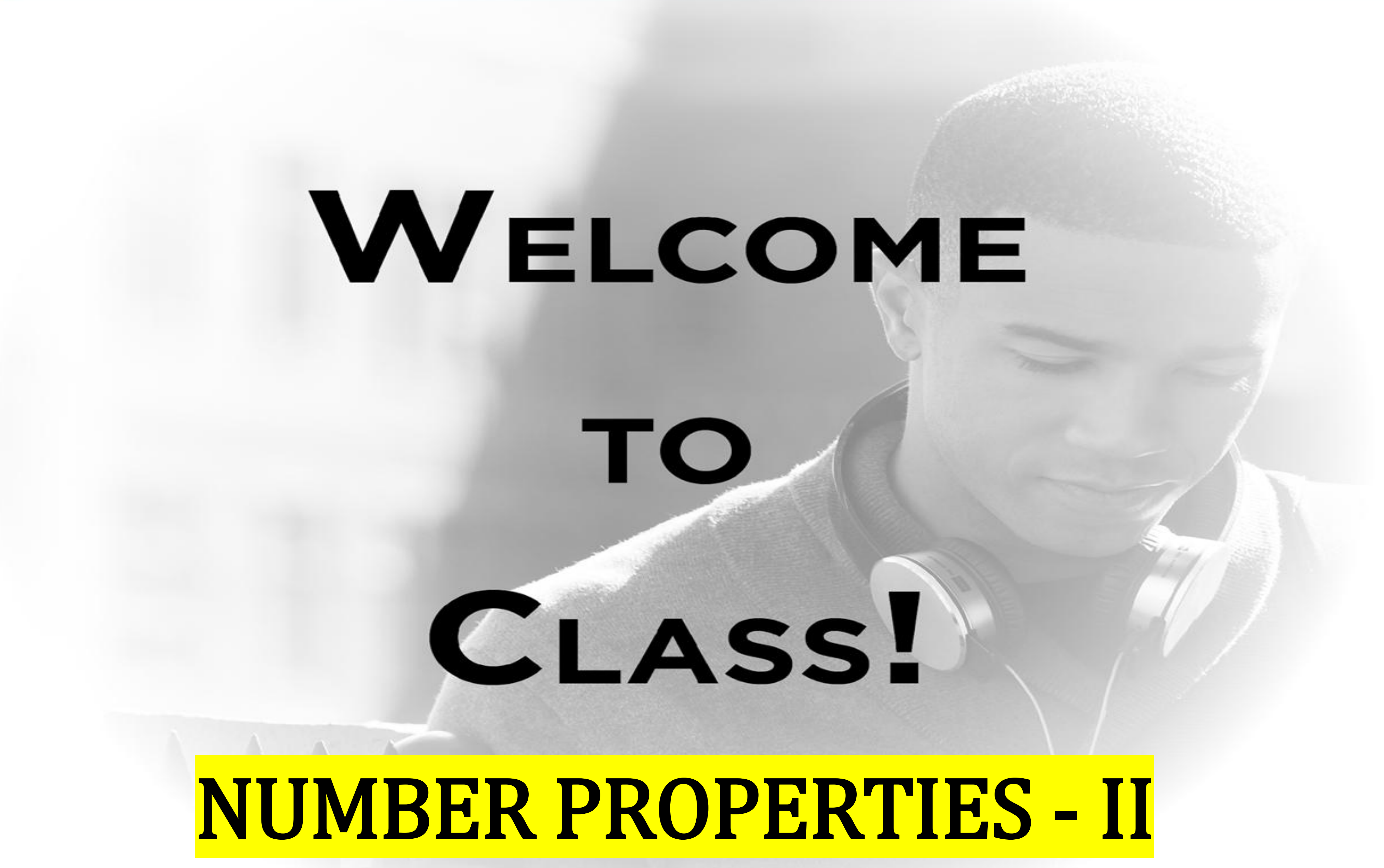

NUMBER PROPERTIES - II

Objectives of the Session

In this session, you will acquire skills required to solve problems based on:

- Multiples & Divisibility
- LCM & GCD
- Remainders
- Units Digit Cyclicity

$\frac{a}{b} = k$

$\boxed{a = bk}$

k is an Integer

$a = bk$

$c = bq$

$a + c = b(k + q)$

Section #1

Multiples & Divisibility

Multiples & Divisibility

An integer x is a multiple of an integer y, means:

- x is evenly divisible by y
- y divides x without leaving a remainder
- y is a factor of x
- $x = y \times n$, where n is an integer $\Rightarrow {}^{x}/_{y}$ is an integer

Example:

20 is a multiple of 4, means:

- 20 is evenly divisible by 4
- 4 divides 20 without leaving a remainder
- 4 is a factor of 20
- $20 = 4 \times 5 \Rightarrow {}^{20}/_{4} = 5$ is an integer

Note

- Every number is a multiple of itself.
- 0 is divisible by all numbers except 0.
- 1 is a factor of all numbers.

Multiples & Divisibility

If x and y are multiples of z, which of the following must be true?

I. $x+y$ is divisible by z

II. $x-y$ is divisible by z

III. xy is divisible by z

- I only
- II only
- III only
- I, II & III
- None

Multiples

Will x+18 also be a multiple of n ?

Positive integer x and 18 are both multiples of positive integer n. If n > 1, which of the following is a possible value for 18+x?

- ○ 41 $\rightarrow x = 23$
- ○ 67 $\Rightarrow x = 49$
- ○ 99 $\Rightarrow x = 81$
- ○ 103 $\Rightarrow x = 85$
- ○ 125 $\Rightarrow x = 107$

$x = \cancel{86}$

$18 = n\,a$

a is an Integer

$18 = n\,a$

1, 2, 3, 6, 9, 18

$\boxed{x = n\,b}$

3. If x, y, and z are positive integers, and x is a multiple of 3, for which of the following must z and 18 have a common factor greater than 1 ?

$x = 3k$

I. $\frac{x}{9} + \frac{y}{6} = \frac{z}{18}$ $\times 18$ → $2x + 3y = z$ → $6k + 3y = z$

$3(2k+y) = z$

II. $\frac{x}{6} + \frac{y}{9} = \frac{z}{18}$ $\times 18$ → $3x + 2y = z$ → $9k + 2y = z$

III. $\frac{x}{6} + \frac{y}{5} = \frac{z}{30}$ $\times 30$ → $5x + 6y = z$ → $15k + 6y = z$

- I only
- II only
- I and II only
- I and III only
- I, II, and III

Multiples

If x and y are positive integers such that y is a multiple of 5 and $3x + 4y = 200$, then x must be a multiple of which of the following?

- ○ 3
- ○ 6
- ○ 7
- ○ 8
- ○ 10

$3x+4y=200$

$y=$ $3(x)+20=200$

Divisibility Rules

A number is divisible by	Rule	Example
2	It's even (i.e., its last digit is even)	1,576
3	Its digits add up to a multiple of 3	8,523 → 8 + 5 + 2 + 3 = 18
4	Its last two digits are divisible by 4	121,532 → 32 ÷ 4 = 8
5	Its last digit is 5 or 0	568,745 , 320
6	Apply the rules of 2 and 3	55,740 → It's even and 5 + 5 + 7 + 4 + 0 = 21
9	Its digits add up to a multiple of 9	235,692 → 2 + 3 + 5 + 6 + 9 + 2 = 27
10	Its last digit is zero	11,130
12	Apply the rules of 3 and 4	3,552 → 3 + 5 + 5 + 2 = 15 and 52 ÷ 4 = 13

$L.C.M(a,b) * H.C.F(a,b) = ab$

Divisibility Rules

If integer x is divisible by 15 but not divisible by 20, then x CANNOT be divisible by which of the following?

- ○ 6
- ○ 10
- ○ 12
- ○ 30
- ○ 150

$15 \rightarrow 3 \,\&\, 5$

$20 \rightarrow \cancel{4} \,\&\, 5$

Divisibility Rules

Will the number be still divisible by 3 if digits are switched ?

How many three-digit integers between 310 and 400, exclusive, are divisible by 3 when the tens digit and the hundreds digit are switched?

- ○ 3
- ○ 19
- ○ 22
- ○ 30
- ○ 90

0 – 100   33

3, 6, 9

10 – 100   30

303 306 309

Section 2

LCM & GCD

LCM

Find the Least Common Multiple of 24 and 36

Conventional Method	Long Division Method	Prime Factorization Method
	2 | 24, 36 2 | 12, 18 3 | 6, 9 2, 3	**Step 1:** Prime factorize 24 & 36 **Step 2:** Select all prime factors with highest powers **Step 3:** Multiply the selected prime factors to get the LCM of 24 and 36.

GCD

Find the Greatest Common Divisor of 24 and 36

Conventional Method	Long Division Method	Prime Factorization Method
	$\begin{array}{r\|c} 2 & 24, 36 \\ \hline 2 & 12, 18 \\ \hline 3 & 6, 9 \\ \hline & 2, 3 \end{array}$	**Step 1:** Prime factorize 24 & 36 **Step 2:** Select only the common prime factors and with the lowest powers **Step 3:** Multiply the selected prime factors to get the GCD of 24 and 36.

LCM & GCD

Recall from the previous examples:

$$24 = 2^3 \times 3^1$$

$$36 = 2^2 \times 3^2$$

$$\mathrm{GCD}(24{,}36) = 2^2 \times 3^1 \text{ and } \mathrm{LCM}(24{,}36) = 2^3 \times 3^2$$

$$\therefore \mathrm{LCM}(24{,}36) \times \mathrm{GCD}(24{,}36) = 2^3 \times 3^2 \times 2^2 \times 3^1 = 24 \times 36$$

$$\mathbf{LCM(a, b) \times GCD(a, b) = a \times b}$$

That is, the product of LCM and GCD of two integers is equal to the product of the integers.

This property ONLY holds true for **two integers**.

LCM & GCD

Mr. Dalton is trying to organize groups for his class's science fair. The class could be divided into 8 groups with an equal number of students in each group or could be divided into 12 groups with an equal number of students in each group. What is the fewest possible number of students in his class?

- ~~12~~
- ~~16~~
- 24
- ~~48~~
- ~~96~~

3 2

12

LCM & GCD

Additional Question-3(MODEL)

(Official One)

What is the greatest number of identical bouquets that can be made out of 21 white and 91 red tulips if no flowers are to be left out? (Two bouquets are identical whenever the number of red tulips in the two bouquets is equal and the number of white tulips in the two bouquets is equal.)

- 3
- 4
- 5
- 6
- 7

13 * 7 ✓

3 * 7 ✓

LCM & GCD

Additional Question-4(DRILL)-2 min

What is the lowest positive integer that is divisible by each of the integers 1 through 7, inclusive?

- ○ 420
- ○ 840
- ○ 1260
- ○ 2540
- ○ 5040

If the positive integer x is a multiple of 4 and the positive integer y is a multiple of 6, then xy must be a multiple of which of the following?

I. 8
II. 12
III. 18

○ II only

○ I and II only

○ I and III only

○ II and III only

○ I, II and III only

worst scenario

$xy = (4m)(6n)$

$xy = 24mn$

$m = n = 1$

$xy = 24$ ✓

Section 3

Remainders

Remainders

What is the remainder when 23 is divided by 5?

We can also represent this result in the following forms.

$$\begin{array}{r|l} & \;\;4 \\ \hline 5 & \;23 \\ & \;20 \\ \hline & \;\;3 \\ \hline \end{array}$$

Form 1:

$$23 = 4 \times 5 + 3$$

$$Dividend = Quotient \times Divisor + Remainder$$

Note that:

$$0 \leq Remainder < Divisor$$

Form 2:

$$\frac{23}{5} = 4 + \frac{3}{5} = 4.6$$

$$\frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor}$$

Note that 0.6 is the decimal representation of $\frac{3}{5}$.

Remainders

For x > y, when positive integer x is divided by positive integer y, the possible remainders can range from 0 to y – 1

Note that when a smaller integer is divided by a larger integer, the quotient will be 0 and the remainder will be the smaller integer.

What is the sum of the remainders of 6 consecutive integers that are each divided by 6 ?

- ○ 0
- ○ 5
- ○ 10
- ○ 15
- ○ 20

y

$0 \le Rem \le y-1$

4 5 6 1

0 1 2 3

$(x)+(x+1)+(x+2)+(x+3)$

$+(x+4)+(x+5)$

~~2~~ ~~3~~

4 5

0, 1, 2, 3, 4, 5

=> 15

When 24 is divided by the positive integer n, the remainder is 4. Which of the following statements about n must be true?

I. n is even

II. n is a multiple of 5

III. n is a factor of 20

○ III only

○ I and II only

○ I and III only

○ II and III only

○ I, II, and III

$R < D$

$\boxed{24 = nq + 4}$

$4 < n$

$nq = 20$

1 20
2 10
4 5
5 4
10 2
20 1

If s and t are positive integers such that $^{s}/_{t}=64.12$, which of the following could be the remainder when s is divided by t?

- 2
- 4
- 8
- 20
- 45

$S = D*Q + R$

$\frac{S}{t} = \frac{D*Q}{t} + \frac{R}{t}$

$= 64 + 0.12$

$\frac{R}{t} = \frac{12}{100}$

$= \frac{3}{25}$

$\frac{R}{D} = \frac{12}{100}$

$N = D*Q + R$

$\frac{N}{D} = \frac{D*Q}{D} + \frac{R}{D}$

$\frac{R}{t} = \frac{\cancel{12}}{\cancel{100}}\ \frac{3}{25}$

$0.12(t) = ?$

$t = \frac{45 \times 100}{\cancel{12}} \quad \frac{1500}{\cancel{4}} \quad 375$

$0.12t = 20$

$t = \frac{\cancel{2000}}{\cancel{12}} = \frac{500}{3}$

= mult of 8 + mult of 8

5. When integer x^3 is divided by 256, the remainder is 0. Which of the following could be the remainder when x is divided by 256 ?

I. 2

II. 16

III. 20

- None
- I only
- II only
- III only
- I, II and III

$8q = 256q + 2$

$x^3 = 256k$

$8q = 256q_1 + 2$

$x^3 = 2^8 k$

$4q_1 = 64q_1 + 1$

$x^3 \Rightarrow 2^9$

$8q = 256q_1 + 20$

$x = 2^3$

$2q = 64q_1 + 5$

$x = 8k$

Section #4

Units Digit Cyclicity

Units Digit Cyclicity

To find the units digit of a product or sum of a series of numbers,
we only need to focus on the product or sum of units' digit of each of the number in the series.
For example,

- Units digit of the product 43×22 ▶ = units digit of $3 \times 2 = 6$.
- Units digit of the product $43 \times 22 \times 79$ ▶ = units digit of $3 \times 2 \times 9$ = units digit of $54 = 4$.

Now,

- units digit of the 7 $= 7$
- units digit of the 7×7 $= 9$
- units digit of the $7 \times 7 \times 7$ $= 3$
- units digit of the $7 \times 7 \times 7 \times 7$ $= 1$
- units digit of the $7 \times 7 \times 7 \times 7 \times 7$ $= 7$
- units digit of the $7 \times 7 \times 7 \times 7 \times 7 \times 7$ $= 9$

Units Digit Cyclicity

The units digit of the product follows a cyclic pattern (7, 9, 3, 1, 7, 9, ...) The cycle repeats after every 4 products, or we can say that the cyclicity of the digit 7 is 4.

$$\text{Units digit of the } 47^{23} = ?$$

Since the cycle of numbers (7, 9, 3, 1) repeats after every four products, 5 such cycles will complete for 47^{20}, and a new cycle will begin for the product 47^{21}.

So, the units digit of 47^{23} will be the third number in the pattern, which is 3.

The same concept can be extended to other digits as well.

Homework Assignment:
Find the cyclicity of digits 1 – 9.

HACK:

You don't have to memorize anything here.

Understand that units digits of the product of a series of same numbers forms a cyclic pattern.

For example, we saw that, for a product of series of numbers ending with 7, the cycle keeps on repeating every 4 products.

You should be able to find such cycles for numbers ending with the other digits as well.

Units Digit Cyclicity

Additional Question-8(APPLY)-2 min

If $n = 33^{43} + 43^{33}$, what is the units digit of n?

- ○ 0
- ○ 2
- ○ 4
- ○ 6
- ○ 8

$\frac{43}{4}$

3 Rem 1

9 Rem 2

7 Rem 3

1 Rem 0

7 + 3

=> 0

Action Items -

Over the course of this week…

1. Revise the lecture recording and make proper notes.

2. Check the 'Homework spreadsheet' and complete the homework.

You'll never change your life until you
change something you do daily –
The secret of your success is found in your
daily routine.
- John C Maxwell

Thank You!

Q & A