The Platform

Thursday, April 2, 2020

Introductions - results

1) Engineering Computer, Electrical, Civil, Biomedical.
   Chemical, Computer Science,
   Material Science, Mechanical, Envir.

   Mathematics (Applied) (6)!
   Statistics
   Physics
   Political Science
   Wildlife
   Economics (Managerial)
   Chemistry (bio)
   Cognitive Science

2) 40 freshman
   25 first gen
   6 transfers!

Concerns:

1) Internet connectivity / Laptop issues/printing
   lack of face-to-face instruction
   not making friends - connection to TA/prof.

2) Motivation, engaging material, focus
   Study schedule, keeping up
   Difficulty, Fair assessment?

Tiger King!

Worksheet 1:

Thm: (Fubini Theorem First Form)
If $f(x,y)$ is continuous throughout the rectangular region
$R: a \leq x \leq b ; c \leq y \leq d$ then

$$\iint_R f(x,y)\, dA = \int_c^d \int_a^b f(x,y)\, dx\, dy$$

$$= \int_a^b \int_c^d f(x,y)\, dy\, dx$$

Problem 1: Evaluate the iterated integrals

a) $\displaystyle\int_\pi^{2\pi} \int_0^\pi (\sin x + \cos y)\, dx\, dy$

$$= \int_\pi^{2\pi} \Big[-\cos x + x\cos y\Big]_0^\pi dy$$

$$= \int_\pi^{2\pi} \Big[(-\cos(\pi) + \pi\cos y) - (-\cos(0) + 0)\Big] dy$$

$$= \int_\pi^{2\pi} (2 + \pi\cos y)\, dy$$

$$= \Big[2y + \pi\sin y\Big]_\pi^{2\pi}$$

$$= (2(2\pi) + \pi\sin(2\pi)) - (2\pi + \pi\sin(\pi))$$

$$= \boxed{2\pi}$$

b) $\int_0^1 \int_0^1 \frac{y}{x^2 y^2 + 1}\, dx\, dy = \int_0^1 \tan^{-1}(xy) \Big|_0^1 dy$

$= \int_0^1 \left(\tan^{-1}(y) - \cancel{\tan^{-1}(0)}^{\,0}\right) dy$

$= \int_0^1 \tan^{-1}(y)\, dy$

$\left[\begin{array}{ll} \text{let } u = \tan^{-1}(y)\, dy & dv = dy \\ du = \frac{1}{1+y^2}\, dy & v = y \end{array} \right]$ Integration by parts!!

$= y \tan^{-1}(y) \Big|_0^1 - \int_0^1 y \cdot \frac{1}{1+y^2}\, dy$

$= \tan^{-1}(1) - \frac{1}{2} \ln(1+y^2) \Big|_0^1$

$= \boxed{\frac{\pi}{4} - \frac{1}{2} \ln 2}$

Problem 2: Find all values of the constant c so that $\int_0^1 \int_0^c (2x+y)\, dx\, dy = 3$

Solve: $\int_0^1 \int_0^c (2x+y)\, dx\, dy$

$= \int_0^1 \left[x^2 + xy\right]_0^c dy$

$= \int_0^1 (c^2 + cy)\, dy$

$= \left[c^2 y + \frac{c}{2} y^2\right]_0^1$

$= c^2 + \frac{c}{2}$

Since $\int_0^1 \int_0^c (2x+y)\,dx\,dy = 3$

$$c^2 + \frac{c}{2} = 3$$

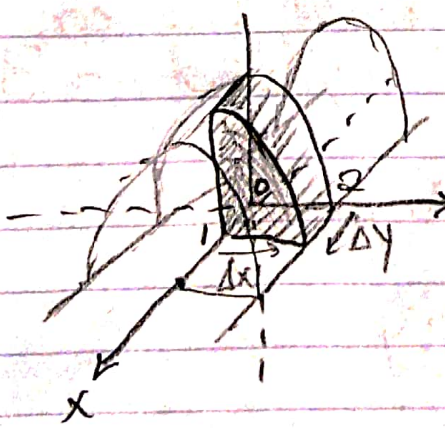
$$c^2 + \tfrac{1}{2}c - 3 = 0$$

$$c = \frac{-\tfrac{1}{2} \pm \sqrt{\tfrac{1}{4} - 4(-3)}}{2}$$

$$= \frac{-1 \pm \sqrt{1+48}}{4} = \frac{-1 \pm 7}{4}$$

$$\boxed{c = -2 \text{ or } \frac{3}{2}}$$

Problem 3: Find the volume of the region bounded above by the surface $z = 4 - y^2$ & below by the rectangle $R: 0 \le x \le 1;\ 0 \le y \le 2$



$$\int_0^2 \int_0^1 (4-y^2)\,dx\,dy$$

$$= \int_0^2 (4-y^2)x\Big|_0^1 dy$$

$$= \int_0^2 (4-y^2)\,dy$$

$$= \left[4y - \frac{y^3}{3}\right]_0^2$$

$$= \boxed{8 - \frac{8}{3}} = \boxed{\frac{16}{3}}$$

Quiz 0 Solution

1) Evaluate:

$$\int_0^1\int_0^2 xy\,dy\,dx = \int_0^1 \frac{x}{2}y^2\Big|_0^2 dx$$

$$= \int_0^1 \frac{x}{2}\cdot 4\,dx = \int_0^1 2x\,dx$$

$$= x^2\Big|_0^1 = \boxed{1}$$

2) Find all constants c such that

$$\int_0^2\int_0^c (4x+y)\,dy\,dx = 8c+4$$

Solve $\int_0^2\int_0^c (4x+y)\,dy\,dx$

$$= \int_0^2 \left[4xy + \frac{y^2}{2}\right]_0^c dx$$

$$= \int_0^2 \left(4cx + \frac{c^2}{2}\right) dx$$

$$= \left[2cx^2 + \frac{c^2}{2}x\right]_0^2$$

$$= 8c + c^2$$

Set equations equal

$$\cancel{8c} + c^2 = \cancel{8c} + 4$$

$$c^2 - 4 = 0$$

$$(c+2)(c-2) = 0 \qquad \boxed{\therefore c = \pm 2}$$