

## Comparison of Titan's Equatorial Dunes to Lambertian Titan Surface Simulations\*

GABRIEL M STEWARD,<sup>1</sup> JASON W. BARNES,<sup>1</sup> WILLIAM MILLER,<sup>1</sup> AND SHANNON MACKENZIE<sup>2</sup>

<sup>1</sup>*University of Idaho, Moscow, Idaho 83844*

<sup>2</sup>*Johns Hopkins University Applied Physics Laboratory, Laurel, Maryland 20723*

### ABSTRACT

**NOTE: Red notes are important! Do not submit the document with any of them remaining!**

**NOTE: Blue notes are placeholders! Do not submit the document with any of them remaining!**

The nature of Titan's surface is poorly understood, largely due to the atmosphere's extensive interference. To handle this interference, we simulate lambertian Titan models using the radiative transfer program SRTC++ at different surface albedos. We then compare these models to real data from Cassini VIMS (Visual and Infrared Mapping Spectrometer) of the HLS (Huygens Landing Site) and equatorial dunes. We confirm that SRTC++ produces reasonable phase functions for lambertian surfaces shrouded by Titan's atmosphere in many, but not all, situations. Furthermore, we find that Titan's dunes act as lambertian surfaces, with all identified deviations correlating to suspected SRTC++ or atmospheric model deficiencies. We also perform a targeted search for an opposition effect on Titan's dunes through VIMS observations and find none, though the effect could conceivably be too narrow to observe.

**Keywords:** KEYWORDS (111) — KEYWORDS (112)

### 1. INTRODUCTION

Titan has one of the least understood surfaces in the entire Solar System, due largely to its thick haze-filled atmosphere that is opaque to most light. While there do exist a handful of atmospheric "windows" through which specific wavelengths of light can pass through relatively unimpeded (Barnes et al. 2007), only limited spectral information on the surface can be gleaned through them. Even within the windows, the thick atmosphere contaminates the relatively small amount of surface information we do receive; transmission is never perfect (Es-sayeh et al. 2023).

To combat this, we turn to radiative transfer models of Titan's atmosphere that predict the influence the atmosphere has on the received signal, allowing for true surface effects to be identified. Many such models have been created over the years, each with their own strengths and weaknesses (Vinatier et al. (2007); Griffith et al. (2012); Xu et al. (2013); Barnes et al. (2018); Rodriguez et al. (2018); Corlies et al. (2021); Rannou et al. (2021); and Es-sayeh et al. (2023), to name a few). These radiative transfer models depend on accurate knowledge of Titan's atmosphere, which is most well characterized at the moon's equatorial regions since that is where the Huygens lander measured the atmosphere (Tomasko et al. 2008). Many surface characterization studies

attempting to filter out the influence of the atmosphere have been performed in the past (Buratti et al. 2006; Soderblom et al. 2009; Kazeminejad et al. 2011; Brossier et al. 2018; Es-sayeh et al. 2023; Solomonidou et al. 2024). However, the majority of them make a notable assumption: that the surface behaves as lambertian; a perfect scatterer with no directly reflected components. Buratti et al. (2006) is a notable exception.

We know that few, if any, surfaces in the Solar System are truly lambertian due to the prevalent opposition effect (Déau et al. 2009; Schröder & Keller 2009). We would expect Titan's surface to exhibit similar non-lambertian behaviors, but first-order analysis of the moon as a whole in the IR (infrared) shows a lambertian surface. (Le Mouélic et al. 2019, 2012). On the other hand, the one observation we have from the ground on Titan, the Huygens lander, did report an opposition effect (Schröder & Keller 2009; Karkoschka et al. 2012); however Huygens landed in a "dark blue" region (Rodriguez et al. 2006), which is an uncommon terrain on Titan's surface (Keller et al. 2008). For Titan as a whole, we have Radar observations that show an opposition effect (Neish et al. 2010; Wye 2011), but given the drastically different length scales probed between radar and IR, we have no reason to believe they would be similar.

In this paper, we seek to demonstrate the degree to which Titan's equatorial dunes exhibit non-lambertian behavior in IR. To that end, we compare lambertian simulations of Ti-

\* Sep, 22, 2025

68 tan with real observations of the dunes, identifying notable  
69 differences and deviations.

70 The vast majority of observations of Titan's surface have  
71 been done by spacecraft visiting Saturn, with the most high-  
72 quality data coming from the Cassini mission. As such,  
73 many images of Titan's surface are taken at unusual view-  
74 ing geometries. This is quite useful, as it allows charac-  
75 terization of Titan's surface from a wide variety of ori-  
76 entations, making it far easier to determine exactly how non-  
77 lambertian a terrain is. Unfortunately, most current radi-  
78 ative transfer models applicable to Titan either assume a plane  
79 parallel atmosphere in their calculations (Griffith et al. 2012;  
80 Es-sayeh et al. 2023), don't consider angle at all (Rannou  
81 et al. 2021), or use a spherical approximation (Corlies et al.  
82 2021). Thus, all these lose accuracy the further the view-  
83 ing geometry is from direct illumination, and would miss  
84 potential non-lambertian effects. To gain the useful infor-  
85 mation contained within observations at non-ideal viewing  
86 geometries, the spherical nature of Titan's atmosphere must  
87 be considered. Thus, to create our lambertian models, we  
88 used SRTC++ (Spherical Radiative Transfer in C++), a ra-  
89 diative transfer code tailored to model Titan in full spherical  
90 geometry at the infrared wavelengths available to Cassini's  
91 VIMS (Visual and Infrared Mapping Spectrometer) instru-  
92 ment (Barnes et al. 2018). Other spherical models do exist  
93 (Xu et al. 2013), but SRTC++ was chosen due to familiarity  
94 with the code.

95 Equipped with a spherical radiative transfer model and at-  
96 mospheric characterization from Huygens, it became possi-  
97 ble to compare models with reality on a scale covering the  
98 entire Cassini mission. As the equatorial regions are the best  
99 characterized atmospherically, we chose to examine the ter-  
100 rain there. While we examined multiple terrain types, it was  
101 determined that only the equatorial dunes had a sufficient  
102 number of reliable observations to draw meaningful conclu-  
103 sions from. We also hand-picked observations from the Huy-  
104 gens Landing Site (HLS) for the sake of comparison, even  
105 though that location was only viewed at a limited number of  
106 viewing angles. For both the dunes and HLS we compiled  
107 VIMS observations from across the entire Cassini mission  
108 over all available viewing geometries, limiting observations  
109 to only those judged to be of sufficient quality.

110 This analysis serves dual purposes—to qualitatively vali-  
111 date the SRTC++ simulation against real data, and to iden-  
112 tify deviations from lambertian behavior on Titan's surface.  
113 To accomplish this, first we outline improvements made to  
114 SRTC++ in Section 2 and report on those the results of those  
115 changes in Section 3. We describe the procedure by which  
116 we gathered our Titan data in Section 4, compare reality to  
117 simulation in Section 5 for both the HLS and dunes, and per-  
118 form a specific search for the opposition effect in Section 6.  
119 Lastly, we conclude in Section 7.

## 120 2. MODEL METHODS

121 **METHODS: Jason's Section.** Brief summary of  
122 SRTC++, citing the previous paper for more details. De-  
123 scribe new SRTC++ modules used, notably Absorption  
124 and the switch to Doose atmosphere. There should be  
125 a comparison figure to note the differences between the  
126 two. Mention the new integration method.

127 **Figs:** comparison between SRTC++ versions.  
128 Will be done by Jason

## 129 3. MODEL RESULTS

130 For our work, we used SRTC++ to create three different  
131 models of a uniformly lambertian Titan, each run with a dif-  
132 ferent surface albedo: 0.0, 0.1, and 0.2. Besides the albedo,  
133 the simulations were set up with identical parameters, as  
134 shown in Figure 1. The virtual detectors were set 10,000 km  
135 away from the lambertian Titan, separated by 5 degrees and  
136 completely encircling the moon. Output was generated at all  
137 eight VIMS IR wavelength windows for Titan's atmosphere:  
138 0.93, 1.08, 1.27, 1.59, 2.01, 2.69, 2.79, and 5.00  $\mu\text{m}$ .

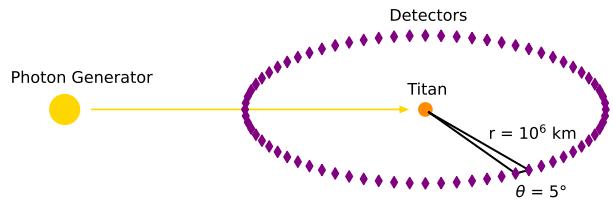
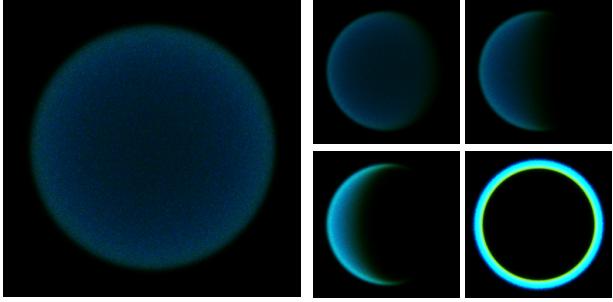


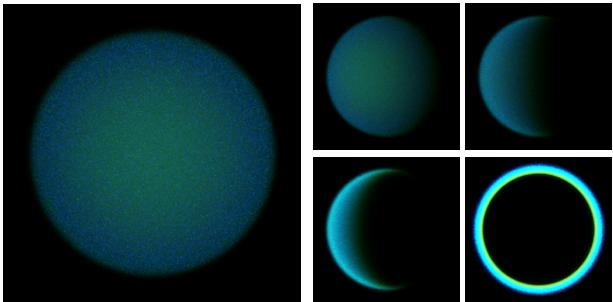
Figure 1. Layout of our SRTC++ simulations. Distances not to scale. Detectors are all equidistant from Titan and angular separation is the same for each one. The yellow arrow represents "photon packets" being shot at Titan. Note that it does not interact with the detector it passes through.

139 Figures 2-4 show the results of the simulations at 0.0, 0.1,  
140 and 0.2 albedo, respectively, colored with 5.00  $\mu\text{m}$  as red,  
141 2.01  $\mu\text{m}$  as green, and 1.27  $\mu\text{m}$  as blue, as done in Barnes  
142 et al. (2005). The results of 0.1 albedo are closest to the  
143 "greenish" pictures of Titan in Barnes et al. (2005), though  
144 naturally without any terrain variation.

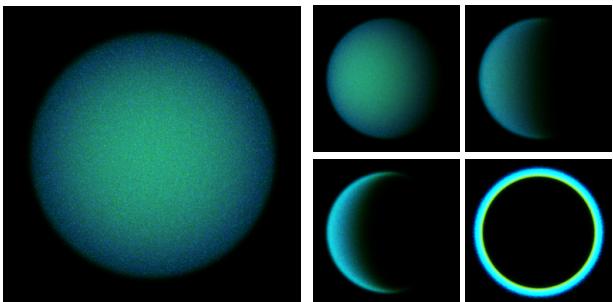
145 The simulations are largely as expected for a lambertian  
146 sphere obscured by a thick atmosphere. We can see what  
147 a lambertian sphere should be without an atmosphere from  
148 Figure 9 in Pont & Koenderink (2007), and the 0.0 albedo  
149 simulation shows us the effect of the atmosphere without any  
150 signal coming from the surface. Both effects should be ac-  
151 tive in the 0.1 and 0.2 albedo simulations, and that is what we  
152 see in the directly illuminated view: a greenish center where  
153 there's minimal atmospheric influence that slowly fades off



**Figure 2.** Simulation results for a lambertian Titan with 0.0 albedo, colored with 5, 2, and 1.3  $\mu\text{m}$  mapped to red, green, and blue, respectively. Left image is viewed at  $0^\circ$  from the incidence angle. Right four images are, clockwise from the top left, at  $35^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $120^\circ$ , respectively. **Animating Version:** [A0.0LambertSimMOVIE.gif](#)



**Figure 3.** Same as Figure 2 but for 0.1 Albedo. **Animating Version:** [A0.1LambertSimMOVIE.gif](#)



**Figure 4.** Same as Figure 2 but for 0.2 Albedo. **Animating Version:** [A0.2LambertSimMOVIE.gif](#)

as the observation angle becomes steeper, at which point the atmosphere takes over with its bluish hue. Note that the coloration is entirely due to atmospheric effects; bluer light is scattered in the atmosphere more readily than redder light. In the raw data, the blue channel is the largest even in the center at 0.1 albedo. It is merely the choice of color balancing that gives Titan its greenish hue; a sensible choice as it allows us to readily identify places with more surface signal than atmospheric. The center of the disc, when viewed from

head-on, is rather uniform with little variation, as it should be.

Views other than direct illumination also behave as expected; the edges are always atmospherically dominated, becoming bluer. Light is more likely to be forward scattered than backscattered in Titan's atmosphere (García Muñoz et al. 2017; Cooper et al. 2025), so the closer the Sun gets to behind Titan, the brighter the edges become. A lambertian surface without an atmosphere would have no light coming from anywhere that was not directly illuminated, but we see a small amount of light released from beyond the terminator due to atmospheric scattering. We can also see the shadow Titan casts on its own atmosphere in the profile views as the signal gets abruptly diminished near the top and bottom of the moon, but the atmosphere above this shadow remains illuminated. This effect is known to happen on Earth as well and is what causes the colors of twilight (Lynch & Livingston 2004).

Notably, the “eclipse” view at 180 degrees is functionally identical in all simulations, which it should be as the surface has little to no influence on the signal at this viewing geometry. There is most certainly interesting science to be done with the simulation at this angle to probe the atmosphere, but that is beyond the scope of this paper.

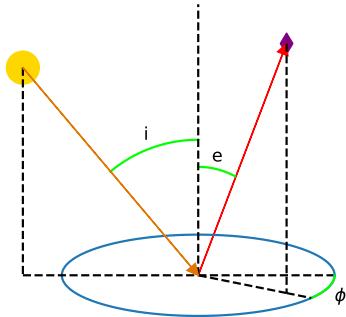
The simulation results notably appear slightly grainy. This is due to the Monte-Carlo nature of SRTC++. If we were to run the simulation for longer, the S/N (signal to noise) would increase, and the overall image result would become smoother. However, given that we end up averaging numerous pixels together for this work, such effort would be wasted.

We ultimately seek to compare these simulations with real data. And while we can qualitatively see that real Titan images do have similar coloration and limb effects (Barnes et al. 2005), the fact that real Titan has numerous different terrain types interferes with any more robust conclusions. As such, we turned these visual models into viewing geometry models. These new models can, given a specific viewing geometry, predict how a lambertian Titan would appear at that geometry, which can then be compared to real data. This effectively makes them bidirectional reflectance distribution functions (BRDFs).

To construct the viewing geometry models, every model pixel was sorted into bins based on viewing geometry, and then their values were averaged. The bins were separated by  $5^\circ$  increments in incidence, emission, and azimuth angle, defined as shown in Figure 5, with  $0^\circ$  azimuth being the forward scattering direction, and  $180^\circ$  being the backscattering. Notably, our models have only three angles, while more traditional BRDFs have four. We do not care about topographic changes in a surface, as such information is very context dependent, so we can treat the sun as always coming in from the

same direction in these models, and thus that angle does not need to be set. As such, we will call these models restricted BRDFs from here on out. This choice also makes it far easier to determine when backscattering or forward scattering is occurring, key indicators of non-lambertian behavior.

Viewing Geometry Model: Incidence 40 Emission 23 Azimuth 34



**Figure 5.** Viewing Geometry Angles used in this paper. Incidence angle is represented by “*i*”, emission angle by “*e*”, and azimuth angle by “ $\phi$ ”. The arrows trace out a path from the sun (yellow circle), to some arbitrary spot on Titan’s surface, to a detector (purple diamond). Diagrams modeled after this one appear in future figures, to give context for the data.

With 3 different albedos and 8 wavelengths, we produced a total of 24 restricted BRDFs ready to be compared with other restricted BRDFs compiled from real VIMS data.

#### 4. OBSERVATIONS AND DATA

Cassini performed over a hundred separate flybys of Titan during its mission (Seal & Bittner 2017), and most of those flybys have observations from Cassini VIMS. Viewing geometries on any single flyby are generally limited in scope, as the spacecraft itself could only examine geometries it personally encountered. Thus, in order to gain a proper understanding of the surface of Titan at a wide range of viewing geometries, observations from as many flybys as possible should be used.

The primary obstacle in properly using all the data is the sheer amount in play; over a hundred flybys, tens of thousands of individual observations, and in each of those hundreds of spectrals, each with hundreds more individual values associated with them. If we wished to make a single global model, this would not be an issue, as an algorithm could easily ingest everything. However, simple inspection of VIMS images reveals that different terrains have different albedos, and our simulations in Figures 2-4 clearly show how much the appearance of the surface changes with albedo. Averaging all terrain types together could smooth out non-

lambertian behavior, or allow a single non-lambertian terrain type to contaminate all the others.

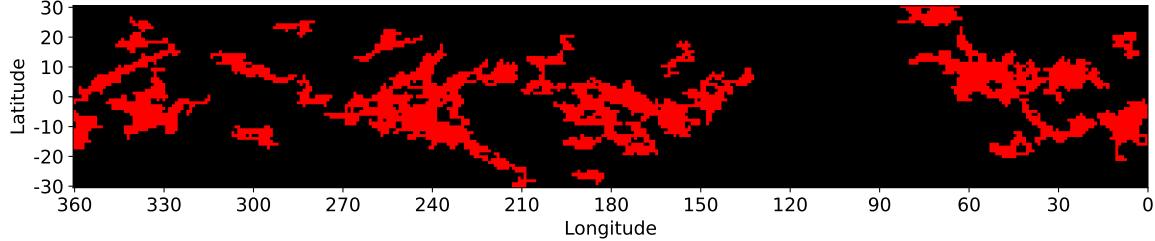
As such, we need to limit our data to single terrain types. We also chose to focus on the equatorial regions between 30° and -30° latitude since that’s where Huygens sampled the atmosphere (Tomasko et al. 2008), making this region the best characterized. We also want terrain types that are viewed from a wide variety of viewing geometries with a large number of reliable observations. We judged that only Titan’s dunes met all these criteria. (The equatorial bright terrain was considered as well, but the results we obtained from it were inconsistent and unreliable. See Appendix A for more information.) In pursuit of this goal, we created a raster mask of Titan’s equatorial dunes in Figure 6.

The creation of the mask began with the Titan terrain map created by Lopes et al. (2020) using radar data. VIMS observations, which are taken in infrared, often don’t match the radar observations (Soderblom et al. 2007), but tend to agree in the bulk of major features; most notably (and importantly for this paper), the dunes have good agreement on a global scale.

The resolution in question for the mask is one pixel per degree on Titan’s surface; 181 in latitude and 360 in longitude. The radar map was scaled down to this resolution. Any pixel that was not clearly or nearly a solid color was replaced with a “Null” pixel; one where we would not harvest data from when using the mask to identify terrain. We erred on the side of caution, more likely to assign “Null” to a pixel than not. Any pixels of different terrains that were adjacent were marked “Null” as well to avoid contamination. After this, we manually removed some areas that notably did not match VIMS data, were too small to be of use, or were known to have different spectral characteristics than other terrains given the same classification. Hotei Regio, Tui Regio, and Southern Xanadu were notable manual exclusions. The original mask had many different terrain types, but we reduced all non-dune terrains to “Null” for this paper.

In addition to the terrain classification in Figure 6, the mask also has a version with a hidden data point: each pixel’s distance to the nearest “Null” pixel in km along Titan’s surface. This allows the mask to be refined: pixels that are close to “Null” pixels can be excluded as likely to have contamination from pointing errors in the VIMS data, which are known to occur (Barnes et al. 2008).

VIMS observations come in the form of files called cubes. Our investigation procedure for these files began with a basic database search; in our specific case, we looked for any cubes that had spectrals in the equatorial regions between 30 and -30° latitude, and also had spectrals of 25 km ground resolution or lower. The database we were using had already filtered out clearly erroneous files, as well as so-called “noodle” images which are only a handful of spectrals in diameter. The cubes



**Figure 6.** Titan equatorial surface terrain mask for dunes, as informed by Lopes et al. (2020) mixed with VIMS observations. Black areas are “Null” pixels as outlined in the text, while the red are dunes. Almost all of the dunes on Titan are included in this mask, though there are a few regions outside the  $30^{\circ}$  and  $-30^{\circ}$  latitude boundaries.

296 themselves were calibrated with the standard VIMS pipeline  
297 (Barnes et al. 2007).

298 Once the cubes were identified, they were ingested and  
299 each individual spectel examined to see if it was satisfactory  
300 according to the provided restrictions; namely, the latitude  
301 and ground resolution limitations. If a spectel passed the test,  
302 it was added to a list. This list could be made with or without  
303 referring to the mask. When the mask was used, only  
304 spectels marked as dunes were cataloged. The spectels were  
305 then judged based on their proximity to a “Null” pixel on the  
306 mask. Two options existed for this: setting a minimum dis-  
307 tance from a “Null” pixel that would be accepted, or setting  
308 an allowed maximum ratio of ground resolution to “Null”  
309 pixel distance. For this work, we set a minimum “Null” dis-  
310 tance of 50 km and a maximum ratio of ground resolution to  
311 “Null” pixel distance of 1/4.

312 When the mask was not used, “Null” values could be ac-  
313 cepted. This was helpful when wanting to look at areas in  
314 or near “Null” pixels, which was the case for the Huygens  
315 Landing Site.

316 We then binned the list of spectels and created a restricted  
317 BRDF out of them in a process identical to that described in  
318 Section 3. Unlike other investigations, we did not subtract off  
319 atmospheric contributions, as our simulations include atmo-  
320 spheric contributions. This saved us considerable processing  
321 difficulties.

322 There are a few limitations to the created restricted BRDFs.  
323 The primary limitation is that certain viewing angles, usu-  
324 ally at the extreme ends of allowed values, do not exist since  
325 Cassini was never in those positions. There was also no  
326 check for interfering clouds during the creation process.

327 Particularly high-resolution cubes can reveal small details  
328 not visible in most views and thus are not reflected in the  
329 mask, such as a handful of observations that can see inter-  
330 dune areas (Barnes et al. 2008). These small details need not  
331 match the behavior of the terrain they are surrounded by, and  
332 could conceivably offset the final model. While the dunes  
333 themselves are known to be relatively homogeneous, the in-  
334 terdune areas vary considerably across Titan (Bonnefoy et al.  
335 2016). Fortunately, the restricted BRDF created for the dunes  
336 exhibited remarkable consistency and order, despite the in-

337 terdunes’ presence. We are unsure precisely why this is—it  
338 could be that a single kind of interdune dominates most of  
339 Titan’s dunes, and all others are insignificant, making the re-  
340 sult orderly. Alternatively, the various interdune types could  
341 simply be drowned out by the sheer amount of data averag-  
342 ing occurring. This relatively smooth result is somewhat re-  
343 markable as the equatorial bright terrain’s restricted BRDF is  
344 decidedly disorderly, despite not having a known equivalent  
345 of interdune interference in place (see Appendix A).

346 Any retrieved albedos from this model will be biased  
347 higher than pure dune sands, given the brighter interdunes  
348 (Bonnefoy et al. 2016). However, as we are examining the  
349 dunes terrain type as a whole, we do not believe this is a  
350 detriment to any of our results.

351 In addition to the dunes, we also made a phase function  
352 for the Huygens Landing Site, as our atmosphere model ultи-  
353 mately draws from observations made by the Huygens lander  
354 (Tomasko et al. 2008). We performed a database search sim-  
355 ilarly to how we made the other models, but we also went in  
356 manually, cleaning up any situations where Cassini reported  
357 the wrong latitude and longitude for the spectels, ensuring  
358 that the data was devoid of any contamination; something we  
359 only had to do since the Huygens Landing Site was such a  
360 small area on the boundary of multiple terrain types. Unfor-  
361 tunately, there were relatively few viewing angles of the HLS  
362 despite it being among the most observed locations on Titan,  
363 which is why examining the dunes as a whole was necessary.

364 In order to counteract the lack of data for both the dunes  
365 and the HLS, we performed tetrahedral interpolation to fill  
366 in as many observation angles as possible in the restricted  
367 BRDFs using the PyVista package (Sullivan & Kaszynski  
368 2019).

## 369 5. MODEL VS DATA COMPARISON

### 370 5.1. Huygens Landing Site

371 We choose to examine the HLS first as its data are simpler,  
372 though it covers significantly fewer viewing geometries than  
373 the dunes. As three-dimensional data are hard to visualize,  
374 we opt to plot individual “skewers” of data at a time, fixing  
375 two of the three viewing geometry angles while allowing the  
376 third to vary. We do this for all restricted BRDFs, be they

377 models or real data. We plot select skewers of the three mod-  
 378 els and HLS data in Figures 7-9.

379 Ignoring the HLS for a moment, the restricted BRDFs  
 380 showcase a distinct shifting of behavior across the various  
 381 wavelengths. The shorter wavelengths have rougher lines,  
 382 while the longer ones are smoother. This is due to the general  
 383 trend of the atmosphere having higher optical depth at shorter  
 384 wavelengths (Es-sayeh et al. 2023), allowing for more scat-  
 385 tering events in more directions, which in turn means more  
 386 simulation time would be required to smooth out all those  
 387 directions.

388 We see characteristic behavioral differences in restricted  
 389 BRDF behavior at the different wavelengths in Figure 7 and  
 390 Figure 8, with the general shape of emission skewers in  
 391 particular changing drastically between 0.93 and 5.00  $\mu\text{m}$ .  
 392 When skewering across incidence or emission, this is com-  
 393 mon; though when skewering across azimuth, the general  
 394 shape is usually similar in all wavelengths, including the  
 395 skewer shown in Figure 9. Keep in mind that across these  
 396 three figures, only a small amount of the total models are  
 397 shown; behavior can change significantly when skewering  
 398 at different locations, though the transitions are always rela-  
 399 tively smooth. These particular skewers were chosen because  
 400 they are the places with the best HLS data visualizations.

401 The HLS data are very well behaved for the most part, in-  
 402 terpolating smoothly and forming lines that correspond rather  
 403 well to the models. In 1.27 and 2.01  $\mu\text{m}$ , the data lines up al-  
 404 most perfectly with the 0.1 albedo prediction, while the oth-  
 405 ers sit somewhere between 0.0 and 0.1 albedo. 1.08  $\mu\text{m}$  hugs  
 406 the 0.0 albedo line, and of all the wavelengths, it shows the  
 407 largest variation, most notable in Figure 7 here, but the vari-  
 408 ation is also visible in other views not shown. We are unsure  
 409 why 1.08  $\mu\text{m}$  in particular does this; we expect 0.93 and 1.08  
 410  $\mu\text{m}$  data to be variable due to the large atmospheric contribu-  
 411 tions, but not for them to be very different from each other.  
 412 Perhaps this is a feature of the HLS itself?

413 Even considering 1.08  $\mu\text{m}$ 's quirks, the HLS appears con-  
 414 sistent with a lambertian surface in all the viewing geome-  
 415 tries we have access to, which admittedly is not very many.  
 416 We chose the best skewers we could for Figures 7-9, and even  
 417 the azimuth skewer only covers about a third of the available  
 418 values. Nothing probes the higher incidence and emission  
 419 angles, and the extremes aren't even approached. This tells  
 420 us nothing about SRTC++'s reliability beyond angles that are  
 421 already probed well by other radiative transfer models. Thus,  
 422 we move to a much larger data set: the dunes.

## 423 5.2. Equatorial Dunes

424 Similarly to the HLS data, we plot a variety of select skew-  
 425 ers for the dunes. We plot more than three as the dunes data  
 426 cover a far wider range of viewing geometries, focusing on  
 427 incidence skewers for Figures 10-12.

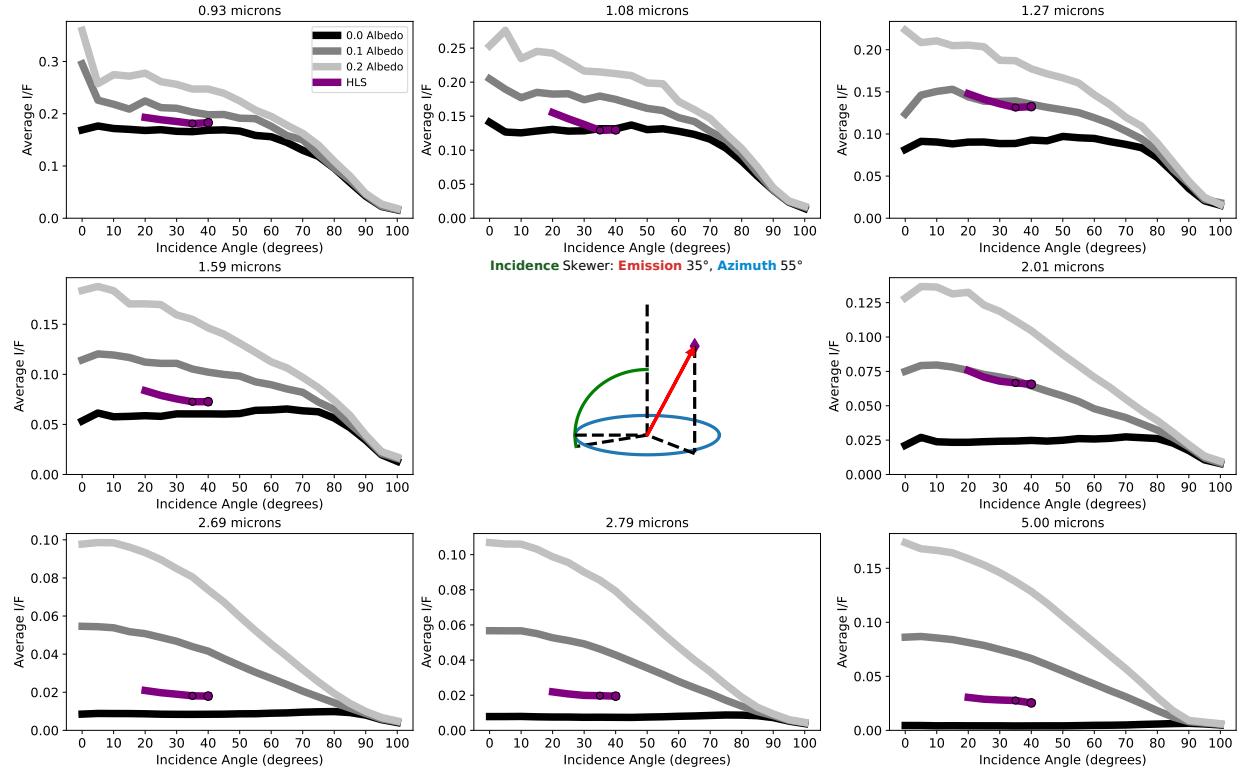
428 We find that most incidence skewers share a similar shape,  
 429 visible in both Figure 10 and Figure 11. Higher incidence  
 430 angles have lower I/F across the board, the exception being  
 431 skewers with high emission and low azimuth, but the dunes  
 432 have no data at those viewing geometries, so we ignore them.  
 433 The dunes data match the shape of the models very well in  
 434 these views, though incidences higher than 80° should be  
 435 taken with a grain of salt, as there is very little data at those  
 436 extreme angles. That said, the points that do exist behave as  
 437 expected in these views.

438 Curiously, the 2  $\mu\text{m}$  has a very noticeable shift in retrieved  
 439 albedo for the dunes between Figure 10 and Figure 11, going  
 440 from hugging the 0.1 albedo model to hanging somewhere  
 441 around 0.05 albedo. 1.27  $\mu\text{m}$  may do this as well, though it  
 442 is harder to tell as the gap between 0.1 and 0.0 albedo models  
 443 is smaller in that window and the data are less consistent.  
 444 We will return to this effect when examining the emission  
 445 skewers, as the effect is far more noticeable there.

446 Most incidence skewers not shown look like Figure 10 and  
 447 Figure 11 in the phase function models, except for those  
 448 where there is no data. The dunes data usually match the  
 449 models in shape rather well, indicating lambertian behavior  
 450 when incidence is considered, with minor and momentary  
 451 deviations for the most part. However, there are exceptions,  
 452 which happen at high emission and high azimuth, exempli-  
 453 fied by Figure 12. Here, the 2.01, 2.69, and 2.79  $\mu\text{m}$  windows  
 454 do not have slopes that match the models. Unfortunately,  
 455 there isn't a very large spread of points at this view, so this  
 456 anomaly is hard to draw conclusions from. We will return  
 457 to this when examining the emission skewers, as they shed  
 458 some light on the anomaly.

459 Figure 12 also makes clear a physical impossibility that  
 460 sometimes crops up in incidence slices: while the 0.93 and  
 461 1.08  $\mu\text{m}$  windows have dune data shapes that match the mod-  
 462 els', the retrieved albedo is below 0.0, an impossibility. This  
 463 may be because SRTC++ does not account for Rayleigh scat-  
 464 tering, which is most relevant at short wavelengths (Es-sayeh  
 465 et al. 2023). Intuitively, one would expect adding Rayleigh  
 466 scattering to make the model brighter, not dimmer, but sub-  
 467 tle effects could be at play—to fully answer this question, we  
 468 would have to implement Rayleigh scattering, which is on  
 469 the list for future improvements to SRTC++.

470 The emission skewers for the phase function models gen-  
 471 erally show gradual increases in brightness with larger emis-  
 472 sion, though 5  $\mu\text{m}$  is a notable exception, along with low in-  
 473 cidence and high azimuth views. Selected views can be seen  
 474 in Figures 13-15. Figure 13 and Figure 14 clearly show that  
 475 for low emission angles, up to around 40°, the data match  
 476 the models pretty well. However, at higher emission angles,  
 477 this fails: Figure 13 shows the 0.93 - 1.59  $\mu\text{m}$  windows dip-  
 478 ping below the 0.0 albedo model at higher emission, while  
 479 2.01 - 2.79  $\mu\text{m}$  do the opposite and sharply tick above the



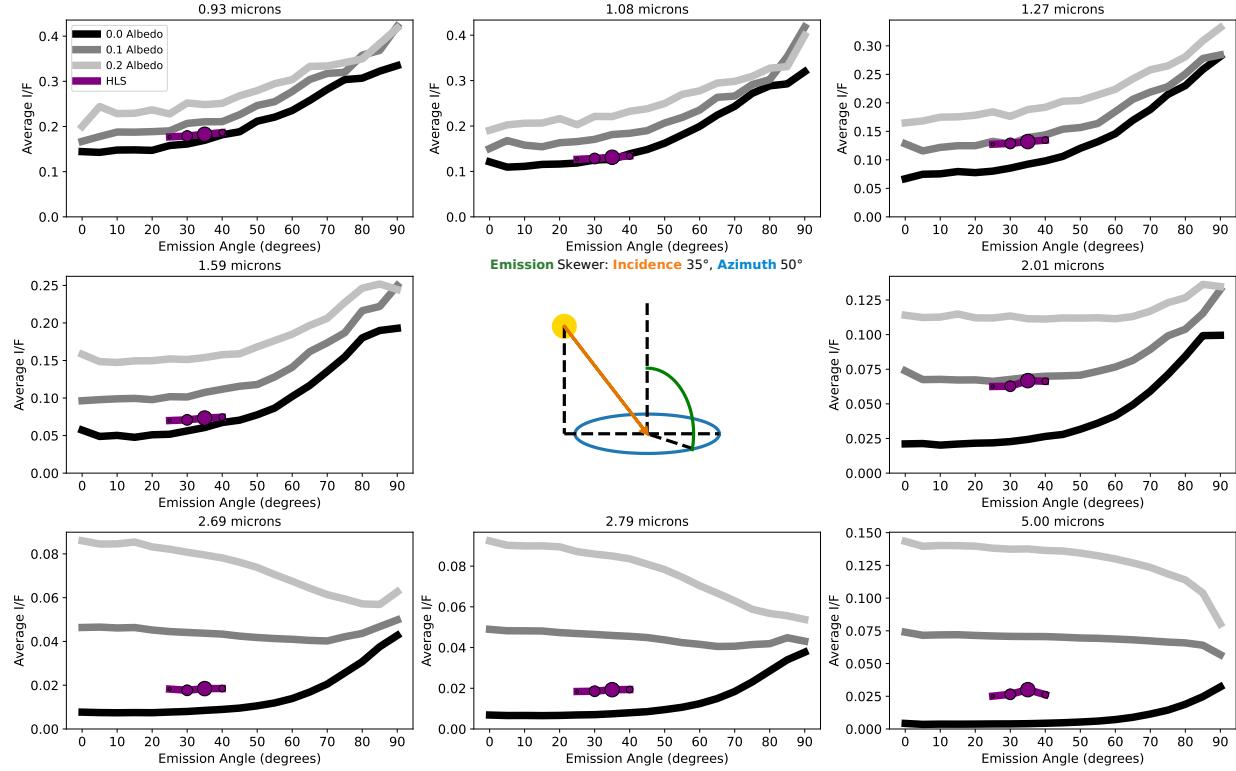
**Figure 7.** Incidence angle skewer: incidence angle versus average I/F comparing phase function models with HLS data with fixed emission and azimuth angles. Showcases all eight wavelengths arranged from shortest to longest, with the central area occupied by a geometry diagram to illustrate the exact situation plotted. Model lines are monochromatic, HLS data are purple. Places with direct HLS observations have dots plotted over the lines, larger dots meaning more observations. Places on the HLS lines without dots are interpolated values. Note that the vertical axis scales with the data; not all wavelengths produce the same average I/F scale.

models. The dramatic brightening of  $2.01 - 2.79 \mu\text{m}$  is just as strong, if not stronger, in Figure 14, showing azimuthal independence of this effect. However, the  $0.93 - 1.59 \mu\text{m}$  dimming below the 0.0 albedo model is not as strong, and doesn't clearly exist at the  $1.27 \mu\text{m}$  and  $1.59 \mu\text{m}$  windows. The dimming effect vanishes entirely in Figure 15, while the brightening effect remains. (Notably, Figure 15 has few data points at high emission, but the fact that it follows the pattern laid out in other nearby skewers gives us confidence that its behavior is not just an interpolation artifact). These brightening and darkening effects explain the inconsistent retrieved albedo and values below 0.0 albedo noted in the incidence skewers; these deviations are emission dependent. Furthermore, the fact that brightening only occurs in  $2.01 - 2.79 \mu\text{m}$  windows and that these are the only windows seen not matching the models in Figure 12 lends further credence that Figure 12's deviations are not just a bad series of observations.

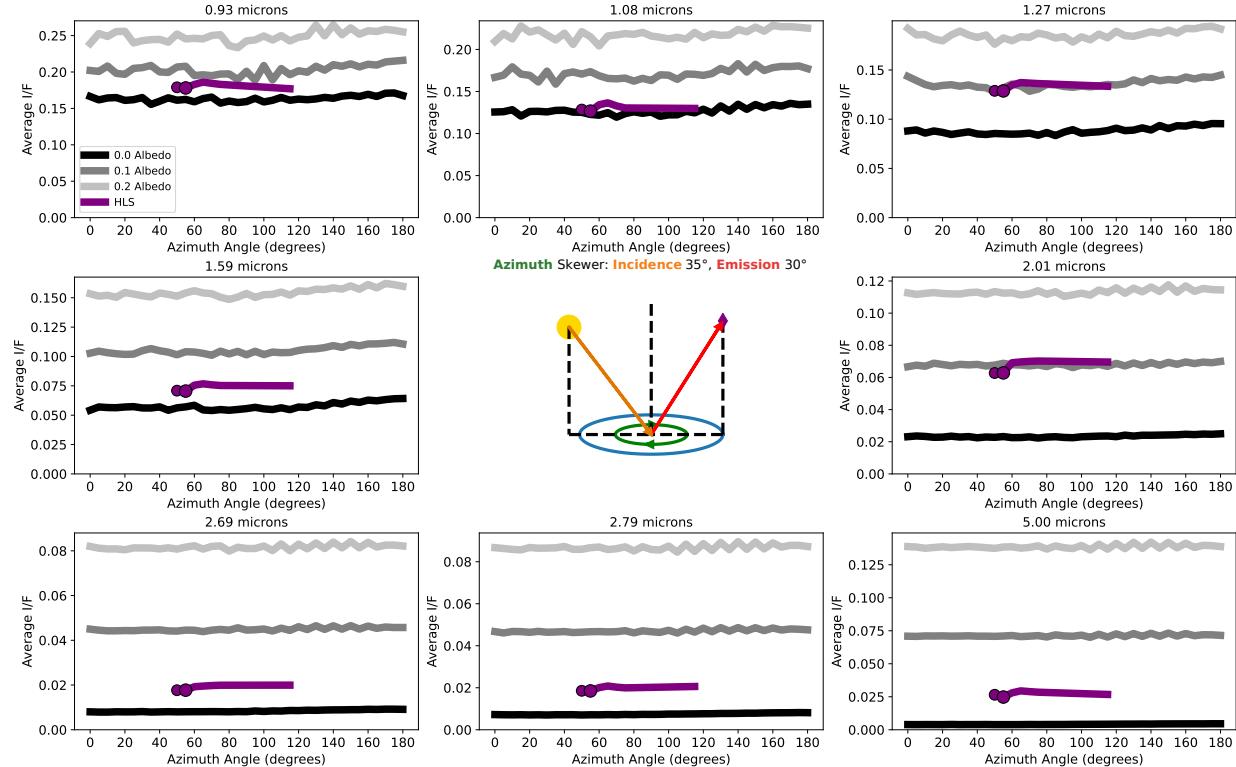
So, how can we explain these discrepancies? There are two primary explanations. First, the model does not accurately account for some atmospheric effect at high emission. While we mentioned Rayleigh Scattering earlier, that will only have an important effect on short wavelengths (Es-sayeh et al. 2023). We suspect the haze model is where the prob-

lem lies, as at higher emission angles, light escaping from the surface has to pass through a lot of atmospheric haze; if our model thinks the haze is thicker than it is in reality, more light will be let through unimpeded as emission rises. It is true that higher incidence angles (up to a point) will also result in a longer path through the haze; however, this will be counteracted by the fact that any spot on the surface of Titan is going to be diffusely illuminated due to light scattering from other directions. Whether this error lies in the atmospheric model or the way the code treats it is unknown at this juncture.

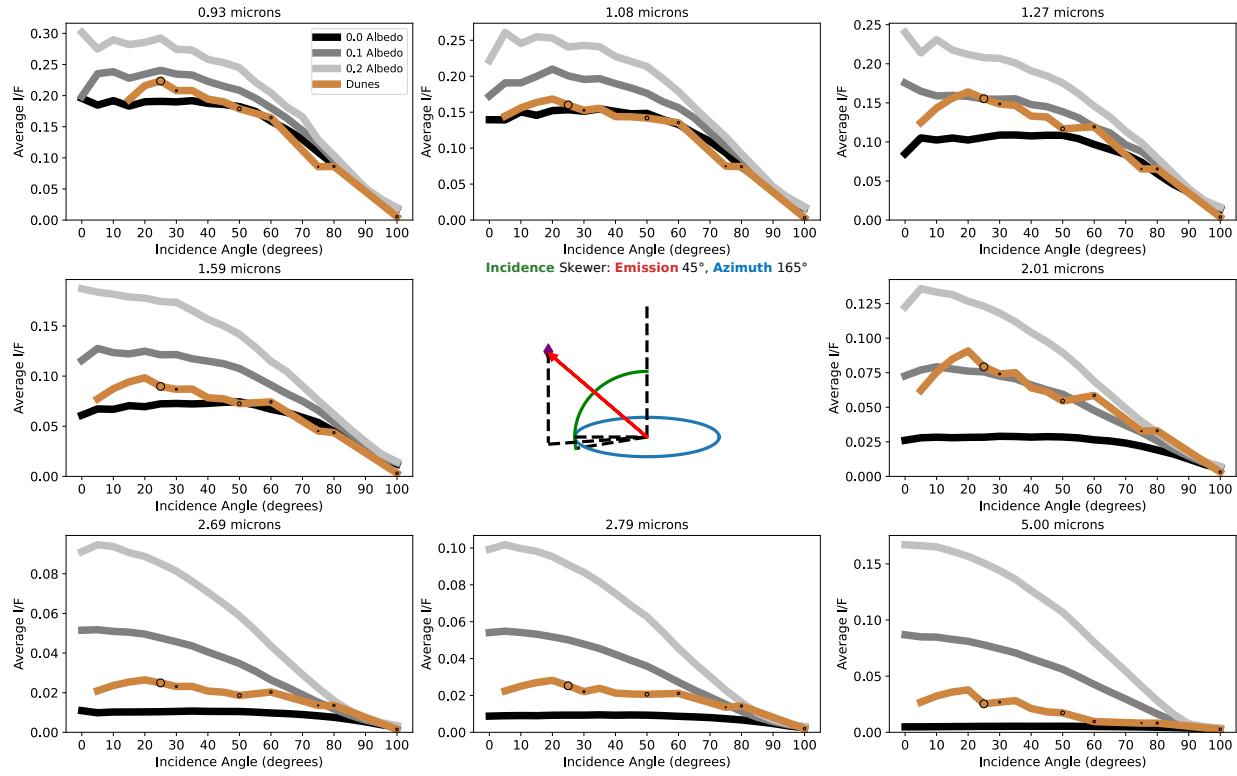
The second explanation is that we're seeing a true non-lambertian effect from the surface. However, this seems unlikely, as the brightening effect appears azimuthally agnostic: no matter what azimuth we point at, the effect exists where we have enough data to see it, and a forward-scattering or backscattering effect would be particularly focused at  $0^\circ$  or  $180^\circ$ , not present everywhere. Topography oriented explanations also seem unlikely, since the effect begins around  $40^\circ$  and only extreme emission angles should be greatly influenced by topography. Not to mention the fact that Titan is rather smooth, topographically speaking, with only a couple km elevation variation on the surface (Corlies et al. 2017).



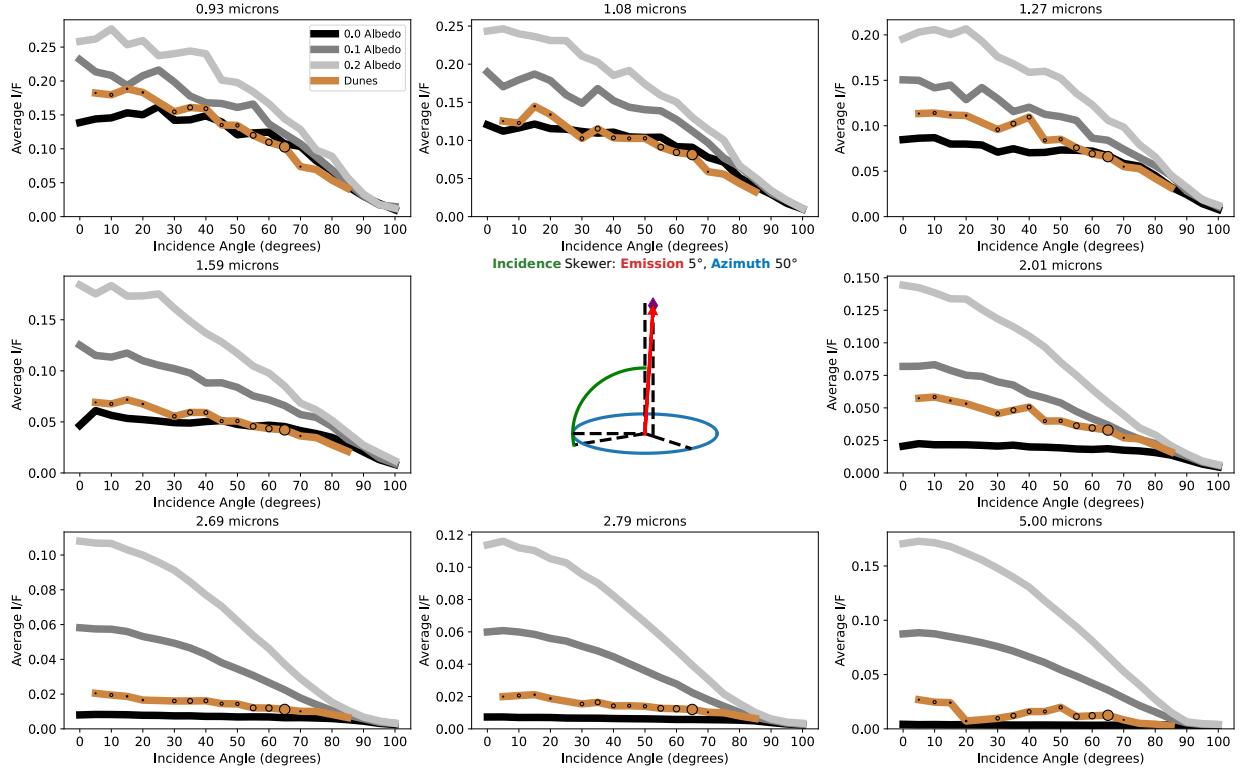
**Figure 8.** Same as Figure 7 but is instead a skewer through the emission angle, with incidence and azimuth held fixed.



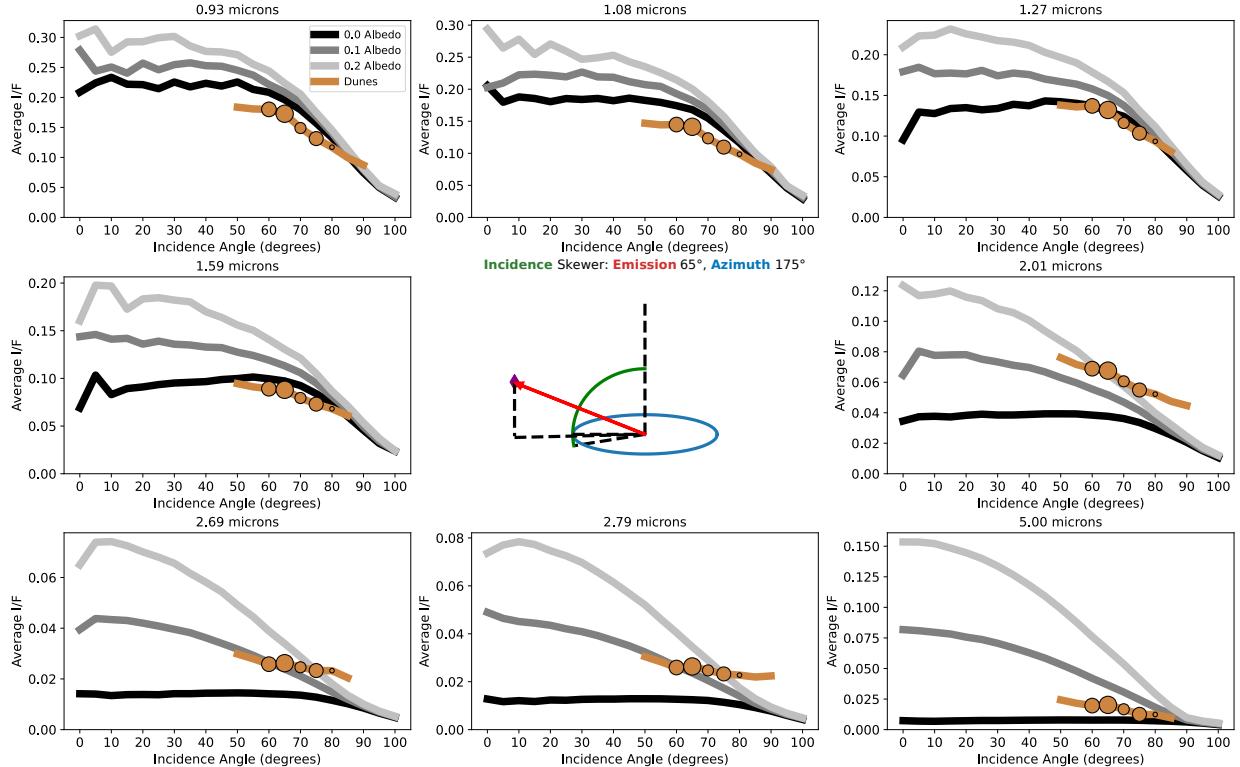
**Figure 9.** Same as Figure 7 but is instead a skewer through the azimuth angle, with incidence and emission held fixed.



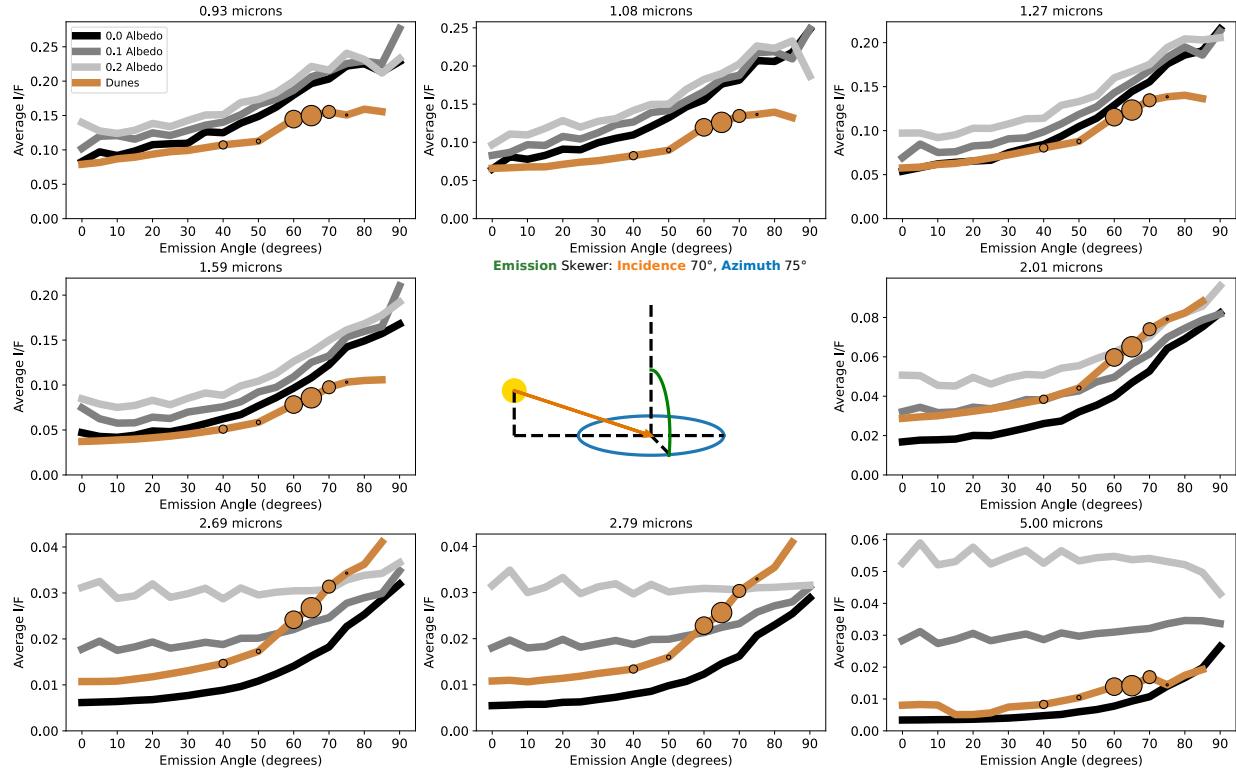
**Figure 10.** Incidence angle skewer: incidence angle versus average I/F comparing phase function models with dunes data with fixed emission and azimuth angles. Showcases all eight wavelengths arranged from shortest to longest, with the central area occupied by a geometry diagram to illustrate the exact situation plotted. Model lines are monochromatic, dunes data are brown. Places with direct dunes observations have dots plotted over the lines, larger dots meaning more observations. Places on the dunes lines without dots are interpolated values. Note that the vertical axis scales with the data; not all wavelengths produce the same average I/F scale.



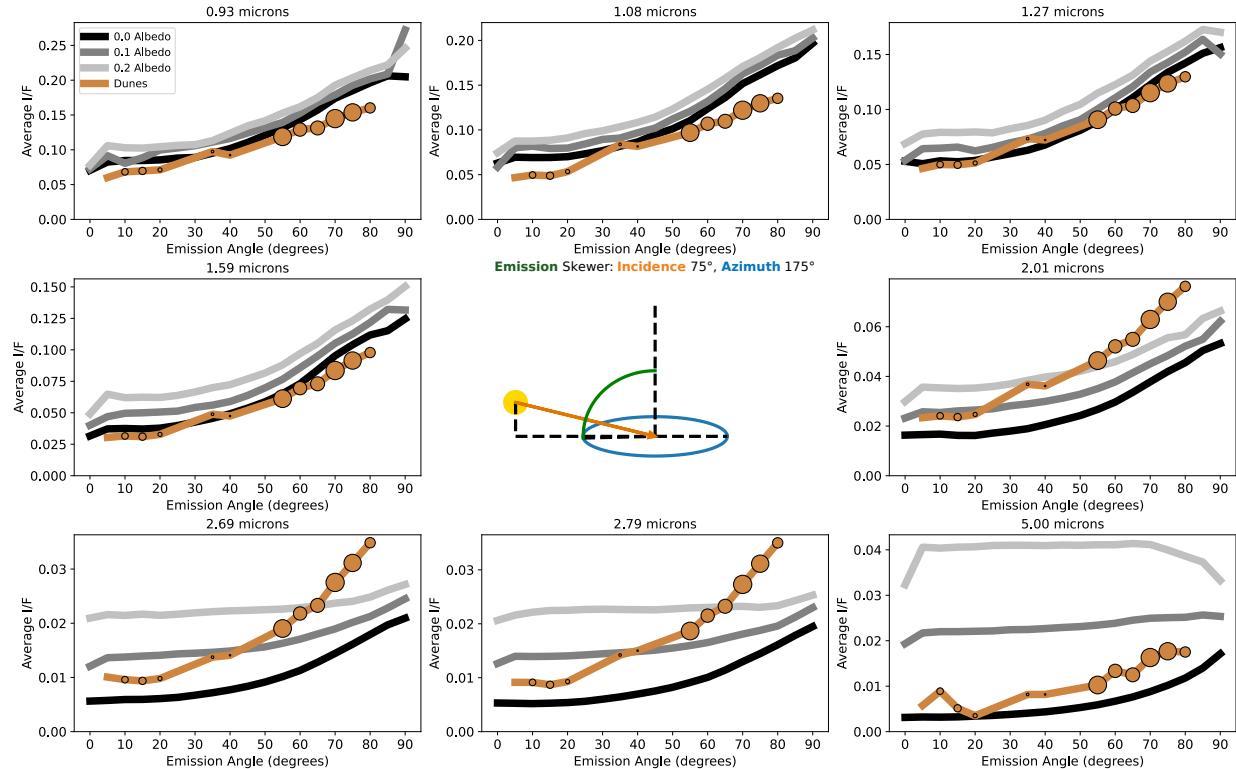
**Figure 11.** Same as Figure 10 but at different fixed emission and azimuth angles.



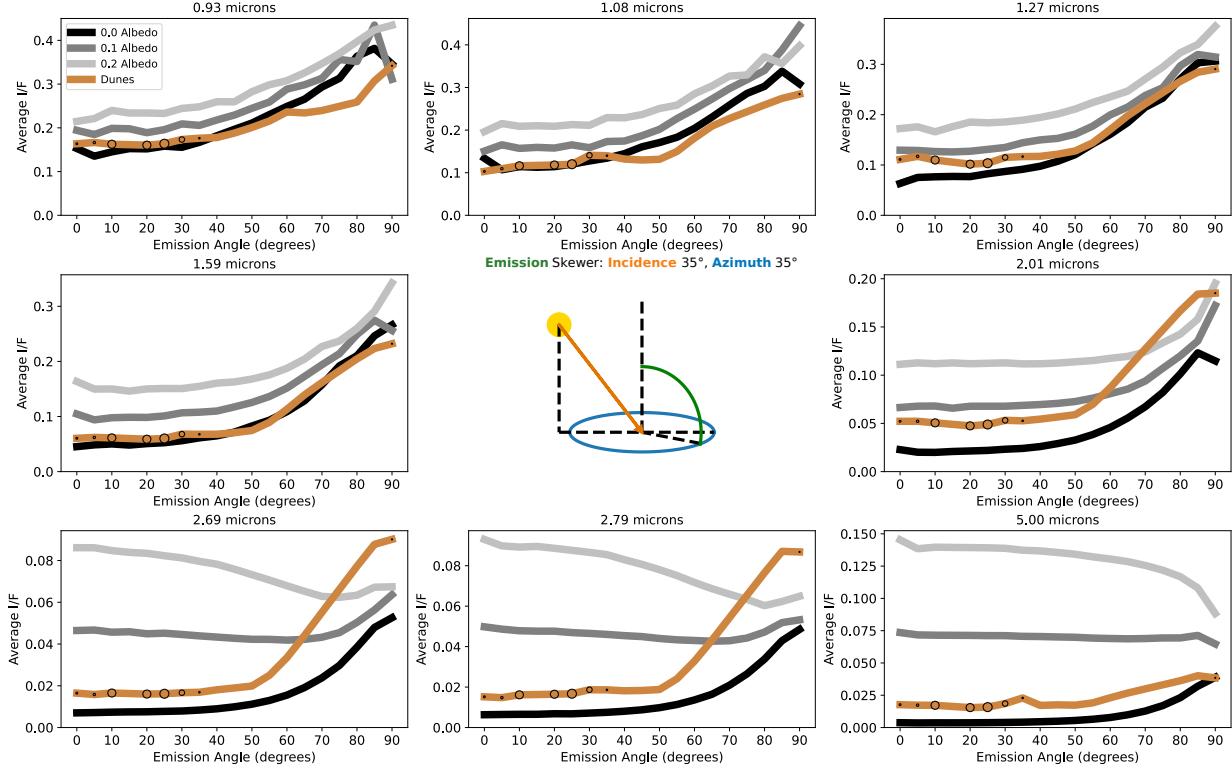
**Figure 12.** Same as Figure 10 but at different fixed emission and azimuth angles.



**Figure 13.** Same as Figure 11 but is instead a skewer through the emission angle, with incidence and azimuth held fixed.



**Figure 14.** Same as Figure 13 but at different fixed incidence and azimuth angles.



**Figure 15.** Same as Figure 13 but at different fixed incidence and azimuth angles.

That said, if we eventually rule out an error in the simulation, we would be forced to consider a surface effect, which would be investigated in future work by testing out various non-lambertian BRDFs for the surface in the SRTC++ models and observing if any of them led to azimuthally agnostic effects.

Despite this clear deviation at high emission, the overall model and data still match remarkably well. Nowhere is this easier shown than the azimuth skewers, which, unlike the incidence and emission skewers, regularly have data in large numbers and data points covering the entire range. In most situations, the result is flat, such as in Figure 16 and Figure 17.

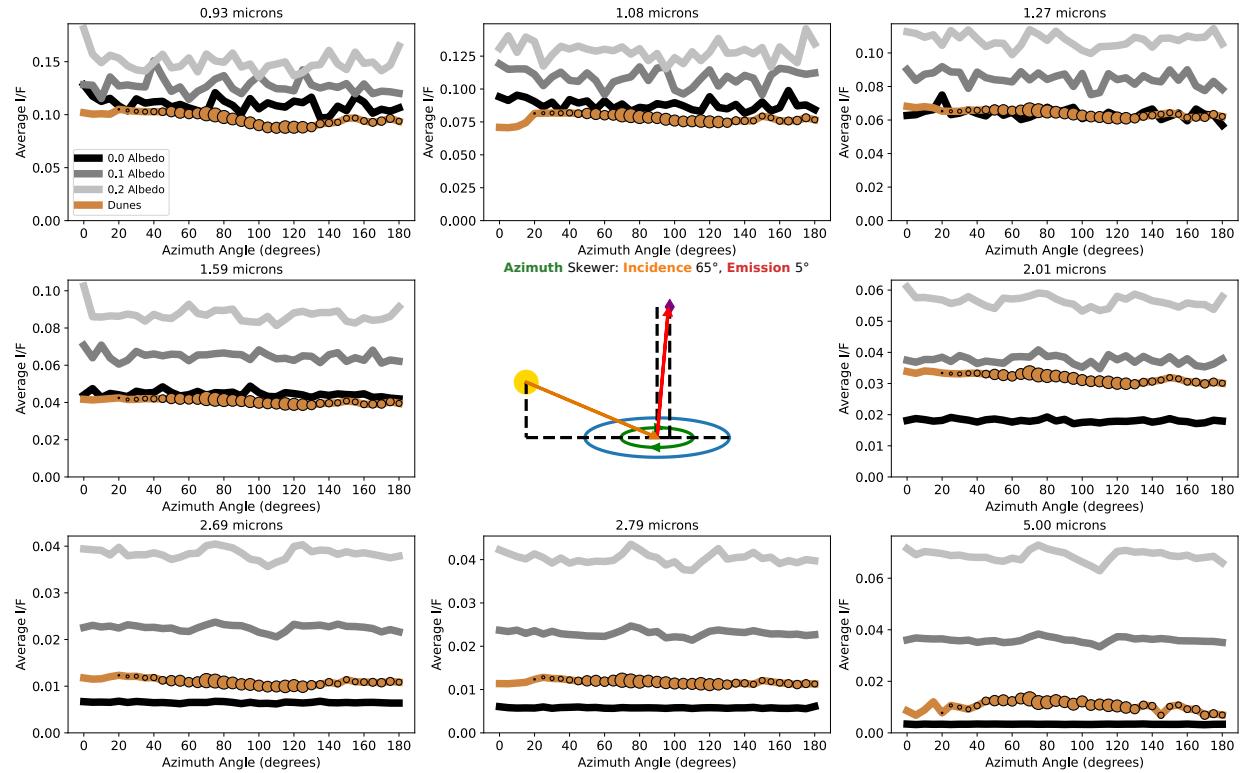
Figure 16 importantly demonstrates in its short wavelength windows that we can still have points below 0.0 albedo even at low emission angles, not just high ones, indicating that there are probably at least two effects influencing this result. However, dark as the data are, they are all flat and uniform, as the models indicate it should be. Most azimuth views are completely flat in both model and real data.

The exception to the flatness is when both incidence and emission are high at the same time, at which point the models predict low azimuth should be brighter than the other azimuths, as can be seen in Figure 18. This represents atmospheric backscatter. Unfortunately, we have very few observations in this region of the phase function, and the interpolation is rather suspect as there aren't other regions with

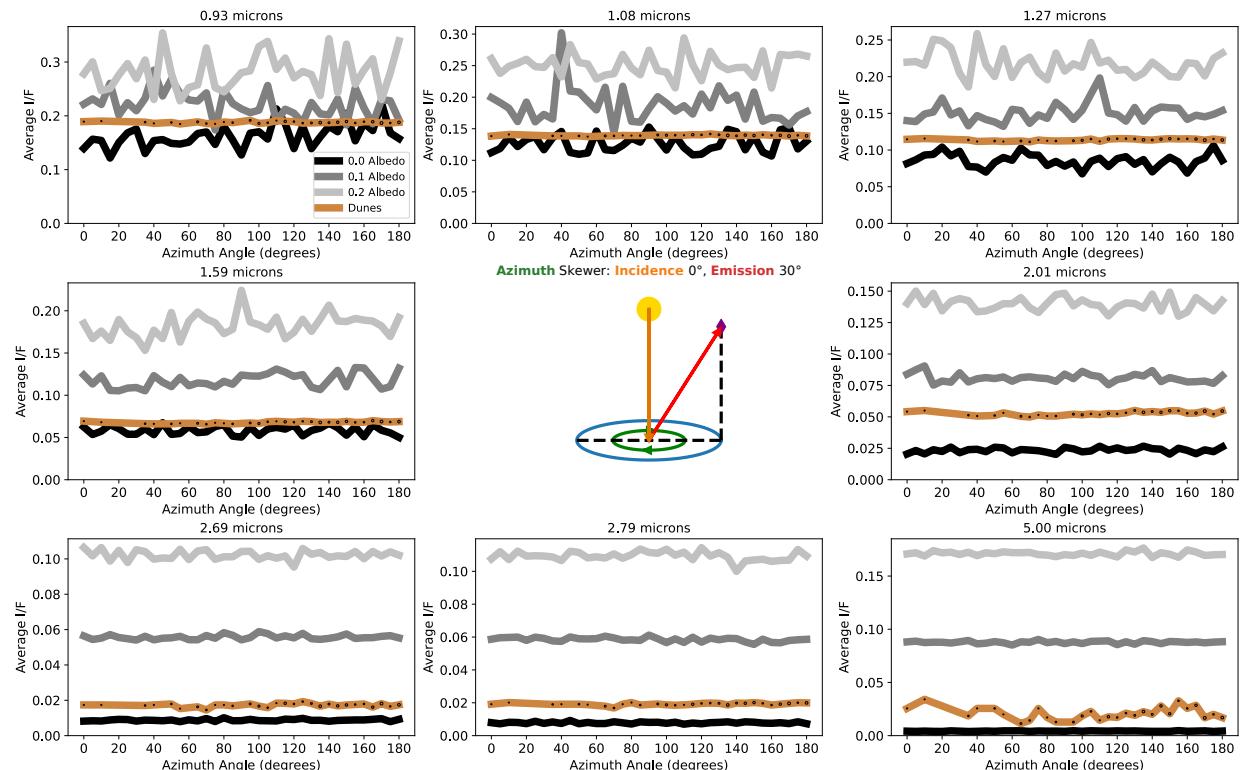
similar behavior as was the case with the emission skewers. Figure 18's interpolation still shows an uptick at low azimuth despite this, which at least suggests the behavior is plausibly accurate. One saving grace is that at such angles, the effects of the atmosphere take over and almost drown out the surface effects, so the dunes themselves are unlikely to have much effect on real observations in the first place. Note that in Figure 18 the different albedo values are all very closely clustered together and nearly identical in shape, corroborating this thought. Even though the  $5\ \mu\text{m}$  window seems to be spread out over albedo, this is merely due to the fact that all the models report very dim I/F values, differing only by 0.05 overall.

In the end, where does this leave us? With the shapes of the dunes data matching the phase function models so closely in incidence and azimuth, it seems reasonable to conclude that Titan acts as a lambertian surface. The primary evidence for non-lambertian behavior comes from the emission skewers, which do not appear to have an azimuthal dependence and is likely an atmospheric effect not accounted for in the model, rather than a true non-lambertian phase function. The other minor deviation in Figure 12 correlates directly with the windows brightening in emission skewers, tying the effects together, lowering our expectations of a non-lambertian effect even further.

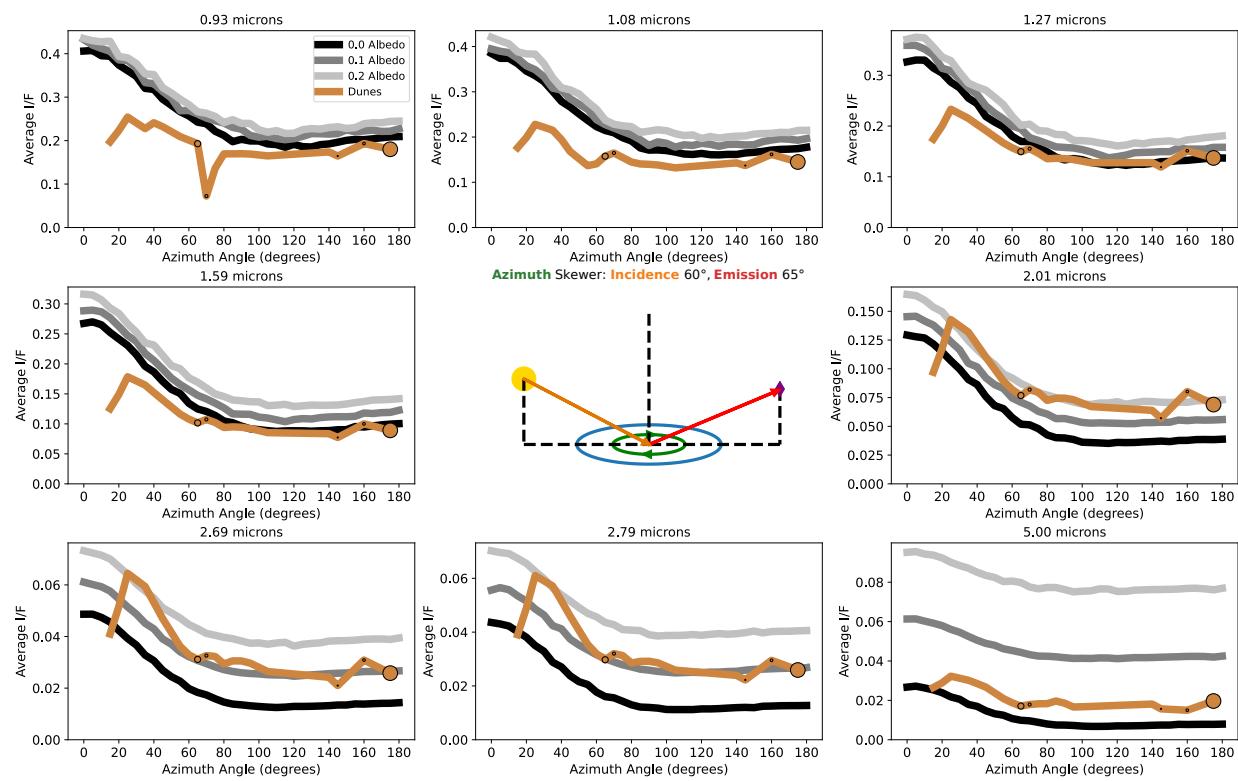
However, we recognize that our method of binning pixels together and averaging them all makes us significantly less



**Figure 16.** Same as Figure 11 but is instead a skewer through the azimuth angle, with incidence and emission held fixed.



**Figure 17.** Same as 16 but at different fixed incidence and emission angles.



**Figure 18.** Same as 16 but at different fixed incidence and emission angles.

580 sensitive to dramatic changes over tiny angular extents, such  
 581 as the sharp central peak often seen in the opposition effect,  
 582 sometimes less than one degree away from opposition (Kulyk  
 583 2008; Schaefer et al. 2008). It is worthwhile to perform a  
 584 more focused check for such an effect.

## 585 6. OPPOSITION EFFECT SEARCH

586 We actively looked for opposition effects in the dune data  
 587 discussed above and found none—these would occur at places  
 588 where incidence and emission were close to each other, and  
 589 azimuth was at or near  $180^\circ$ . We unfortunately had very little  
 590 data exactly at these points, but if the opposition effect was  
 591 somewhat broad on Titan, we would have expected to still see  
 592 some "humps" in the data that would not have been replicated  
 593 in the models. This was not the case. Forward scattering is  
 594 not a component of the opposition effect, though it would  
 595 be similarly placed in the phase functions, just at  $0^\circ$  azimuth  
 596 instead. We also do not observe it in our skewers.

597 However, we would not have noticed an extremely sharp  
 598 opposition effect, as it could conceivably only matter at an-  
 599 gles extremely close to direct opposition; that is, near  $0^\circ$  inci-  
 600 dence,  $0^\circ$  emission,  $180^\circ$  azimuth. Our dunes data only has  
 601 a handful of points around this region, and it is at the very  
 602 border of the phase function models, so looking at skewers  
 603 is rather unhelpful. Instead, we went back to the original  
 604 VIMS cube files and looked for ones of the dunes that had  
 605 viewing geometries within  $1^\circ$  of opposition. There was pre-  
 606 cisely one cube in our data set that met this criterion: cube  
 607 1574127168\_1 from flyby T37. We then took the data from  
 608 this cube and plotted its I/F versus the sun-to-spacecraft an-  
 609 gle, which is a measure of how close each pixel was to oppo-  
 610 sition. The result is Figure 19.

611 While there does appear to be a spike in Figure 19, it hap-  
 612 pens at around  $3^\circ$  and vanishes as we approach actual oppo-  
 613 sition. We can identify this feature with a slightly brighter  
 614 section of the dunes in the lower left of the cube, near the  
 615 central non-dune strip. We must consider the possibility that  
 616 this could be a pointing error; Cassini is known to be off by a  
 617 degree at times (Barnes et al. 2008). Due to the stretching of  
 618 the cube, the brighter section is within one geographical de-  
 619 gree of the pixels labeled closest to opposition. We examined  
 620 other VIMS views of the nearby geography from different  
 621 flybys in Figure 20 and found that the dunes consistently get  
 622 slightly brighter in that direction (north), regardless of view-  
 623 ing geometry, so this is most likely a persistent feature and  
 624 not a very narrow opposition effect.

625 Curiously, there appears to be a slight uptick in bright-  
 626 ness toward direct opposition in the three shortest wavelength  
 627 windows, though this is within the average intensity of other  
 628 dunes pixels, so it does not constitute any opposition effect.  
 629 Furthermore, we must contend with the fact that the opposi-  
 630 tion effect is all but removed by diffuse lighting and indirect

631 viewing angles from scattering (Schröder & Keller 2008),  
 632 which certainly occurs at the lower wavelength windows,  
 633 making it so we would not expect to see any kind of signal  
 634 here. That said, this is not true at other windows.  $5.00\ \mu\text{m}$   
 635 in particular sees significant unimpeded sunlight even when  
 636 it is near sunset (Barnes et al. 2018). If there were an opposi-  
 637 tion effect being hidden by the atmosphere, we would expect  
 638 to see it in  $5.00\ \mu\text{m}$ , but we do not. Figure 19's  $5.00\ \mu\text{m}$   
 639 window has some of the least evidence of a spike out of all  
 640 windows.

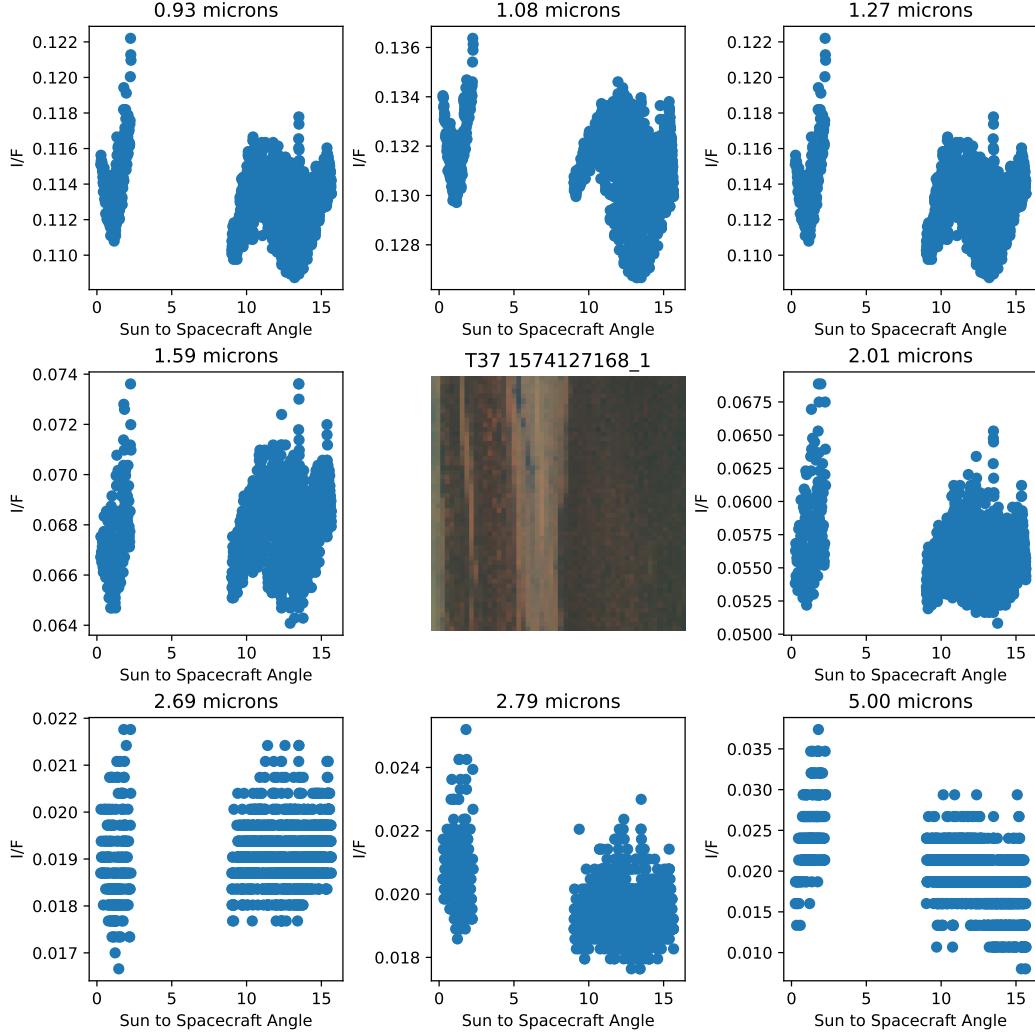
641 With the spike at  $3^\circ$  explained, we find the increase in  
 642 brightness with proximity to opposition appears vaguely lin-  
 643 ear, which is expected (Kulyk 2008), though in a few win-  
 644 dows the spread of intensities is such that no brightening  
 645 trend can be discerned. With this, we conclude our search,  
 646 declaring that we observe no opposition effect whatsoever in  
 647 the dunes. However, we cannot say for certain that there is no  
 648 opposition effect—this is a single observation, and we have no  
 649 way of knowing if VIMS pointing was in error. The closest  
 650 point to true opposition is reported as  $0.25^\circ$  degrees away;  
 651 assuming this is correct, even it does not entirely rule out a  
 652 narrow and sharp opposition effect, as these can be confined  
 653 to within  $0.1^\circ$  (Kulyk 2008; Schaefer et al. 2008). Huygens  
 654 observed the opposition effect spike to begin around  $0.2^\circ$   
 655 (Karkoschka et al. 2012), so we have no reason to suspect  
 656 our observations are sufficient to detect it. The observation  
 657 angle simply does not exist for the dunes; we can only say  
 658 that if the dunes have an opposition effect, it is a narrow one  
 659 with little-to-no broad component.

## 660 7. SUMMARY AND CONCLUSION

661 There were two primary purposes to this paper: to validate  
 662 the SRTC++ simulation against real data, and to identify how  
 663 Lambertian Titan's dunes were.

664 On the validation front, SRTC++ in general gave phase  
 665 function trends that matched the real data, but clearly didn't  
 666 always get the correct albedo, as evidenced by situations  
 667 where the recovered albedo is below 0.0, an impossibility.  
 668 As these moments primarily occur at short wavelengths, this  
 669 could be the influence of Rayleigh Scattering. Alternatively,  
 670 or perhaps additionally, its source could be a mistake in the  
 671 characterization of atmospheric haze—the deviations along  
 672 emission skewers imply there's certainly something missing  
 673 there.

674 Despite these caveats, SRTC++ still produces smooth lines  
 675 that stand alongside the real data in most incidence and az-  
 676 imuth skewers, only with inconsistent albedo results. As  
 677 such, the simulations are still useful for characterizing Ti-  
 678 tan's surface, particularly looking for clear deviations from  
 679 the lambertian ideal. It is telling, then, that we found almost  
 680 none. There is no evidence of an opposition effect and no ev-  
 681 idence of a forward scattering component. The only poten-



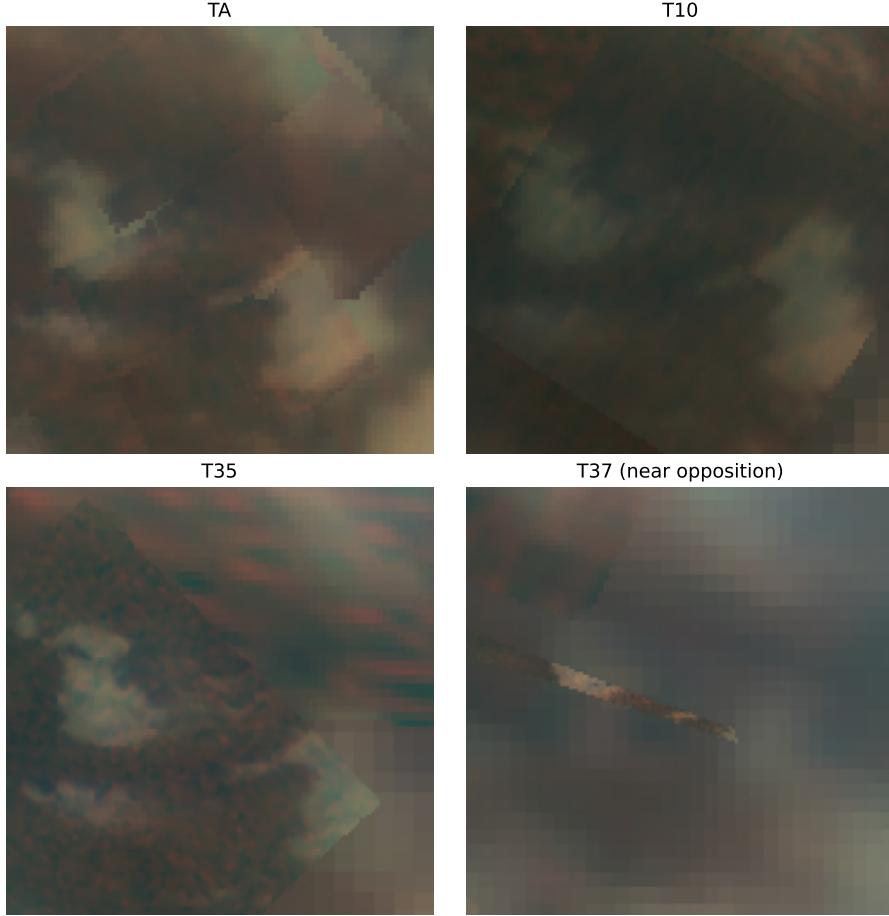
**Figure 19.** Cube 1574127168\_1 from flyby T37 with dunes pixels separated out and plotted by opposition proximity and I/F in all eight windows. The cube itself is plotted in the center with a color scheme of red  $5.00\ \mu\text{m}$ , green  $2.01\ \mu\text{m}$ , and blue  $1.27\ \mu\text{m}$ . Note that the image is significantly stretched;; the actual surface of Titan covered by this image is significant and greatly extended in the horizontal direction. The gap in the middle of the points exists due to those pixels not being dune pixels. Note that most of the points follow a generally linear trend, with the exception of a single spike at around  $3^\circ$  from opposition. We can visually see some somewhat brighter dunes pixels in the central image in the upper right, which no doubt cause this spike. The physical location of the pixels closest to true opposition is in the upper left region.

tial deviation is the curious dimming and brightening some windows exhibit at high emission angles—but as this effect is azimuthally agnostic, it is most likely not a surface effect.

Which leads us to say Titan’s dunes are lambertian surfaces, or very close to it. This is not an unexpected result, for Earth’s sand is also generally lambertian (Hapke & van Hoen 1963), though Earth’s sand exhibits an opposition effect within two degrees of ideal (Wise & Mars 2022) which would have been detectable were this the case on Titan. However, it is thought that the width of an opposition effect narrows the further an object is from the sun (Karkoschka et al. 2012), so we cannot say that we should have seen an opposition effect on the dunes. Furthermore, Earth’s sand and Titan’s dunes are characteristically different, as most of Earth’s

deserts are quite bright, while Titan’s dunes are the darkest solid terrain on its surface.

Future work would include improving SRTC++ to emulate the atmosphere properly at high emission—or, if that were to prove impossible, investigating what other effects might cause the inaccuracies at high emission. If we cannot track down a specific issue with SRTC++ through this, we will investigate unusual surface BRDFs and see if any of them can produce an azimuthally agnostic effect when seen through the atmosphere. This work could also be turned to the specular lakes at Titan’s north pole; while the atmosphere is not as well characterized there, we can use the relatively simple lake phase functions to probe where specifically it’s poorly characterized. Other terrain types on Titan could also be



**Figure 20.** Cylindrical projections of Titan near the opposition observation in flybys TA-, T10, and T35, with the opposition flyby itself (T37) there for comparison. Cylindrical projections made in a manner similar to Barnes et al. (2009), but from singular flybys only. While in T37 itself the observations are not of high enough quality to see, the others show that the dunes get slightly brighter to the north, consistent with a shift in dune properties, not an opposition effect.

examined—it would just require a manual rather than automated approach, as the other terrains are not as distinct as the dunes. Preliminary investigations on this front can be found in the appendix.

#### 714 Acknowledgements:

715 We wish to acknowledge the pyVIMS code (Seignovert  
 716 et al. 2023), even though we did not end up using it in our  
 717 final analysis; it was extremely helpful for preliminary inves-  
 718 tigations and early code.

719 The BRDFs created for this research are available by re-  
 720 quest in the form of numpy arrays.

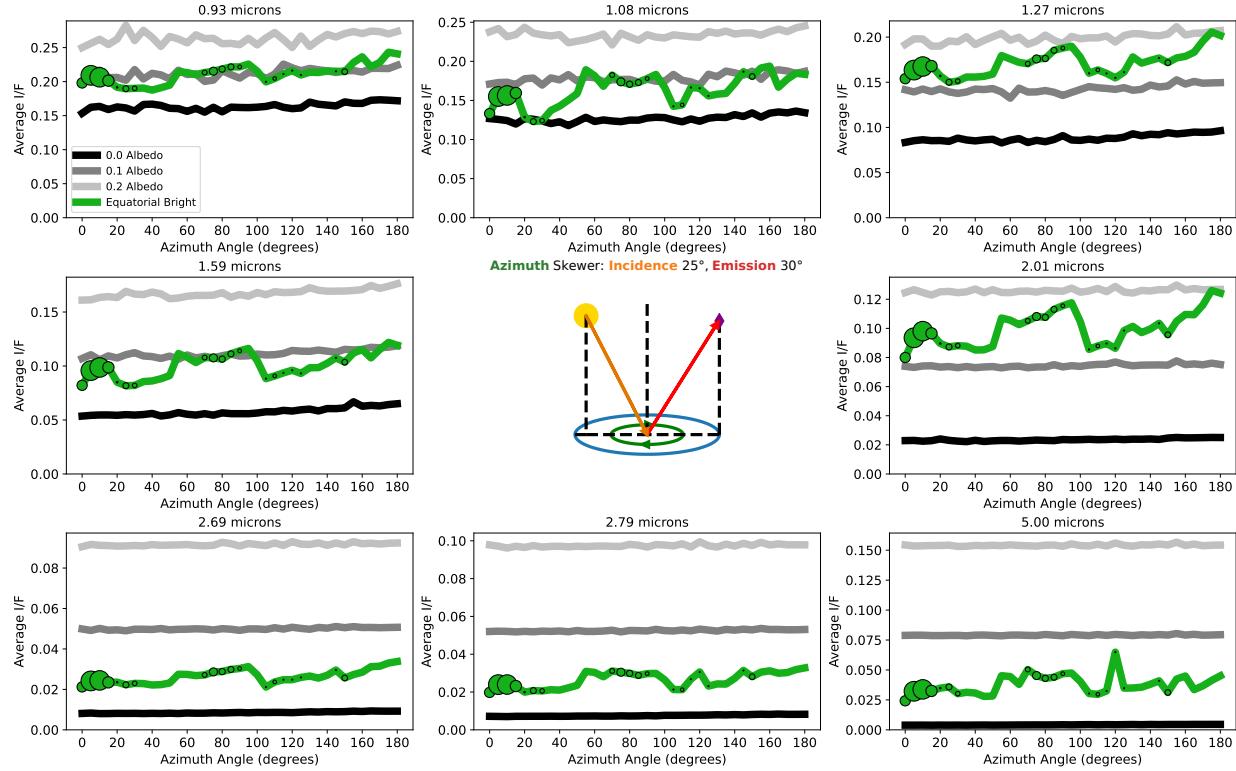
721 All authors are funded by grant #80NSSC22K0340 from  
 722 the NASA Cassini Data Analysis Program.

723

## APPENDIX

### A. EQUATORIAL BRIGHT TERRAIN

725 Like the dunes, the equatorial bright terrain (known as "plains" in radar maps) is extensive and has lots of observations over  
 726 many viewing geometries. Originally, we had planned to treat it alongside the dunes and use both to bolster our conclusions.  
 727 However, as can be seen in Figure 21, the equatorial bright terrain was far less well-behaved than the dunes.



**Figure 21.** Azimuth angle skewer: azimuth angle versus average I/F comparing phase function models with equatorial bright data with fixed incidence and emission angles. Showcases all eight wavelengths arranged from shortest to longest, with the central area occupied by a geometry diagram to illustrate the exact situation plotted. Model lines are monochromatic, equatorial bright data are green. Places with direct equatorial bright observations have dots plotted over the lines, larger dots meaning more observations. Places on the equatorial bright lines without dots are interpolated values. Note that the vertical axis scales with the data; not all wavelengths produce the same average I/F scale.

728 If the inconsistency in the equatorial bright terrain was predictable, then we would likely have continued to try to draw con-  
 729 clusions from it. However, as we can see in Figure 21, the observations have large stretches where the points are in line, before  
 730 suddenly jumping up or down. This forces us to conclude that what is labeled as equatorial bright terrain is actually multiple  
 731 different terrain types with their own phase functions. This is not just a distinction between flat areas and mountains—while VIMS  
 732 has a hard time differentiating those, the radar data used to make the terrain maps had a separate "hummocky" terrain type that  
 733 was not included in the equatorial bright terrain mask.

734 In general, the equatorial bright terrain would follow the lines of the restricted BRDFs, and it too exhibits the deviations at  
 735 high emission, further suggesting that the deviation is in the atmospheric model and not a surface effect. But any other deviations  
 736 in the equatorial bright terrain we noted could not have anything else said about them, and even the level to which the terrain  
 737 followed the models' shapes was suspect due to the clear albedo inconsistencies in single skewers. A dedicated investigation will  
 738 likely need to be devoted to this terrain in the future, perhaps even extending it beyond the equatorial regions if that is deemed  
 739 reasonable.

740 It is curious that the dunes do not exhibit this behavior, despite it being known that different areas in the dunes have different  
 741 interdune terrains under them (Bonnefoy et al. 2016). One would expect, since the dunes form on top of various terrains, that these  
 742 interdunes would at least produce a lesser version of the oscillations seen in the equatorial bright terrain, but we do not observe

743 that. Perhaps a single interdune terrain type dominates, or perhaps the placement of the dunes allows for effective averaging out  
 744 of the variations.

## REFERENCES

- 745 Barnes, J. W., MacKenzie, S. M., Lorenz, R. D., & Turtle, E. P.  
 746 2018, *The Astronomical Journal*, 156, 247,  
 747 doi: [10.3847/1538-3881/aae519](https://doi.org/10.3847/1538-3881/aae519)
- 748 Barnes, J. W., Brown, R. H., Turtle, E. P., et al. 2005, *Science*, 310,  
 749 92, doi: [10.1126/science.1117075](https://doi.org/10.1126/science.1117075)
- 750 Barnes, J. W., Brown, R. H., Soderblom, L., et al. 2007, *Icarus*,  
 751 186, 242, doi: [10.1016/j.icarus.2006.08.0219](https://doi.org/10.1016/j.icarus.2006.08.0219)
- 752 —. 2008, *Icarus*, 195, 400, doi: [10.1016/j.icarus.2007.12.006](https://doi.org/10.1016/j.icarus.2007.12.006)
- 753 Barnes, J. W., Soderblom, J. M., Brown, R. H., et al. 2009,  
 754 *Planetary and Space Science*, 57, 1950–1962,  
 755 doi: [10.1016/j.pss.2009.04.013](https://doi.org/10.1016/j.pss.2009.04.013)
- 756 Bonnefoy, L. E., Hayes, A. G., Hayne, P. O., et al. 2016, *Icarus*,  
 757 270, 222–237, doi: [10.1016/j.icarus.2015.09.014](https://doi.org/10.1016/j.icarus.2015.09.014)
- 758 Brossier, J. F., Rodriguez, S., Cornet, T., et al. 2018, *Journal of  
 759 Geophysical Research: Planets*, 123, 1089–1112,  
 760 doi: [10.1029/2017je005399](https://doi.org/10.1029/2017je005399)
- 761 Buratti, B., Sotin, C., Brown, R., et al. 2006, *Planetary and Space  
 762 Science*, 54, 1498, doi: [10.1016/j.pss.2006.06.015](https://doi.org/10.1016/j.pss.2006.06.015)
- 763 Cooper, C. A., Robinson, T. D., Barnes, J. W., Mayorga, L. C., &  
 764 Robinthal, L. 2025, Extreme Forward Scattering Observed in  
 765 Disk-Averaged Near-Infrared Phase Curves of Titan, *arXiv*,  
 766 doi: [10.48550/ARXIV.2507.00924](https://doi.org/10.48550/ARXIV.2507.00924)
- 767 Corlies, P., Hayes, A. G., Birch, S. P. D., et al. 2017, *Geophysical  
 768 Research Letters*, 44, doi: [10.1002/2017gl075518](https://doi.org/10.1002/2017gl075518)
- 769 Corlies, P., McDonald, G. D., Hayes, A. G., et al. 2021, *Icarus*,  
 770 357, 114228, doi: [10.1016/j.icarus.2020.114228](https://doi.org/10.1016/j.icarus.2020.114228)
- 771 Déau, E., Dones, L., Rodriguez, S., Charnoz, S., & Brahic, A.  
 772 2009, *Planetary and Space Science*, 57, 1282,  
 773 doi: [10.1016/j.pss.2009.05.005](https://doi.org/10.1016/j.pss.2009.05.005)
- 774 Es-sayeh, M., Rodriguez, S., Coutelier, M., et al. 2023, *The  
 775 Planetary Science Journal*, 4, 44, doi: [10.3847/PSJ/acbd37](https://doi.org/10.3847/PSJ/acbd37)
- 776 García Muñoz, A., Lavvas, P., & West, R. A. 2017, *Nature  
 777 Astronomy*, 1, doi: [10.1038/s41550-017-0114](https://doi.org/10.1038/s41550-017-0114)
- 778 Griffith, C. A., Doose, L., Tomasko, M. G., Penteado, P. F., & See,  
 779 C. 2012, *Icarus*, 218, 975, doi: [10.1016/j.icarus.2011.11.034](https://doi.org/10.1016/j.icarus.2011.11.034)
- 780 Hapke, B., & van Hoen, H. 1963, *Journal of Geophysical  
 781 Research*, 68, 4545–4570, doi: [10.1029/jz068i015p04545](https://doi.org/10.1029/jz068i015p04545)
- 782 Karkoschka, E., Schröder, S. E., Tomasko, M. G., & Keller, H. U.  
 783 2012, *Planetary and Space Science*, 60, 342–355,  
 784 doi: [10.1016/j.pss.2011.10.014](https://doi.org/10.1016/j.pss.2011.10.014)
- 785 Kazeminejad, B., Atkinson, D. H., & Lebreton, J.-P. 2011, *The  
 786 Astrophysical Journal Letters*, 47, 1622–1632,  
 787 doi: [10.1016/j.asr.2011.01.019](https://doi.org/10.1016/j.asr.2011.01.019)
- 788 Keller, H., Grieger, B., Küppers, M., et al. 2008, *Planetary and  
 789 Space Science*, 56, 728–752, doi: [10.1016/j.pss.2007.11.020](https://doi.org/10.1016/j.pss.2007.11.020)
- 790 Kulyk, I. 2008, *Planetary and Space Science*, 56, 386–397,  
 791 doi: [10.1016/j.pss.2007.11.011](https://doi.org/10.1016/j.pss.2007.11.011)
- 792 Le Mouélic, S., Cornet, T., Rodriguez, S., et al. 2012, *Planetary  
 793 and Space Science*, 73, 178–190, doi: [10.1016/j.pss.2012.09.008](https://doi.org/10.1016/j.pss.2012.09.008)
- 794 —. 2019, *Icarus*, 319, 121–132, doi: [10.1016/j.icarus.2018.09.017](https://doi.org/10.1016/j.icarus.2018.09.017)
- 795 Lopes, R. M. C., Malaska, M. J., Schoenfeld, A. M., et al. 2020,  
 796 *Nature Astronomy*, 4, 228, doi: [10.1038/s41550-019-0917-6](https://doi.org/10.1038/s41550-019-0917-6)
- 797 Lynch, D. K., & Livingston, W. 2004, *Color and light in nature*  
 798 (Cambridge Univ. Press)
- 799 Neish, C. D., Lorenz, R. D., Kirk, R. L., & Wye, L. C. 2010,  
 800 *Icarus*, 208, 385–394, doi: [10.1016/j.icarus.2010.01.023](https://doi.org/10.1016/j.icarus.2010.01.023)
- 801 Pont, S. C., & Koenderink, J. J. 2007, Perception amp;  
 802 Psychophysics, 69, 459–468, doi: [10.3758/bf03193766](https://doi.org/10.3758/bf03193766)
- 803 Rannou, P., Coutelier, M., Rivière, E., et al. 2021, *The  
 804 Astrophysical Journal*, 922, 239,  
 805 doi: [10.3847/1538-4357/ac2904](https://doi.org/10.3847/1538-4357/ac2904)
- 806 Rodriguez, S., Le Mouélic, S., Sotin, C., et al. 2006, *Planetary and  
 807 Space Science*, 54, 1510–1523, doi: [10.1016/j.pss.2006.06.016](https://doi.org/10.1016/j.pss.2006.06.016)
- 808 Rodriguez, S., Le Mouélic, S., Barnes, J. W., et al. 2018, *Nature  
 809 Geoscience*, 11, 727–732, doi: [10.1038/s41561-018-0233-2](https://doi.org/10.1038/s41561-018-0233-2)
- 810 Schaefer, B. E., Rabinowitz, D. L., & Tourtellotte, S. W. 2008, *The  
 811 Astronomical Journal*, 137, 129–144,  
 812 doi: [10.1088/0004-6256/137/1/129](https://doi.org/10.1088/0004-6256/137/1/129)
- 813 Schröder, S., & Keller, H. 2008, *Planetary and Space Science*, 56,  
 814 753–769, doi: [10.1016/j.pss.2007.10.011](https://doi.org/10.1016/j.pss.2007.10.011)
- 815 —. 2009, *Planetary and Space Science*, 57, 1963–1974,  
 816 doi: [10.1016/j.pss.2009.03.012](https://doi.org/10.1016/j.pss.2009.03.012)
- 817 Seal, D., & Bittner, M. 2017, in 2017 IEEE Aerospace Conference  
 818 (IEEE), 1–12, doi: [10.1109/aero.2017.7943848](https://doi.org/10.1109/aero.2017.7943848)
- 819 Seignovert, B., Mouélic, S. L., Heslar, M., et al. 2023, PyVIMS,  
 820 Zenodo, doi: [10.5281/ZENODO.4708004](https://doi.org/10.5281/ZENODO.4708004)
- 821 Soderblom, L. A., Kirk, R. L., Lunine, J. I., et al. 2007, *Planetary  
 822 and Space Science*, 55, 2025, doi: [10.1016/j.pss.2007.04.014](https://doi.org/10.1016/j.pss.2007.04.014)
- 823 Soderblom, L. A., Brown, R. H., Soderblom, J. M., et al. 2009,  
 824 *Icarus*, 204, 610, doi: [10.1016/j.icarus.2009.07.033](https://doi.org/10.1016/j.icarus.2009.07.033)
- 825 Solomonidou, A., Malaska, M., Lopes, R., et al. 2024, *Icarus*, 421,  
 826 116215, doi: [10.1016/j.icarus.2024.116215](https://doi.org/10.1016/j.icarus.2024.116215)
- 827 Sullivan, C. B., & Kaszynski, A. A. 2019, *Journal of Open Source  
 828 Software*, 4, 1450, doi: [10.21105/joss.01450](https://doi.org/10.21105/joss.01450)
- 829 Tomasko, M. G., Doose, L., Engel, S., et al. 2008, *Planetary and  
 830 Space Science*, 56, 669, doi: [10.1016/j.pss.2007.11.019](https://doi.org/10.1016/j.pss.2007.11.019)
- 831 Vinatier, S., Bezard, B., Fouchet, T., et al. 2007, *Icarus*, 188,  
 832 120–138, doi: [10.1016/j.icarus.2006.10.031](https://doi.org/10.1016/j.icarus.2006.10.031)
- 833 Wise, J. E., & Mars, J. C. 2022, *Remote Sensing*, 14, 5020,  
 834 doi: [10.3390/rs14195020](https://doi.org/10.3390/rs14195020)

835 Wye, L. C. 2011, Radar scattering from Titan and Saturn's icy  
836 satellites using the Cassini spacecraft (stanford university)

837 Xu, F., West, R. A., & Davis, A. B. 2013, Journal of Quantitative  
838 Spectroscopy and Radiative Transfer, 117, 59,  
839 doi: [10.1016/j.jqsrt.2012.10.013](https://doi.org/10.1016/j.jqsrt.2012.10.013)