Let $f(x)$ be a curve on $\mathbb{R}^1$ and let it be at least D^1-type. Let A be a point such that $A \notin \Gamma_A$. Find all the possible tangents on f such that they pass through any $A \notin \Gamma_A$?

$A(x_A, y_A)$

$t \dots y = kx + b \Rightarrow y - y_0 = k(x - x_\alpha)$

$A \in \Gamma_t$

$y - f(x_\alpha) = f'(x_\alpha)(y - x_\alpha)$ for some $(x_\alpha, f(x_\alpha))$.

$y_A - f(x_\alpha) = f'(x_\alpha)(x_A - x_\alpha)$

$y_A - f(x_\alpha) = f'(x_\alpha) \cdot x_\alpha + f(x_\alpha) \cdot x_A$

$f'(x_\alpha)(x_\alpha - x_A) - f(x_\alpha) + y_A = 0$

$Dx \cdot f'(x_\alpha) - f(x_\alpha) + y_A = 0$

Ans: Depends on f, lets examine

I] f is a polynomial of second degree or higher

a) quadratic

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

$(x_\alpha - x_A) f'(x_\alpha) - f(x_\alpha) + y_A = 0$

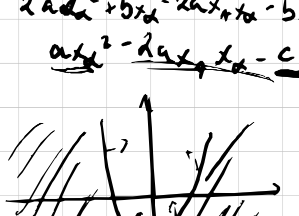
$(x_\alpha - x_A)(2ax_\alpha + b) - ax_\alpha^2 - bx_\alpha - c + y_A = 0$

$2ax_\alpha^2 + bx_\alpha - 2ax_A x_\alpha - bx_A - ax_\alpha^2 - bx_\alpha - c + y_A = 0$

$ax_\alpha^2 - 2ax_A x_\alpha - c + y_A = 0$

$x_\alpha = \frac{2ax_A \pm \sqrt{4a^2x_A^2 - 4a(y_A - c - bx_A)}}{2a}$

No inside



$4a^2x_A^2 - 4a(y_A - bx_A - c)$

Has solutions in a lot of places!

All outside the curve!!

b) cubic

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$(x_\alpha - x_A) f'(x_\alpha) - f(x_\alpha) + y_A = 0$

$(x_\alpha - x_A)(3ax_\alpha^2 + 2bx_\alpha + c) - ax_\alpha^3 - bx_\alpha^2 - cx - d = 0$

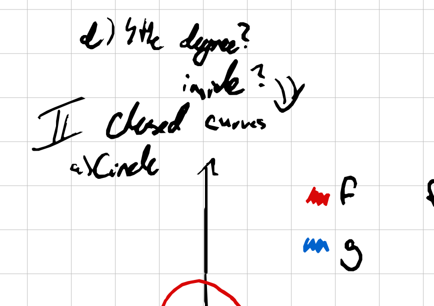
cubic!! 1 or 3 solutions

c) 4th degree?

inside?

II Closed curves

a) Circle



$f(x) = \sqrt{1 - x^2}$

$g(x) = -\sqrt{1 - x^2}$

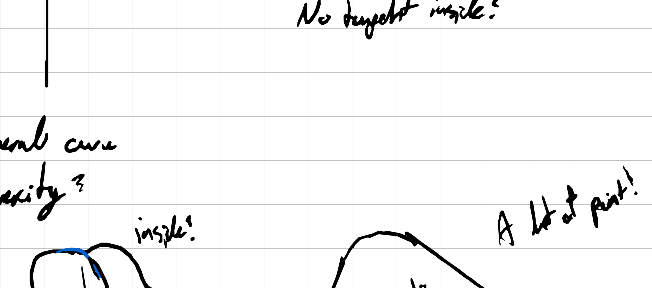
If $x_A \notin \langle -1, 1 \rangle$ 2 different tangents (up and down)

Otherwise 2 same (up up, down down)

No tangent inside?

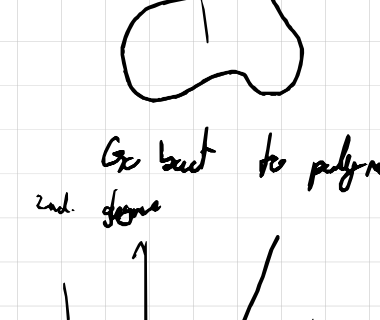
b) General curve

Convexity?

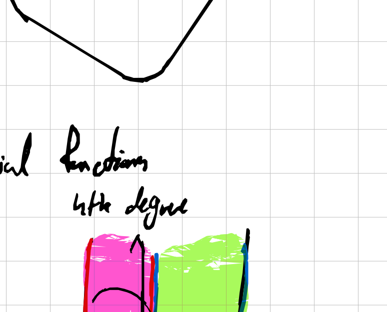


Go back to polynomial functions

2nd degree

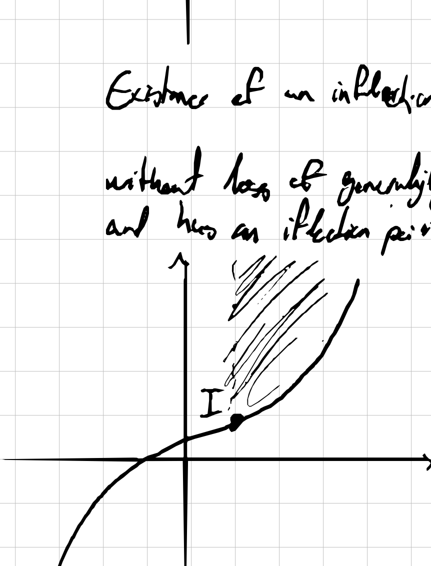


4th degree

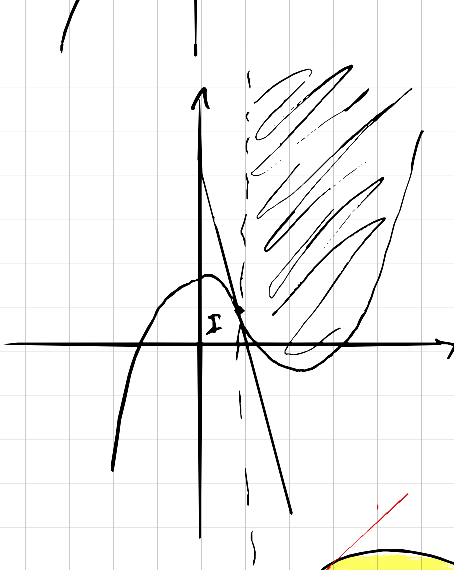


Existence of an inflection point implying solution always exists?

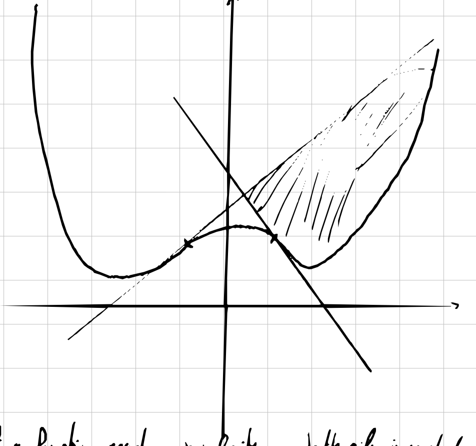
without loss of generality let f be a C^2 curve such that its concave down and has an inflection point becoming concave up



Let S denote the inner shading of the concave up part of f. Let E be the inner shading covered by rolling tangents of the concave down part. If we graph some its not hard to see that we are lacking E. What we want ideally $S \cap E = S$ or $S = E$ or colloquially E covers S fully. What we see is that we need tangent at point I call it $t(I)$ to be lower than f in other words $t(I) < f(\langle I_x, \infty))$

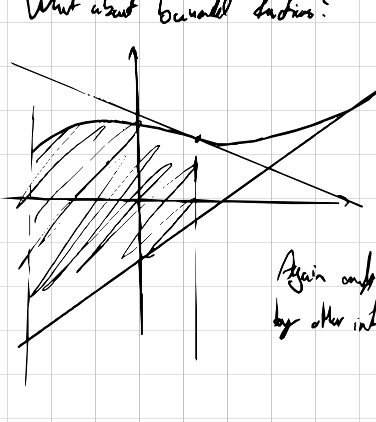


A function being concave up means that that the function bends inwards and the inflection point's tangent therefore presents as the minimum for any concave up function. Analogous argument holds for concave up to concave down. For functions with multiple inflection points when we look at restrictions on the functions between them. * Assuming the function reaches $\pm$ infinity.



If a function reaches $+$ infinity on both sides inner shade gets reduced by some amount dictated by tangent at other inflection points

What about bounded functions?



Again only a part dictated by other inflection points