class assignments. It is used to evaluate the agreement between K and C. It is defined as:

$$\mathrm{AMI} = \frac{\mathrm{MI} - E[\mathrm{MI}]}{\mathrm{mean}(H(K), H(C)) - E[\mathrm{MI}]}.$$

$H(X)$ is the entropy function, representing the amount of uncertainty for a partition set, defined as:

$$H(X) = -\sum_{i=1}^{|X|} \frac{|X_i|}{M} \log\left(\frac{|X_i|}{M}\right),$$

Where $M = |X|$, so that $\frac{|X_i|}{M}$ is the probability that an object picket at random from X falls into class X_i.
MI is the mutual information score, defined as:

$$\mathrm{MI} = \sum_{i=1}^{|K|} \sum_{j=1}^{|V|} \frac{|K_i \cap C_j|}{N} \log\left(\frac{N|K_i \cap C_i|}{|K_i||C_j|}\right),$$

and $E[\mathrm{MI}]$ is its expected value, and is discounted in the mathematical formulation of AMI to correct the effect of agreement solely due to chance between clusterings, similar to the way ARI corrects RI.

Purity score is defined as $\frac{1}{N}\sum_{i=1}^{|K|} \max_{C_i \in C} |K_i \cap C_i|$, and is a measure of the extent to which clusters in K contain points from a single class of C.

Finally, BCubed evaluates the precision and recall for every object in the clustering. The precision of an object indicates how many other objects in the same cluster belong to the same category as the object. The recall of an object reflects how many objects of the same category are assigned to the same cluster. Formally, let $L(o_i)$ be the category of object o_i, $(1 \leq i \leq N)$ according to ground truth C, and $K(o_i)$ be the cluster of object o_i, according to the clustering algorithm class assignments K. Then, for two objects o_i and o_j, $(1 \leq i, j \leq n$, $i \neq j)$, the correctness of the relation between object o_i and o_j in clustering K is equal to 1 if and only if $L(o_i) = L(o_j) \iff C(o_i) = C(o_j)$ (otherwise it is equal to 0). Thus Bcubed precision and recall are defined as follows:

$$\text{Precision BCubed} = \frac{\sum_{i=1}^{N} \frac{\sum_{o_j: i \neq j, C(o_i)=C(o_j)} \text{Correctness}(o_i, o_j)}{|\{o_j: i \neq j, C(o_i) = C(o_j)\}|}}{N},$$

$$\text{Recall BCubed} = \frac{\sum_{i=1}^{N} \frac{\sum_{o_j: i \neq j, L(o_i)=L(o_j)} \text{Correctness}(o_i, o_j)}{|\{o_j: i \neq j, L(o_i) = L(o_j)\}|}}{N}.$$

The script `results/metrics.py` has been used to evaluate these metrics.

```
# File: results/metrics.py

import sklearn.metrics as mtr
import numpy as np
import bcubed

def ARI_score(true_labels, predicted_labels):
  return mtr.adjusted_rand_score(true_labels, predicted_labels)

def AMI_score(true_labels, predicted_labels):
  return mtr.adjusted_mutual_info_score(true_labels, predicted_labels)

def PURITY_score(true_labels, predicted_labels):
  contingency_matrix = mtr.cluster.contingency_matrix(true_labels,
predicted_labels)
  return np.sum(np.amax(contingency_matrix, axis=0)) /
np.sum(contingency_matrix)

def BCubed_Precision_score(true_labels, predicted_labels):
  ldict = {}
  cdict = {}
  for i in range(len(true_labels)):
    ldict[i] = set([true_labels[i]])
    cdict[i] = set([predicted_labels[i]])
  return bcubed.precision(cdict, ldict)

def BCubed_Recall_score(true_labels, predicted_labels):
  ldict = {}
  cdict = {}
  for i in range(len(true_labels)):
    ldict[i] = set([true_labels[i]])
    cdict[i] = set([predicted_labels[i]])
  return bcubed.recall(cdict, ldict)

def print_metrics(true_labels, elaborated_labels, message):
  print(f'{message}:')
  print(f'Purity: {PURITY_score(true_labels, elaborated_labels):.4f}')
  print(f'ARI: {ARI_score(true_labels, elaborated_labels):.4f}')
  print(f'AMI: {AMI_score(true_labels, elaborated_labels):.4f}')
  print(f'BCubed Precision: {BCubed_Precision_score(true_labels,
elaborated_labels):.4f}')
  print(f'BCubed Recall: {BCubed_Recall_score(true_labels,
elaborated_labels):.4f}\n')
```

These metrics have been used in the experimental analysis here presented to evaluate the efficiency of DBSCAN algorithm and compare it to the efficiency of the federated versions of DBSCAN previously proposed[1]. The comparison between DBSCAN and its federated versions is the first goal of the present analysis. The second goal of the analysis is to study how performances vary when some clients are excluded from the learning process in horizontal federated DBSCAN. Specifically, we distinguish active clients from passive clients. Active clients share information with the server, and are engaged in the training process; passive clients, on the other hand, do not share any knowledge with the server, but are notified by the server at the conclusion of the learning process with the resulting clustering. The metrics presented above have been evaluated both for the active and the passive components of the learning system. The algorithm was tested with increasing passive clients percentages (10%, 20%, 30%). For each percentage, five tests have been run, each one with a different random clients selection, such that it was possible to compute the mean value (μ) and the standard deviation (σ) of the metrics in each of the cases.

The following scripts have been used to retrieve the results.

```
# File: results/HF_results.py

from scipy.io import arff
from sklearn.preprocessing import MinMaxScaler
from sklearn.cluster import DBSCAN
from urllib.request import urlopen
import numpy as np
import pandas as pd
import json

from plot import plot2D
import metrics as mtr

# results_folder = 'banana'
results_folder = 's-set1'

start_port = 5000
N_clients = 10
url = 'http://localhost:8080/?action=start'
urlopen(url)

dataset = []
active_dataset = []
passive_dataset = []
labels = []
active_labels = []
passive_labels = []
true_labels = []
active_true_labels = []
passive_true_labels = []
passive_clients = False
```

[1] Note that when estimating the efficiency of horizontal federated DBSCAN, the metrics have been evaluated referring to the concatenation of the clients datasets and class assignments.

```
for port in range(start_port, start_port + N_clients):
  url = f'http://localhost:{port}/?action=results'
  response = urlopen(url)
  data_json = json.loads(response.read())
  res_dataset = data_json['dataset']
  res_labels = data_json['labels']
  res_true_labels = data_json['true_labels']
  passive = data_json['passive']
  dataset += res_dataset
  labels += res_labels
  true_labels += res_true_labels
  if passive:
    passive_clients = True
    passive_dataset += res_dataset
    passive_labels += res_labels
    passive_true_labels += res_true_labels
  else:
    active_dataset += res_dataset
    active_labels += res_labels
    active_true_labels += res_true_labels

# MIN_POINTS = 4 # banana
MIN_POINTS = 15 # s-set1
# EPSILON = 0.03 # banana
EPSILON = 0.03 # s-set1
clustering = DBSCAN(eps = EPSILON, min_samples = MIN_POINTS)
dbscan_labels = clustering.fit_predict(np.array(dataset))

plot2D(points = np.array(dataset), labels = np.array(true_labels), folder =
results_folder, message = 'Ground Truth')
plot2D(points = np.array(dataset), labels = dbscan_labels, folder =
results_folder, message = 'DBSCAN')
plot2D(points = np.array(dataset), labels = np.array(labels), folder =
results_folder, message = 'Federated DBSCAN')
plot2D(points = np.array(active_dataset), labels = np.array(active_labels),
folder = results_folder, message = 'Federated DBSCAN - Active')
if passive_clients:
  plot2D(points = np.array(passive_dataset), labels =
np.array(passive_labels), folder = results_folder, message = 'Federated DBSCAN
- Passive’)

mtr.print_metrics(true_labels, dbscan_labels, 'DBSCAN')
mtr.print_metrics(true_labels, labels, 'Federated DBSCAN')
mtr.print_metrics(active_true_labels, active_labels, 'Federated DBSCAN -
Active')
if passive_clients:
  mtr.print_metrics(passive_true_labels, passive_labels, 'Federated DBSCAN -
Passive’)
```

```
# File: results/VF_results.py

from scipy.io import arff
from sklearn.preprocessing import MinMaxScaler
from sklearn.cluster import DBSCAN
from urllib.request import urlopen
import numpy as np
import pandas as pd
import json

from plot import plot2D
import metrics as mtr

# results_folder = ‘3MC'
results_folder = 'aggregation'
```

```
dataset_dir = '../datasets'
# dataset_file = ‘3MC.arff’
dataset_file = 'aggregation.arff'
dataset_path = f'{dataset_dir}/{dataset_file}'
dataset = arff.loadarff(dataset_path)
df = pd.DataFrame(dataset[0])

true_labels = df['class'].tolist()
true_labels = np.array([-1 if label == b'noise' else int(label) for label in
true_labels])
del df['class']
X_original = np.array(df.values)
min_max_scaler = MinMaxScaler()
X = min_max_scaler.fit_transform(X_original)

# MIN_POINTS = 4 # 3MC
MIN_POINTS = 6 # aggregation
# EPSILON = 0.1 # 3MC
EPSILON = 0.04 # aggregation
clustering = DBSCAN(eps = EPSILON, min_samples = MIN_POINTS)
dbscan_labels = clustering.fit_predict(X)

url = 'http://localhost:8080/?action=start'
urlopen(url)
url = 'http://localhost:5000/?action=results'
response = urlopen(url)
data_json = json.loads(response.read())
labels = np.array(data_json['labels'])

plot2D(points = X, labels = true_labels, folder = results_folder, message =
"Ground Truth")
plot2D(points = X, labels = dbscan_labels, folder = results_folder, message =
"DBSCAN")
plot2D(points = X, labels = labels, folder = results_folder, message =
f'Federated DBSCAN')

mtr.print_metrics(true_labels, dbscan_labels, 'DBSCAN')
mtr.print_metrics(true_labels, labels, 'Federated DBSCAN')
```

Finally, the script `results/plot.py` has been used to plot the results.

```
# File: results/plot.py

from matplotlib import pyplot as plt
import matplotlib.cm as cm
import numpy as np

def plot2D(points: np.ndarray, labels: np.array, folder: str, message: str):
  int_labels = [int(label) for label in labels]
  color_range = cm.rainbow(np.linspace(0, 1, np.max(np.unique(int_labels))+1))
  colors = []
  count_outliers = 0

  for label in int_labels:
    if label == -1:
      count_outliers += 1
      colors.append([0, 0, 0, 1])
    else:
      colors.append(color_range[label])
```

```
 plt.scatter(points[:, 0], points[:, 1], color = colors, marker = "o")
  plt.title(f'{message} - '
        f'{len(int_labels)} Points - '
        f'{len(np.unique(int_labels)) - (1 if count_outliers > 0 else 0)}
Clusters - '
        f'{count_outliers} Outliers')
  plt.savefig(f'{folder}/{message}.png')
  plt.clf()
```

5.2 Results

Table 2 shows the results obtained by vertical federated DBSCAN, compared to those obtained by DBSCAN[2]. Figures 7 - 10 graphically represents the resulting clusterings for visual inspection (black points denote outliers).

	DBSCAN		VERTICAL FEDERATED DBSCAN	
Dataset	**aggregation.arff**	**3MC.arff**	**aggregation.arff**	**3MC.arff**
AMI	0,9675	1,0000	0,9808	1,0000
ARI	0,9779	1,0000	0,9866	1,0000
Purity	0,9911	1,0000	0,9949	1,0000
BCubed Precision	0,9856	1,0000	0,9902	1,0000
BCubed Recall	0,9678	1,0000	0,9849	1,0000

Tab. 2: vertical federated DBSCAN resulting metrics compared to DBSCAN ones

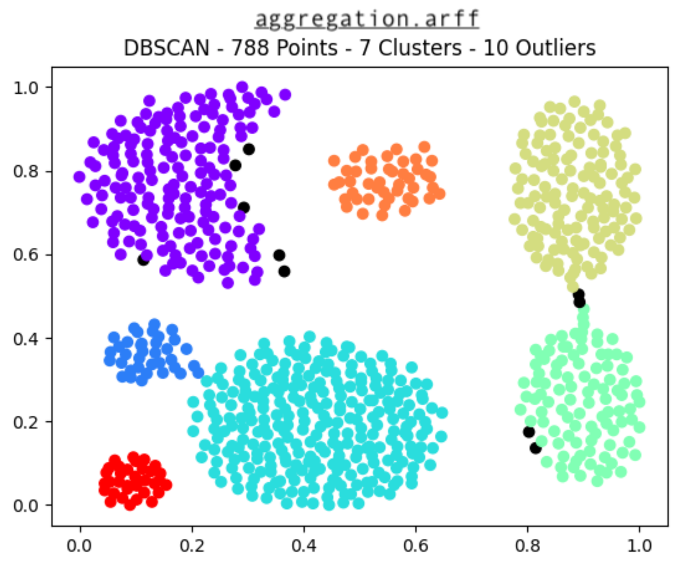

Fig. 7: `aggregation.arff` resulting clustering obtained by the application of DBSCAN

[2] When training the model for `aggregation.arff` dataset, $MinPts$ was set to 6, and Eps to 0.04; when training the model for `3MC.arff` dataset, $MinPts$ was set to 4, and Eps to 0.1.

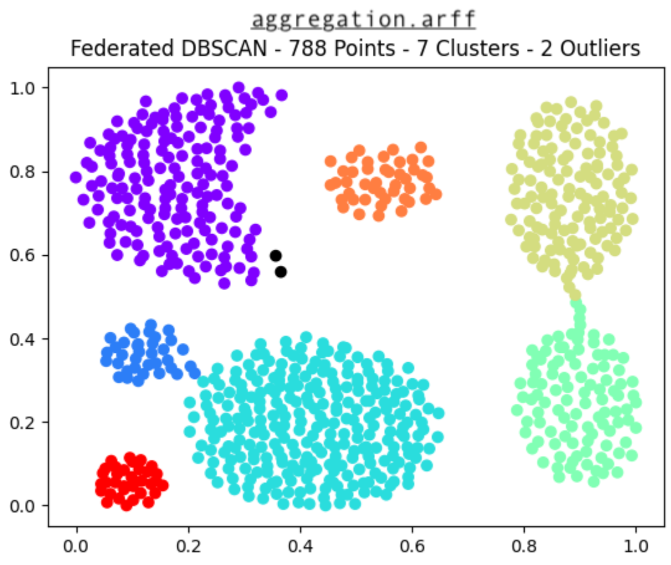

Fig. 8: `aggregation.arff` resulting clustering obtained by the application of vertical federated DBSCAN

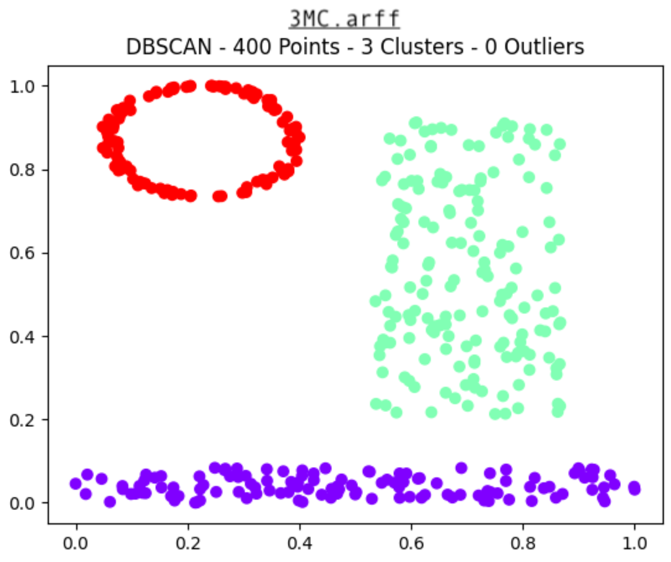

Fig. 9: `3MC.arff` resulting clustering obtained by the application of DBSCAN

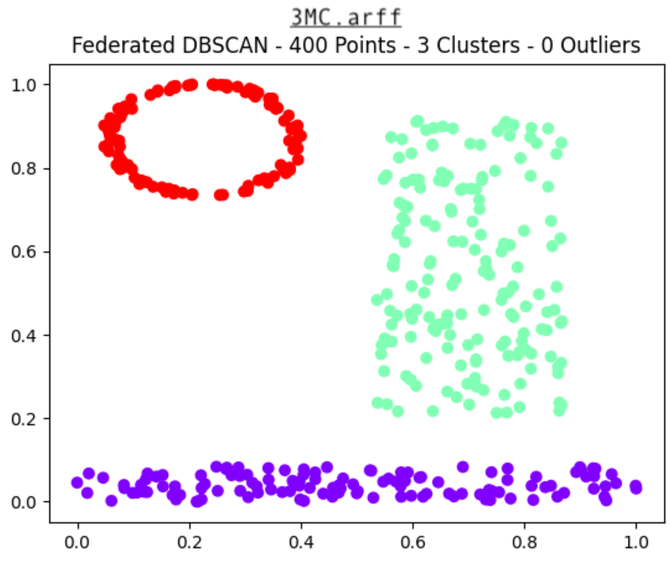

Fig. 10: `3MC.arff` resulting clustering obtained by the application of vertical federated DBSCAN

High-valued metrics prove the effectiveness and efficiency of both algorithms in modelling the input datasets. In comparing vertical federated DBSCAN to DBSCAN no performance degradation emerges. On the contrary, the federated approach presents slightly better performance in processing `aggregation.arff` dataset. This is reasonably due to the fact that DBSCAN neighborhoods have different shapes from vertical federated DBSCAN ones. In fact, consider for simplicity a bidimensional feature space. When running DBSCAN, the neighborhood of a point is the circle with radius *Eps* centered in that point. Conversely, when running vertical federated DBSCAN, as a consequence of the feature distribution across clients, the neighborhood of a point results to be the square with side 2*Eps* centered in that point. Thus, vertical federated DBSCAN neighborhoods are wider than DBSCAN ones, and when processing `aggregation.arff`, some points labeled as outliers by DBSCAN are instead included in the right cluster by vertical federated DBSCAN, improving performances.

Table 3 shows the results obtained by horizontal federated DBSCAN, compared to those obtained by DBSCAN[3]. Figures 11 - 14 graphically represents the resulting clusterings for visual inspection (black points denote outliers).

	STANDARD DBSCAN		**HORIZONTAL FEDERATED DBSCAN**	
Dataset	**banana.arff**	**s-set1.arff**	**banana.arff**	**s-set1.arff**
AMI	0,9881	0,9615	0,9956	0,9316
ARI	0,9956	0,9600	0,9984	0,9136
Purity	0,9996	0,9740	1,0000	0,9522
BCubed Precision	0,9993	0,9716	1,0000	0,9451
BCubed Recall	0,9954	0,9411	0,9983	0,8916

Tab. 3: horizontal federated DBSCAN resulting metrics compared to DBSCAN ones

[3] When training the model for `banana.arff` dataset, $MinPts$ was set to 4, while Eps and L were set to 0.03; when training the model for `s-set1.arff` dataset, $MinPts$ was set to 15, while Eps and L were set to 0.03.

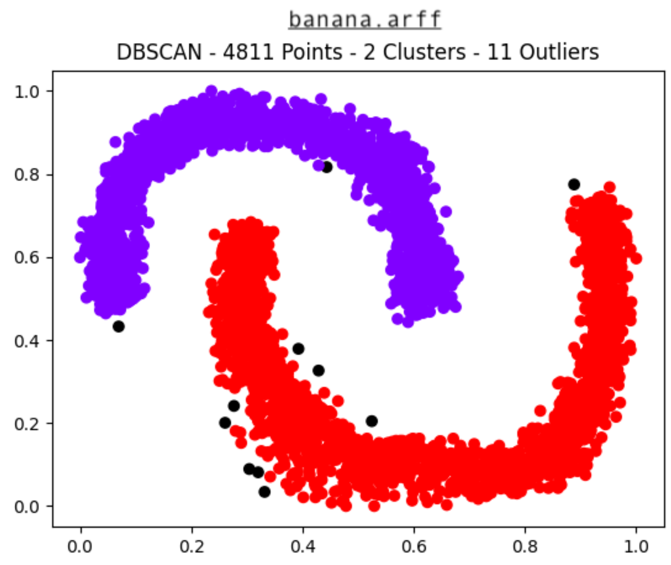

Fig. 11: `banana.arff` resulting clustering obtained by the application of DBSCAN

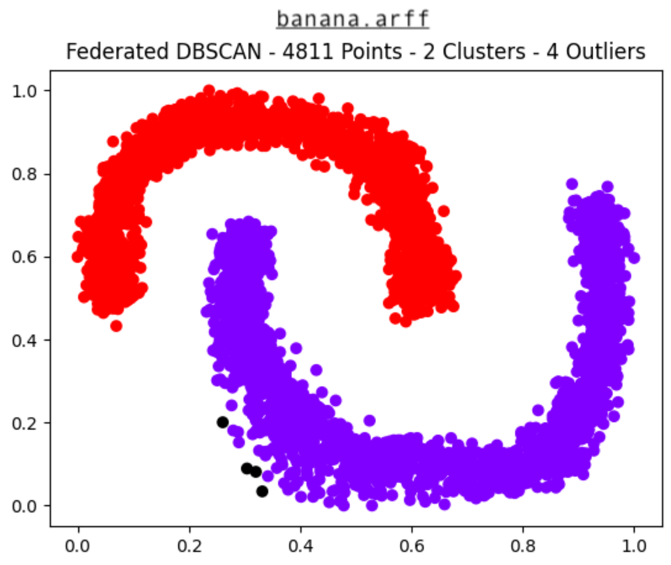

Fig. 12: `banana.arff` resulting clustering obtained by the application of horizontal federated DBSCAN

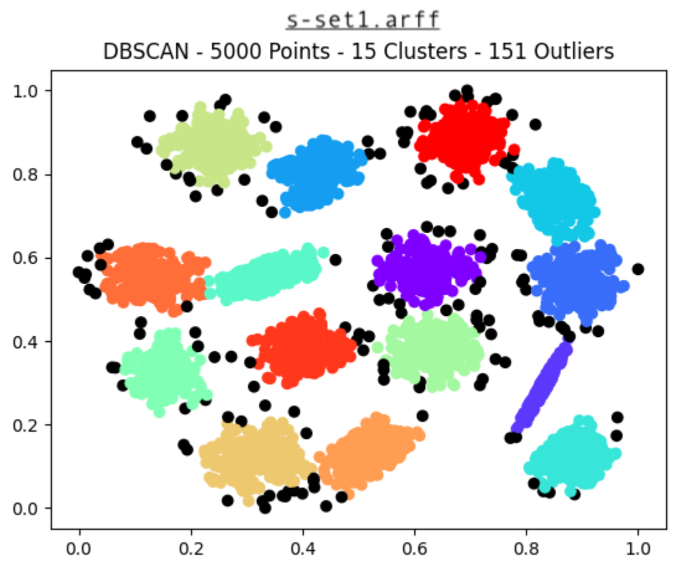

Fig. 13: `s-set1.arff` resulting clustering obtained by the application of DBSCAN

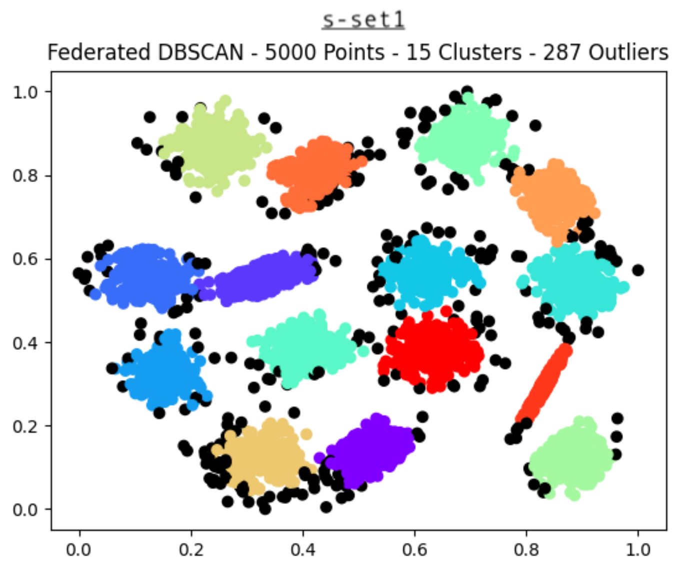

Fig. 14: `s-set1.arff` resulting clustering obtained by the application of horizontal federated DBSCAN

In this case too, high scores in metrics evaluations show the effectiveness and efficiency of both algorithms in processing the input datasets. The slight differences between the scores are reasonably due to the different geometric meaning of the two parameters *Eps* and *L*, both set to the same value in testing the algorithm. In fact, *Eps* is the length of the radius of the circular neighborhoods of the points in the datasets, while *L* is the granularity of the grid partitioning of the dataset. In the densest parts of the datasets this difference does not emerge, but in the less dense ones (such as the clusters contours), this difference leads to different outliers detection, thus improving or decreasing the performances depending on the clusters shapes.

Finally, Tables 4 - 6 and Figures 15 - 19 show the results obtained when testing horizontal federated DBSCAN over `banana.arff` dataset with an increasing percentage of passive clients.

	Test Round		I	II	III	IV	V	μ	σ
10%	AMI	Overall	0,9947	0,9947	0,9894	0,9912	0,9947	0,9929	0,0025
		Active	0,9952	0,9952	0,9884	0,9923	0,9952	0,9933	0,0030
		Passive	0,9904	0,9904	1,0000	0,9830	0,9904	0,9908	0,0060
	ARI	Overall	0,9980	0,9980	0,9961	0,9968	0,9980	0,9974	0,0009
		Active	0,9982	0,9982	0,9956	0,9973	0,9982	0,9975	0,0011
		Passive	0,9960	0,9960	1,0000	0,9921	0,9960	0,9960	0,0028
	Purity	Overall	1,0000	1,0000	1,0000	0,9998	1,0000	1,0000	0,0001
		Active	1,0000	1,0000	1,0000	0,9998	1,0000	1,0000	0,0001
		Passive	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,0000
	BCubed Precision	Overall	1,000	1,000	1,0000	0,9996	1,000	0,9999	0,0002
		Active	1,0000	1,0000	1,0000	0,9996	1,0000	0,9999	0,0002
		Passive	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,0000
	BCubed Recall	Overall	0,9979	0,9979	0,9959	0,9967	0,9979	0,9973	0,0009
		Active	0,9982	0,9982	0,9954	0,9972	0,9982	0,9974	0,0012
		Passive	0,9959	0,9959	1,0000	0,9918	0,9959	0,9959	0,0029

Tab. 4: horizontal federated DBSCAN resulting metrics when testing the algorithm with 10% passive clients percentage.

	Test Round		I	II	III	IV	V	μ	σ
20%	**AMI**	**Overall**	0,8665	0,9473	0,9508	0,9779	0,9826	0,9450	0,0466
		Active	0,9289	0,9651	0,9492	0,9936	0,9936	0,9661	0,0282
		Passive	0,7721	0,8984	0,9683	0,9345	0,9569	0,9060	0,0795
	ARI	**Overall**	0,8886	0,9769	0,9765	0,9913	0,9933	0,9653	0,0436
		Active	0,9527	0,9844	0,9745	0,9975	0,9975	0,9813	0,0187
		Passive	0,7504	0,9473	0,9851	0,9666	0,9766	0,9252	0,0987
	Purity	**Overall**	0,9985	0,9979	0,9996	0,9979	0,9988	0,9985	0,0007
		Active	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,0000
		Passive	0,9969	0,9917	1,0000	0,9938	0,9990	0,9963	0,0035
	BCubed Precision	**Overall**	0,9983	0,9970	0,9994	0,9978	0,9984	0,9982	0,0009
		Active	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,0000
		Passive	0,9952	0,9908	1,0000	0,9922	0,9981	0,9953	0,0039
	BCubed Recall	**Overall**	0,8834	0,9762	0,9758	0,9913	0,9934	0,9640	0,0458
		Active	0,9503	0,9836	0,9735	0,9974	0,9974	0,9804	0,0196
		Passive	0,7402	0,9472	0,9856	0,9673	0,9775	0,9236	0,1035

Tab. 5: horizontal federated DBSCAN resulting metrics when testing the algorithm with 20% passive clients percentage

	Test Round		I	II	III	IV	V	μ	σ
30%	AMI	Overall	0,9797	0,7093	0,9783	0,7830	0,6854	0,8271	0,1432
		Active	0,9886	0,7310	0,9899	0,7764	0,7390	0,8450	0,1328
		Passive	0,9651	0,6880	0,9599	0,8049	0,6254	0,8087	0,1545
	ARI	Overall	0,9913	0,6435	0,9905	0,7643	0,6747	0,8129	0,1685
		Active	0,9959	0,6673	0,9965	0,7618	0,7341	0,8311	0,1546
		Passive	0,9806	0,6171	0,9768	0,7723	0,5868	0,7867	0,1889
	Purity	Overall	0,9996	0,9985	0,9996	0,9998	0,9967	0,9988	0,0013
		Active	0,9994	0,9997	0,9994	0,9997	1,0000	0,9996	0,0003
		Passive	1,0000	0,9986	1,0000	1,0000	0,9896	0,9976	0,0045
	BCubed Precision	Overall	0,9992	0,9977	0,9992	0,9996	0,9942	0,9980	0,0022
		Active	0,9992	0,9995	0,9992	0,9995	1,0000	0,9995	0,0003
		Passive	1,0000	0,9974	1,0000	1,0000	0,9817	0,9958	0,0080
	BCubed Recall	Overall	0,9909	0,6492	0,9901	0,7525	0,6630	0,8091	0,1702
		Active	0,9959	0,6749	0,9964	0,7499	0,7207	0,8276	0,1562
		Passive	0,9797	0,6185	0,9757	0,7612	0,5827	0,7836	0,1894

Tab. 6: horizontal federated DBSCAN resulting metrics when testing the algorithm with 30% passive clients percentage

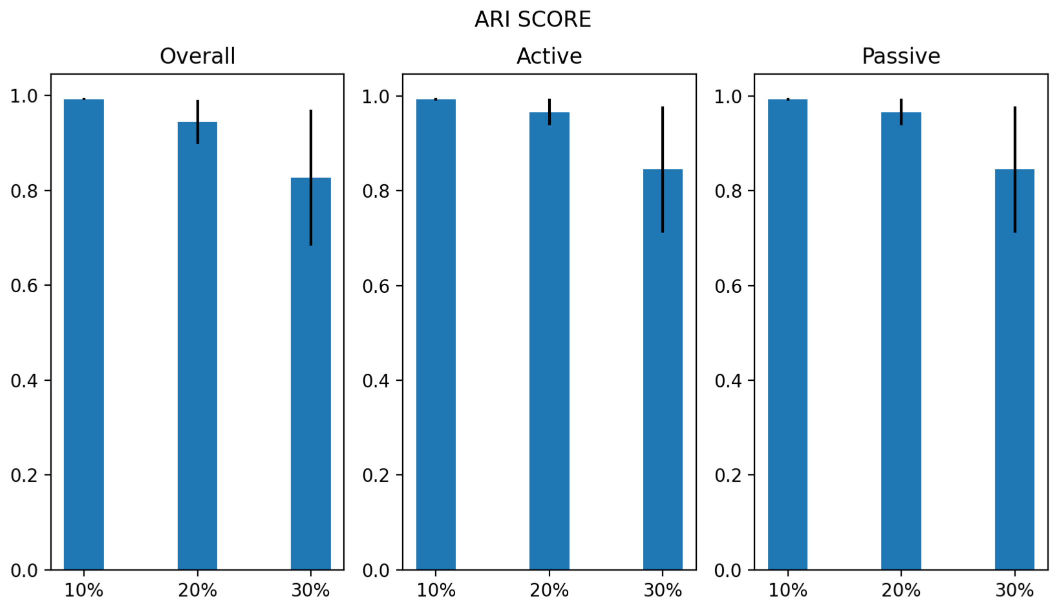

Fig. 15: horizontal federated DBSCAN ARI scores when testing the algorithm with increasing passive clients percentages

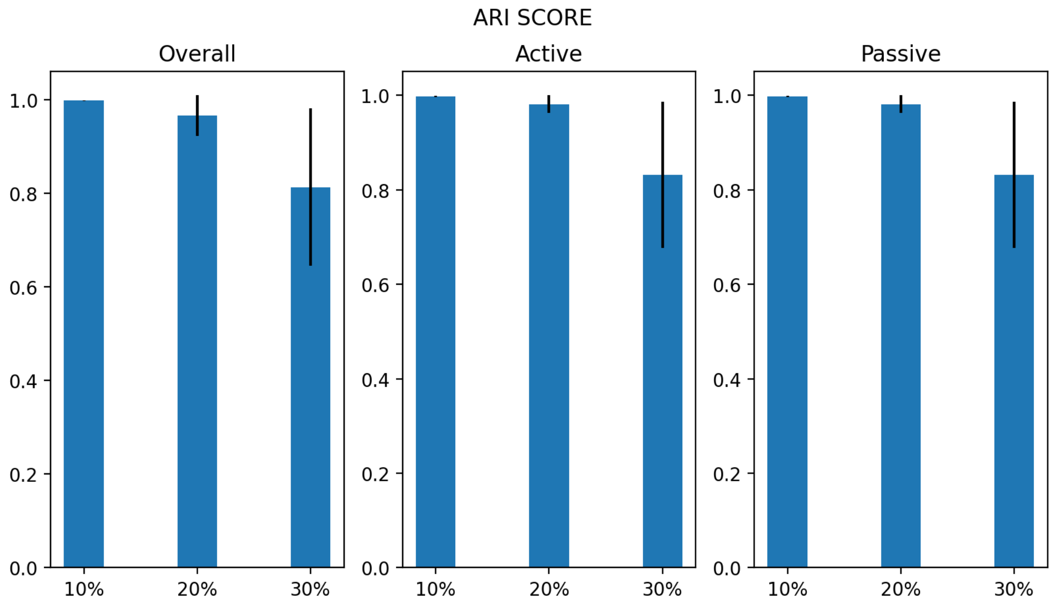

Fig. 16: horizontal federated DBSCAN AMI scores when testing the algorithm with increasing passive clients percentages

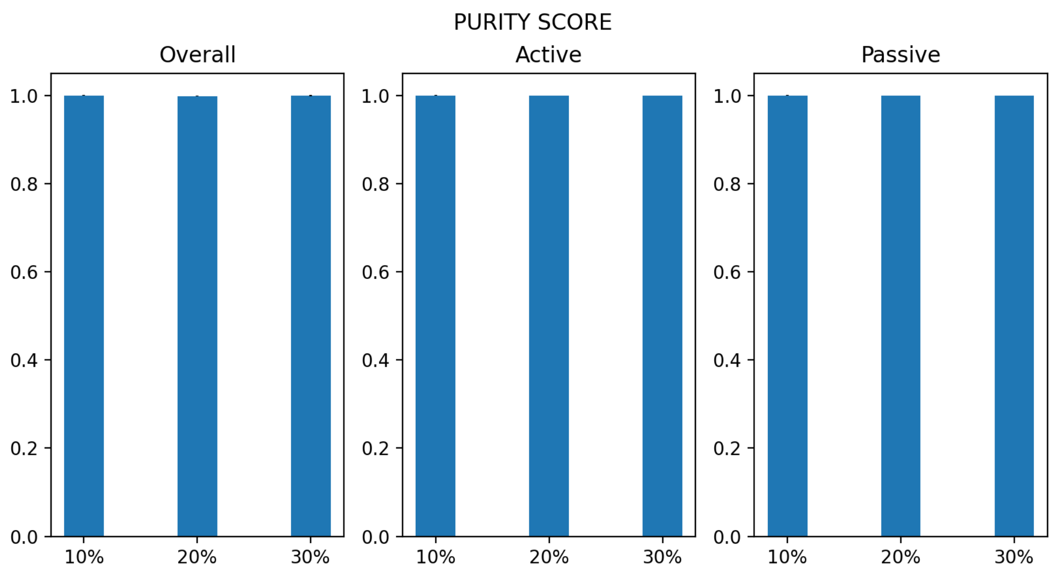

Fig. 17: horizontal federated DBSCAN Purity scores when testing the algorithm with increasing passive clients percentages

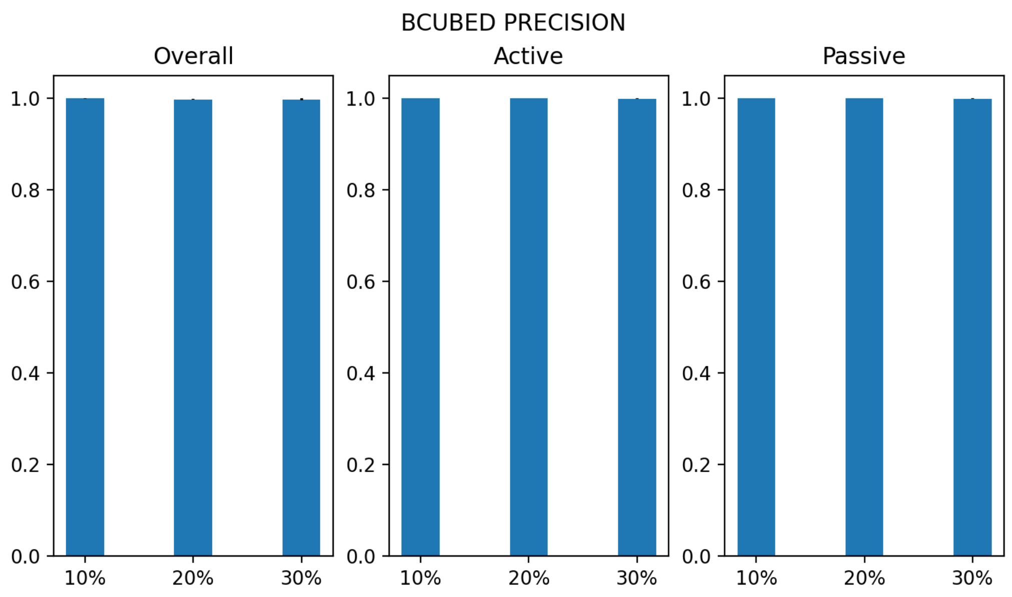

Fig. 18: horizontal federated DBSCAN BCubed Precision scores when testing the algorithm with increasing passive clients percentages

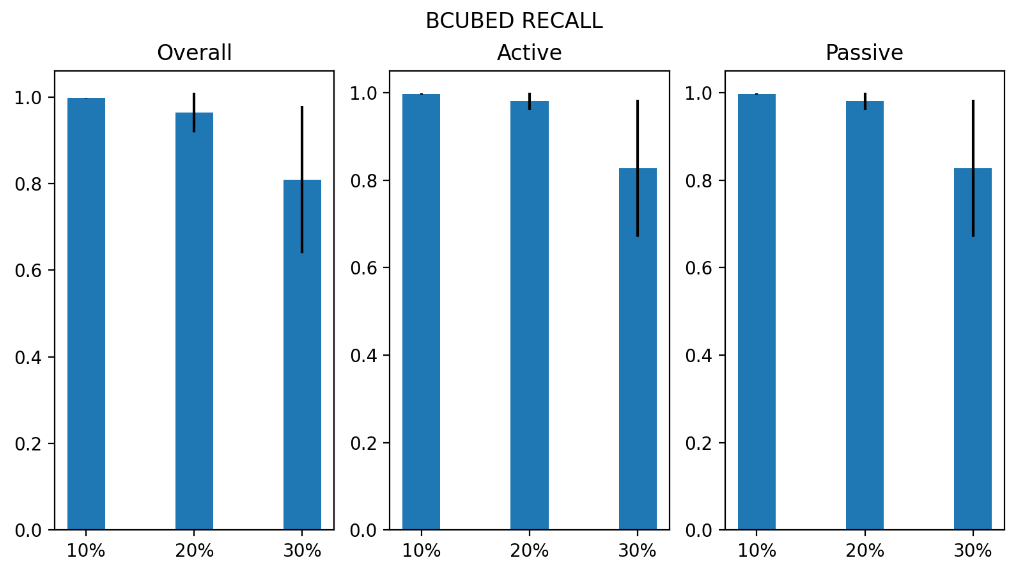

Fig. 19: horizontal federated DBSCAN BCubed Recall scores when testing the algorithm with increasing passive clients percentages

As could be expected, performances rapidly degrade when increasing the passive clients percentage. Nevertheless, no big differences can be spotted comparing the results obtained when testing the algorithm with a 10% passive clients percentage to those obtained in normal conditions. With a 20% passive clients percentage, scores start decreasing, but still retain high values (>0.95 for most of them, >0.9 for all of them) and a quite low standard deviation (<0.05 for most of them, <0.1 for all of them). Further increasing the passive client percentage to 30% leads to such a big loss of information that ARI, AMI and BCubed recall scores fall down to ~0.8, and their standard deviations rise up to ~0.15. Of course this result establish an upper bound to the amount of lack of information that the algorithm is able to support.

5.3 Conclusions

In this thesis, federated versions of DBSCAN algorithm have been introduced both for horizontally and vertically partitioned data. At first, a brief overview of DBSCAN algorithm and federated learning has been given. Then, the proposed solution for horizontal and vertical federated DBSCAN have been described. These versions of the well-known algorithm have the great advantage of preserving privacy, avoiding raw data sharing, when dealing with distributed databases. An implementation of both algorithms has then been presented, using Python programming language. Each federated version was finally tested using two different datasets, and different metrics were evaluated. The scores were then compared to those given by the application of DBSCAN. The federated versions proved to be equally efficient, giving slightly different but consistent results. Furthermore, the horizontal federated version was tested in the hypothesis of an increasing number of passive clients. Scores retained high values up to 20% passive clients, proving the algorithm to be quite resistant to raw data loss.

6. References

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