Math 21D, Spring 2020 – Homework 2

Friday, April 3

These problems could appear on Quiz 1 (Thursday, April 9)

1. Evaluate the integral:

   (a) $\displaystyle\int_0^{\pi}\int_0^{x} x\sin y\,dy\,dx$ (b) $\displaystyle\int_0^{1}\int_0^{y^2} 3y^3e^{xy}\,dx\,dy$

2. Integrate the function $f(x,y)=\dfrac{x}{y}$ over the quadrilateral in the first quadrant bounded by the lines $y=x$, $y=2x$, $x=1$, and $x=2$.

3. Write an equivalent double integral with the order of integration reversed (you may find it helpful to sketch the region of integration):

   (a) $\displaystyle\int_0^{2}\int_{y-2}^{0} dx\,dy$ (c) $\displaystyle\int_0^{\ln 2}\int_{e^y}^{2} dx\,dy$

   (b) $\displaystyle\int_0^{1}\int_{1-x}^{1-x^2} dy\,dx$ (d) $\displaystyle\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\,dy\,dx$

4. Reverse the order of integration and evaluate the integral:

   (a) $\displaystyle\int_0^{2}\int_x^{2} 2y^2\sin xy\,dy\,dx$ (c) $\displaystyle\int_0^{3}\int_{\sqrt{x/3}}^{1} e^{y^3}\,dy\,dx$

   (b) $\displaystyle\int_0^{2}\int_0^{4-x^2} \frac{xe^{2y}}{4-y}\,dy\,dx$ (d) $\displaystyle\int_0^{1/16}\int_{y^{1/4}}^{1/2} \cos 16\pi x^5\,dx\,dy$

5. Find the volume of the solid whose base is the region in the xy-plane bounded by the parabola $y=x^2$ and the line $y=3x$ and whose top is given by the plane $z=x+4$.

6. Find the volume of the solid cut from the first octant by the surface $z=4-x^2-y$.

7. ($\star$) Evaluate the (improper) double integral:

   (a) $\displaystyle\int_1^{\infty}\int_{e^{-x}}^{1} \frac{1}{x^3y}\,dy\,dx$ (b) $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(y^2+1)}\,dx\,dy$

8. ($\star$) Evaluate the integral $\displaystyle\int_0^{2} \tan^{-1}\pi x-\tan^{-1}x\,dx$. (*Hint*: Write this as a double integral and change the order of integration.)

9. ($\star$) What region R in the xy-plane minimizes the value of $\displaystyle\iint_R x^2+y^2-9\,dA$?