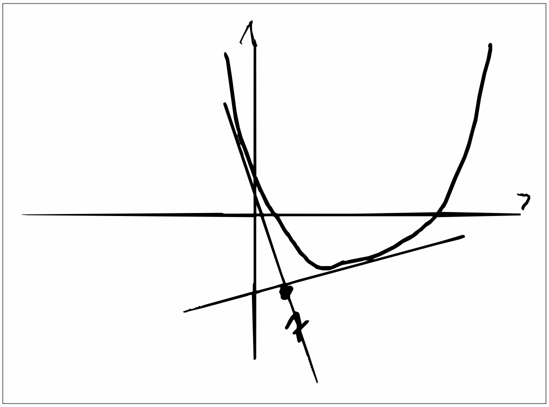
1 Intro



- Initial quadratic
- polynomial?
- trig?
- closed curve?
- Any differential curve?

Scene 1.1

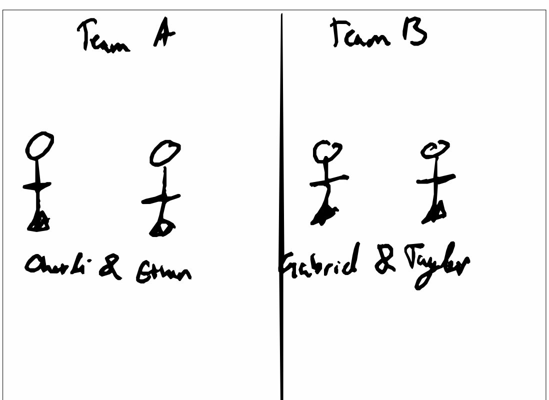
Initial problem

$y = x^2 - 4x + 3$, $A(2, -2)$

What about other points?

Scene 1.2

Problem progression



Scene 1.3

Introducing our teams

Competition and example

Geometry/generalisation and Topology

Scene 1.4

Clear goal?

$E \subset \mathbb{R}^2$ such that there are tangent passes thorugh a point $P \in E$.

2 Team A - Polynomial

$\frac{d}{dx}(P(x))$ is easy!

Scene 2.1

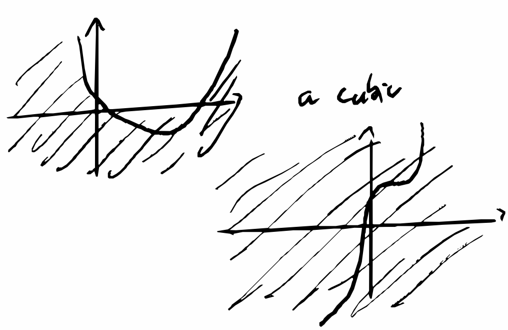
Why polynomials?

$(x_\alpha - x_A) f'(x_\alpha) - f(x_\alpha) + y_A = 0$

Scene 2.2.

Finding the formula

quadratic turn cubic



Scene 2.3

Graphs for which x,y solutions exist

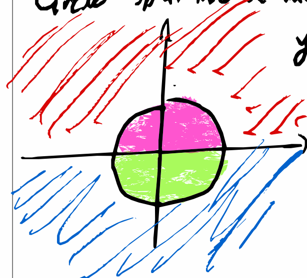
Odd parity - $\mathbb{R}^2$

Even parity - Not "inside"?

$(n-1) a x_\alpha^n$ main term

Circle - split into 2 functions

$y = \sqrt{1-x^2}$ ~ parabola?



Closed shapes are empty!

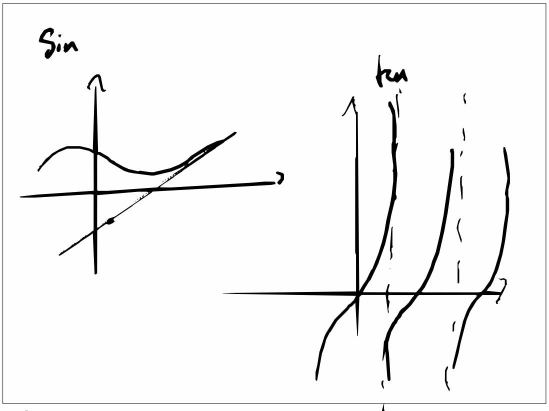
Scene 2.4

Circle works as a parabola?

Inside/outside

Where do the solutions originate (which equation)

2 Team A – trig and non convex

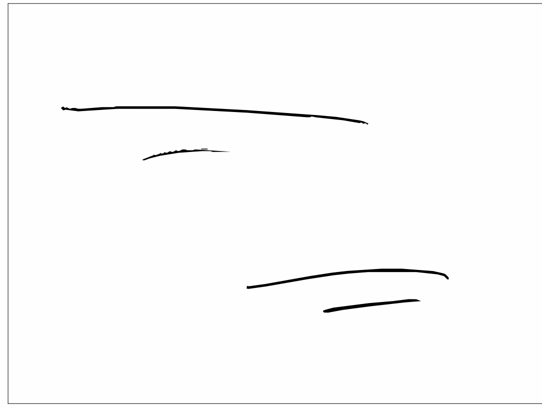


Scene 2.5

Trig

Sin, Cos -> always infinite solutions (taylor series allusion)

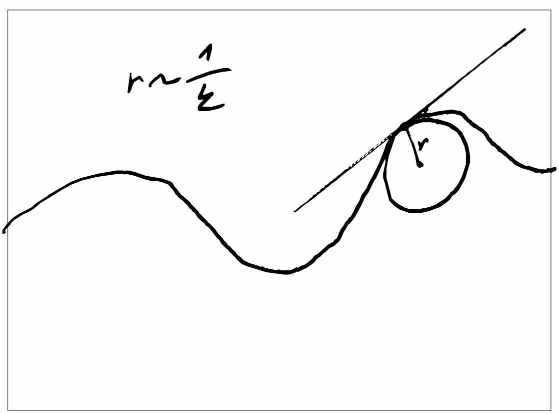
tan -> good question, needs a different method



Scene 2.6

Any function without it's interior has a tangent! Are we missing something?

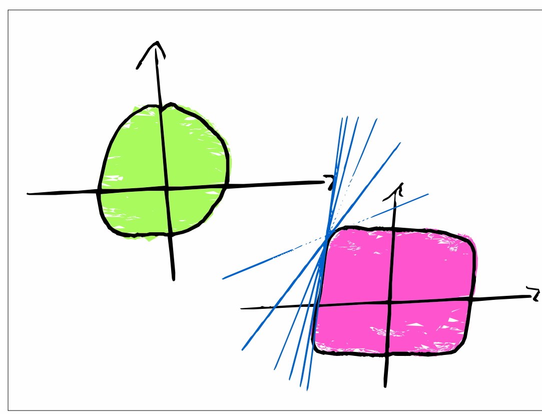
3 Team B



Scene 3.1

Osculating circles and curvature

Find form "where" we can draw tangents to those circle

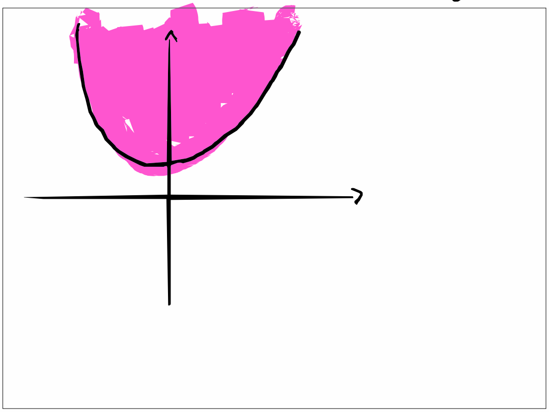


Scene 3.2

Circle - simplest form   360° spin

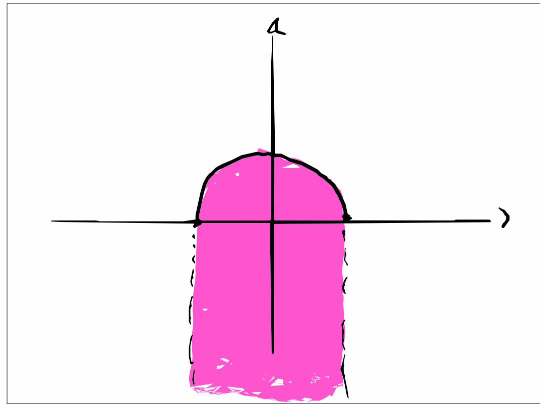
- tangent can't come from inside   IMT!
- can be from anywhere else
- rolling tangent -> convex

3 Team B



Scene 3.3

Solutions everywhere outside!



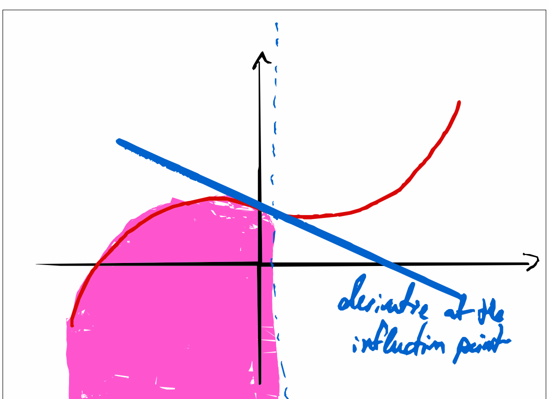
Scene 3.4

Convexity revisited?

180° spins around „inner shadow"

Solutions outside exist always

^ inner shadow revisit parabola



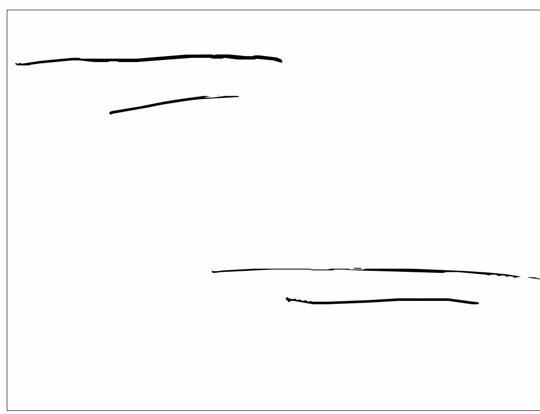
Scene 3.5 2 scenes!!

Complicated curves

Inflection point shows complete clearance

and at least 1 solution for any point

Arcus sin, hm....

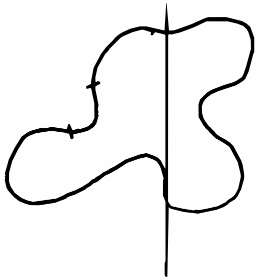


Scene 3.6 ⟺ with 2

I think that's everything covered

And without numbers!

3 Team B



Scene 3.7

Revisiting any curve idea above

*guarantees solution exists

Minimises shadow area idk

Split the function to inflection points, and find our area without a solution, again rolling tangents (not necessary)

4 conclusion

3/5 tasks proven computations get harder

- Doesn't mean there isn't a computational solution just that any D^1 curve is incredibly hard to write down with symbols (or closed form)

= $(x_2 - x_A) f'(x_a) - f(x_a)$ ... $= 0$

- easy for initial task

Scene 4.1

Team A recap

5/5 tasks proven

- way harder than using the equation and checking for solution (Team A)
- Requires a D^2 curve. D^1 curve idk
- rolling tangents and inner shadows
- really overkill it:

Scene 4.2

Team B recap

- Initial quadratic
  ↳ HS back, equation from A
- Any polynomial
  ↳ Quadratic, add proof at HS and A
- Trig
  ↳ sin HS, rest is uhh, A
- Closed curve and any D1
  ↳ uni, B

Scene 4.3

Level of question in my opinion