

Let $f(x)$ be a curve on \mathbb{R}^1 and let it be at least D^1 -type. Let A be a point such that $A \notin \Gamma_A$. Find all the possible tangents on P such that they pass through any $A \notin \Gamma_A$?

$A(x_A, y_A)$
 $f \dots y = kx + b \Rightarrow y - y_0 = k(x - x_0)$
 $A \in \Gamma_A$
 $y - f(x_0) = f'(x_0)(x - x_0)$ for some $(x_0, f(x_0))$.
 $y_A - f(x_0) = f'(x_0)(x_A - x_0)$
 $y_A - f(x_0) = f'(x_0) \cdot x_A + P(x_0) \cdot x_A$
 $f'(x_0)(x_A - x_0) - P(x_0) \cdot x_A + y_A = 0$.
 $D_x f'(x_0) - f(x_0) + y_A = 0$
 Ans: Depends on P , like convex

I] P is a polynomial of second degree or higher

a) quadratic

$P(x) = ax^2 + bx + c$
 $f'(x) = 2ax + b$

$(x_A - x_0)f'(x_0) - f(x_0) + y_A = 0$

$(x_A - x_0)(2ax_0 + b) - ax_0^2 - bx_0 - c + y_A = 0$

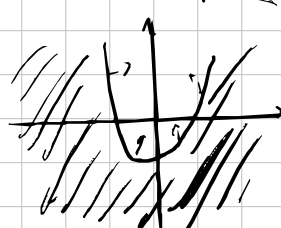
$2ax_0^2 + bx_0 - 2ax_0x_A - bx_0x_A - cx_0 + y_A = 0$

$ax_0^2 - 2ax_Ax_0 - bx_Ax_0 - c + y_A = 0$

$x_0 = \frac{2ax_A \pm \sqrt{4a^2x_A^2 - 4abx_A - 4c + 4y_A}}{2a}$

$4a^2x_A^2 - 4abx_A - 4c + 4y_A$
 has solutions in a lot of places!
 All outside the curve!!

No inside



b) cubic

$f(x) = ax^3 + bx^2 + cx + d$
 $f'(x) = 3ax^2 + 2bx + c$

$(x_A - x_0)f'(x_0) - f(x_0) + y_A = 0$

$(x_A - x_0)(3ax_0^2 + 2bx_0 + c) - ax_0^3 - bx_0^2 - cx_0 - d + y_A = 0$

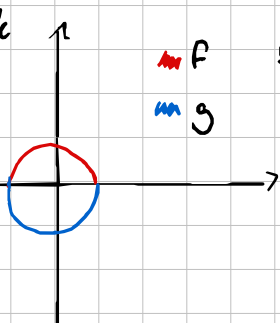
cubic!! 1 or 3 solutions

c) 4th degree?

inside?

II closed curves

a) Circle



$f(x) = \sqrt{1-x^2}$

$g(x) = -\sqrt{1-x^2}$

If $x_A \in (-1, 1)$ 2 different tangents (up and down)

otherwise 2 same (up pp, down down)

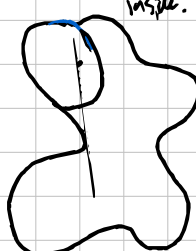
No tangent inside?



b) General curve

Convexity?

inside!



No points

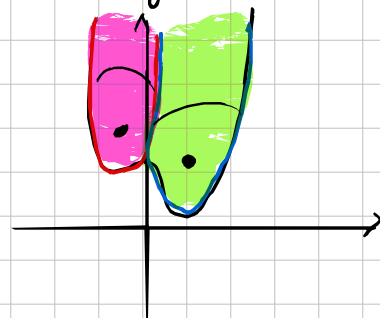
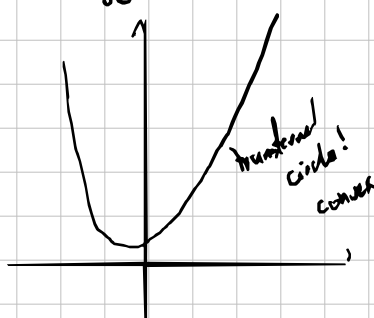
A lot of points!



Go back to polynomial functions

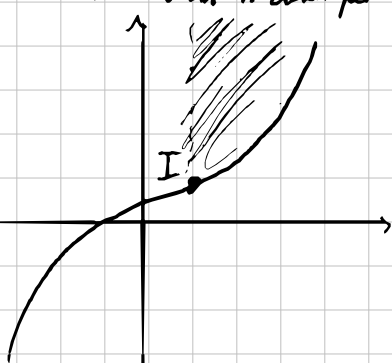
2nd degree

4th degree

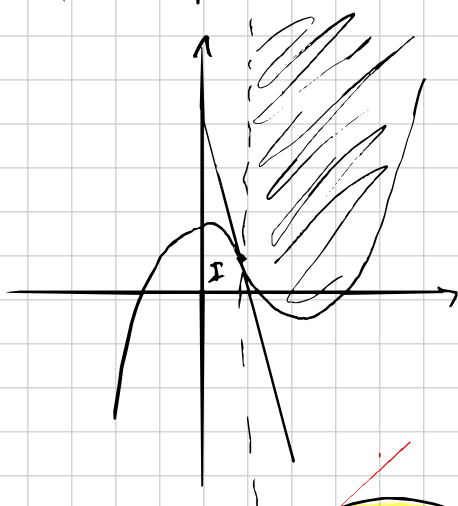


Existence of an inflection point implying solution always exists?

without loss of generality let f be a C^2 curve such that it is concave down and has an inflection point becoming concave up



Let S denote the inner shadow of the concave up part of f . Let E be the inner shadow covered by rolling tangents of the concave down part. If we graph some it's not hard to see that we are increasing E . What we want ideally $S \cap E = S$ or $S = E$ or colloquially E covers S fully. what we see is that we need tangent at point I call it $t(I)$ to be lower than f , in other words $t(I) \in P([I, \infty))$



A function being concave up means that that the function bends upwards and the inflection point's tangent therefore presents as the minimum for any concave up function. Analogous argument holds for concave up to concave down. For functions with multiple inflection points, when we look at restrictions on the functions between them.

