Università Di Pisa
Dipartimento Di Ingegneria Dell'Informazione

Corso Di Laurea Triennale In
Ingegneria Informatica

Federated Dbscan

Relatore:
Prof. Francesco Marcelloni

Laureando:
Gabriele Marino

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1. DBSCAN

1.1 Introduction

Clustering algorithms are used to solve identification problems in spatial databases. DBSCAN (Density Based Spatial Clustering of Applications with Noise) is a clustering algorithm which deals with good efficiency with the main problems that rise when applying clustering algorithms to large spatial databases: minimal requirements of domain knowledge to determine the input parameters and discovery of clusters with arbitrary shape.

1.2 A Density Based Notion of Clusters

DBSCAN relies on a density-based notion of clusters. The algorithm is based on the key idea is that for each point of a cluster the neighborhood of a given radius has to contain at least a minimum number of points, where both the radius of the neighbourhood and the minimum number of points depend on the specific cluster.

Let D be a database of points of some n-dimensional space E, and let $dist(p, q)$ be a distance function between two points of D. The following definitions hold.

Definition 1: (Eps-neighborhood of a point) The *Eps-neighborhood* of a point p, denoted by $N_{Eps}(p)$, is defined by $N_{Eps}(p) = \{q \in D \mid dist(p, q) \leq Eps\}$.

Definition 2: (directly density-reachable) A point p is *directly density-recheable* from a point q wrt. Eps, $MinPts$ if:

- $p \in N_{Eps}(q)$, and
- $|N_{Eps}(q)| \geq MinPts$ (core point condition).

Definition 3: (density-reachable) A point p is *density-recheable* from a point q wrt. Eps, $MinPts$ if there is a chain of points $p_1, \ldots, p_n$, such that $p_1 = q$, $p_n = p$ and p_{i+1} is directly reachable from p_i.

Definition 4: (density-connected) A point p is *density-connected* from a point q wrt. Eps, $MinPts$ if there is a point o such that both, p and q are density reachable from o wrt. Eps, $MinPts$.

Definition 5: (cluster) Let D be a database of points. A *cluster* C wrt. Eps, $MinPts$ is a non-empty subset of D satisfying the following conditions:

- $\forall\, p, q$: if $p \in C$ and q is density-reachable fro p wrt. Eps, $MinPts$, then $q \in C$. (Maximality)
- $\forall\, p, q \in C$: p is density-connected to q wrt. Eps, $MinPts$. (Connectivity)

Definition 6: (noise) Let $C_1, \ldots, C_k$ be the clusters of the database D wrt. Eps_i, $MinPts_i$, $i = 1, \ldots, k$. Then the *noise* is the set of points in the database D not belonging to any cluster C_i.

DBSCAN algorithm is designed to discover the clusters and the noise in a spatial database according to definitions 5 and 6, given Eps and $MinPts$.

1.3 The Algorithm

DBSCAN uses global values for Eps and $MinPts$ for all clusters. Good candidates for these parameter values are those specifying the density of the "thinnest" cluster of the database, i.e. the lowest density which is not considered to be noise.

Initially, all points in the database D are marked as "**unvisited**". DBSCAN randomly selects an unvisited point p, marks it as "**visited**" and checks the core point condition. If not, p is marked as a "**noise**" point. Otherwise, a new cluster C is created for p, and all the points in its neighborhood are added to a candidate set, N. Then, the points in N that do not belong to any cluster are iteratively added to C. Furthermore, for a point p' in N that carries the label "**unvisited**", DBSCAN marks it as "**visited**" and check its core point condition. If the condition holds, all points in the Eps-neighborhood of p' are added to N. The loop continues adding points to C until N reach emptiness. At this time, cluster C is completed. To compute the next cluster, DBSCAN randomly selects an "**unvisited**" point from the remaining ones, until all points are visited.

The following pseudocode describes DBSCAN algorithm.

```
DBSCAN

Input
D: database containing n points
Eps: the radius parameter
MinPts: the neighborhood density threshold

Output
A set of density-based clusters
```

Method

- Mark each point as *unvisited*
- **do:**
  - Randomly select an *unvisited* point p
  - Mark p as *visited*
  - **If** the Eps-neighborhood of p has at least $MinPts$ points:
    - Create a new cluster C
    - Add p to C
    - Let N be the set of points in the Eps-neighborhood of p
    - **For** each point p' in N:
      - **If** p' is *unvisited*:
        - Mark p' as *visited*
        - **If** the Eps-neighborhood of p' has at least $MinPts$ points: add those points to N
      - **If** p' is not yet a member of any cluster: add p' to C
    - Output C
  - **Else:** mark p as *noise*
- **Until** no point is *unvisited*

2. Federated Learning

2.1 Introduction

Federated learning deals with the possibility to fuse together data from different organizations. This is crucial in real-world situations, where with the exception of a few industries, most fields have only limited data or poor-quality data. Still, in many situations, it is very difficult to break the barriers between data sources. This is due to industry competition, privacy security and complicated administrative procedures. In fact, as a result of new data regulations and privacy laws, we are forbidden to collect, fuse and process data from different places. Federated learning is a possible solution for this challenge.

2.2 A Categorization of Federated Learning

To define what federated learning is, consider N data owners $\{P_1, \ldots, P_N\}$ with their respective data $\{D_1, \ldots, D_N\}$, and let M_{SUM} be the model trained by $D = D_1 \cup \ldots \cup D_N$. A federated-learning system is a learning process in which the data owners collaboratively train a model M_{FED} without exposing their own data to others. The accuracy of M_{FED}, V_{FED}, should be as close as possible to that of M_{SUM}, V_{SUM}. Formally, let δ be a non-negative real number; if $|V_{FED} - V_{SUM}| < \delta$, the federated learning algorithm is said to have δ-accuracy loss.

The definition given above for federated learning systems is quite general. Federated learning systems can be coarsely categorized based on the data partitioning scheme, i.e. how data are distributed across the various data owners. To introduce this categorization, let F_i be the feature space and I_i the sample ID space of the data D_i. Then, we can distinguish *horizontal federated learning* from *vertical federated learning* as follows.

In horizontal federated learning the dataset is said to be *sample-partitioned*, and the following relations hold:

$$F_i = F_j, \qquad I_i \neq I_j, \qquad \forall\, D_i, D_j\colon i \neq j,$$

whilst in vertical federated learning the dataset is *feature-partitioned*:

$$F_i \neq F_j, \qquad I_i = I_j, \qquad \forall\, D_i, D_j\colon i \neq j.$$

3. Federated DBSCAN

3.1 Horizontal Federated DBSCAN

In presenting the algorithm we refer to a set of clients willing to train a federated learning model interacting with a trustworthy server.

The main idea of the algorithm is the partitioning of each local feature space with a fixed granularity. The domain of definition of the features and the granularity of the grid are known and are the same for each client. This approach allow each client to share with the server only the number of points within the non-empty cells of the grid, preventing raw data sharing and thus preserving privacy.

The following pseudocode describes horizontal federated DBSCAN.

Horizontal Federated DBSCAN

Input
L: the granularity of the cells
$MinPts$: the cell density threshold

Method
Each client m
- Compute grid for its local dataset, based on L
- Evaluate the number of points in each non-empty cell and transmits this information to the server

Server
- **For** each cell c:
  - Compute N_c, the overall number of points in the cell c, obtained as the sum of the contributions from all clients
  - **If** $N_C \geq MinPts$: Mark c as a *dense* cell
- Evaluate clustering by expanding a cluster along adjacent dense cells
- Transmit to each client information about cluster membership of each dense cell

Each client m
- **For** each cell c:
  - **If** c is *dense*: assign all the points in c to the cluster the cell belongs to, as assigned by the server
  - **Else:**
    - **If** at least one of the cells adjacent to c is dense: assign each point in c to the cluster of the nearest adjacent dense cell
    - **Else:** Mark the points in the cell as outliers

3.2 Vertical Federated DBSCAN

As for the horizontal federated version, we still refer to a set of clients willing to train a federated learning model interacting with a trustworthy server.

The main idea of the algorithm is to locally compute a neighborhood matrix and share this matrix with the server, thus protecting raw data. The server then aggregates such local neighborhood matrices in a global neighborhood matrix, considering two points as neighbors if and only if they are neighbors for each client. Finally, the server executes an adapted version of the classical DBSCAN algorithm and share the results with the clients.

The following pseudocode describes vertical federated DBSCAN.

<u>Vertical Federated DBSCAN</u>

Input
Eps: neighborhood radius
$MinPts$: the neighborhood density threshold

Method (Vertical DBSCAN)
<u>Server</u>
- Share with client Eps parameter

<u>Each client m</u>
// Let N be the number of points of the database
- **For** $i \in \{1,\dots,N\}$:
  - **For** $j \in \{1,\dots,N\}$:
    - **If** distance(x_i, x_j) < Eps: $Sim_m[i,j] = Sim_m[j,i] = 1$
- Send Sim_m matrix to server

<u>Server</u>
- **For** $i \in \{1,\dots,N\}$:
  - **For** $j \in \{1,\dots,N\}$:
    - **If** $Sim_m[i,j] = Sim_m[j,i] = 1$ for each client m: $Sim[i,j] = Sim[j,i] = 1$
- Q = Server_DBSCAN(Sim, $MinPts$)
- Send the Q vector to the clients

<u>Server_DBSCAN</u>

Input
Sim: global neighborhood matrix
$MinPts$: the neighborhood density threshold

Method
- **For** each row i in Sim:
  - **If** i is *visited*: **continue**
  - **Else:**
    - Mark i as *visited*
    - $NumPts$ = sum(i) // Sum of cells on the i-row equal to 1

- **If** $NumPts < MinPts$: mark i as *noise*
- **Else:**
  - $c = newCluster$
  - $toVisit = \{j : Sim[i,j] = 1\}$
  - Expand_Cluster(Sim, i, $toVisit$, c, $MinPts$)
- Let $Q = [q_1, \ldots, q_N]$ be the vector of cluster assignment of the points in the database
- **Return** Q

Expand_Cluster

Input
Sim: global neighborhood matrix
i: current row of the matrix, related to the i–th point
c: current cluster
$toVisit$: vector of points to visit
$MinPts$: the neighborhood density threshold

Method
- $q_i = c$ // Add i to cluster c
- **For** j in $toVisit$:
  - **If** j is *unvisited:*
    - Mark j as *visited*
    - $NumPts_j = \mathrm{sum}(j)$ // Sum of cells on the j–row equal to 1
    - **If** $NumPts_j \geq MinPts$: $toVisit = toVisit \cup \{k : Sim[j,k] = 1\}$
  - **If** j is not member of any cluster: $q_j = c$ // Add j to cluster c

4. Python Implementation

4.1 Introduction

A Python implementation of the horizontal and vertical federated DBSCAN is here proposed. Flask and Flask-RESTful frameworks and HTTP post method have been used to handle the clients-server network communication.

The following function allow server and clients to communicate with each other.

```
# File: utils.py

import requests
import concurrent.futures
from typing import Dict, List

def send_post(url: str, data: Dict):
  r = requests.post(f'http://{url}', json = data)
  return r.status_code, r.json()

def process_http_posts(clients: List, data: Dict):
  with concurrent.futures.ThreadPoolExecutor() as executor:
    futures = [executor.submit(send_post, c, data) for c in clients]
    concurrent.futures.wait(futures)

  results = []
  failures = []


  for future in futures:
    failure = future.exception()
    if failure is not None:
      failures.append(failure)
    else:
      result = future.result()
      results.append(result[1])

  return results, failures
```

4.2 Horizontal Federated DBSCAN

The file `HF_DBSCAN/fd_dbscan.py` defines the methods of the federated server and client.

```
# File: HF_DBSCAN/fd_dbscan.pt

import math
import numpy as np
from typing import List, Dict
from scipy.spatial import distance
```

```python
def get_all_neighbors(cell: tuple):
  diag_coord = [(x - 1, x, x + 1) for x in cell]
  cartesian_product = [[]]
  for pool in diag_coord:
    cartesian_product = [(x + [y]) for x in cartesian_product for y in pool]

  neighbors = []
  for prod in cartesian_product:
    differential_coord = 0
    for i in range(len(prod)):
      if prod[i] != cell[i]:
        differential_coord += 1
    if differential_coord == 1:
      neighbors.append(tuple(prod))
  return neighbors

class FDBSCAN_Client():

  def initialize(self, params: Dict):
    self.__dataset = params['dataset']
    self.__L = params['L']
    self.__labels = []
    self.__training_completed = False

  def get_dataset(self):
    return self.__dataset

  def get_results(self):
    if self.__training_completed:
      dataset = np.array(self.__dataset)
      labels = np.array([self.__labels])
      return np.concatenate((dataset, labels.T), axis = 1)
    else:
      return []

  def get_points(self, floor: bool = False):
    dimension = len(self.__dataset[0])
    points = []
    for row in self.__dataset:
      if floor:
        points.append(tuple(math.floor(row[i] / self.__L) for i in
range(dimension)))
      else:
        points.append(tuple(row[i] for i in range(dimension)))
    return points

  def compute_local_update(self):

    cells = np.array(self.get_points(floor = True))
    dimensions = len(cells[0])

    max_cell_coords = []
    min_cell_coords = []
    for i in range(dimensions):
      max_cell_coords.append(np.amax(cells[:, i]))
      min_cell_coords.append(np.amin(cells[:, i]))

    shifts = np.zeros(dimensions)
    for i in range(dimensions):
      if min_cell_coords[i] < 0:
        shifts[i] = -1 * min_cell_coords[I]

    shifted_dimensions = ()
    for i in range(dimensions):
      shifted_dimensions += (int(max_cell_coords[i] + 1 + shifts[i]), )
```

```
    count_matrix = np.zeros(shifted_dimensions)
    for cell in cells:
      shifted_cell_coords = ()
      for i in range(dimensions):
        shifted_cell_coords += (int(cell[i] + shifts[i]),)
      count_matrix[shifted_cell_coords] += 1

    non_zero = np.where(count_matrix > 0)
    non_zero_indexes = []
    for i in range(len(non_zero)):
      for j in range(len(non_zero[i])):
        if i == 0:
          non_zero_indexes.append((int(non_zero[i][j]), ))
        else:
          non_zero_indexes[j] += (int(non_zero[i][j]), )

    dict_to_return = {}
    for index in non_zero_indexes:
      shifted_index = ()
      for i in range(len(index)):
        shifted_index += (int(index[i] - shifts[i]), )
      dict_to_return[shifted_index] = count_matrix[index]

    return dict_to_return

  def assign_points_to_cluster(self, cells: List, labels: List):

    points = self.get_points()

    dense_cells = []
    for row in cells:
      dense_cells.append(tuple(row))

    while len(points) > 0:
      actual_point = points.pop(0)
      actual_cell = tuple(math.floor(actual_point[I] / self.__L) for i in
range(len(actual_point)))
      outlier = True
      if actual_cell in dense_cells:
        self.__labels.append(labels[dense_cells.index(actual_cell)])
      else:
        min_dist = float('inf')
        cluster_to_assign = -1
        check_list = get_all_neighbors(actual_cell)
        for check_cell in check_list:
          if check_cell in dense_cells:
            cell_mid_point = tuple(cell_coord * self.__L + self.__L/2 for
cell_coord in check_cell)
            actual_dist = distance.euclidean(actual_point, cell_mid_point)
            if actual_dist < min_dist:
              min_dist = actual_dist
              cluster_to_assign = labels[dense_cells.index(check_cell)]
            outlier = False
        self.__labels.append(cluster_to_assign)

    self.__training_completed = True

class FDBSCAN_Server():

  def initialize(self, params: Dict):
    self.__MIN_POINTS = params['MIN_POINTS']
    self.__running = False
```

```
  def get_running(self):
    return self.__running

  def run(self, value: bool = True):
    self.__running = value

  def compute_clusters(self, contribution_map: Dict):

    key_list = list(contribution_map.keys())
    value_list = list(contribution_map.values())

    n_cells = len(key_list)
    visited = np.zeros(n_cells)
    clustered = np.zeros(n_cells)
    cells = []
    labels = []
    cluster_ID = 0

    while 0 in visited:
      curr_index = np.random.choice(np.where(np.array(visited) == 0)[0])
      curr_cell = key_list[curr_index]
      visited[curr_index] = 1

      num_points = value_list[curr_index]
      if num_points >= self.__MIN_POINTS:
        cells.append(curr_cell)
        labels.append(cluster_ID)
        clustered[curr_index] = 1

        list_of_cells_to_check = get_all_neighbors(curr_cell)
        while len(list_of_cells_to_check) > 0:
          neighbor = list_of_cells_to_check.pop(0)
          neighbor_index = key_list.index(neighbor) if neighbor in key_list
else ""
          if neighbor in key_list and visited[neighbor_index] == 0:
            visited[neighbor_index] = 1
            if value_list[neighbor_index] >= self.__MIN_POINTS:
              list_of_cells_to_check += get_all_neighbors(neighbor)
            if clustered[neighbor_index] == 0:
              cells.append(neighbor)
              labels.append(cluster_ID)
              clustered[neighbor_index] = 1
        cluster_ID += 1

    return cells, labels
```

The server and client interface are defined in the files `HF_DBSCAN/fd_server.py` and `HF_DBSCAN/fd_client.py`.

```
# File: HF_DBSCAN/fd_server.py

import random
from flask import Flask, request
from flask_restful import Resource, Api
from typing import Dict, List

from utils import process_http_posts
from fd_dbscan import FDBSCAN_Server
```

```python
def training(clients: List, server: FDBSCAN_Server):

  data = {'action': 'compute_local_update'}
  # clients.pop(0) # Missing client simulation
  local_updates, failures = process_http_posts(clients, data)
  contribution_map = {}
  for i in range(len(local_updates)):
    local_update = local_updates[i]
    for string_key, value in local_update.items():
      tuple_key = eval(string_key)
      if tuple_key in contribution_map:
        contribution_map[tuple_key] += value
      else:
        contribution_map[tuple_key] = value

  cells, labels = server.compute_clusters(contribution_map)

  data = {'action': 'assign_points_to_cluster', 'cells': cells, 'labels':
labels}
  process_http_posts(clients, data)

def connect(json_data: Dict, clients: List, client_ids: List, running: bool):
  code = 200
  if (not running):
    client_id = json_data.get('client_id')
    address = json_data.get('address')
    client_ids.append(client_id)
    clients.append(address)
    message = {'message': f'Client {client_id} Connected'}
  else:
    message = {'message': 'Unable to connect: a training process is runnning'}
    code = 500
  return message, code

def run_server(host: str, port: int, MIN_POINTS: int):

  clients = []
  client_ids = []

  params = {
    'MIN_POINTS': MIN_POINTS
  }
  server = FDBSCAN_Server()
  server.initialize(params)

  class FederatedServer(Resource):

    def get(self):
      action = request.args.get('action')
      if (action == 'start'):
        server.run();
        training(clients, server)
        server.run(False);
        return {'message': 'Training completed'}
      else:
        return {'message': 'Hello world from server', 'clients': clients}

    def post(self):
      json_data = request.get_json()
      action = json_data.get('action')
      if (action == 'connect'):
        return connect(json_data, clients, client_ids, server.get_running())
      else:
        return {'message': 'Action not Supported'}
```

```python
    app = Flask(__name__)
    api = Api(app)
    api.add_resource(FederatedServer, '/')
    app.run(host = host, port = port)
```

```python
# File: HF_DBSCAN/fd_client.py

from typing import List
from flask import Flask, request
from flask_restful import Resource, Api

from utils import send_post
from fd_dbscan import FDBSCAN_Client

def run_client(client_id: str, server_url: str, host: str, port: int, dataset: 
List, L: float):

  address = host + f':{port}'
  send_post(server_url, {'action': 'connect','client_id': client_id, 
'address': address})

  params = {
    'dataset': dataset,
    'L': L
  }
  client = FDBSCAN_Client()
  client.initialize(params)

  class FederatedClient(Resource):

    def get(self):
      action = request.args.get('action')
      if (action == 'dataset'):
        dataset = client.get_dataset().tolist()
        return {'rows': len(dataset), 'dataset': dataset}
      elif (action == 'results'):
        results = client.get_results();
        if len(results) > 0:
          return {'results': results.tolist()}
        else:
          return {'message': 'Training not completed', 'results': []}
      else:
        return {'message': 'Hello World from Client', 'client_id': client_id}

    def post(self):
      json_data = request.json
      action = json_data.get('action')
      if (action == 'compute_local_update'):
        result = client.compute_local_update()
        to_return = {}
        for tuple_key in result:
          string_key = ','.join([str(coord) for coord in tuple_key])
          to_return[string_key] = result[tuple_key]
        return to_return
      elif (action == 'assign_points_to_cluster'):
        cells = json_data.get('cells')
        labels = json_data.get('labels')
        client.assign_points_to_cluster(cells, labels)
      else:
        return {'message': 'Action not Supported'}

  app = Flask(client_id)
  api = Api(app)
  api.add_resource(FederatedClient, '/')
  app.run(host = host, port = port)
```

Finally, `HF_DBSCAN/main_server.py` runs the server and `HF_DBSCAN/main_client.py` runs the client.

```
# File: HF_DBSCAN/main_server.py

from fd_server import run_server

host = "127.0.0.1"
port = 8080
MIN_POINTS = 4 # banana
# MIN_POINTS = 15 # s-set1

run_server(host, port, MIN_POINTS)
```

```
# File: HF_DBSCAN/main_client.py

from sklearn.model_selection import StratifiedKFold
from sklearn.preprocessing import MinMaxScaler
from scipy.io import arff

from fd_client import run_client

def generate_dataset_chunks(X: np.array, Y: List, n_splits: int, shuffle: bool
= True):
  if (n_splits == 1):
    return [X]
  skf = StratifiedKFold(n_splits = n_splits, shuffle = shuffle)
  dataset_chunks = []
  for train_index, test_index in skf.split(X, Y):
    dataset_chunks.append(X[test_index])
  return dataset_chunks

def prepare_dataset(num_clients: int):
  dataset_dir = '../datasets'
  dataset_file = 'banana.arff'
  # dataset_file  = 's-set1.arff'
  dataset_path = f'{dataset_dir}/{dataset_file}'
  dataset = arff.loadarff(dataset_path)
  df = pd.DataFrame(dataset[0])

  Y = df['class'].tolist()
  Y = np.array([-1 if y == b'noise' else int(y) for y in Y])
  del df['class']
  X_original = np.array(df.values)
  min_max_scaler = MinMaxScaler()
  X = min_max_scaler.fit_transform(X_original)
  dataset_chunks = generate_dataset_chunks(X, Y, num_clients)
  return dataset_chunks

server_url = "127.0.0.1:8080"
host = "127.0.0.1"
start_port = 5000
N_clients = 10
dataset_chunks = prepare_dataset(N_clients)
L = 0.03 # banana
# L = 0.03 # s-set1
```

```
threads = []
for cli in range(N_clients):
  client_id = uuid.uuid4().hex
  port = start_port + cli
  thread_obj = Thread(target = run_client, args = (client_id, server_url,
host, port, dataset_chunks[cli], L))
  threads.append(thread_obj)
  thread_obj.start()

for t in threads:
  t.join()
```

4.3 Vertical Federated DBSCAN

As for the horizontal federated DBSCAN implementation, the file `VF_DBSCAN/fd_dbscan.py` defines the methods of the federated server and client, their interfaces are defined in the files `VF_DBSCAN/fd_server.py` and `VF_DBSCAN/fd_client.py`, and the `HF_DBSCAN/main_server.py` and `HF_DBSCAN/main_client.py` files run them.

```
# File: VF_DBSCAN/fd_dbscan.py

import numpy as np
from typing import List, Dict
from scipy.spatial import distance

class FDBSCAN_Client():

  def initialize(self, params: Dict):
    self.__dataset = params['dataset']
    self.__labels = []
    self.__training_completed = False

  def get_dataset(self):
    return self.__dataset

  def get_results(self):
    if self.__training_completed:
      dataset = np.array(self.__dataset)
      labels = np.array([self.__labels])
      return np.concatenate((dataset, labels.T), axis = 1)
    else:
      return []

  def get_points(self):
    dimension = len(self.__dataset[0])
    points = []
    for row in self.__dataset:
      points.append(tuple(row[i] for i in range(dimension)))
    return points
```

```
  def compute_neighborhood_matrix(self, epsilon: float):
    points = self.get_points()
    matrix = []
    for i in range(len(points)):
      row = []
      for j in range(len(points)):
        if distance.euclidean(points[i], points[j]) <= epsilon:
          row.append(1)
        else:
          row.append(0)
      matrix.append(row)
    return matrix

  def update_labels(self, labels: List):
    self.__training_completed = True
    self.__labels = labels

class FDBSCAN_Server():

  def initialize(self, params: Dict):
    self.__MIN_POINTS = params['MIN_POINTS']
    self.__EPSILON = params['EPSILON']
    self.__running = False

  def get_running(self):
    return self.__running

  def run(self, value: bool = True):
    self.__running = value

  def get_epsilon(self):
    return self.__EPSILON

  def DBSCAN(self, global_matrix: np.ndarray):

    N = len(global_matrix)
    visited = np.zeros(N)
    labels = np.zeros(N)
    labels -= 1
    cluster_ID = 0

    for i in range(N):
      if visited[i]:
        continue
      else:
        visited[i] = 1
        num_points = np.sum(global_matrix[i])
        if num_points >= self.__MIN_POINTS:
          labels[i] = cluster_ID
          to_visit = np.where(global_matrix[i] == 1)[0].tolist()
          for j in to_visit:
            if visited[j] == 0:
              visited[j] = 1
              num_neighbors = np.sum(global_matrix[j])
              if num_neighbors >= self.__MIN_POINTS:
                to_visit += np.where(global_matrix[j] == 1)[0].tolist()
            if labels[j] == -1:
              labels[j] = cluster_ID
          cluster_ID += 1

    return labels
```

```python
# File: VF_DBSCAN/fd_server.py

import numpy as np
from flask import Flask, request
from flask_restful import Resource, Api
from typing import Dict, List

from utils import process_http_posts
from fd_dbscan import FDBSCAN_Server

def training(clients: List, server: FDBSCAN_Server):

  data = {'action': 'compute_neighborhood_matrix', 'epsilon':
server.get_epsilon()}
  results, failures = process_http_posts(clients, data)

  N = len(results[0]['matrix'])
  global_matrix = np.zeros((N, N))
  for i in range(len(results)):
    matrix = np.array(results[i]['matrix'])
    global_matrix += matrix
  global_matrix = np.where(global_matrix < len(results), 0, 1)

  Q = server.DBSCAN(global_matrix)

  data = {'action': 'update_labels', 'labels': Q.tolist()}
  process_http_posts(clients, data)
  return

def connect(json_data: Dict, clients: List, client_ids: List, running: bool):
  code = 200
  if (not running):
    client_id = json_data.get('client_id')
    address = json_data.get('address')
    client_ids.append(client_id)
    clients.append(address)
    message = {'message': f'Client {client_id} Connected'}
  else:
    message = {'message': 'Unable to connect: a training process is runnning'}
    code = 500
  return message, code

def run_server(host: str, port: int, MIN_POINTS: int, EPSILON: float):

  clients = []
  client_ids = []

  params = {
    'MIN_POINTS': MIN_POINTS,
    'EPSILON': EPSILON
  }
  server = FDBSCAN_Server()
  server.initialize(params)

  class FederatedServer(Resource):

    def get(self):
      action = request.args.get('action')
      if (action == 'start'):
        server.run();
        training(clients, server)
        server.run(False);
        return {'message': 'Training completed'}
      else:
        return {'message': 'Hello world from server', 'clients': clients}
```

```
    def post(self):
      json_data = request.get_json()
      action = json_data.get('action')
      if (action == 'connect'):
        return connect(json_data, clients, client_ids, server.get_running())
      else:
        return {'message': 'Action not Supported'}

  app = Flask(__name__)
  api = Api(app)
  api.add_resource(FederatedServer, '/')
  app.run(host = host, port = port)
```

```
# File: VF_DBSCAN/fd_client.py

from fd_dbscan import FDBSCAN_Client

def run_client(client_id: str, server_url: str, host: str, port: int, dataset:
List):

  address = host + f':{port}'
  send_post(server_url, {'action': 'connect','client_id': client_id,
'address': address})

  params = {
    'dataset': dataset,
  }
  client = FDBSCAN_Client()
  client.initialize(params)

  class FederatedClient(Resource):

    def get(self):
      action = request.args.get('action')
      if (action == 'dataset'):
        dataset = client.get_dataset().tolist()
        return {'rows': len(dataset), 'dataset': dataset}
      elif (action == 'results'):
        results = client.get_results();
        if len(results) > 0:
          return {'results': results.tolist()}
        else:
          return {'message': 'Training not completed'}
      else:
        return {'message': 'Hello World from Client', 'client_id': client_id}

    def post(self):
      json_data = request.json
      action = json_data.get('action')
      if (action == 'compute_neighborhood_matrix'):
        epsilon = json_data.get('epsilon')
        neighborhood_matrix = client.compute_neighborhood_matrix(epsilon)
        return {'matrix': neighborhood_matrix}
      elif (action == 'update_labels'):
        labels = json_data.get('labels')
        client.update_labels(labels)
        return
      else:
        return {'message': 'Action not Supported'}

  app = Flask(client_id)
  api = Api(app)
  api.add_resource(FederatedClient, '/')
  app.run(host = host, port = port)
```

```
# File: VF_DBSCAN/main_server.py

from fd_server import run_server

host = "127.0.0.1"
port = 8080

# MIN_POINTS = 4 # 3MC
MIN_POINTS = 6 # aggregation
# EPSILON = 0.1 # 3MC
EPSILON = 0.04125 # aggregation

run_server(host, port, MIN_POINTS, EPSILON)
```

```
# File: VF_DBSCAN/main_client.py

import numpy as np
import pandas as pd
import uuid
from threading import Thread
from sklearn.preprocessing import MinMaxScaler
from scipy.io import arff

from fd_client import run_client

def generate_dataset_chunks(X: np.array, num_clients: int):
  dataset_chunks = []
  features_list = [i for i in range(len(X[0]))]
  for i in range(num_clients):
    dataset_chunks.append(X[:, features_list[i::num_clients]])
  return dataset_chunks

def prepare_dataset(num_clients: int):
  dataset_dir = '../datasets'
  # dataset_file = '3MC.arff'
  dataset_file = 'aggregation.arff'
  dataset_path = f'{dataset_dir}/{dataset_file}'
  dataset = arff.loadarff(dataset_path)
  df = pd.DataFrame(dataset[0])

  Y = df['class'].tolist()
  Y = np.array([-1 if y == b'noise' else int(y) for y in Y])
  del df['class']
  X_original = np.array(df.values)
  min_max_scaler = MinMaxScaler()
  X = min_max_scaler.fit_transform(X_original)
  dataset_chunks = generate_dataset_chunks(X, num_clients)
  return dataset_chunks

server_url = "127.0.0.1:8080"
host = "127.0.0.1"
start_port = 5000
N_clients = 2
dataset_chunks = prepare_dataset(N_clients)

threads = []
for cli in range(N_clients):
  client_id = uuid.uuid4().hex
  port = start_port + cli
  thread_obj = Thread(target = run_client, args = (client_id, server_url,
host, port, dataset_chunks[cli]))
  threads.append(thread_obj)
  thread_obj.start()

for t in threads:
  t.join()
```

5. Results

5.1 Horizontal Federated DBSCAN

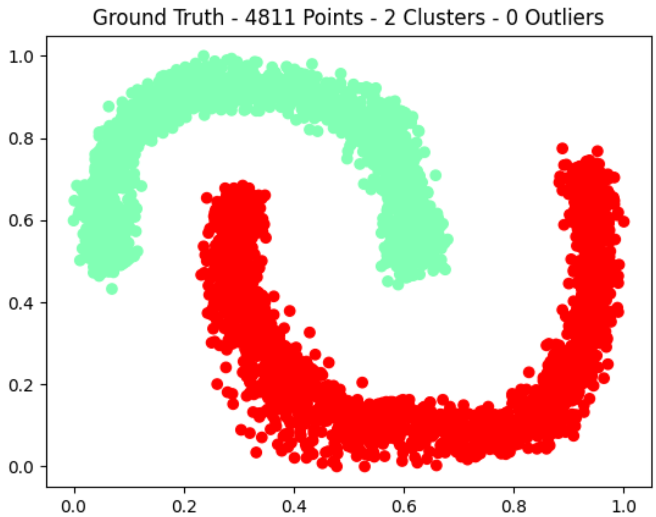
The following script has been used to evaluate the accuracy of the horizontal federated DBSCAN algorithm.

```
# File: results/HF_results.py

from scipy.io import arff
from sklearn.preprocessing import MinMaxScaler
from sklearn.cluster import DBSCAN
from urllib.request import urlopen
import numpy as np
import pandas as pd
import json

from plot import plot2D

results_folder = 'banana'
# results_folder = 's-set1'
dataset_dir = '../datasets'
dataset_file = 'banana.arff'
# dataset_file = 's-set1.arff'
dataset_path = f'{dataset_dir}/{dataset_file}'
dataset = arff.loadarff(dataset_path)
df = pd.DataFrame(dataset[0])

true_labels = df['class'].tolist()
true_labels = np.array([-1 if label == b'noise' else int(label) for label in
true_labels])
del df['class']
X_original = np.array(df.values)
min_max_scaler = MinMaxScaler()
X = min_max_scaler.fit_transform(X_original)
plot2D(points = X, labels = true_labels, folder = results_folder, message =
"Ground Truth")

MIN_POINTS = 4 # banana
# MIN_POINTS = 15 # s-set1
EPSILON = 0.03 # banana
# EPSILON = 0.0325 # s-set1
clustering = DBSCAN(eps = EPSILON, min_samples = MIN_POINTS)
dbscan_labels = clustering.fit_predict(X)
dbscan_labels = [int(label) for label in dbscan_labels]
plot2D(points = X, labels = dbscan_labels, folder = results_folder, message =
"DBSCAN")
```

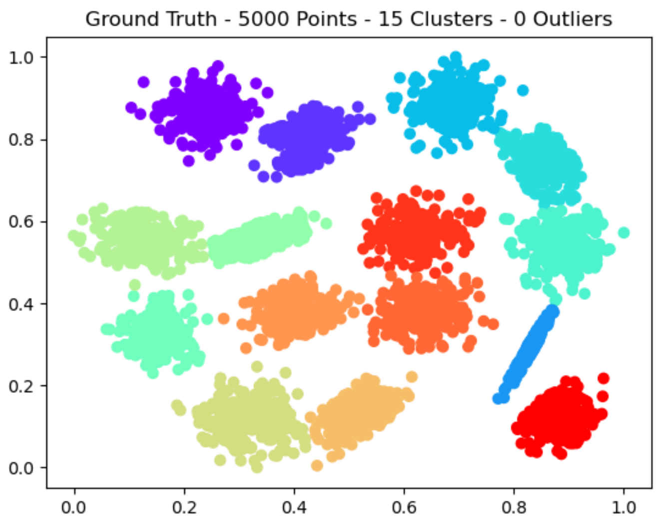
```
start_port = 5000
N_clients = 10
url = f'http://localhost:8080/?action=start'
urlopen(url)
for port in range(start_port, start_port + N_clients):
  url = f'http://localhost:{port}/?action=results'
  response = urlopen(url)
  data_json = json.loads(response.read())
  result = np.array(data_json['results'])
  # if len(result) == 0:
  #   continue
  if port == start_port:
    joined_result = result
  else:
    joined_result = np.concatenate((joined_result, result))
joined_points = joined_result[:,:2]
joined_labels = joined_result[:,2]
joined_labels = [int(label) for label in joined_labels]
plot2D(points = joined_points, labels = joined_labels, folder =
results_folder, message = f'Federated DBSCAN')
# plot2D(points = joined_points, labels = joined_labels, folder =
results_folder, message = f'Federated DBSCAN - Missing Client')
```

To test the algorithm two different datasets have been used: `dataset/banana.arff` and `dataset/s-set1.arff`, scattered below.

`banana.arff`



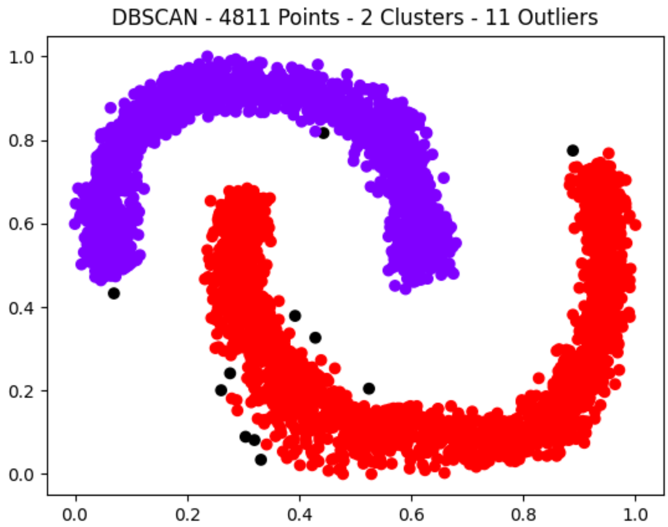
`s-set1.arff`



Running the script the following results were obtained for the two datasets. Standard DBSCAN is compared to horizontal federated DBSCAN. The number of clients in the simulation is set to 10.

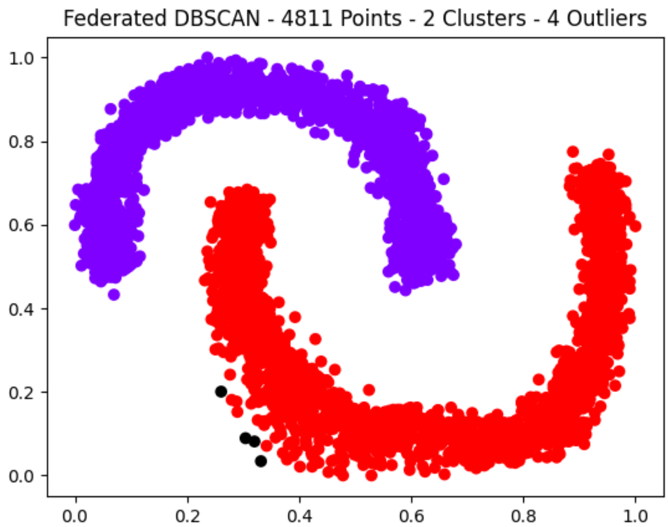
banana.arff

DBSCAN
$(Eps = 0.03,\ MinPts = 4)$



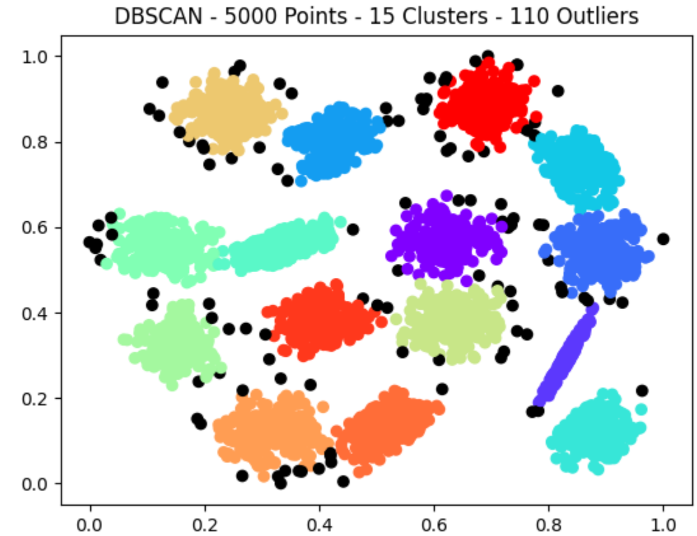
banana.arff

Horizontal Federated DBSCAN
$(L = 0.03,\ MinPts = 4)$



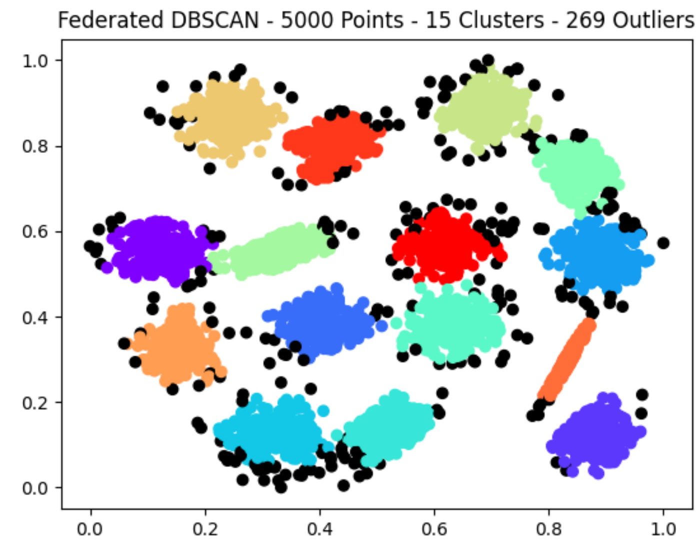
s-set1.arff

DBSCAN
($Eps = 0.0325$, $MinPts = 15$)



s-set1.arff

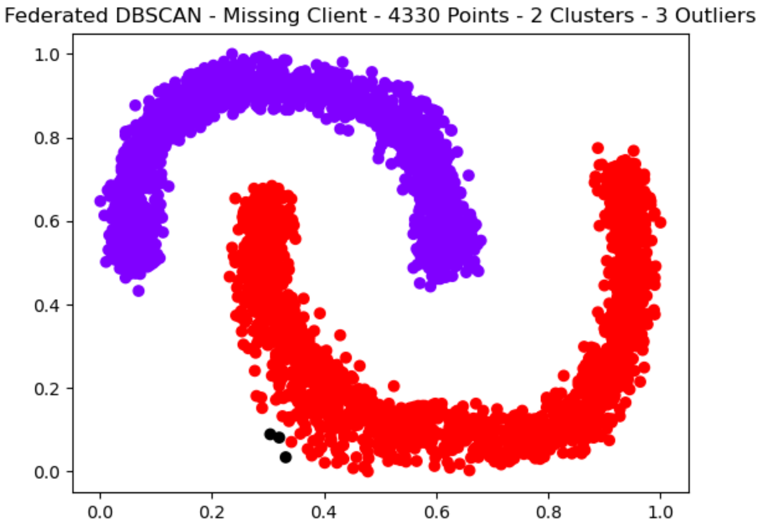
Horizontal Federated DBSCAN
($L = 0.03$, $MinPts = 15$)



A further experiment has then been conducted, running the algorithm in the hypothesis of a missing client. The following results were obtained.

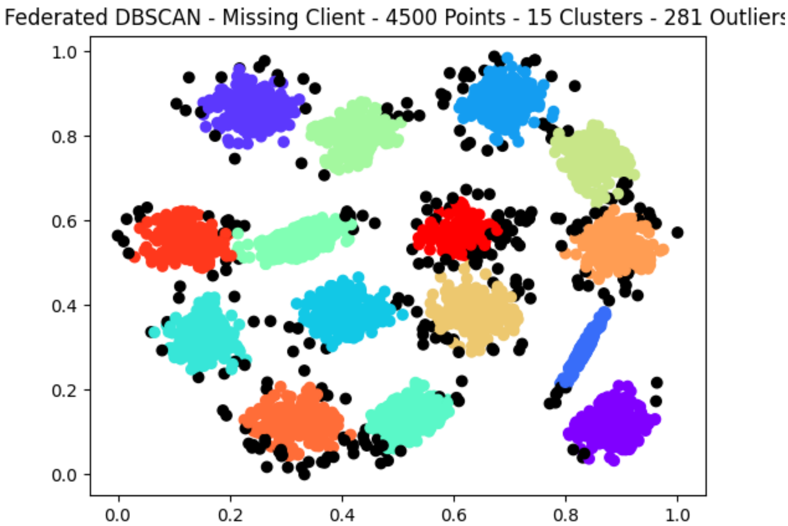
banana.arff

Horizontal Federated DBSCAN
Missing Client Hypothesis
$(L = 0.03, \ MinPts = 4)$



s-set1.arff

Horizontal Federated DBSCAN
Missing Client Hypothesis
$(L = 0.03, \ MinPts = 15)$



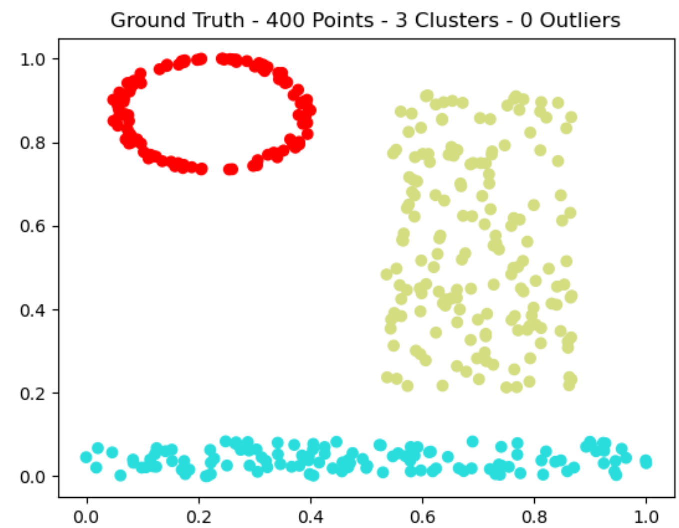
5.2 Vertical Federated DBSCAN

The following script has been used to evaluate the accuracy of the vertical federated DBSCAN algorithm.

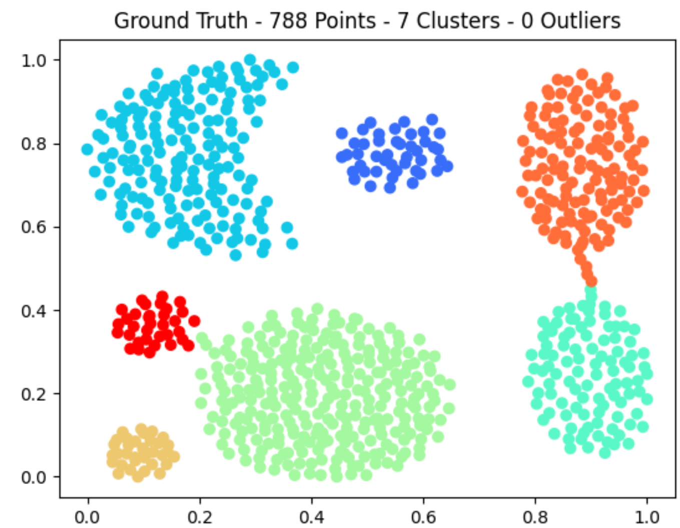
```
# File: results/VF_results.py

from scipy.io import arff
from sklearn.preprocessing import MinMaxScaler
from sklearn.cluster import DBSCAN
from urllib.request import urlopen
import numpy as np
import pandas as pd
import json

from plot import plot2D

# results_folder = '3MC'
results_folder = 'aggregation'
dataset_dir = '../datasets'
# dataset_file = '3MC'
dataset_file = 'aggregation.arff'
dataset_path = f'{dataset_dir}/{dataset_file}'
dataset = arff.loadarff(dataset_path)
df = pd.DataFrame(dataset[0])

true_labels = df['class'].tolist()
true_labels = np.array([-1 if label == b'noise' else int(label) for label in
true_labels])
del df['class']
X_original = np.array(df.values)
min_max_scaler = MinMaxScaler()
X = min_max_scaler.fit_transform(X_original)
plot2D(points = X, labels = true_labels, folder = results_folder, message =
"Ground Truth")

# MIN_POINTS = 4 # 3MC
MIN_POINTS = 6 # aggregation
# EPSILON = 0.1 # 3MC
EPSILON = 0.04125 # aggregation
clustering = DBSCAN(eps = EPSILON, min_samples = MIN_POINTS)
dbscan_labels = clustering.fit_predict(X)
dbscan_labels = [int(label) for label in dbscan_labels]
plot2D(points = X, labels = dbscan_labels, folder = results_folder, message =
“DBSCAN")

start_port = 5000
N_clients = 2
url = f'http://localhost:8080/?action=start'
urlopen(url)
for port in range(start_port, start_port + N_clients):
  url = f'http://localhost:{port}/?action=results'
  response = urlopen(url)
  data_json = json.loads(response.read())
  result = np.array(data_json['results'])
  if port == start_port:
    joined_result = result
  else:
    joined_result = np.insert(joined_result, 1, result[:, 0], axis = 1)
joined_points = joined_result[:,:2]
joined_labels = joined_result[:,2]
joined_labels = [int(label) for label in joined_labels]
plot2D(points = joined_points, labels = joined_labels, folder =
         results_folder, message = f'Federated DBSCAN')
```

To test the algorithm two different datasets have been used: `dataset/3MC.arff` and `dataset/aggregation.arff`, scattered below.

`3MC.arff`



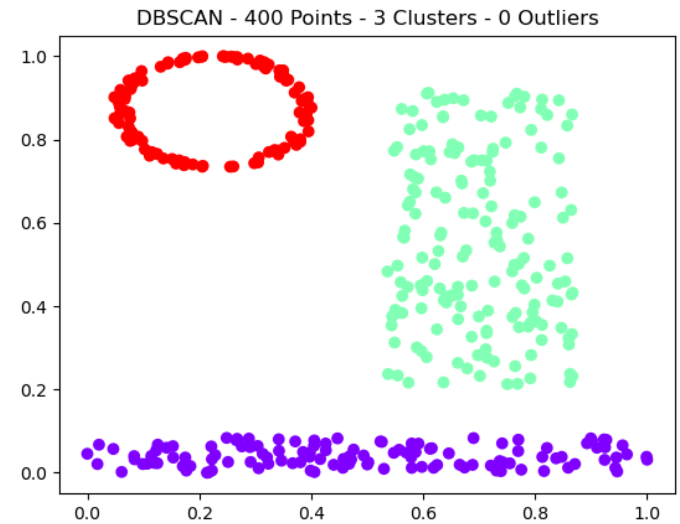
`aggregation.arff`



Running the script the following results were obtained for the two datasets.
Standard DBSCAN is compared to vertical federated DBSCAN

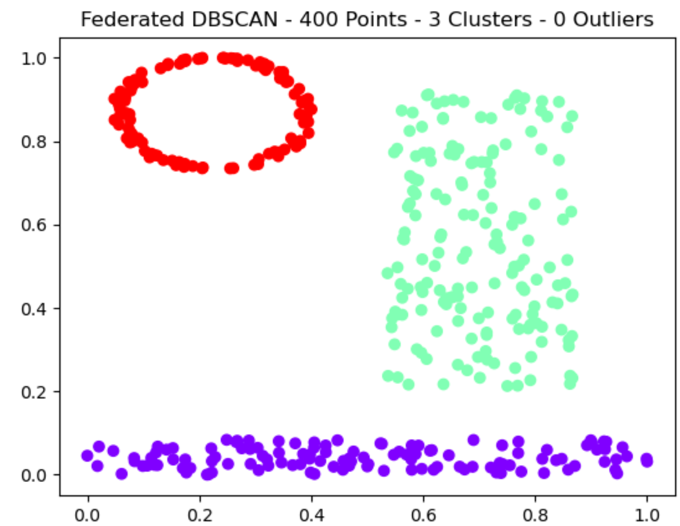
3MC.arff

DBSCAN
$(Eps = 0.1\,,\ \ MinPts = 4)$



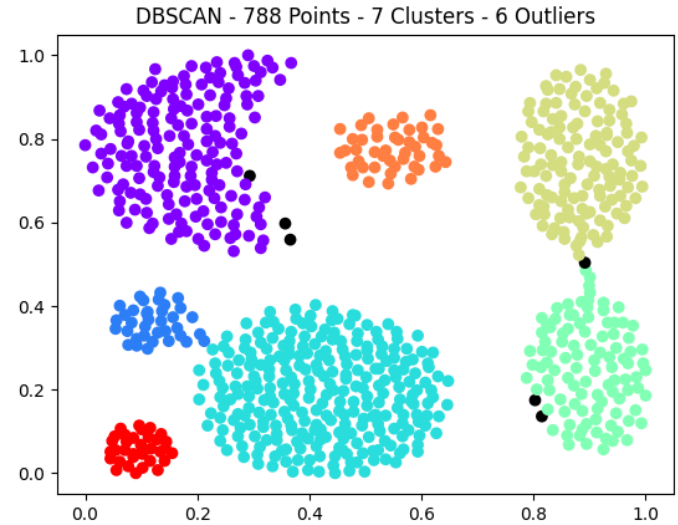
3MC.arff

Vertical Federated DBSCAN
$(Eps = 0.1\,,\ \ MinPts = 4)$



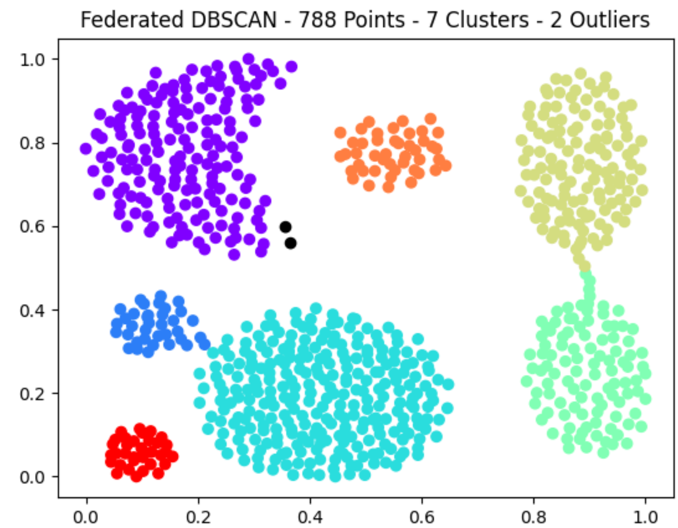
aggregation.arff

DBSCAN
$(Eps = 0.04125, \ MinPts = 6)$



aggregation.arff

Vertical Federated DBSCAN
$(Eps = 0.04125, \ MinPts = 6)$



5.3 Conclusions

At first, a brief overview of DBSCAN algorithm and federated learning as been given. Then, horizontal and vertical federated learning versions of DBSCAN have been introduced. These versions of the well-known algorithm have the great advantage of preserving privacy, avoiding raw data sharing, when dealing with distributed databases. An implementation of both algorithms has then been presented, using Python programming language.

Each federated version was finally tested using two different datasets, and results were compared to those given by the application of standard DBSCAN. The federated versions proved to be equally efficient, giving slightly different but consistent results. Furthermore, the horizontal federated version was tested with the hypothesis of a missing client (out of ten clients), and proved to be resistant to raw data loss.

6. References

[1] An introduction to clustering algorithms:

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[2] A detailed description of DBSCAN:

Martin Ester, Hans-Peter Kriegel, Jörg Sander, and Xiaowei Xu. 1996. A density-based algorithm for discovering clusters in large spatial databases with noise. In Proceedings of the Second International Conference on Knowledge Discovery and Data Mining (KDD'96). AAAI Press, 226–231.

[3] A comprehensive overview of Federated Learning:

Qiang Yang, Yang Liu, Tianjian Chen, and Yongxin Tong. 2019. Federated Machine Learning: Concept and Applications. ACM Trans. Intell. Syst. Technol. 10, 2, Article 12 (January 2019), 19 pages.