

UNIVERSITÀ DI PISA DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

CORSO DI LAUREA TRIENNALE IN INGEGNERIA INFORMATICA

FEDERATED DBSCAN

RELATORE:

PROF. FRANCESCO MARCELLONI

LAUREANDO:

GABRIELE MARINO

CONTENTS

1. DBSCAN1.1 Introduction1.2 A Density Based Notion of Clusters1.3 The Algorithm	3
2. Federated Learning2.1 Introduction	
3. Federated DBSCAN 3.1 Horizontal Federated DBSCAN	_
4. Python Implementation 4.1 Introduction 4.2 Horizontal Federated DBSCAN 4.3 Vertical Federated DBSCAN	13 13 21
5. Experimental Analysis5.1 Experimental Setup5.2 Results5.3 Conclusions	27 31 37
6. References	41

1. DBSCAN

1.1 Introduction

Clustering algorithms are used to solve identification problems in spatial databases. DBSCAN (Density Based Spatial Clustering of Applications with Noise) is a clustering algorithm which deals with good efficiency with the main problems that rise when applying clustering algorithms to large spatial databases: minimal requirements of domain knowledge to determine the input parameters and discovery of clusters with arbitrary shape.

For a gentle introduction to clustering algorithms refer to [1]. For a detailed description of DBSCAN refer to [2].

1.2 A Density Based Notion of Clusters

DBSCAN relies on a density-based notion of clusters. The algorithm is based on the key idea is that for each point of a cluster the neighborhood of a given radius has to contain at least a minimum number of points, where both the radius of the neighbourhood and the minimum number of points depend on the specific cluster.

Let D be a database of points of some n-dimensional space E, and let dist(p,q) be a distance function between two points of D. The following definitions hold.

Definition 1: (Eps-neighborhood of a point) The *Eps-neighborhood* of a point p, denoted by $N_{Eps}(p)$, is defined by $N_{Eps}(p) = \{q \in D \mid dist(p,q) \leq Eps\}$.

Definition 2: (directly density-reachable) A point *p* is *directly density-reachable* from a point *q* wrt. *Eps* and *MinPts* if:

- $p \in N_{Eps}(q)$, and
- $|N_{Eps}(q)| \ge MinPts$ (core point condition).

Definition 3: (density-reachable) A point p is *density-reachable* from a point q wrt. Eps and MinPts if there is a chain of points $p_1, ..., p_n$, such that $p_1 = q$, $p_n = p$ and p_{i+1} is directly reachable from p_i .

Definition 4: (density-connected) A point p is *density-connected* from a point q wrt. Eps and MinPts if there is a point o such that both, p and q are density reachable from o wrt. Eps and MinPts.

Definition 5: (cluster) Let D be a database of points. A *cluster* C wrt. Eps and MinPts is a non-empty subset of D satisfying the following conditions:

- $\forall p, q$: if $p \in C$ and q is density-reachable fro p wrt. Eps and MinPts, then $q \in C$. (Maximality)
- $\forall p, q \in C$: p is density-connected to q wrt. Eps and MinPts. (Connectivity)

Definition 6: (noise) Let $C_1, ..., C_k$ be the clusters of the database D wrt. Eps_i , $MinPts_i$, i = 1, ..., k. Then the *noise* is the set of points in the database D not belonging to any cluster C_i .

DBSCAN algorithm is designed to discover the clusters and the noise in a spatial database according to definitions 5 and 6, given *Eps* and *MinPts*.

1.3 The Algorithm

DBSCAN uses global values for *Eps* and *MinPts* for all clusters. Good candidates for these parameter values are those specifying the density of the "thinnest" cluster of the database, i.e. the lowest density which is not considered to be noise.

Initially, all points in the database D are marked as "unvisited". DBSCAN randomly selects an unvisited point p, marks it as "visited" and checks the core point condition. If not, p is marked as a "noise" point. Otherwise, a new cluster C is created for p, and all the points in its neighborhood are added to a candidate set, N. Then, the points in N that do not belong to any cluster are iteratively added to C. Furthermore, for a point p' in N that carries the label "unvisited", DBSCAN marks it as "visited" and check its core point condition. If the condition holds, all points in the Eps-neighborhood of p' are added to N. The loop continues adding points to C until N reaches emptiness. At this time, cluster C is completed. To compute the next cluster, DBSCAN randomly selects an "unvisited" point from the remaining ones, until all points are visited.

The following pseudocode describes DBSCAN algorithm.

DBSCAN

Input

D: database containing n points

Eps: the radius parameter

MinPts: the neighborhood density threshold

Output

A set of density-based clusters

Method

- Mark each point as unvisited
- do
 - Randomly select an *unvisited* point p
 - Mark p as visited
 - If the Eps-neighborhood of p has at least MinPts points:
 - Create a new cluster C
 - Add p to C
 - Let N be the set of points in the Eps-neighborhood of p
 - For each point p' in N:
 - If p' is unvisited:
 - Mark p' as visited
 - If the Eps-neighborhood of p' has at least MinPts points: add those points to N
 - ullet **If** p' is not yet a member of any cluster: add p' to C
 - Output C
 - Else: mark p as noise
- Until no point is unvisited

2. Federated Learning

2.1 Introduction

Federated learning deals with the possibility to fuse together data from different organizations. This is crucial in real-world situations, where with the exception of a few industries, most fields have only limited data or poor-quality data. Still, in many situations, it is very difficult to break the barriers between data sources. This is due to industry competition, privacy security and complicated administrative procedures. In fact, as a result of new data regulations and privacy laws, we are forbidden to collect, fuse and process data from different places. Federated learning is a possible solution for this challenge.

Federated learning is widely overviewed in [3].

2.2 A Categorization of Federated Learning

To define what federated learning is, consider N data owners $\{P_1, ..., P_N\}$ with their respective data $\{D_1, ..., D_N\}$, and let M_{SUM} be the model trained by $D = D_1 \cup ... \cup D_N$. A federated-learning system is a learning process in which the data owners collaboratively train a model M_{FED} without exposing their own data to others. The accuracy of M_{FED} , V_{FED} , should be as close as possible to that of M_{SUM} , V_{SUM} . Formally, let δ be a non-negative real number; if $|V_{FED} - V_{SUM}| < \delta$, the federated learning algorithm is said to have δ -accuracy loss.

Federated learning systems can be coarsely categorized based on the data partitioning scheme, i.e. how data are distributed across the various data owners. To introduce this categorization, let F_i be the feature space and I_i the sample ID space of the data D_i . Then, we can distinguish horizontal federated learning from vertical federated learning as follows.

In horizontal federated learning the dataset is said to be *sample-partitioned*, and the following relations hold:

$$F_i = F_j$$
, $I_i \neq I_j$, $\forall D_i, D_j$: $i \neq j$,

whilst in vertical federated learning the dataset is feature-partitioned:

$$F_i \neq F_j$$
, $I_i = I_j$, $\forall D_i, D_j$: $i \neq j$.

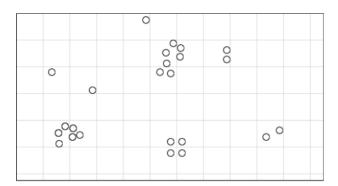
3. Federated DBSCAN

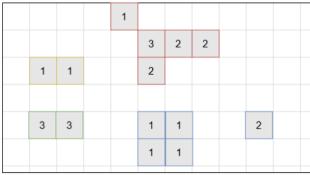
3.1 Horizontal Federated DBSCAN

In presenting the algorithm we refer to a set of clients willing to train a federated learning model interacting with a trustworthy server.

The key idea of the algorithm is the partitioning of each local feature space with a fixed granularity. The domain of definition of the features and the granularity of the grid are known and are the same for each client. This approach allows each client to share with the server only the number of points within the non-empty cells of the grid, preventing raw data sharing and thus preserving privacy.

The picture below describes the idea: the left picture represents a local dataset for a given client with the superimposed grid partitioning, supposing a two dimensional feature space, while the right one represents the information shared with the server, that is the number of points in each cell.





The following pseudocode describes horizontal federated DBSCAN.

Horizontal Federated DBSCAN

Input

L: the granularity of the cells
MinPts: the cell density threshold

Method

Each client m

- ullet Compute grid for its local dataset, based on L
- Evaluate the number of points in each non-empty cell and transmits this information to the server

```
Server
For each cell c:

Compute N<sub>c</sub>, the overall number of points in the cell c,
obtained as the sum of the contributions from all clients
If N<sub>C</sub> ≥ MinPts: Mark c as a dense cell

Evaluate clustering by expanding a cluster along adjacent dense cells
Transmit to each client information about cluster membership of each dense cell
Each client m
For each cell c:

If c is dense: assign all the points in c to the cluster the cell belongs to, as assigned by the server
Else:

If at least one of the cells adjacent to c is dense: assign each point in c to the cluster of the nearest adjacent
```

3.2 Vertical Federated DBSCAN

dense cell

As for the horizontal federated version, we still refer to a set of clients willing to train a federated learning model interacting with a trustworthy server.

• Else: Mark the points in the cell as outliers

The key idea of the algorithm is to locally compute a neighborhood matrix and share this matrix with the server, thus protecting raw data. The server then aggregates such local neighborhood matrices in a global neighborhood matrix, considering two points as neighbors if and only if they are neighbors for each client. Finally, the server executes an adapted version of the classical DBSCAN algorithm and share the results with the clients.

The following pseudocode describes vertical federated DBSCAN.

```
Vertical Federated DBSCAN

Input

Eps: neighborhood radius

MinPts: the neighborhood density threshold

Method (Vertical DBSCAN)

Server

• Share with client Eps parameter

Each client m

// Let N be the number of points of the database

• For i \in \{1, ..., N\}:

• For j \in \{1, ..., N\}:

• If distance(x_i, x_j) < Eps: Sim_m[i, j] = Sim_m[j, i] = 1
```

```
\bullet Send Sim_m matrix to server
Server
• For i \in \{1, ..., N\}:
   • For j \in \{1, ..., N\}:
      • If Sim_m[i,j] = Sim_m[j,i] = 1 for each client m: Sim[i,j] = Sim[j,i] = 1
• Q = Server\_DBSCAN(Sim, MinPts)
• Send the Q vector to the clients
Server DBSCAN
Input
Sim: global neighborhood matrix
MinPts: the neighborhood density threshold
Method
• For each row i in Sim:
   • If i is visited: continue
   • Else:

    Mark i as visited

      • NumPts = sum(i) // Sum of cells on the i-row equal to 1
      • If NumPts < MinPts: mark i as noise
      • Else:
         \bullet c = newCluster
         • toVisit = \{j : Sim[i, j] = 1\}
         • Expand_Cluster(Sim, i, toVisit, c, MinPts)
• Let Q = [q_1, \dots, q_N] be the vector of cluster assignment of the
 points in the database
• Return Q
```

Expand_Cluster

```
Input
```

Sim: global neighborhood matrix
i: current row of the matrix, related to the i-th point
c: current cluster
toVisit: vector of points to visit
MinPts: the neighborhood density threshold

Method

- $q_i = c$ // Add i to cluster c
- For j in toVisit:
 - **If** *j* is unvisited:
 - Mark j as visited
 - $NumPts_i = sum(j)$ // Sum of cells on the j-row equal to 1
 - If $NumPts_j \ge MinPts$: $toVisit = toVisit \cup \{k : Sim[j,k] = 1\}$
 - If j is not member of any cluster: $q_i = c$ // Add j to cluster c

4. Python Implementation

4.1 Introduction

A Python implementation of the horizontal and vertical federated DBSCAN is here proposed. Flask and Flask-RESTful frameworks and HTTP post method have been used to handle the clients-server network communication.

The following function allow server and clients to communicate with each other.

```
# File: utils.py
import requests
import concurrent.futures
from typing import Dict, List
def send_post(url: str, data: Dict):
    r = requests.post(f'http://{url}', json = data)
  return r.json(), r.status_code
def process_http_posts(clients: List, data: Dict):
  with concurrent futures. ThreadPoolExecutor() as executor:
    futures = [executor.submit(send_post, c, data) for c in clients]
    concurrent futures wait (futures)
  results = []
  failures = []
  for future in futures:
    failure = future.exception()
    if failure is not None:
      failures.append(failure)
    else:
      result = future.result()
      results.append(result[0])
  return results, failures
```

4.2 Horizontal Federated DBSCAN

The file HF_DBSCAN/fd_dbscan.py defines the methods of the federated server and client.

```
# File: HF_DBSCAN/fd_dbscan.py
import math
import numpy as np
from typing import List, Dict, Tuple, Callable
from scipy.spatial import distance
```

```
def get_all_neighbors(cell: tuple):
 diag_coord = [(x - 1, x, x + 1)] for x in cell] cartesian_product = [[]]
  for pool in diag_coord:
    cartesian_product = [(x + [y]) \text{ for } x \text{ in cartesian_product for } y \text{ in pool}]
  neighbors = []
  for prod in cartesian_product:
    differential_coord
    for i in range(len(prod)):
      if prod[i] != cell[i]:
        differential_coord += 1
    if differential_coord == 1:
      neighbors.append(tuple(prod))
  return neighbors
class FDBSCAN Client():
 def initialize(self, params: Dict):
    self.__dataset = params['dataset']
    self.__L = params['L']
self.__labels = []
    self.__true_labels = params['true_labels']
    self.__passive = True
 def get_dataset(self):
    return self.__dataset
 def get_labels(self):
    return self.__labels, self.__true_labels
 def is passive(self):
    return self.__passive
 def __get_points(self, floor: bool = False):
    dimension = len(self.\__dataset[0])
    points = []
    for row in self.__dataset:
      if floor:
        points.append(tuple(math.floor(row[i] / self.__L) for i in
range(dimension)))
      else:
        points.append(tuple(row[i] for i in range(dimension)))
    return points
 def compute_local_update(self):
    self.__passive = False
    cells = np.array(self.__get_points(floor = True))
    dimensions = len(cells[0])
    max_cell_coords = []
    min_cell_coords = []
    for i in range(dimensions):
      max_cell_coords.append(np.amax(cells[:, i]))
      min_cell_coords.append(np.amin(cells[:, i]))
    shifts = np.zeros(dimensions)
    for i in range(dimensions):
      if min_cell_coords[i] < 0:
        shifts[i] = -1 * min_cell_coords[i]
```

```
shifted dimensions = ()
    for i in range(dimensions):
      shifted_dimensions += (int(max_cell_coords[i] + 1 + shifts[i]), )
    count_matrix = np.zeros(shifted_dimensions)
    for cell in cells:
      shifted_cell_coords = ()
      for i in range(dimensions):
        shifted cell coords += (int(cell[i] + shifts[i]),)
      count_matrix[shifted_cell_coords] += 1
    non_zero = np.where(count_matrix > 0)
    non_zero_indexes = []
    for i in range(len(non_zero)):
      for j in range(len(non_zero[i])):
   if i == 0:
          non_zero_indexes.append((int(non_zero[i][j]), ))
        else:
          non_zero_indexes[j] += (int(non_zero[i][j]), )
    dict_to_return = {}
    for index in non_zero_indexes:
      shifted_index = ()
      for i in range(len(index)):
        shifted_index += (int(index[i] - shifts[i]), )
      dict_to_return[shifted_index] = count_matrix[index]
   return dict_to_return
 def assign points to cluster(self, cells: List, labels: List):
    points = self.__get_points()
    dense_cells = [tuple(row) for row in cells]
    self.__labels = [-1] * len(points)
    cell_points_dic = {}
    for actual_point in points:
      actual_cell = tuple(math.floor(actual_point[i] / self.__L) for i in
range(len(actual_point)))
        cell_points_dic[actual_cell].append(actual_point)
        cell_points_dic[actual_cell] = [actual_point]
    cells = list(cell_points_dic.keys())
    dense_cells_index = dense_cells.index
    points_index = points.index
    for cell in cells:
      current_cell_points = cell_points_dic[cell]
      if (cell in dense_cells):
        cluster = labels[dense_cells_index(cell)]
        self.__assign_labels_to_points(points_index, current_cell_points,
cluster)
        nearest_adjacent_cell = self.__locate_nearest_adjacent_cell(cell,
dense_cells, current_cell_points)
        if (nearest_adjacent_cell is not None):
          cluster = labels[dense_cells_index(nearest_adjacent_cell)]
          self.__assign_labels_to_points(points_index, current_cell_points,
cluster)
```

```
_locate_nearest_adjacent_cell(self, cell: Tuple, dense_cells: List,
current_cell_points: List):
    adjacent_cells = get_all_neighbors(cell)
    selected_adjacent_cell = None
    min_dist = float('inf')
for adjacent_cell in adjacent_cells:
      if (adjacent_cell in dense_cells):
        adjacent_cell_mid_point = tuple(cell_coord * self.__L + self.__L/2 for
cell coord in adjacent cell)
        current_min_dist = min([distance.euclidean(adjacent_cell_mid_point,
cell_point) for cell_point in current_cell_points])
         if (current_min_dist < min_dist):</pre>
           selected_adjacent_cell = adjacent_cell
    return selected_adjacent_cell
def __assign_labels_to_points(self, points_index: Callable, cell_points:
List, cluster: int):
    labels = self.__labels
    for point in cell_points:
      labels[points_index(point)] = cluster
class FDBSCAN Server():
 def initialize(self, params: Dict):
    self.__MIN_POINTS = params['MIN_POINTS']
    self.__running = False
 def get_running(self):
    return self.__running
 def run(self, value: bool = True):
    self.___running = value
 def compute_clusters(self, contribution_map: Dict):
    key_list = list(contribution_map.keys())
    value_list = list(contribution_map.values())
    n_cells = len(key_list)
    visited = np.zeros(n_cells)
    clustered = np.zeros(n_cells)
    cells = []
labels = []
    cluster_ID = 0
    while 0 in visited:
      curr_index = np.random.choice(np.where(np.array(visited) == 0)[0])
curr_cell = key_list[curr_index]
      visited[curr_index]
      num_points = value_list[curr_index]
      if num_points >= self.__MIN_POINTS:
        cells.append(curr_cell)
        labels.append(cluster_ID)
        clustered[curr_index] =
        list of cells to check = get all neighbors(curr cell)
        while len(list_of_cells_to_check) > 0:
           neighbor = list_of_cells_to_check.pop(0)
           neighbor_index = key_list.index(neighbor) if neighbor in key_list
else ""
```

```
if neighbor in key_list and visited[neighbor_index] == 0:
    visited[neighbor_index] = 1
    if value_list[neighbor_index] >= self.__MIN_POINTS:
        list_of_cells_to_check += get_all_neighbors(neighbor)
    if clustered[neighbor_index] == 0:
        cells.append(neighbor)
        labels.append(cluster_ID)
        clustered[neighbor_index] = 1
    cluster_ID += 1
```

The server and client interfaces are defined in file HF_DBSCAN/fd_server.py and HF_DBSCAN/fd_client.py.

```
# File: HF_DBSCAN/fd_server.py
import random
from flask import Flask, request
from flask_restful import Resource, Api
from typing import Dict, List
from utils import process_http_posts
from fd_dbscan import FDBSCAN_Server
def training(clients: List, server: FDBSCAN_Server, clients selection seed:
int, missing_clients_percentage: int):
  import random
  random.seed(clients_selection_seed)
  n_{clients} = len(clients)
 n_{clients_{to_{select}}} = int(n_{clients} * (100 - missing_clients_percentage) /
  selected_clients_idx = random.sample(range(n_clients), n_clients_to_select)
  selected_clients = [c for cli, c in enumerate(clients) if cli in
selected_clients_idx]
  print('Training started')
  print(f'Selected clients: {selected_clients_idx}')
  data = {'action': 'compute_local_update'}
  local_updates, failures = process_http_posts(selected_clients, data)
  contribution_map = {}
  for i in range(len(local_updates)):
    local_update = local_updates[i]
    for string_key, value in local_update.items():
      tuple_key = eval(string_key)
      if tuple_key in contribution_map:
        contribution_map[tuple_key] += value
        contribution_map[tuple_key] = value
  cells, labels = server.compute_clusters(contribution_map)
 data = {'action': 'assign_points_to_cluster', 'cells': cells, 'labels':
labels}
  process_http_posts(clients, data)
```

```
def connect(json data: Dict, clients: List, client ids: List, running: bool):
  if (not running):
    client_id = json_data.get('client_id')
    address = json_data.get('address')
    client_ids.append(client_id)
    clients.append(address)
    message = {'message': f'Client {client_id} Connected'}
 else:
    code = 500
    message = {'message': 'Unable to connect: a training process is runnning'}
 return message, code
def run_server(host: str, port: int, MIN_POINTS: int, clients_selection_seed:
int, missing_clients_percentage: int):
  clients = []
  client_ids = []
  params = {
    'MIN POINTS': MIN POINTS
  server = FDBSCAN_Server()
  server.initialize(params)
 class FederatedServer(Resource):
    def get(self):
      action = request.args.get('action')
      if (action == 'start'):
        server.run();
        training(clients, server, clients_selection_seed,
missing_clients_percentage)
        server.run(False);
        return {'message': 'Training completed'}
        return {'message': 'Hello world from server', 'clients': clients}
    def post(self):
      json_data = request.get_json()
      action = json_data.get('action')
if (action == 'connect'):
        return connect(json_data, clients, client_ids, server.get_running())
        return {'message': 'Action not Supported'}, 201
  app = Flask(__name__)
  api = Api(app)
  api.add_resource(FederatedServer, '/')
  app.run(host = host, port = port)
# File: HF_DBSCAN/fd_client.py
from typing import List
from flask import Flask, request
from flask_restful import Resource, Api
from utils import send_post
from fd dbscan import FDBSCAN Client
```

```
def run_client(client_id: str, server_url: str, host: str, port: int, dataset:
List, L: float, true_labels: List):
  address = host + f':{port}'
  send_post(server_url, {'action': 'connect', 'client_id': client_id,
'address': address})
  params = {
     'dataset': dataset,
    'L': L,
    'true_labels': true_labels
  client = FDBSCAN_Client()
  client.initialize(params)
  class FederatedClient(Resource):
    def get(self):
      action = request.args.get('action')
if (action == 'results'):
    labels, true_labels = client.get_labels();
    return {'passive': client.is_passive(), 'dataset':
client.get_dataset().tolist(), 'labels': labels, 'true_labels':
true_labels.tolist()}
      else:
         return {'message': 'Hello World from Client', 'client_id': client_id}
    def post(self):
      json_data = request.json
      action = json_data.get('action')
      if (action == 'compute_local_update'):
         result = client.compute_local_update()
         to return = {}
         for tuple_key in result:
           string_key = ','.join([str(coord) for coord in tuple_key])
           to_return[string_key] = result[tuple_key]
         return to_return
      elif (action == 'assign_points_to_cluster'):
         cells = json_data.get('cells')
         labels = json_data.get('labels')
         client.assign_points_to_cluster(cells, labels)
         return {'message': 'Action not Supported'}, 201
  app = Flask(client_id)
  api = Api(app)
  api.add_resource(FederatedClient, '/')
app.run(host = host, port = port)
```

Finally, HF_DBSCAN/main_server.py runs the server and HF_DBSCAN/main_client.py runs the client.

```
# File: HF_DBSCAN/main_server.py

from fd_server import run_server

host = "127.0.0.1"
port = 8080
# MIN_POINTS = 4 # banana
MIN_POINTS = 15 # s-set1

clients_selection_seed = 1
missing_client_percentage = 10
```

```
run_server(host, port, MIN_POINTS, clients_selection_seed,
missing_client_percentage)
```

```
# File: HF DBSCAN/main client.py
import numpy as np
import pandas as pd
import uuid
from typing import List
from threading import Thread
from sklearn.model_selection import StratifiedKFold
from sklearn preprocessing import MinMaxScaler
from scipy.io import arff
from fd_client import run_client
def generate_dataset_chunks(X: np.array, Y: List, n_splits: int):
  if (n_{splits} == 1):
   return [X]
  skf = StratifiedKFold(n_splits = n_splits)
  dataset\_chunks = []
  true_labels = []
  for train_index, test_index in skf.split(X, Y):
   dataset_chunks.append(X[test_index])
    true_labels.append(Y[test_index])
  return dataset_chunks, true_labels
def prepare_dataset(num_clients: int):
  dataset_dir = '../datasets'
# dataset_file = 'banana.arff'
  dataset_file = 's-set1.arff'
dataset_path = f'{dataset_dir}/{dataset_file}'
  dataset = arff.loadarff(dataset_path)
  df = pd.DataFrame(dataset[0])
  Y = df['class'].tolist()
  Y = np.array([-1 if y == b'noise' else int(y) for y in Y])
  del df['class']
  X_original = np.array(df.values)
  min_max_scaler = MinMaxScaler()
  X = min_max_scaler.fit_transform(X_original)
  dataset_chunks, true_labels = generate_dataset_chunks(X, Y, num_clients)
  return dataset_chunks, true_labels
server_url = "127.0.0.1:8080"
host = "127.0.0.1"
start_port = 5000
N_clients = 10
dataset_chunks, true_labels = prepare_dataset(N_clients)
\# L = 0.03 \# banana
L = 0.03 \# s-set1
threads = []
for cli in range(N_clients):
  client_id = uuid.uuid4().hex
  port = start_port + cli
  thread_obj = Thread(target = run_client, args = (client_id, server_url,
host, port, dataset_chunks[cli], L, true_labels[cli]))
  threads.append(thread_obj)
  thread_obj.start()
for t in threads:
  t.join()
```

4.3 Vertical Federated DBSCAN

As for the horizontal federated DBSCAN implementation, the file VF_DBSCAN/fd_dbscan.py defines the methods of the federated server and client, their interfaces are defined in the files VF_DBSCAN/fd_server.py and VF_DBSCAN/fd_client.py, and the VF_DBSCAN/main_server.py and VF_DBSCAN/main_client.py files run them.

```
# File: VF_DBSCAN/fd_dbscan.py
import numpy as np
from typing import List, Dict
from scipy.spatial import distance
class FDBSCAN Client():
  def initialize(self, params: Dict):
    self.__dataset = params['dataset']
self.__labels = []
  def get results(self):
    return self.__labels
  def __get_points(self):
    dimension = len(self.__dataset[0])
    points = []
    for row in self.__dataset:
      points.append(tuple(row[i] for i in range(dimension)))
    return points
  def compute neighborhood matrix(self, epsilon: float):
    points = self.__get_points()
n_points = len(points)
    matrix = [[0] * n_points for i in range(n_points)]
    for i in range(n_points):
       for j in range(n_points):
   if (distance.euclidean(points[i], points[j]) < epsilon):</pre>
           matrix[i][j] = 1
    return matrix
  def update_labels(self, labels: List):
    self.__labels = labels
class FDBSCAN Server():
  def initialize(self, params: Dict):
    self.__MIN_POINTS = params['MIN_POINTS']
    self.__EPSILON = params['EPSILON']
    self.__running = False
  def get running(self):
    return self.__running
  def run(self, value: bool = True):
    self. running = value
  def get epsilon(self):
    return self.__EPSILON
```

```
def DBSCAN(self, global_matrix: np.ndarray):
    N = len(global_matrix)
    visited = np.zeros(N)
    labels = np.zeros(N)
    labels -=
    cluster_ID = 0
     for i in range(N):
      if visited[i]:
       else:
         visited[i] = 1
         num_points = np.sum(global_matrix[i])
         if num_points >= self.__MIN_POINTS:
    labels[i] = cluster_ID
            to_visit = np.where(global_matrix[i] == 1)[0].tolist()
            cluster_ID = self.__expand_cluster(to_visit, visited, global_matrix,
labels, cluster_ID)
    return labels
def __expand_cluster(self, to_visit: List, visited: np.array,
global_matrix:np.ndarray, labels: np.array, cluster_ID: int):
    for j in to_visit:
        if visited[j] == 0:
         visited[j] = 1
         num_neighbors = np.sum(global_matrix[j])
         if num_neighbors >= self.__MIN_POINTS:
            to_visit += np.where(global_matrix[j] == 1)[0].tolist()
       if labels[j] == -1:
         labels[j] = cluster_ID
    cluster_ID += 1
     return cluster_ID
```

```
# File: VF_DBSCAN/fd_server.py
import numpy as np
from flask import Flask, request
from flask restful import Resource, Api
from typing import Dict, List
from utils import process_http_posts
from fd_dbscan import FDBSCAN_Server
def training(clients: List, server: FDBSCAN_Server):
   data = {'action': 'compute_neighborhood_matrix', 'epsilon':
server.get_epsilon()}
  results, failures = process_http_posts(clients, data)
 N = len(results[0]['matrix'])
  global_matrix = np.zeros((N, N))
  for i in range(len(results)):
    matrix = np.array(results[i]['matrix'])
    global_matrix += matrix
  global_matrix = np.where(global_matrix < len(results), 0, 1)</pre>
  Q = server.DBSCAN(global_matrix)
  data = {'action': 'update_labels', 'labels': Q.tolist()}
  process_http_posts(clients, data)
```

```
def connect(json data: Dict, clients: List, client ids: List, running: bool):
  if (not running):
    client_id = json_data.get('client_id')
    address = json_data.get('address')
    client_ids.append(client_id)
    clients.append(address)
    message = {'message': f'Client {client_id} Connected'}
 else:
    code = 500
    message = {'message': 'Unable to connect: a training process is runnning'}
  return message, code
def run_server(host: str, port: int, MIN_POINTS: int, EPSILON: float):
  clients = []
  client_ids = []
  params = {
    'MIN POINTS': MIN_POINTS,
    'EPSILON': EPSILON
  server = FDBSCAN_Server()
  server.initialize(params)
 class FederatedServer(Resource):
    def get(self):
      action = request.args.get('action')
      if (action == 'start'):
        server.run();
        training(clients, server)
        server.run(False);
        return {'message': 'Training completed'}
        return {'message': 'Hello world from server', 'clients': clients}
    def post(self):
      json_data = request.get_json()
action = json_data.get('action')
      if (action == 'connect'):
        return connect(json_data, clients, client_ids, server.get_running())
        return {'message': 'Action not Supported'}, 201
  app = Flask(__name__)
  api = Api(app)
  api.add_resource(FederatedServer, '/')
  app.run(host = host, port = port)
# File: VF_DBSCAN/fd_client.py
from typing import List
from flask import Flask, request
from flask_restful import Resource, Api
from utils import send_post
```

from fd_dbscan import FDBSCAN_Client

```
# File: VF DBSCAN/fd client.py
def run_client(client_id: str, server_url: str, host: str, port: int, dataset:
List):
  address = host + f':{port}'
  send_post(server_url, {'action': 'connect','client_id': client_id,
'address': address})
  params = {
    'dataset': dataset,
  client = FDBSCAN_Client()
  client.initialize(params)
  class FederatedClient(Resource):
    def get(self):
      action = request.args.get('action')
      if (action == 'results'):
        labels = client.get_results();
        return {'labels': labels}
        return {'message': 'Hello World from Client', 'client_id': client_id}
    def post(self):
      json_data = request.json
      action = json_data.get('action')
      if (action == 'compute_neighborhood_matrix'):
    epsilon = json_data.get('epsilon')
        neighborhood_matrix = client.compute_neighborhood_matrix(epsilon)
        return {'matrix': neighborhood_matrix}
      elif (action == 'update_labels'):
   labels = json_data.get('labels')
        client.update_labels(labels)
        return {'message': 'Action not Supported'}, 201
  app = Flask(client_id)
  api = Api(app)
  api.add_resource(FederatedClient, '/')
  app.run(host = host, port = port)
# File: VF_DBSCAN/main_server.py
from fd server import run server
```

```
# File: VF_DBSCAN/main_server.py

from fd_server import run_server

host = "127.0.0.1"
port = 8080

# MIN_POINTS = 4 # 3MC
MIN_POINTS = 6 # aggregation
# EPSILON = 0.1 # 3MC
EPSILON = 0.04 # aggregation

run_server(host, port, MIN_POINTS, EPSILON)
```

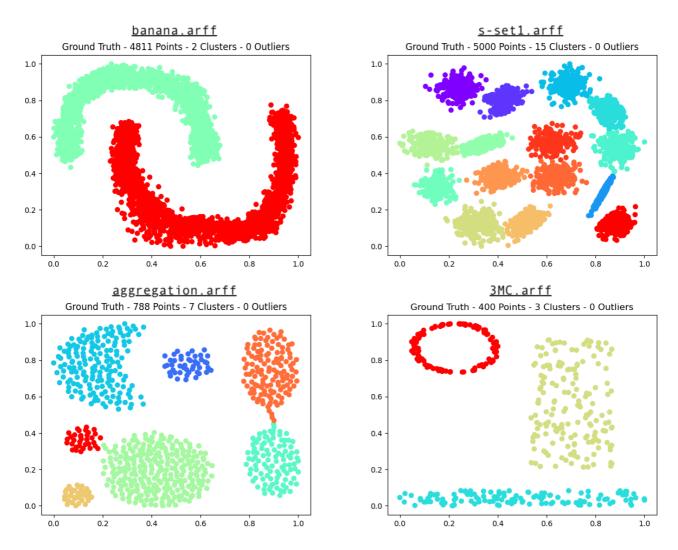
```
# File: VF_DBSCAN/main_client.py
import numpy as np
import pandas as pd
import uuid
from threading import Thread
from sklearn.preprocessing import MinMaxScaler
from scipy.io import arff
from fd_client import run_client
def generate_dataset_chunks(X: np.array, num_clients: int):
  dataset_chunks = []
  features_list = [i for i in range(len(X[0]))]
  for i in range(num_clients):
    dataset_chunks.append(X[:, features_list[i::num_clients]])
  return dataset_chunks
def prepare_dataset(num_clients: int):
  dataset_dir = '../datasets'
# dataset_file = '3MC.arff'
dataset_file = 'aggregation.arff'
dataset_path = f'{dataset_dir}/{dataset_file}'
  dataset = arff.loadarff(dataset_path)
  df = pd.DataFrame(dataset[0])
  Y = df['class'].tolist()
  Y = np.array([-1 if y == b'noise' else int(y) for y in Y])
  del df['class']
  X_original = np.array(df.values)
  min_max_scaler = MinMaxScaler()
  X = min_max_scaler.fit_transform(X_original)
  dataset chunks = generate dataset chunks(X, num clients)
  return dataset_chunks
server_url = "127.0.0.1:8080"
host = "127.0.0.1"
start_port = 5000
N_clients =
dataset_chunks = prepare_dataset(N_clients)
threads = []
for cli in range(N_clients):
  client_id = uuid.uuid4().hex
  port = start_port + cli
  thread_obj = Thread(target = run_client, args = (client_id, server_url,
host, port, dataset_chunks[cli]))
  threads.append(thread_obj)
  thread_obj.start()
for t in threads:
  t.join()
```

5. Experimental Analysis

5.1 Experimental Setup

The horizontal and vertical federated versions of DBSCAN presented have been tested using the datasets described in the following table and represented below.

	HORIZONTAI DBS		VERTICAL FEDERATED DBSCAN		
Dataset	banana.arff s-set1.arff a		aggregation.arff	3MC.arff	
Attributes	2	2	2	2	
Points	4811	5000	788	400	
Clusters	2	15	7	3	
Outliers	0 0		0	0	



Five different metrics have been used to evaluate the efficiency of the algorithms: ARI and AMI scores, purity score, BCubed precision score and BCubed recall score, defined in the script results/metrics.py.

```
# File: results/metrics.py
import sklearn.metrics as mtr
import numpy as np
import bcubed
def ARI_score(true_labels, predicted_labels):
  return mtr.adjusted rand score(true labels, predicted labels)
def AMI_score(true_labels, predicted_labels):
  return mtr.adjusted mutual info score(true labels, predicted labels)
def PURITY score(true labels, predicted labels):
  contingency matrix = mtr.cluster.contingency matrix(true labels,
predicted labels)
  return np.sum(np.amax(contingency_matrix, axis=0)) /
np.sum(contingency_matrix)
def BCubed_Precision_score(true_labels, predicted_labels):
  ldict = {}
  cdict = {}
  for i in range(len(true_labels)):
    ldict[i] = set([true_labels[i]])
    cdict[i] = set([predicted_labels[i]])
  return bcubed.precision(cdict, ldict)
def BCubed_Recall_score(true_labels, predicted labels):
  ldict = {}
  cdict = {}
  for i in range(len(true_labels)):
    ldict[i] = set([true_labels[i]])
    cdict[i] = set([predicted_labels[i]])
  return bcubed.recall(cdict, ldict)
def print_metrics(true_labels, elaborated_labels, message):
  print(f'{message}:')
  print(f'Purity: {PURITY_score(true_labels, elaborated_labels):.4f}')
  print(f'ARI: {ARI_score(true_labels, elaborated_labels):.4f}')
  print(f'AMI: {AMI_score(true_labels, elaborated_labels):.4f}')
  print(f'BCubed Precision: {BCubed_Precision_score(true_labels,
elaborated_labels):.4f}')
  print(f'BCubed Recall: {BCubed_Recall_score(true_labels,
elaborated_labels):.4f}\n')
```

The following scripts have been used to retrieve the results.

```
# File: results/HF_results.py

from scipy.io import arff
from sklearn.preprocessing import MinMaxScaler
from sklearn.cluster import DBSCAN
from urllib.request import urlopen
import numpy as np
import pandas as pd
import json
```

```
from plot import plot2D
import metrics as mtr
# results folder = 'banana'
results_folder = 's-set1'
start_port = 5000
N \text{ clients} = 10
url = 'http://localhost:8080/?action=start'
urlopen(url)
dataset = []
active_dataset = []
passive_dataset = []
labels = []
active_labels = []
passive_labels = []
true_labels = []
active_true_labels = []
passive_true_labels = []
passive_clients = False
for port in range(start_port, start_port + N_clients):
   url = f'http://localhost:{port}/?action=results'
  response = urlopen(url)
  data_json = json.loads(response.read())
res_dataset = data_json['dataset']
  res_labels = data_json['labels']
  res_true_labels = data_json['true_labels']
  passive = data_json['passive']
  dataset += res_dataset
  labels += res_labels
  true_labels += res_true_labels
  if passive:
    passive_clients = True
    passive_dataset += res_dataset
    passive_labels += res_labels
    passive_true_labels += res_true_labels
  else:
    active_dataset += res_dataset
    active_labels += res_labels
    active_true_labels += res_true_labels
# MIN_POINTS = 4 # banana
MIN_POINTS = 15 # s-set1
# EPSILON = 0.03 # banana
EPSILON = 0.03 # s-set1
clustering = DBSCAN(eps = EPSILON, min_samples = MIN_POINTS)
dbscan_labels = clustering.fit_predict(np.array(dataset))
plot2D(points = np.array(dataset), labels = np.array(true_labels), folder =
results_folder, message = 'Ground Truth')
plot2D(points = np.array(dataset), labels = dbscan_labels, folder =
results_folder, message = 'DBSCAN')
plot2D(points = np.array(dataset), labels = np.array(labels), folder =
results_folder, message = 'Federated DBSCAN')
plot2D(points = np.array(active_dataset), labels = np.array(active_labels),
folder = results_folder, message = 'Federated DBSCAN - Active')
if passive_clients:
plot2D(points = np.array(passive_dataset), labels =
np.array(passive_labels), folder = results_folder, message = 'Federated DBSCAN
- Passive')
```

```
mtr.print_metrics(true_labels, dbscan_labels, 'DBSCAN')
mtr.print_metrics(true_labels, labels, 'Federated DBSCAN')
mtr.print_metrics(active_true_labels, active_labels, 'Federated DBSCAN -
Active')
if passive_clients:
    mtr.print_metrics(passive_true_labels, passive_labels, 'Federated DBSCAN -
Passive')
```

```
# File: results/VF_results.py
from scipy.io import arff
from sklearn.preprocessing import MinMaxScaler from sklearn.cluster import DBSCAN from urllib.request import urlopen
import numpy as np
import pandas as pd
import json
from plot import plot2D
import metrics as mtr
# results folder = '3MC'
results_folder = 'aggregation'
dataset_dir = '../datasets'
# dataset_file = '3MC.arff'
dataset_file = 'aggregation.arff'
dataset_path = f'{dataset_dir}/{dataset_file}'
dataset = arff.loadarff(dataset_path)
df = pd.DataFrame(dataset[0])
true_labels = df['class'].tolist()
true_labels = np.array([-1 if label == b'noise' else int(label) for label in
true_labels])
del df['class']
X_original = np.array(df.values)
min max scaler = MinMaxScaler()
X = min_max_scaler.fit_transform(X_original)
# MIN POINTS = 4 # 3MC
MIN_POINTS = 6 # aggregation
# EPSILON = 0.1 # 3MC
EPSILON = 0.04 # aggregation
clustering = DBSCAN(eps = EPSILON, min samples = MIN POINTS)
dbscan labels = clustering.fit predict(X)
url = 'http://localhost:8080/?action=start'
urlopen(url)
url = 'http://localhost:5000/?action=results'
response = urlopen(url)
data_json = json.loads(response.read())
labels = np.array(data_json['labels'])
plot2D(points = X, labels = true_labels, folder = results_folder, message =
"Ground Truth")
plot2D(points = X, labels = dbscan_labels, folder = results_folder, message =
"DBSCAN")
plot2D(points = X, labels = labels, folder = results_folder, message =
f'Federated DBSCAN')
mtr.print_metrics(true_labels, dbscan_labels, 'DBSCAN')
mtr.print_metrics(true_labels, labels, 'Federated DBSCAN')
```

Finally, the script results/plot.py was used to plot the results.

```
# File: results/plot.py
from matplotlib import pyplot as plt
import matplotlib.cm as cm
import numpy as np
def plot2D(points: np.ndarray, labels: np.array, folder: str, message: str):
   int_labels = [int(label) for label in labels]
   color_range = cm.rainbow(np.linspace(0, 1, np.max(np.unique(int_labels))+1))
  colors = []
  count outliers = 0
  for label in int labels:
    if label == -1:
      count_outliers += 1
      colors.append([0, 0, 0, 1])
    else:
      colors.append(color_range[label])
  f'{len(np.unique(int_labels)) - (1 if count_outliers > 0 else 0)}
         f'{count_outliers} Outliers')
  plt.savefig(f'{folder}/{message}.png')
  plt.clf()
```

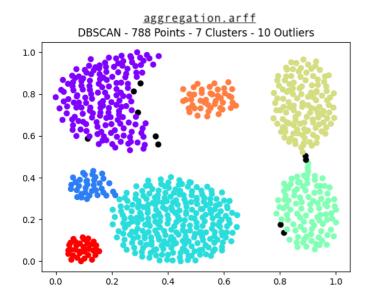
In a further experiment some of the clients were excluded from the learning process when testing horizontal federated DBSCAN over banana arff dataset. Thus, active clients, whose datasets were used in training the model, were distinguished by passive clients. Passive clients, who did not share any knowledge with the server, were notified by the server at the conclusion of the learning process with the resulting clustering. Statistics have been tracked both for the active and the passive components of the learning system. The algorithm was tested with increasing passive clients percentages (10%, 20%, 30%). For each percentage, five tests have been run, each one with a different random client selection, such that it was possible to compute the mean value (μ) and the standard deviation (σ) of the metrics specified above in each of the cases.

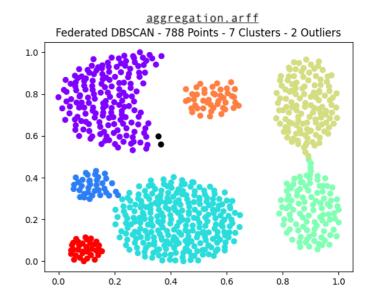
5.2 Results

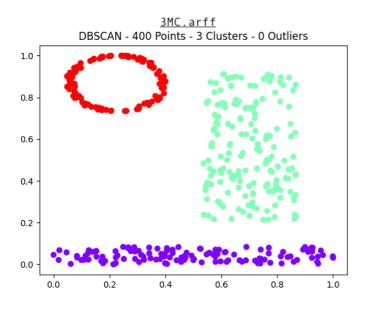
The following table shows the results obtained when testing vertical federated DBSCAN, compared to those obtained by the application of the standard version of the algorithm. The plots graphically represents the resulting clustering.

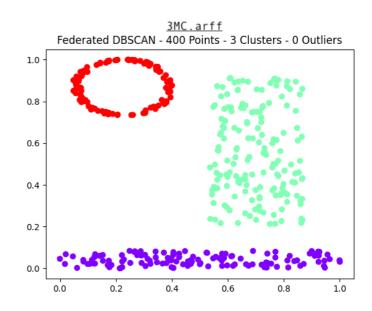
When training the model for aggregation arff dataset, MinPts was set to 6, and Eps to 0.04; when training the model for 3MC arff dataset, MinPts was set to 4, and Eps to 0.1.

	VERTICAL I DBS		STANDARD DBSCAN		
Dataset	aggregation.arff	3MC.arff	aggregation.arff	3MC.arff	
AMI	0.9808	1.0000	0.9675	1.0000	
ARI	0.9866	1.0000	0.9779	1.0000	
Purity	0.9949	1.0000	0.9911	1.0000	
BCubed Precision	0.9902	1.0000	0.9856	1.0000	
BCubed Recall	0.9849	1.0000	0.9678 1.0000		





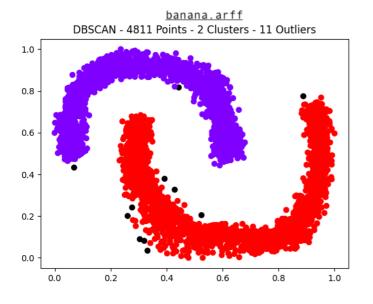


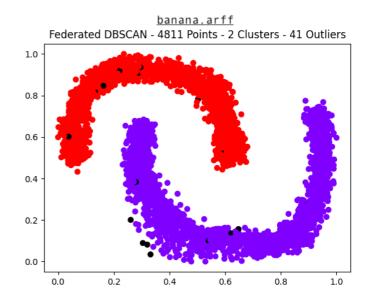


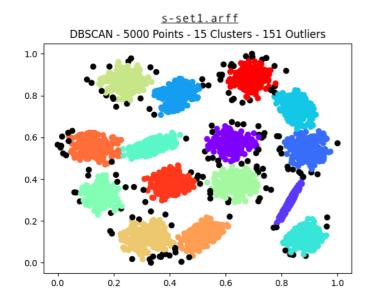
The following table shows the results obtained when testing horizontal federated DBSCAN, compared to those obtained by the application of the standard version of the algorithm. The plots graphically represents the resulting clustering. No passive clients are introduced.

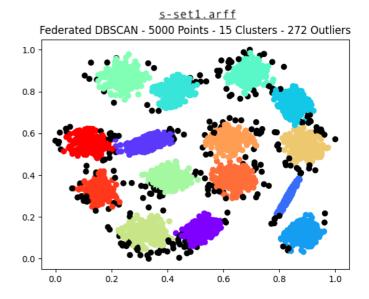
When training the model for banana.arff dataset, *MinPts* was set to 4, and *Eps* to 0.03; when training the model for s-set1.arff dataset, *MinPts* was set to 15, and *Eps* to 0.03.

		L FEDERATED CAN	STANDARD DBSCAN		
Dataset	banana.arff	s-set1.arff	banana.arff s-set1.ar		
AMI	0.9622	0.9622 0.9335		0.9615	
ARI	0.9828	0.9175	0.9956	0.9600	
Purity	0.9973	0.9512	0.9996	0.9740	
BCubed Precision	0.9963	0.9469	0.9993	0.9716	
BCubed Recall	0.9831	0.9411	0.9954 0.9411		







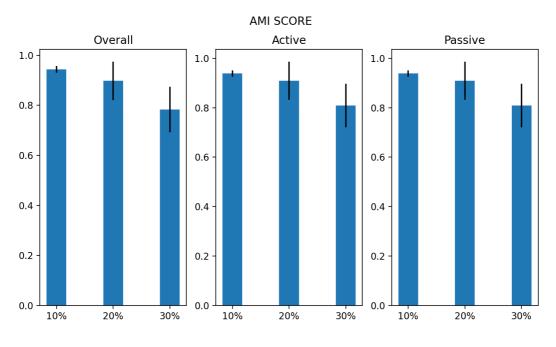


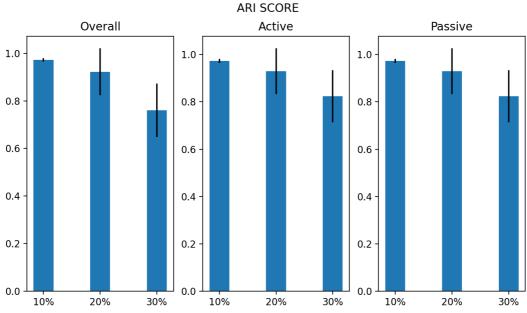
Finally, the following tables and charts show the results obtained when testing horizontal federated DBSCAN over banana.arff dataset with an increasing percentage of passive clients.

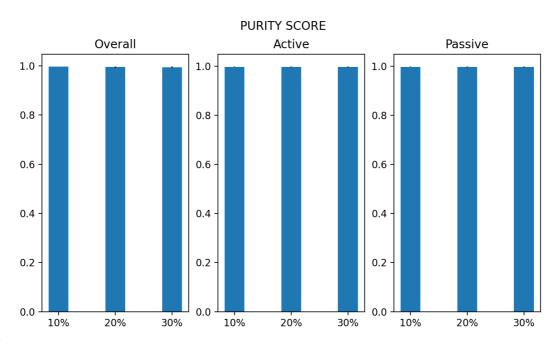
			I	II	III	IV	V	μ	σ
		Overall	0,9262	0,9541	0,9386	0,9613	0,9386	0,9438	0,0139
	AMI	Active	0,9254	0,9368	0,9359	0,9603	0,9359	0,9389	0,0129
		Passive	0,9431	0,8889	0,9707	0,9706	0,9707	0,9488	0,0355
		Overall	0,9645	0,9777	0,97	0,9824	0,97	0,9729	0,0071
	ARI	Active	0,964	0,9836	0,968	0,9818	0,968	0,9731	0,009
		Passive	0,9691	0,9259	0,9878	0,9874	0,9878	0,9716	0,0268
	Purity	Overall	0,9956	0,9971	0,9973	0,9971	0,9973	0,9969	0,0007
10%		Active	0,9952	0,9975	0,9975	0,997	0,9975	0,9969	0,001
		Passive	1	0,9938	0,9979	0,9979	0,9979	0,9975	0,0023
		Overall	0,995	0,9957	0,9963	0,9961	0,9963	0,9959	0,0005
	BCubed Precision	Active	0,9945	0,9965	0,9964	0,996	0,9964	0,996	0,0008
		Passive	1	0,9896	0,9972	0,9972	0,9972	0,9962	0,0039
		Overall	0,964	0,9783	0,9697	0,9827	0,9697	0,9729	0,0075
	BCubed Recall	Active	0,9636	0,984	0,9677	0,9822	0,9677	0,973	0,0094
		Passive	0,9676	0,929	0,9876	0,9876	0,9876	0,9719	0,0255

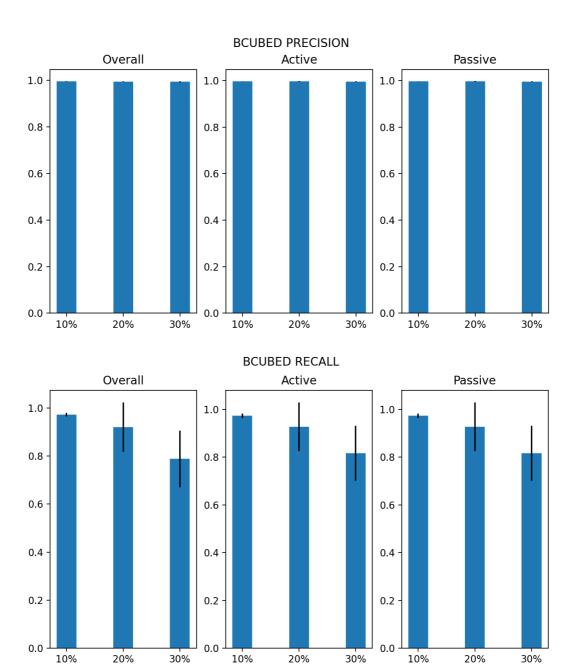
			I	II	III	IV	V	μ	σ
		Overall	0,9496	0,9213	0,9179	0,7618	0,938	0,8977	0,0771
	AMI	Active	0,9656	0,9241	0,9189	0,778	0,9624	0,9098	0,0767
		Passive	0,8974	0,9174	0,9144	0,7131	0,8678	0,862	0,0855
		Overall	0,9757	0,961	0,9592	0,747	0,9694	0,9225	0,0983
	ARI	Active	0,9848	0,9617	0,9599	0,7572	0,9826	0,9292	0,0969
		Passive	0,9399	0,9583	0,956	0,7122	0,918	0,8969	0,1045
		Overall	0,9958	0,9973	0,9942	0,995	0,9927	0,995	0,0017
20%	Purity	Active	0,9974	0,9982	0,9943	0,9979	0,9977	0,9971	0,0016
		Passive	0,9896	0,9938	0,9938	0,9834	0,9844	0,989	0,005
		Overall	0,9946	0,9962	0,9936	0,9937	0,9923	0,9941	0,0014
	BCubed Precision	Active	0,9966	0,9973	0,9938	0,997	0,9966	0,9963	0,0014
		Passive	0,9864	0,9925	0,9929	0,9815	0,9802	0,9867	0,0059
		Overall	0,9762	0,9607	0,9584	0,7362	0,9697	0,9202	0,1031
	BCubed Recall	Active	0,9851	0,9612	0,9592	0,7462	0,983	0,9269	0,1017
		Passive	0,9417	0,9592	0,9553	0,7034	0,9188	0,8957	0,1086

			I	II	III	IV	V	μ	σ
30%	AMI	Overall	0,8293	0,7066	0,7566	0,7066	0,9194	0,7837	0,091
		Active	0,8911	0,7363	0,7653	0,7363	0,9167	0,8091	0,0878
		Passive	0,7504	0,6709	0,7392	0,6709	0,9317	0,7526	0,1068
	ARI	Overall	0,8649	0,703	0,7527	0,703	0,9598	0,7967	0,1126
		Active	0,9296	0,7377	0,7576	0,7377	0,9571	0,8239	0,1097
		Passive	0,7579	0,6669	0,7423	0,6669	0,9665	0,7601	0,1228
	Purity	Overall	0,9919	0,9946	0,9921	0,9946	0,9971	0,9941	0,0021
		Active	0,997	0,9973	0,9944	0,9973	0,9976	0,9967	0,0013
		Passive	0,9882	0,9882	0,9896	0,9882	0,9958	0,99	0,0033
	BCubed Precision	Overall	0,9919	0,9933	0,9917	0,9933	0,996	0,9932	0,0017
		Active	0,9959	0,9963	0,9934	0,9963	0,9967	0,9957	0,0013
		Passive	0,9851	0,9867	0,9984	0,9867	0,9943	0,9902	0,0058
	BCubed Recall	Overall	0,86	0,6899	0,7422	0,6899	0,9595	0,7883	0,1183
		Active	0,9274	0,7258	0,7468	0,7258	0,9564	0,8164	0,1153
		Passive	0,7491	0,6539	0,7326	0,6539	0,9673	0,7514	0,1284









5.3 Conclusions

At first, a brief overview of DBSCAN algorithm and federated learning has been given. Then, horizontal and vertical federated learning versions of DBSCAN have been introduced. These versions of the well-known algorithm have the great advantage of preserving privacy, avoiding raw data sharing, when dealing with distributed databases. An implementation of both algorithms has then been presented, using Python programming language.

Each federated version was finally tested using two different datasets, and results were compared to those given by the application of standard DBSCAN. The federated versions proved to be equally efficient, giving slightly different but consistent results. Furthermore, the horizontal federated version

was tested in the hypothesis of an increasing number of passive clients, and demonstrated to be resistant to raw data loss, up to 20% of the whole dataset.

6. References

[1] An introduction to clustering algorithms:

Jiawei Han, Micheline Kamber, and Jian Pei. 2011. Data Mining: Concepts and Techniques (3rd. ed.). Morgan Kaufmann Publishers Inc., San Francisco, CA, USA.

[2] A detailed description of DBSCAN:

Martin Ester, Hans-Peter Kriegel, Jörg Sander, and Xiaowei Xu. 1996. A density-based algorithm for discovering clusters in large spatial databases with noise. In Proceedings of the Second International Conference on Knowledge Discovery and Data Mining (KDD'96). AAAI Press, 226–231.

[3] A comprehensive overview of Federated Learning

Qiang Yang, Yang Liu, Tianjian Chen, and Yongxin Tong. 2019. Federated Machine Learning: Concept and Applications. ACM Trans. Intell. Syst. Technol. 10, 2, Article 12 (February 2019), 19 pages.