WELCOME TO CLASS!

NUMBER PROPERTIES-I

Objectives of the Session

In this session, you will acquire skills required to solve problems based on:

- Basic Properties of Numbers
- Prime & Composite Numbers
- Prime Factorization
- Number of Factors
- Fractions & Decimals

Section #1

Basic Properties of Numbers

Basic Properties of Numbers

On the GMAT, the word real number/number refers to all numbers that exist on a number line, namely:

- Positive Integers
- Negative Integers
- Zero
- Positive Fractions/Decimals
- Negative Fractions/Decimals and
- Irrational Numbers

Integers

What do you mean by distinct integers ?

If x and y are distinct negative integers greater than -10, what is the greatest possible product of x and y ?

- ○ 2
- ○ 4
- ○ 72
- ○ 81
- ○ 90

The product of two integers is 36 and their sum is 13. What is the positive difference of the two numbers ?

- 1
- 4
- 5 ✓
- 7
- 9

$x + y = 13$

$xy = 36$

36

6 6

12 3

4 9

Odd & Even Numbers

Even numbers are numbers that can be expressed as a multiple of 2 or 2n, where ‘n’ is an integer.

2, 4, 6…are all even since they can be expressed as 2n

▶ $2 \times 1, 2 \times 2, 2 \times 3\ldots$

$-2, -4, -6$…are all even since they can be expressed as 2n

▶ $2 \times -1, 2 \times -2, 2 \times -3\ldots$

<u>0 is even</u> since 0 can be expressed as 2n

▶ 2×0

Odd numbers are numbers that can be expressed as $2n \pm 1$, where ‘n’ is an integer.

3, 5, 7… are all odd since they can be expressed as $2n \pm 1$

▶ $2 \times 1 + 1, 2 \times 2 + 1, 2 \times 3 + 1\ldots$

$-3, -5, -7$… are all odd since they can be expressed as $2n \pm 1$

▶ $2 \times -1 - 1, 2 \times -2 - 1, 2 \times -3 - 1\ldots$

Even and Odd

Quick Quiz :

- Even × Even = E
- Odd × Odd = O
- Even × Odd = E
- Even + Even = E
- Even – Even = E
- Odd + Odd = E
- Odd – Odd = E
- Even + Odd = O
- Even – Odd = O
- Even ÷ Even = ~~Not det~~ Can't be concluded

Must Be True

- Choose numbers for the variables that fit any conditions in the problem.
- Eliminate any answers that don't match what you need to find.
- Plug in new numbers that have different characteristics.
- Repeat until one answer remains.

1. If a and b are distinct positive integers, and a is even, then which of the following must also be even?

$a = 2$	$a = 2$
$b = 3$	$b = 4$

$a = 2k$

- ~~$2(a + b) - 3$~~ =) odd
- ~~$(a - b) + 2$~~
- ~~$a + b - 1$~~
- ~~$a - b$~~
- $ab - 2$

$E - 2 = \text{Even}$

Even and Odd

If m is an even integer, v is an odd integer, and m > v > 0, which of the following represents the number of even integers less than m and greater than v ?

- $\frac{m-v}{2} - 1$
- $\frac{m-v-1}{2}$
- $\frac{m-v}{2}$
- $m - v - 1$
- $m - v$

$\frac{2}{2} = 1$

m − v

Even − odd

m = 6

v = 3

# of Even

3 < < 6

1

Even and Odd

Additional Question-2(DRILL)

Have you tried "SIMPLIFICATION" ?

If r and s are integers and rs + r is odd, which of the following must be even?

always

- ○ r
- ○ s
- ○ r + s
- ○ rs - r
- ○ $r^2 + s$

6 - 3 => 3

9 + 2

r = 3 s = 2

2·3 + 3 => 9

r (s+1) => odd

odd odd

Section #2

Prime & Composite Numbers

Prime & Composite Numbers

A prime number is a positive integer that is divisible by only 1 and itself.

- 2, 3, 5, 7, 11, 13, 17… are all primes.
- 2 is the first prime number.
- 2 is also the only even prime number.

In essence, prime numbers are positive integers that are made up only of 1 and themselves.

- All the rest of the positive integers (except 1) are called **composite numbers**, and they are made up of prime numbers.
- 4, 6, 8, 9, 12, 14, 15… are all composite numbers.
- *1 is neither prime nor composite*

Prime & Composite Numbers

What is the sum of the distinct prime numbers between 50 and 60?

○ 104

○ 108

○ 110

○ 112 ✓

○ 116

53, ~~57~~, 59

Prime & Composite Numbers

Is there any commonality among all prime numbers greater than 2 ?

If x and y are different prime numbers, each greater than 2, which of the following must be true?

I. $x+y \neq 91$
II. $x-y$ is an even integer
III. x/y is not an integer

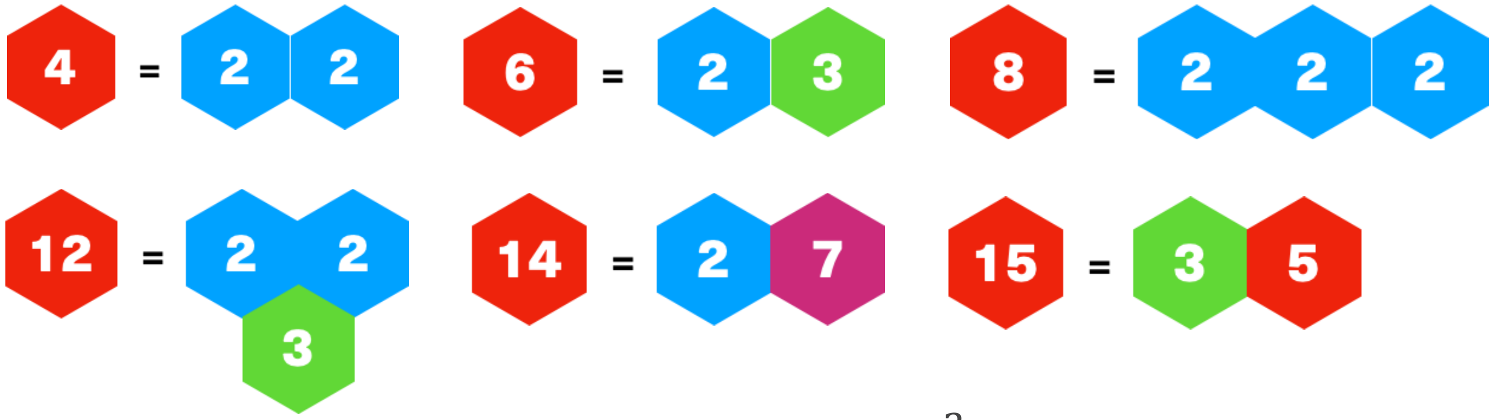
- ○ II only
- ○ I and II only
- ○ I and III only
- ○ II and III only
- ○ I, II and III

Section #3

Prime Factorization

Prime Factorization

Expressing a composite number as a product of the prime numbers it is made up of, is called prime factorization.



$$4 = 2^2$$
$$6 = 2^1 \times 3^1$$
$$8 = 2^3$$
$$10 = 2^1 \times 5^1$$
$$12 = 2^2 \times 3^1$$

Prime Factorization

What is the sum of the distinct prime factors of 72 ?

- ○ 2
- ○ 3
- ○ 5
- ○ 8
- ○ 13

What are the clues to the concept this problem is testing?

1. If a, b, c, and d are integers such that $1 < a < b < c < d$ and the product of a, b, c, and d is 330, what is the value of c^b ?

- 32
- 121
- 125
- 243
- 2,048

$2 * 3 * 5 * 11$

5^3

What answer is too obvious?

4. How many different ~~positive~~ prime factors of $7^{21} - 7^{19}$ are there?

- One
- Two
- Three ✓
- Four
- More than four

$7^{19}(7^2 - 1)$

$7^{19} * 2^4 * 3$

$\frac{100}{x} = \text{Integer}$

100

Facts

$\frac{100}{2}$

Prime Factorization

HACK:
While dealing with a large number, it's common for test-takers to panic.

Instead, acknowledge that working with large numbers might be challenging.

Break the large number down into smaller components for better insights and understanding.

So, the next time you encounter a large number, consider exploring its prime factorization for a clearer understanding

A bag contains game pieces worth 3, 5, and 11 points, respectively. During a turn, a player selects up to 10 pieces at random from the bag. The player's score for the turn is found by taking the product of the point values of each selected piece. If Jake's score for a turn is 12,375, how many total pieces did he select?

- 2
- 3
- 4
- 5
- 6

2 -> Even -> Unit's place
3 -> Sum of the digits
4 -> last 2 digits
5 -> 0 or 5
8 -> last 3 digits
6 -> Both 2 & 3
9 ->
11 -> Sum of odd − Sum of even = 0 or 11

7

12, 15, 36
(4, 3) (3, 5) (9, 4)

$x + y + z \le 10$

34 − 6 => 28

$3^x * 5^y * 11^z = 12375$

$5 * 2475$

5 495

$5^2 * 495$

$45 * 11$

$=) 5^3 * 3^2 * 11$

Section #4

Number of Factors

Number of Factors

A factor or a divisor is a number that evenly — without leaving a remainder — divides another number.

Let us list the factors of 36.

$$36 = 2 \times 2 \times 3 \times 3$$

$$or, 36 = 2^{\boxed{2}} \times 3^{\boxed{2}}$$

$$\downarrow \qquad \downarrow$$

$$(2+1) = 3 \qquad (2+1) = 3$$

$$\downarrow$$

$$3 \times 3 = 9$$

36 =	1	×	36
36 =	2	×	18
36 =	3	×	12
36 =	4	×	9
36 =	6	×	6

So, the number 36 has got 9 factors, namely 1, 2, 3, 4, 6, 9, 12, 18, 36.

Is there any other way to get the number of factors of a given number?

Number of Factors

How to calculate the number of factors of any number?

Factorize a number x into its prime factors.

1. If $x = a^m$, where a is a prime number, then

$$\text{Number of Factors} = (m+1)$$

2. If $x = a^m \times b^n$, where a, b are distinct prime numbers

$$\text{Number of Factors} = (m+1) \times (n+1)$$

3. If $x = a^m \times b^n \times c^p$, where $a, b, and\ c$ are distinct prime numbers

$$\text{Number of Factors} = (m+1) \times (n+1) \times (p+1)$$

4. If $x = a^m \times b^n \times c^p \times d^q$, where $a, b, c, and\ d$ are distinct prime numbers

$$\text{Number of Factors} = (m+1) \times (n+1) \times (p+1) \times (q+1)$$

Number of Factors

Which of the following integers has the greatest number of factors ?

- 8 → 4
- 51 → $17 * 3 = 2 * 2 = 4$
- 75 → $5^2 * 3 = 3 * 2 = 6$
- 118 → $59 * 2 = 4$
- 121 ⇒ 11^2 ⇒ 3

Squares & Cubes

A perfect square is an integer that can be expressed as the product of an integer by itself.

$$1 = 1 \times 1 = 1^2$$

$$25 = 5 \times 5 = 5^2$$

$$144 = 12 \times 12 = 12^2 = 2^4 \times 3^2$$

A perfect square *always have odd number of factors. Power of the prime factors are always a multiple of 2.*

Similarly, a perfect cube is an integer that can be expressed as the triple product of an integer by itself.

$$1 = 1 \times 1 \times 1 = 1^3$$

$$27 = 3 \times 3 \times 3 = 3^3$$

$$216 = 6 \times 6 \times 6 = 2^3 \times 3^3$$

For a perfect cube, *Power of the prime factors are always a multiple of 3.*

2. If y is the least positive integer such that 5,880 multiplied by y is the square of an integer, then which of the following is the value of y ?

- 2
- 3
- 5
- 15
- 30 ✓

$5880 * y = N^2$

$\sqrt{5880 * y} = N$

$588 * 10$

$84 * 7 * 2 * 5$

$2^2 * 3 * 7^2 * 2 * 5$

$(2^3 * 3 * 5 * 7^2) * y \rightarrow 2 * 3 * 5$

Section #5

Fractions & Decimals

Place Value

Each digit in a number can be associated to a name that describes its position.
For example, in the number 123.45

- 3 represents the units/ones digit ▶ the place value of 3 is $3 \times 1 = 3$
- 2 represents the tens digit ▶ the place value of 2 is $2 \times 10 = 20$
- 1 represents the hundreds digit ▶ the place value of 1 is $1 \times 100 = 100$
- 4 represents the ten<u>ths</u> digit ▶ the place value of 4 is $4 \times \frac{1}{10} = 0.4$
- 5 represents the hundred<u>ths</u> digit ▶ the place value of 5 is $5 \times \frac{1}{100} = 0.05$

The number 123.45 can be formed by adding the place values.
That is,
123.45 = 1 "hundreds" + 2 "tens" + 3 "ones" + 4 "one-tenths" + 5 "one-hundredths"

$$123.45 = 1 \times 100 + 2 \times 10 + 3 \times 1 + 4 \times \frac{1}{10} + 5 \times \frac{1}{100} = 100 + 20 + 3 + 0.4 + 0.05$$

2

2.463 → 2

2.763

Nearest hundredths place

2.76

2.768

2.77

Place Value

Additional Question-4(MODEL)

When dealing with a difficult
fraction on the GMAT
think of simplifying the
fraction to an easier form
check if you can bring the
denominator in the form
of powers of 10.

for example, $\frac{7}{25}$ can be easily represented in decimal form when it is of the form
$\frac{7\times4}{25\times4} = \frac{28}{100} = 0.28$

What is the thousandths digit in the decimal equivalent of $\frac{53}{5000}$?

- ○ 0
- ○ 1
- ○ 3
- ○ 5
- ○ 6

$\frac{53}{5000} \times \frac{2}{2}$

$\frac{106}{10^4}$

$\Rightarrow 0.0106$

Place Value

How many integers n greater than 10 and less than 100 are such that, if the digits of n are reversed, the resulting integer is $n+9$?

- ○ 5
- ○ 6
- ○ 7
- ○ 8
- ○ 9

n

$n = 10a + b$

$10b + a$

$10b + a - 10a - b = 9$

$\cancel{10a + b - 10b - a = 9}$

$9b - 9a = 9$

$\boxed{b - a = 1}$

12
23
34
45

23
34

12 21
~~13~~ ~~31~~
14 41
15 51
16
17

Place Value

Additional Question-6(DRILL)-2min

If X is the hundredths digit in the decimal 0.1X, and Y is the thousandths digit in the decimal 0.02Y , where X and Y are non-zero digits, which of the following is closest to the greatest possible value of $\frac{0.1X}{0.02Y}$?

- ○ 4
- ○ 5
- ○ 6
- ○ 9
- ○ 10

$$\frac{0.19}{0.021} \Rightarrow \frac{19}{2.1}$$

$$\frac{19}{2} \Rightarrow 9.5$$

$$\cancel{\frac{10X}{2Y}}$$

$$10\left(\frac{1X}{2Y}\right)$$

$$\frac{10 * 19}{21} \Rightarrow \frac{190}{21}$$

Terminating and Non-Terminating Decimals

Any decimal that has only a finite number of nonzero digits is a terminating decimal.

Example 1:

$0.82 = \frac{82}{100}$, $5.096 = \frac{5096}{1000}$ are terminating decimals.

Notice that denominator of all the above fractions are powers of 10.

<u>If the denominator of a fraction can be expressed as powers of 10, the decimal representation of the fraction will terminate.</u>

Example 2:

$\frac{21}{40}$ can be expressed as $\frac{21}{2^3 \times 5}$

Can the denominator of this fraction be expressed as power of 10?

Since the denominator has only 2's and 5's in it, we can multiply both numerator and denominator by 5^2 to get $\frac{21}{2^3 \times 5} \times \frac{5^2}{5^2} = \frac{21 \times 25}{2^3 \times 5^3} = \frac{21 \times 25}{1000}$

So, $\frac{21}{2^3 \times 5}$ can be expressed as a terminating decimal.

Terminating and Non-Terminating Decimals

On the other hand, if the denominator of a fraction cannot be expressed as powers of 10, the decimal representation of the fraction will not terminate.

Example:

$\frac{21}{360}$ can be expressed as $\frac{21}{2^3 \times 5 \times 3^2}$

$\frac{21}{2^3 \times 5 \times 3^2}$ can be reduced to $\frac{7}{2^3 \times 5 \times 3}$

Since the denominator of the reduced fraction still has a 3 in it, which doesn't cancel out with the numerator, we cannot express the denominator in powers of 10.

So, decimal representation of $\frac{21}{360}$ will not terminate.

$$\frac{21}{360} = 0.0583333\ldots$$

It will be a recurring, non-terminating decimal.

In general, a reduced fraction $\frac{x}{y}$ can be expressed as a terminating decimal, if and only if the denominator does not contain any prime factors other than 2 or 5.

Terminating and Non-Terminating Decimals

Additional Question-7(APPLY)

Any decimal that has only a finite number of nonzero digits is a terminating decimal. For example, 24, 0.82, and 5.096 are three terminating decimals. Which of the following fractions will result in a terminating decimal?

I. $\frac{42}{13}$

II. $\frac{42}{16}$

III. $\frac{42}{25}$

IV. $\frac{42}{70}$ $\Rightarrow \frac{\cancel{7}\times 6}{\cancel{7}\times 10}$

A. I & IV only
B. II, III & IV only
C. III & IV only
D. I, II, III & IV
E. None

Terminating and Non-Terminating Decimals

Additional Question-8(DRILL)-2min

Which of the following fractions has a decimal equivalent that is a terminating decimal?

- ○ $\frac{10}{189}$
- ○ $\frac{15}{196}$
- ○ $\frac{16}{225}$
- ○ $\frac{25}{144}$
- ○ $\frac{39}{128}$ $=\frac{\quad}{2^7}$

Rounding

Rounding off is simplifying a number by keeping its value closer to the original.
For example, to round the decimal **1.3457 to nearest tenth**,
drop the extra decimal places after tenth.
1.3457
If the first dropped digit is 5 or more, the last digit that you keep must be rounded up by 1.
If the first dropped digit is less than 5, do nothing.
Since the first dropped digit is 4, which is less than 5,
1.3457 rounded to nearest tenth is 1.3

Similarly, to round the decimal **1.3457 to nearest hundredth**,
drop the extra decimal places after hundredth.
1.3457
If the first dropped digit is 5 or more, the last digit that you keep must be rounded up by 1.
If the first dropped digit is less than 5, do nothing.
Since the first dropped digit in 1.3457 is 5, 4 is rounded up by 1 to get 1.35

Rounding

If n=2.0453 and $n*$ is the decimal obtained by rounding n to the nearest hundredth, what is the value of $n* - n$?

- ○ – 0.0053
- ○ –0.0003
- ○ 0.0007
- ○ 0.0047
- ○ 0.0153

$n^* = 2.0500$

$n = 2.0453$

Action Items -

Over the course of this week…

1. Revise the lecture recording and make proper notes.

2. Check the 'Homework spreadsheet' and complete the homework.

Success is no accident.
It is hard work, perseverance, learning,
studying, sacrifice, and most of all, love of what
you are doing or learning to do.

- Pelé

THANK YOU!

Q & A