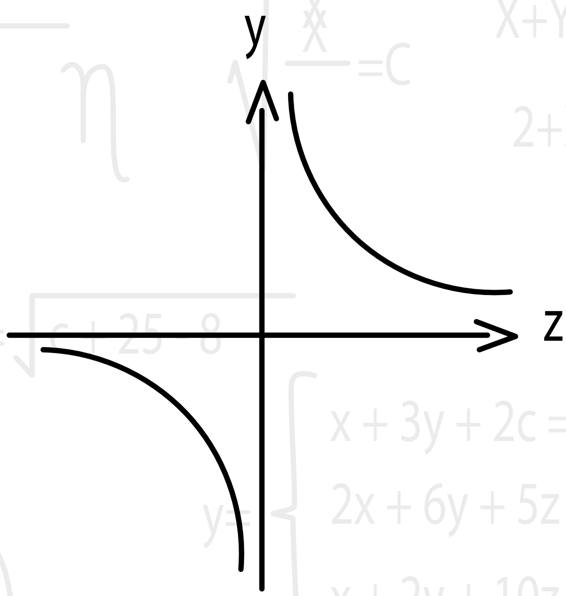
SSS3

MATHEMATICS



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Contents

THEME 01

Numbers and Numeration.

Algebraic Process.

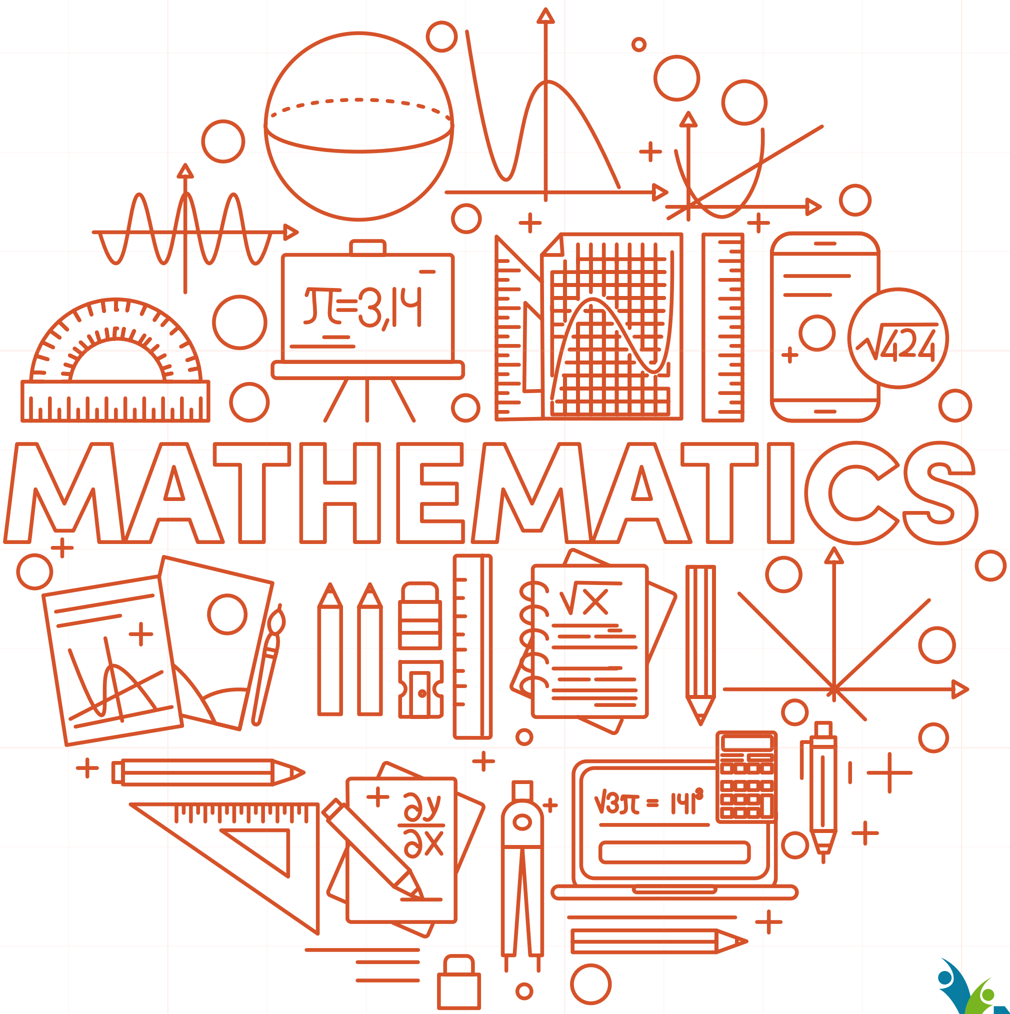
Geometry.

Introductory Calculus.

SURDS

PERFORMANCE OBJECTIVES

1. Differentiate between Rational and Irrational numbers leading to definition of surds
2. Perform and solve problems on addition, subtraction, multiplication and division of surdic numbers
3. Solve problems involving conjugates of binomial surds
4. Relate surds to trigonometric ratios.



IRRATIONAL NUMBERS

Irrational numbers ≠ Ratio/Fraction

2 = 2/1 or 2:1	Rational number
1.7 = 17/10 or 17:10	Rational number
$\sqrt{9}$ = 3 or 3:1	Rational number

1.3333... = recurring decimal
= 4/3 or 4:3

7/0 cannot be defined	Irrational number
$\sqrt{2}$ = 1.4142135... (non-recurring)	Irrational number

Recurring decimals e.g. 1.333... .333
2.173173 .173

Terminating decimals 1/2 = 0.5
1/8 = 0.125

DRILL

Is $\sqrt{25}$ a rational number ?

True/False

What then is a surd ?

A surd, simply put, is an irrational number which is ALSO a square root. So, a surd MUST have two properties: It must be a square root and must be irrational.

$\sqrt{8}$ = surd
$\sqrt{9}$ = 3 not surd.

RULES OF SURDS

Rule 1: Multiplication

$\sqrt{m \times n} = \sqrt{m} \times \sqrt{n}$

Example

a. $\sqrt{6} = \sqrt{2 \times 3} = \sqrt{2} \times \sqrt{3}$

b. $\sqrt[2]{3} \times \sqrt[4]{5} = 2 \times 4 \times \sqrt{3} \times \sqrt{5}$

$= 8 \times \sqrt{15}$

$= 8\sqrt{15}$

Rule 2: Division

$$\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$$

Example

a. $\sqrt{\frac{7}{8}} = \frac{\sqrt{7}}{\sqrt{8}}$

b. $\frac{m}{\sqrt{n}} = \frac{m}{\sqrt{n}} = \frac{\sqrt{n}}{\sqrt{n}}$

Rule 3: Rationalization

$$\sqrt{\frac{11}{16}} = \frac{\sqrt{11}}{\sqrt{16}} = \frac{\sqrt{11}}{4}$$

$$= \frac{m\sqrt{n}}{n}$$

Note

$$\sqrt{n}\ x\ \sqrt{n} = (\sqrt{n})^2 = n$$

Example:

$$\frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}}\ x\ \frac{\sqrt{7}}{\sqrt{7}}$$

$$= \frac{2\sqrt{7}}{7}$$

Rule 4: Addition/Subtraction of Surds

Note

$m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$

Also,

$m\sqrt{a} - n\sqrt{a} = (m-n)\sqrt{a}$

Like Surds

We can only add or subtract like surds. What are like surds?

Like surds are surds that have the same number in the root. For instance, 2 root 3 and 5 root 3 are like surds while 2 root 2 and 5 root 3 are not like surds, because of the difference in the roots.

$2\sqrt{3}$ and $5\sqrt{3}$ = Like Surds

$2\sqrt{2}$ and $5\sqrt{2}$ ≠ Unlike Surds

E.g.

* $3\sqrt{6} + 4\sqrt{6} = (3 + 4)\sqrt{6}$
* $15\sqrt{8} - 18\sqrt{8} = (15 - 18)\sqrt{8}$

DRILL

$2\sqrt{3}$ and $3\sqrt{2}$ are like surds

True/False

Answer:

False; This is because the roots of $\sqrt{3}$ and $\sqrt{2}$ does not have the same number. So, they are unlike surds.

SIMPLIFICATION OF SURDS

Example 1:

Simplify $\sqrt{32}$

$$\sqrt{32} = \sqrt{(2 \times 16)}$$

$$\sqrt{16} \times \sqrt{2} = 4 \times \sqrt{2}$$

$$= 4\sqrt{2}$$

Example 2:

Simplify $\sqrt{\dfrac{12}{121}}$

$$= \sqrt{\frac{12}{121}} = \frac{\sqrt{(3x4)}}{11} = \frac{\sqrt{3} \times \sqrt{4}}{11} = \frac{\sqrt{3} \times 2}{11}$$

$$= \frac{2\sqrt{3}}{11}$$

CONJUGATION OF BINOMIAL SURDS

-What is a binomial surd?
A binomial surd is the sum or difference of two surds or a surd and a rational number. For instance,

Binomial surd	• $2\sqrt{3} + 3\sqrt{5}$
Binomial surd	• $1 - \sqrt{3}$
Not a binomial surd	• $2\sqrt{3} + \sqrt{5} - \sqrt{7}$

Binomial surd	Conjugate
$2 + \sqrt{5}$	$2 - \sqrt{5}$
$3\sqrt{15} + 2\sqrt{7}$	$3\sqrt{15} - 2\sqrt{7}$

NOTE: Binomial surd x Its conjugate = Rational number

E.g. $(2 + \sqrt{5}) \times (2 - \sqrt{5})$

$= 2(2 - \sqrt{5}) + \sqrt{5}(2 - \sqrt{5})$

$= 4 - 2\sqrt{5} + 2\sqrt{5} - 5$

$=$ $-2\sqrt{5}$ will cancel out $+ 2\sqrt{5}$

$= 4 - 5 = -1$

Shorter method of multiplying conjugate using difference of two squares

$(a + b)(a - b) = a^2 - b^2$

If we have

$(2 + \sqrt{5})\ (2 - \sqrt{5})$,

Let $2 = a$, $\sqrt{5} = b = 22 - (\sqrt{5})2 = 4 - 5 = 1$

Example: Solve $\dfrac{(2 + \sqrt{3})}{(3\sqrt{2} - 2\sqrt{5})}$

Solution: $\dfrac{(2 + \sqrt{3})}{(3\sqrt{2} - 2\sqrt{5})} = \dfrac{(2 + \sqrt{3})}{(3\sqrt{2} - 2\sqrt{5})} \times \dfrac{(3\sqrt{2} + 2\sqrt{5})}{(3\sqrt{2} \times 2\sqrt{5})}$

$$= \frac{2(3\sqrt{2} + 2\sqrt{5}) + \sqrt{3}(3\sqrt{2} + 2\sqrt{5})}{(3\sqrt{2})^2 - (2\sqrt{5})^2}$$

$$= \frac{(2 + \sqrt{3})(3\sqrt{2} + 2\sqrt{5})}{(3\sqrt{2} - 2\sqrt{5})(3\sqrt{2} + 2\sqrt{5})}$$

Let's solve the question piece by piece:

Numerator: $2 \times 3\sqrt{2} = 6\sqrt{2}$

$2 \times 2\sqrt{5} = 4\sqrt{5}$

$\sqrt{3} \times 3\sqrt{2} = 3 \times \sqrt{(3 \times 2)} = 3\sqrt{6}$

$\sqrt{3} \times 2\sqrt{5} = 2 \times \sqrt{(3 \times 5)} = 2\sqrt{15}$

Denominator: $(3\sqrt{2})^2 = 3 \times 3 \times \sqrt{2} \times \sqrt{2}$

$9 \times 2 = 18$

$(2\sqrt{5})2 = 2 \times 2 \times \sqrt{5} \times \sqrt{5}$

$= 4 \times 5 = 20$

So, we will have,

$$= \frac{6\sqrt{2} + 4\sqrt{5} + 3\sqrt{6} + 2\sqrt{15}}{18 - 20} = \frac{6\sqrt{2} + 4\sqrt{5} + 3\sqrt{6} + 2\sqrt{15}}{-2}$$

SURDS AND TRIGONOMETRIC RATIOS

From our knowledge of Trigonometry,

θ	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined

Example:

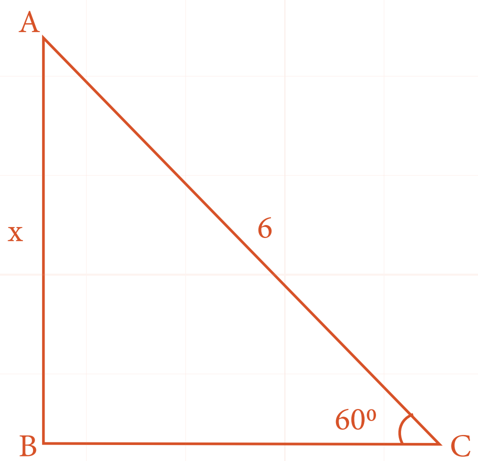
1. Evaluate (Sin 45^0 + Sin 30^0) in surd form.

- $\text{Sin } 45^0 = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $\text{Sin } 30^0 = \frac{1}{2}$

So, $\text{Sin } 45^0 + \text{Sin } 30^0 = \frac{\sqrt{2}}{2} + \frac{1}{2}$

$$= \frac{\sqrt{2} + 1}{2} = \frac{1 + \sqrt{2}}{2}$$

2. Find the length of x in simplified surd form



Using **SOH CAH TOA**

- **Sin ϴ** = $\frac{\text{opp}}{\text{hyp}}$ • **Cos ϴ** = $\frac{\text{Adj}}{\text{hyp}}$ • **Tan ϴ** = $\frac{\text{Opp}}{\text{Adj}}$

Sin $\mathbf{60}^0 = \frac{\mathbf{AB}}{\mathbf{AC}} = \frac{\mathbf{x}}{\mathbf{6}}$

Therefore, $x = 6 \sin 60^0$

Recall, Sin $60^0 = \frac{\sqrt{3}}{2}$, we will have:

$$x = 6 \times \frac{\sqrt{3}}{2} = \frac{6\sqrt{3}}{2}$$

$$x = 3\sqrt{3}$$

SUMMARY

So far, we have learnt how to

1. **Differentiate between rational and irrational numbers**
2. **Perform basic operations on surd numbers**
3. **Solve problems involving conjugate of binomial surds**
4. **Relate surds to trigonometric ratios**

INTERACTIVE ASSESSMENT QUESTIONS

1. Classify the expressions below into “Surds” and “not surds”

A $\sqrt{9}$

B $\sqrt{15}$
$2\sqrt{3}$

C $1/5$

D 0

2. Match each question to its correct answer

A	$4\sqrt{10}$	$2\sqrt{3} + 4\sqrt{3}$
B	$\frac{2}{3}\sqrt{3}$	Simplify $3\sqrt{8}$
C	$6\sqrt{6}$	$17\sqrt{2} - 15\sqrt{3}$
D	$6\sqrt{2}$	$5\sqrt{10} + 2\sqrt{10} - 3\sqrt{3}$
E	$2\sqrt{2}$	Simplify $\frac{2}{\sqrt{3}}$

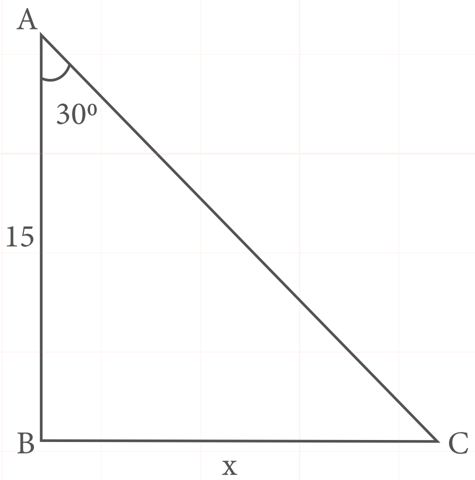
1. Classify the expressions below into “Surds” and “not surds”

   A $\quad 2 + \sqrt{5}$ is the conjugate of $5 + \sqrt{2}$

   B $\quad 2\sqrt{2} - 3\sqrt{7}$ is a binomial surd.

2. Match each question to its correct answer

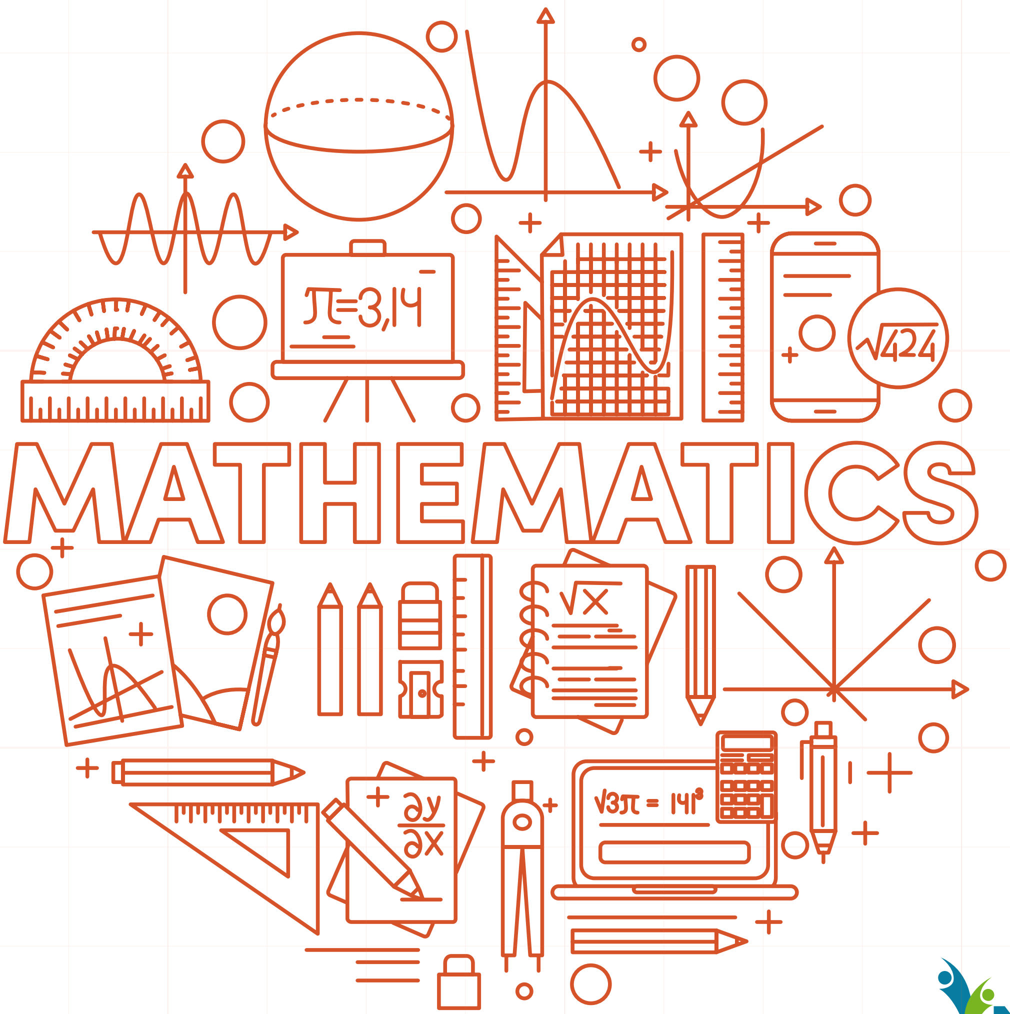
3. Choose whether the following is true or false



MATRICES AND DETERMINANTS

PERFORMANCE OBJECTIVES

1. Define matrix
2. Identify matrix notations and different types of matrices
3. Perform and solve problems on addition and subtraction of matrices
4. Perform multiplication, inverse and the transpose of a matrix
5. Calculate the determinant of a matrix
6. Use matrix to solve simultaneous equations.



LESSON

DEFINITION OF A MATRIX

Matrix is the rectangular arrangement of numbers, symbols and expressions into rows and columns. The arrangement from left to right is called ROWS while the arrangement from up to down is called COLUMNS

Fig 1.1 

Fig 1.2 $0 \quad 0 \quad 0 \quad 0$

Fig 1.3 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

The arrangement above is called a **MATRIX**

MATRIX NOTATION

A Matrix is denoted by an uppercase letter, say A, while its elements are denoted by small letters.
For instance

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

b_{11} means first row, first column
b_{12} means first row, second column
b_{13} means first row, third column
b_{21} means second, first column

To know the order of Matrix, we call it by the numbers of rows/columns contained therein. For instance, consider the following matrices below.

A. $\begin{bmatrix} 1 & 2 & 9 & 6 \end{bmatrix}$ B. $\begin{bmatrix} 1 \\ 2 \\ 9 \\ 6 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 2 \\ 6 & 9 \end{bmatrix}$

For A., the order is 1 x 4 because we have one row and four columns therein. For B., the order is 4 x 1 because there are four rows and one column.

TYPES OF MATRICES

1. Square Matrix

A square matrix has the same number of rows and columns.
E.g. 2 by 2, 3 by 3, etc.

$$\begin{bmatrix} 3 & 1 \\ 9 & 2 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & 0 \\ 9 & -2 & -3 \\ 1 & 8 & 5 \end{bmatrix}$$

2. Identity Matrix

An identity matrix is a square matrix that has 1 on its diagonal and 0 as the other elements. It is represented by the letter “I”

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Zero Matrix

This type of matrix has all its elements as zero. It is also called a Null Matrix

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4. Diagonal Matrix

This is a square matrix whose elements that are not its principal diagonal is zero

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

The determinant of a diagonal matrix is the product of the values in the principal diagonal. For instance, the determinant of the diagonal matrix above is

8 x –1 x 15 = –120

5. Triangular Matrix

This is a type of square matrix in which all entries either above or below the main diagonal is zero.

$$\begin{bmatrix} 7 & 7 & 9 \\ 0 & 8 & 4 \\ 0 & 0 & 5 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 7 & 0 & 0 \\ 4 & 8 & 0 \\ 9 & 7 & 5 \end{bmatrix}$$

ADDITION/SUBTRACTION OF MATRIX

You can only add or subtract two matrices of the same order.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Then $\mathbf{A} + \mathbf{B} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$

E.g. If,

$$\mathbf{A} = \begin{bmatrix} 1 & 9 & 8 \\ 2 & 7 & 0 \\ 11 & 8 & -1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 7 & 5 & -2 \\ 8 & 1 & 4 \\ 0 & 3 & 0 \end{bmatrix}$$

Find a. $\mathbf{A} + \mathbf{B}$ b. $\mathbf{A} - \mathbf{B}$

a. **A + B**

$$= \begin{bmatrix} 1+7 & 9+5 & 8+(-2) \\ 2+8 & 7+1 & 0+4 \\ 11+0 & 8+3 & -1+0 \end{bmatrix} = \begin{bmatrix} 8 & 14 & 6 \\ 6 & 8 & 4 \\ 11 & 11 & -1 \end{bmatrix}$$

b. **A – B**

$$= \begin{bmatrix} 1-7 & 9-5 & 8-(-2) \\ 2-8 & 7-1 & 0-4 \\ 11-0 & 8-3 & -1-0 \end{bmatrix} = \begin{bmatrix} -6 & 4 & 10 \\ -6 & 6 & -4 \\ 11 & 5 & -1 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

Two types of Multiplication

(1) Multiplication by a constant (scalar multiplication)

(2) Multiplication by another Matrix

(a) Scalar Multiplication

E.g.

If A $\begin{bmatrix} 7 & 0 \\ -3 & 5 \end{bmatrix}$

find 4A

Solution:

4A = 4 x A

$$4 \times \begin{bmatrix} 7 & 0 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 28 & 0 \\ -12 & 20 \end{bmatrix}$$

(b) Matrix – Matrix Multiplication

For you to be able to multiply matrix A by matrix B, i.e. A x B, the number of the column in A must be equal to the number of rows in B.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

The order of the resultant matrix C will take the form of the number of rows in matrix A by the number of columns in matrix B. For instance, if since matrix A has two rows and two columns, while matrix B has two rows and three columns, then matrix C will have two rows and three columns. So,

$$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

STEP 1

$$C_{11} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{X} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$C_{11} = a_{11} \times b_{11} + a_{12} \times b_{12}$$

STEP 2

C_{12} = First row in A and second column B

$$C_{12} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{X} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$C_{12} = (a_{11} \times b_{12}) + (a_{12} \times b_{22})$$

Therefore,

$C_{13} = (a_{11} \times b_{13}) + (a_{12} \times b_{23})$

Let's move to the second row

C_{21} = second row in A and second column in B

$$C_{21} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{X} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$C_{21} = (a_{21} \times b_{11}) + (a_{22} \times b_{21})$$

NEXT

$C_{22} = (a_{21} \times b_{12}) + (a_{22} \times b_{22})$

AND FINALLY,

$C_{23} = (a_{21} \times b_{13}) + (a_{22} \times b_{23})$

EXAMPLE:

If **A** = $\begin{bmatrix} 9 & -1 \\ 2 & 0 \\ 3 & 6 \end{bmatrix}$ and **B** = $\begin{bmatrix} 7 & -1 & 3 \\ -3 & 4 & 9 \end{bmatrix}$

Find (i) A x B (ii) B x A (iii) is A x B = B x A?s

SOLUTION

A x **B** = $\begin{bmatrix} 9 & -1 \\ 2 & 0 \\ 3 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & -1 & 3 \\ -3 & 4 & 9 \end{bmatrix}$

The highest number of rows to columns is 3 and 3 so,
The resultant matrix will have 3 rows and 3 columns.

C = $\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$

$$\begin{bmatrix} (9 \times 7)+(-1 \times -3) & (9 \times -1)+(-1 \times 4) & (9 \times 3)+(4 \times 9) \\ (2 \times 7)+(0 \times -3) & (2 \times -1)+(0 \times 4) & (2 \times 3)+(0 \times 9) \\ (3 \times 7)+(6 \times -3) & (3 \times -1)+(6 \times 4) & (3 \times 3)+(6 \times 9) \end{bmatrix}$$

$$= \begin{bmatrix} (63+3) & -9+(-4) & 27+36 \\ 14+(-3) & -2+0 & 6+0 \\ 21+(-18) & -3+24 & 9+54 \end{bmatrix}$$

$$= \begin{bmatrix} 66 & -13 & 63 \\ 11 & -2 & 6 \\ 3 & 21 & 63 \end{bmatrix}$$

If you try solving B X A on your own, you will notice that B X A is not equal to A x B. So, we need to be careful which matrix comes first when multiplying.

TRANSPOSE OF A MATRIX

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The transpose of A, A^T, will have its rows swapped with its columns and vice versa.

For instance,

Then, $\mathbf{A} = \begin{bmatrix} 4 & 2 & 9 \\ 7 & -1 & 3 \end{bmatrix}$ Then, $\mathbf{A^T} = \begin{bmatrix} 4 & 7 \\ 2 & -1 \\ 9 & 3 \end{bmatrix}$

DETERMINANT OF A 2 x 2 MATRIX

Given a matrix

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The determinant A, or det A is (a x d) – (b x c) = ad - bc

$$\mathbf{R} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Blue means positive, Red means negative. The determinant of a matrix will result in a constant.

Example: find the determinant of the matrix.

$$\mathbf{R} = \begin{bmatrix} 1 & 8 \\ -9 & 5 \end{bmatrix}$$

$$\begin{aligned} \text{Det A or } |A| &= (1 \times 5) - (8 \times -9) \\ & 5 - (-72) \\ &= 5 + 72 \\ &= 77 \end{aligned}$$

INVERSE OF A MATRIX

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Finding the inverse of a matrix requires three simple steps:
After finding the determinant

- Swap the positions of a and d
- Put negatives in front of b and c
- Divide everything by the determinant (ad - bc)

The inverse of a matrix A is denoted as A^{-1}

Note: $A \times A^{-1} = A^{-1} \times A = I$

Example: Find the inverse of;

$$\mathbf{A} = \begin{bmatrix} 2 & 9 \\ 4 & 1 \end{bmatrix}$$

Solution:

Determinant = (2 x 1) – (9 x 4)

= 2 – 36

= –34

STEP 1: Swap the position a and d

$$\begin{bmatrix} 1 & \\ & 2 \end{bmatrix}$$

STEP 2: Put negatives in front of b and d

$$\begin{bmatrix} 1 & -9 \\ -4 & 2 \end{bmatrix}$$

STEP 3: Divide by the determinant

$$\mathbf{A}^{-1} = \frac{\begin{bmatrix} 1 & -9 \\ -4 & 2 \end{bmatrix}}{-34} \quad \text{or} \quad \frac{-1}{34}\begin{bmatrix} 1 & -9 \\ -4 & 2 \end{bmatrix}$$

This is the inverse. To check if we are correct, we multiply it by the original matrix A That is, A^{-1} x A

$$= \frac{-1}{34}\begin{bmatrix} 2 & -9 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 9 \\ 4 & 1 \end{bmatrix}$$

$$= \frac{-1}{34}\begin{bmatrix} (1 \text{ x } 2)+(-9 \text{ x } 4) & (1 \text{ x } 9)+(-9 \text{ x } 1) \\ (-4 \text{ x } 2)+(2 \text{ x } 4) & (-4 \text{ x } 9)+(-4 \text{ x } 1) \end{bmatrix}$$

$$= \frac{-1}{34}\begin{bmatrix} 2-36 & 9-9 \\ -8+8 & -36+2) \end{bmatrix}$$

$$= \frac{-1}{34}\begin{bmatrix} -34 & 0 \\ 0 & -34) \end{bmatrix}$$

$$= \frac{-1}{34}\begin{bmatrix} -34/-34 & 0/-34 \\ 0/-34 & -34/-34 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$,which is the identity element **I**.

NOTE: WE CANNOT FIND THE INVERSE OF A SINGULAR MATRIX. THIS IS BECAUSE THE DETERMINANT OF A SINGULAR MATRIX IS = 0

USING MATRIX TO SOLVE SIMULTANEOUS EQUATIONS

Example: Solve the simultaneous equation using matrix

$2x + y = 5$

$3x + y = 7$

Solution

$2x + y = 5$

$3x + y = 7$

Can be written in matrix form as:

Soln, $\underset{\mathbf{A}}{\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}} \underset{\mathbf{X}}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{\mathbf{B}}{\begin{bmatrix} 5 \\ 7 \end{bmatrix}}$

Now, if we have A x X = B

To find x, we say B/A, which is also A^{-1} X B

In matrix, A^{-1} is called the inverse of A. So, we need to find the inverse of A.

Recall, if

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{Then, } \mathbf{A}^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{\text{Det}}$$

So, if we have

$$= \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$\begin{aligned} \text{Det } \mathbf{A} &= (2 \times 1) - (1 \times 3) \\ &= 2 - 3 \\ &= -1 \end{aligned}$$

$$\text{So, } \mathbf{A}^{-1} = \frac{\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}}{-1} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

Let's move on ....., So, $x = A^{-1} \times B$

Soln, $= \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (-1 \times 5)+(1 \times 7) \\ (3 \times 5)+(-2 \times 7) \end{bmatrix} = \begin{bmatrix} -5 + 7 \\ 15 - 14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Therefore, $\mathbf{x} = 2$, $\mathbf{y} = 1$

SUMMARY

So far, we have learnt how to

1. Define Matrix
2. Identify Matrix Notations
3. Identify different types of Matrices
4. Perform and solve problems on addition and subtraction of Matrices
5. Perform Multiplication of Matrix
6. Find the Transpose of a Matrix
7. Calculate the Determinant of a Matrix
8. Find the Inverse of a Matrix
9. Use Matrix to solve Simultaneous equation

INTERACTIVE ASSESSMENT QUESTIONS

1. Which of the following statements are true?

A. A matrix is a circular arrangement of numbers, symbols and expression

B. The arrangement from left to right is called ROWS

C. The arrangement from left to right is called COLUMNS

D. A matrix is a rectangular arrangement of numbers, symbols and letters.

E. The arrangement from up to down is called COLUMNS

1. The order of matrix C is _ by _

$$\mathbf{C} = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$$

2. What type of Matrix is this?

$$\mathbf{C} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A	Diagonal Matrix
B	Triangular Matrix
C	Middle Matrix
D	Zero Matrix

3. Solve the matrix below. Then match the options to their corresponding correct answer.

$$\begin{bmatrix} 3 & 12 & 5 \\ 13 & 16 & 7 \\ 8 & -4 & -9 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 18 \\ 2 & -5 & -10 \\ 4 & 3 & 0 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & -7 & -9 \end{bmatrix}$$

A	4
B	17
C	2
D	21
E	9
F	11

Choose the correct option.

If, **B** = $\begin{bmatrix} 1 & 0 & 1 \\ 9 & -8 & 3 \\ 7 & 2 & 1 \end{bmatrix}$ Then, $\mathbf{B}^{\mathbf{T}}$ =

A = $\begin{bmatrix} 1 & 9 & 7 \\ 0 & -8 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ **B** = $\begin{bmatrix} 1 & 0 & 1 \\ 7 & 2 & 1 \\ 9 & -8 & 3 \end{bmatrix}$

C = $\begin{bmatrix} 1 & 3 & 1 \\ 0 & -8 & 2 \\ 1 & 9 & 7 \end{bmatrix}$ **D** = $\begin{bmatrix} 1 & 9 & 7 \\ 1 & 3 & 1 \\ 0 & -8 & 2 \end{bmatrix}$

If **M** = $\begin{bmatrix} -1 & -1 \\ 8 & 9 \end{bmatrix}$ Then the inverse of **M**, $\mathbf{M}^{-1}$ is,

A = $\begin{bmatrix} -1 & -1 \\ 8 & 9 \end{bmatrix}$ **B** = $\begin{bmatrix} 9 & -1 \\ 8 & 1 \end{bmatrix}$ **C** = $\begin{bmatrix} -9 & -1 \\ 8 & 1 \end{bmatrix}$

Solve the following simultaneous equation using matrix and find Det **A** , **x** and **y**

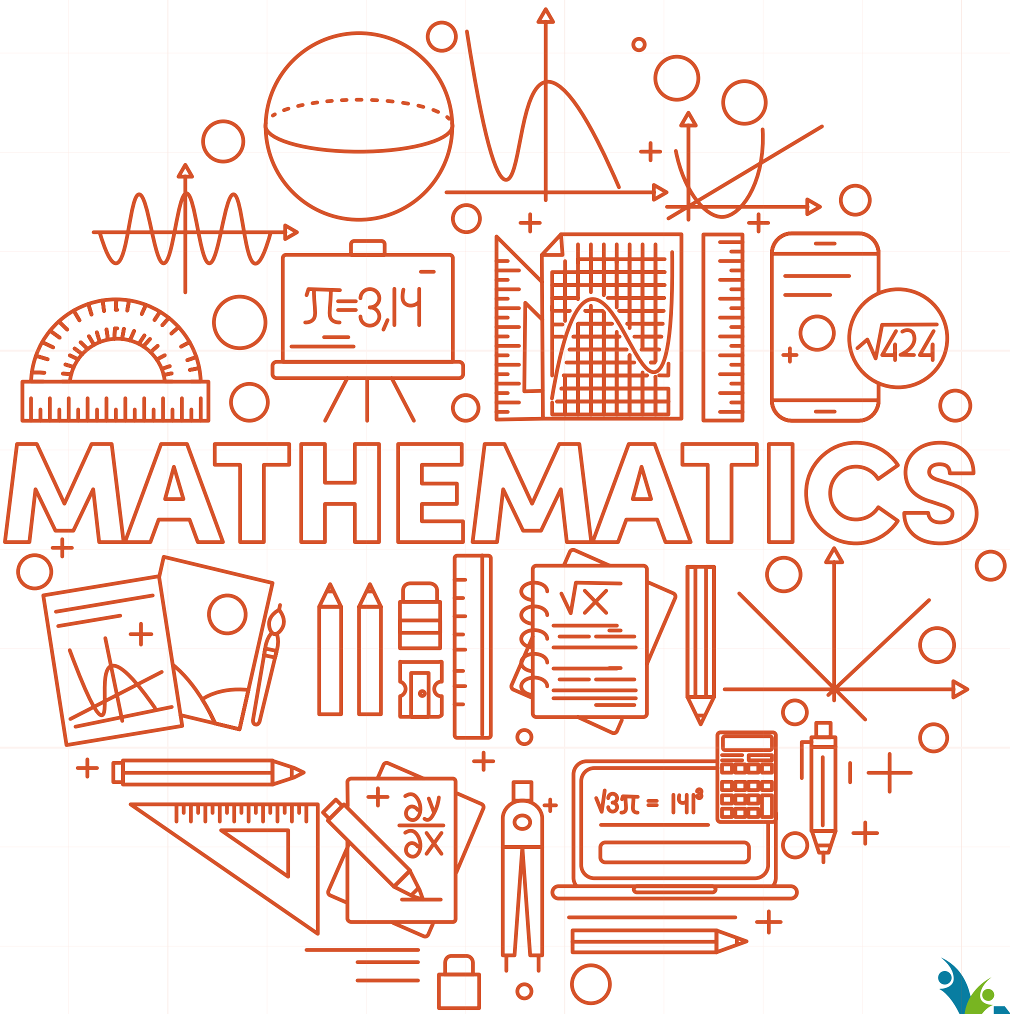
$x - 3y = 7, \quad x + y = 11$

Det **A** = _ , **x** = _ and **y**= _

LOGARITHM

PERFORMANCE OBJECTIVES

1. State the laws of logarithm
2. Use logarithm tables to solve problems involving calculations



LESSON

LAWS OF LOGARITHM

Recall, in indices,

If $10^1 = 10$, then $\log_{10}10 = 1$

If $10^2 = 100$, then $\log_{10}100 = 2$

Also, if $a^n = x$, then $\log_a x = n$

Consider; $\text{Log}_a x$

x is called the "Argument", a is called the "Base"

Also, if we have

23.4567

The brown part is called the characteristic while the blue part is called the mantissa

LOGARITHM LAW CHART

01	$\log_a x + \log_a y = \text{Log}_a xy$
02	$\log_a x - \log_a y = \log_a x/y$
03	$\log_a x^y = y\log_a x$
04	$1 = \text{Log}_a a$
05	$0 = \text{Log}_a 1$

VERIFICATION OF LAWS OF LOGARITHM

Law 1: $\log_a x + \log_a y = \text{Log}_a xy$

E.g: $\log_2 6 = \log_2(3 \times 2) = \log_2 3 + \log_2 2$ – *Here, multiplication becomes addition.*

Law 1: $\log_a x + \log_a y = \text{Log}_a xy$

E.g: $\log_2 6 = \log_2(3 \text{ X } 2) = \log_2 3 + \log_2 2$ – *Here, multiplication becomes addition*

Law 2: $\log_a x - \log_a y = \log_a x/y$

E.g: $\log_3 8/5 = \log_3 8 - \log_3 5$ – Here, division becomes subtraction

Law 3: $\log_a xy = y\log_a x$

E.g: $\log_3 2^5 = 5\log_3 2$ – Here, the power becomes the coefficient.

Subsequently, $\log_a \sqrt{x} = \log_a x^{\frac{1}{y}}$

$= 1/y \ \log_a x$

Also, $\log_a 1/x = \log_a x^{-1}$

$= -1 \times \log_a x$

$= -\log_a x$

E.g: $\log_3 1/5 = \log_3 5^{-1}$

$= -\log_3 5$

Law 4: $\log_a a=1$ $\{ \text{ since } a^1 = 0\}$

E.g: $\log_2 2 =1$

Law 5: $\log_a 1 = 0$ $\{\text{since } a^0 = 1 \}$

E.g: $\log_5 1 = 0$

USING LOGARITHM TABLES FOR CALCULATION

In order to find the logarithm of numbers, we need to have a log table. For this lesson, we would be using logarithm table 10. A snippet is shown below:

LOGARITHM SOLUTION CHART

											MEAN DIFFERENCE								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	37
12	1792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18

E.g: Find (a) log 215.2; (b) log 1.083

Solution

(a) Find log 215.2

Step 1: Choose the correct logarithm table. To find log ax you will need $\log_a$ table. Most common log tables are base 10 logarithms. Now, $\log_{10}215.2$ needs a $\log_{10}$ table which is shown above.

Step 2: Ignoring the decimal places,

- Look for the row labelled with the first two digits (21)
- Look for the column labelled with the third digit (5)
- Look at the intersection of (i) and (ii) which is 3324

An illustration is shown in the table on the next page.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	37
12	1792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18

Step 3: Look at the mean difference table. Still on the row from step 2, slide your finger over to the column in the table marked with the next digit of the number you are looking up. In this case, from row 21 column 5, slide your finger in the same row to the column that has heading 2.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	37
12	1792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18

The number in the intersection is 4.

Step 4: Add up the answer you got in Step 2 with Step 3. We will have 3324 + 4 = 3328

Step 5: Prefix a decimal point

.3328

Step 6: Look at the characteristic part, i.e. the digits at the left-hand side to the equation. There are three digits in the characteristic. Subtract one, i.e. 3-1 = 2

Step 7: Write 2 at the left side of the decimal in the answer you got in Step 5. = 2.3328

This is your final answer.

Therefore, log 215.2 = 2.3328

Example 2: Find log 1.083

Solution

Ignoring the decimals, the digits are 1083.

So, we look at the intersection of row 10, column 8. An illustration is shown below:

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	37
12	1792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18

The value gotten is **0334**

Next, from the same row, we slide our fingers to the mean difference section under column 3. An illustration is shown below.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	37
12	1792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18

The value gotten in the intersection is 12.

So, we add it to the value gotten from the previous step

0334 + 12 = 0346

Next, we prefix a decimal point .0346

We look at the characteristic in the original question = 1.083

There is only one digit there. 1-1=0

So, we will put 0 at the left-hand side of the decimal gotten

i.e. **0.0346**, This is our answer.

Therefore, log 1.083 = 0.0346

Note that if you are asked to find the log of more than four digits, you should round it up to four digits for easy calculation. For example, if you were told to find log 36.4886, there are six digits in all. We have to make it four digits so we approximate it to two decimal places which is 36.49. We can easily find the logarithm of 36.49.

FINDING THE ANTILOG OF LOGARITHM NUMBERS

We have learnt how to use logarithm table to find the logarithm of numbers. Our knowledge is not complete if we do not know how to find the antilog of numbers. For instance, if log 215.2 is 2.3328, then the antilog of 2.3328 is 215.2. Now, how do we do that using the antilog table?

First, we need to look at the antilog table. An illustration is given below:

ANTILOG SOLUTION CHART

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
20	1585	1589	1592	1596	1600	1603	1607	1611	614	1616	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3

Example: If log x = 2.1965, find x

To find x, we need to find the antilog of 2.1965

In finding the antilog,

Step 1: Look at the mantissa. The digits in the mantissa is .1965

Just the way we did in logarithm, we look for the intersection of row .19 column 6, just the way we did it when finding the logarithm of a number. An illustration is shown below

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
20	1585	1589	1592	1596	1600	1603	1607	1611	614	1616	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3

Step 3: Look at the mean difference table. Still on the row from step 2, slide your finger over to the column in the table marked with the next digit of the number you are looking up. In this case, from row 21 column 5, slide your finger in the same row to the column that has heading 2.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
20	1585	1589	1592	1596	1600	1603	1607	1611	614	1616	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3

The number in the intersection is 4.

Step 4: Add up the answer you got in Step 2 with Step 3. We will have 3324 + 4 = 3328

Step 5: Prefix a decimal point

.3328

Step 6: Look at the characteristic part, i.e. the digits at the left-hand side to the equation. There are three digits in the characteristic. Subtract one, i.e. 3-1 = 2

Step 7: Write 2 at the left side of the decimal in the answer you got in Step 5. = 2.3328

This is your final answer.

Therefore, log 215.2 = 2.3328

Example 2: Find log 1.083

Solution

Ignoring the decimals, the digits are 1083.

So, we look at the intersection of row 10, column 8. An illustration is shown below:

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1 324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	15 56	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	614	1616	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3

The value gotten is .1570

Step 2: Next, we slide our fingers from row .19 up to the mean difference section and stop at column 5. The value gotten is 2. An illustration is given below.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	13 90	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	614	1616	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3

Step 3: We add the value gotten in Step 1 to the one gotten in step 2

1570 + 2 = 1572= .1572

Step 4: Next, look at the characteristic of the original question. The number there is 2. We add 1 to 2 (since in logarithm, we subtracted, we will have to add 1 in antilog.)

2 + 1 = 3

Step 5: Move the decimal three places to the right. We will have 157.2

This is our final answer

Note: While moving the decimal in step 5, if the movement exceed toe digit we have, we add zeros to the empty place.

For instance, if we have .7876

And we want to move it to six decimal places to the right, it becomes .787600

USING LOG TABLES TO SOLVE PROBLEMS

1. Use log tables to calculate:

$$\mathbf{X} = \frac{78.4}{253.4}$$

Soln

$$\text{Log } x = \log \frac{78.4}{253.4}$$

$= \log 78.4 - \log 253.4$

$= 1.8943 - 2.4038$

$= (1 + 0.8943) - (2 + 0.4038)$

$= (1 + .8943) - (2 + .4038)$

$= (1 - 2) + (.8943 - .4038)$

$= -1 + .4095$

$= \bar{1}.4095$

Since $\log x = \bar{1}.4095$, then x = antilog of $\bar{1}.4095$

The antilog of 4095 is .3094

Looking at the characteristic we have $\bar{1}$ which means –1. We add 1 (Remember in logarithm, we subtracted 1, so in antilog, we will do the opposite which is to add 1.

That is, $-1 + 1 = 0$.

This means we are moving the decimal point 0 times to the right.

Our final answer will be **0.3094.**

Now to confirm if we are correct, punch the original question in your calculator and you will find out that you get the same answer.

2. Use log tables to calculate:

$$\mathbf{X} = \frac{2.535}{785}$$

Soln

Log x = log 2.534 – log 785

= 0.4036 – 2.8948

= (0 + .4036) – (2 + .8948)

Collect like terms

(0 – 2) + (.4036 - .8948)

= – 2 – .4910

If you look closely at the answer here, you will see that there is a minus sign in the middle. This is not proper. We have to replace the minus sign in the middle with a plus sign so that we can go on in our calculations. To do this, we put a (+1 –1) in the middle. Look below:

–2+1–1– **.4910**

Since –1+1 = 0 it did not change the answer. We only put it there in order to change the minus sign to plus. Grouping the expression above, we will have

(–2–1) (+1–.4910)

= –3+ .509

= $\bar{3}.509$

Therefore, log x = 3.509

X = Antilog of 3.509 or $\log^{-1} \bar{3}.509$

To find the antilog of $\bar{3}.509$ we look at the intersection of row 50 column 9 in the antilog table.

It will give us .3228

There is no other digit remaining at the right-hand side so we stop there.

Next, we look at the characteristic which has $\bar{3}$.

We add 1 which is –3 + 1 = –2.

-2 means we move the decimal two times to the left.

Our final answer will be 0.003228

Therefore, 2.535/785 = **0.003228**

If you punch the same numbers in your calculator, you will get the answer to e 0,003229 which is very close to the answer.

SUMMARY

So far, we have learnt how to

1. Verify laws of logarithm
2. Use logarithm tables to solve problems involving calculations

INTERACTIVE ASSESSMENT QUESTIONS

1. Simplify $\log_{10}3 + \log_{10}5$

A $\quad$ $\text{Log}_{10}15$

B $\quad$ $\text{Log}_{10}8$

C $\quad$ $\text{Log}_{10}35$

D $\quad$ $\text{Log}_{10}3.5$

E $\quad$ $\text{Log}_{10}2$

2. Simplify log 3y – log y

A Log $3y^2$
B Log 2y
C Log 3
D Log 4y

3. $\text{Log}_5 5^2$ can be written as

A Log 10
B 2
C 1
D 25

4. If log x = 1. 1257, then the value of x is,

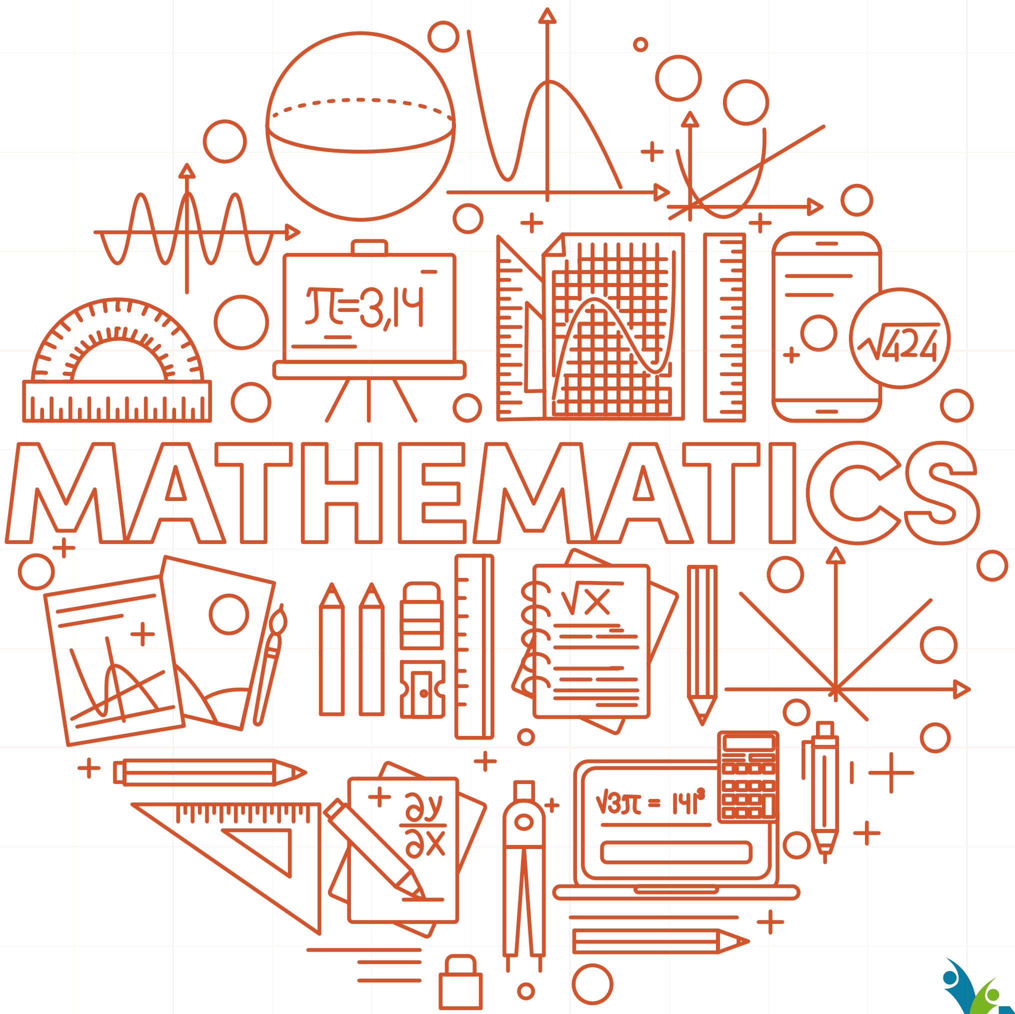
A 1.336
B 13.36
C 12.96
D 1.296
E 11.257

5. Use log tables to solve $\mathbf{X} = \dfrac{12.56}{4.097}$

ARITHMETIC OF FINANCE

PERFORMANCE OBJECTIVES

1. Calculate simple interest given the principal, rate and time
2. Calculate compound interest using the formula
3. Determine the depreciation value of an item
4. Calculate the annuities of a given problem
5. Compute the amortization of a given problem
6. Solve further problems in capital market using logarithm table



LESSON

INTRODUCTION

How do banks or financial institutions make money? Well, in many ways. However, in this topic we will be looking at a way – through interest on loans and investments

Say Mr. Kenneth, a businessman, goes to the bank to apply for a loan. He gets a loan of #100,000 to pay back after a period of time. Now, if he is to pay the money back, he cannot pay the same #100 000. The bank will charge him a percentage of the amount he borrowed. So, when he is returning the money, he will return the initial capital plus the extra charge on top. The extra is called the interest.

There are two types of interest viz: (a) simple interest (b) compound interest

SIMPLE INTEREST

Recall, simple interest is the product of the principal, rate and time.

i.e. **S.I = P x R x T**

For instance, if you are asked to find the interest of #20 000 on 5% for two years, your interest will be:

$$\mathbf{I} = \frac{20\ 000 \times 5 \times 2}{100} \qquad \text{Hence } \mathbf{I} = \#2\ 000$$

COMPOUND INTEREST

Unlike simple interest where interest is calculated at the end of the time period, compound interest is calculated each year till the time elapses.

The amount at the end of the time period (A) or future value is the sum of the compound interest and the principal

The amount is calculated as

$$\mathbf{A} = \mathbf{P}\left(1 + \frac{r}{n}\right)^{nt}$$

Therefore the compound interest is given as

C.I = A – P

So, to calculate compound interest, first find the amount (A), then subtract the principal from your amount.

Where P = initial principal (initial deposit or loan amount)

r= annual interest rate (decimal)

n= number of times the interest is compounded per unit time.

t= the timeframe the money is invested or borrowed for.

Example: If an amount of #10 000 is deposited into a savings account at an annual interest rate of 6%, compounded monthly, the value of the investment after 10 years can be calculated as follows:

P= initial deposit = #10 000

r= interest rate = 6% = 6/100 = 0.06

n= number of times the interest is compounded per time. Since it is compounded monthly and we have 12 months in a year, then n=12

t= time= 10 years

Putting these parameters into the equation, we will have:

$$\mathbf{A} = \mathbf{P}\left(1 + \frac{r}{n}\right)^{nt} \qquad \mathbf{A} = \mathbf{10\ 000}\left(1 + \frac{0.06}{12}\right)^{12 \times 10}$$

$A = 10\ 000(1 + 0.005)^{120}$ $= 10\ 000(1.005)120$ $=10\ 000(1.819396734)$

$=$ **18193.97**

This is the future value of the money invested, also called the amount. To get the compound interest, subtract the principal from the amount, that is:

#18 193.97 - #10 000

= **#8 193.97**

DEPRECIATION

Sola buys a phone for **#100,000** (one hundred thousand naira). Two years later, she goes to a phone store to price her used phone. The price of her phone now costs **#58,000.** You will notice that the value of the phone has reduced due to use, hence the reduction in price. The decrease is termed depreciation. Depreciation is associated with tangible assets like Land, machinery, building furniture, etc.

$$\text{Depreciation Expense} = \frac{\text{Asset Cost-Salvage Value}}{\text{Useful Life span of Asset}}$$

Cost of the asset is the purchasing price of the asset, i.e. the amount that was paid for the asset. Salvage value is the value of the asset at the end of its usefulness. Useful life of asset represent the time frame the asset is used or expected to be used.

Now, can we calculate the depreciation of Sola's phone?

$$\text{Depreciation Expense} = \frac{\text{Asset Cost-Salvage Value}}{\text{Useful Life span of Asset}}$$

$$= \frac{100\ 000-58\ 000}{2} = \frac{42\ 000}{2}$$

$= 21{,}000$ naira

This means that every year, the value of Sola's phone reduces by **#21,000**

Let's look at one more example:

Example:

Company A purchases a machine for #500 000 with an estimated salvage value of #40 000 and a useful life of 10 years

$$\text{Depreciation Expense} = \frac{\text{Asset Cost-Salvage Value}}{\text{Useful Life span of Asset}}$$

$$= \frac{50\ 000 - 40\ 000}{10} \qquad = \mathbf{\#46\ 000}$$

Therefore, company A would depreciate the machine at the amount of **#46 000** monthly

ANNUITY

Annuity is a financial product that entitles a person to certain cash flows at equal time intervals. These products are created by financial institutions primarily life insurance companies, to provide regular income to a client.

In other words, an annuity is a sequence of payments paid or received at equal intervals of time.

Annuities are calculated in two basic ways:

Present Value: This is the amount you pay in order to receive payments in batches later in the future

Future Value: This is the amount you are to receive in the future in batches

$$\text{Present value (P.V)} = \frac{1 - (1 + i)^{-n}}{i} \times R$$

$$\text{Future value (F.V)} = \frac{(1 + i)^{n} - 1}{i} \times R$$

Calculate the present value of a 12-year ordinary annuity of #10 000 per year. The interest rate is 6 percent.

Solution

$$\text{Present value (P.V)} = \frac{1 - (1 + 6\%)^{-12}}{6\%} \times 10{,}000$$

$$= \frac{1 - (1.06)^{-12}}{0.06} \times 10{,}000 = \frac{0.49693636}{0.06} \times 10{,}000$$

$$= \frac{0.530306364}{0.06} \times 10{,}000 = \#8.38384394 \times 10\,000$$

= #83838.44

AMORTIZATION

Amortization refers to the reduction of a debt or loan over time by paying the same amount each period, usually monthly.

$$\mathbf{A} = \mathbf{P}\left(\frac{i(1+i)^{n}}{(1+i)^{n} - 1}\right)$$

A= Periodic payment amount
P= amount of principal, net of initial payment, meaning “subtract any down payment”

i= period interest rate
n= total number of payments
Example: What is the monthly payment for a #4,600 two year loan with an APR(annual payment rate) of 8.25%

Solution

P= #4,600
i= (8.25/12)% = 0.006875(since we are looking for monthly payment, we have to divide the annual rate by 12)
n= 2yrs X 12 = 24 months (since it is monthly)

$$\mathbf{A} = \mathbf{P}\left(\frac{i(1+i)^n}{(1+i)^n - 1}\right)$$

$$\mathbf{A} = \mathbf{4,600}\left(\frac{0.006875(1+0.006875)^{24}}{(1+0.006875)^{24}-1}\right) = \left(\frac{31.625 \text{ X } 1.1787}{0.1787}\right)$$

$$= \left(\frac{37.2763875}{0.1787}\right) = \mathbf{\#208.59}$$

USE OF LOG TABLES TO SOLVE PROBLEMS IN CAPITAL MARKET

Do you know that log tables can be used to solve these questions we have had? Alright! Let's use log table to solve the example we had on compound interest.

This was our solution:

$$A = \mathbf{P}\left(1 + \frac{r}{n}\right)^{nt} \qquad A = \mathbf{10{,}000}\left(1 + \frac{0.06}{12}\right)^{12 \times 10}$$

$$= 10\,000(1 + 0.005)^{120}$$

$$= 10\,000(1.005)^{120}$$

$$= 10\,000(1.819396734)$$

$$= \mathbf{18193.97}$$

Using log tables to perform the calculation (say, from line 4)

A = 10 000(1.005)120

Log A = log 10 000(1.005)120

= log 10000 + log1.005120 (using law 1)

= log 10000 + [120 log 1.005] (from law 3)

= 4 + 0.2599

Log A = 4.2599, hence A = log-1 4.2599 or the antilog of 4.2599 = 18200 which is the approximate value of **18193.97**

MALEZI

EMPOWERING THOSE WHO CARE

SUMMARY

So far, we have learnt how to

1. Calculate simple interest
2. Calculate compound interest using the formula
3. Determine the depreciation value of an item
4. Calculate the annuities of a given problem
5. Compute the amortization of a given problem
6. Solve further problems in capital market using logarithm table

INTERACTIVE ASSESSMENT QUESTIONS

1. The simple interest on #5,000 for 3 years at 2%interest is

A #500
B #750
C #90,000
D #150
E #300

2. What is the compound interest of #12,000 compounded biannually at the rate of 5% per year for three years?

A #1,916.32
B #13,916.32
C #12,000
D #1,000

3. Mrs. Martins bought a new pair of shoes at #15,000. Three years later she sells the shoe for #6,000, Calculate the depreciation expense of the shoe.

   A #6,000
   B #3,000
   C #5,000
   D #9,000

4. How much money must be deposited now at 6% interest compounded quarterly in order to be able to withdraw #3 000 at the end of each quarter year for two years? (Approximate your answer to the nearest ten)

   A #22460.259
   B #22460
   C #22450
   D #22450.77

5. Halima borrows #200,000 at 10% annual interest on a 5 year loan. What is her monthly payment? Approximate your answer to two decimal places.

   A 4249.03
   B 2025814.35
   C 121.59

THEME 02

Numbers and Numeration.

Algebraic Process.

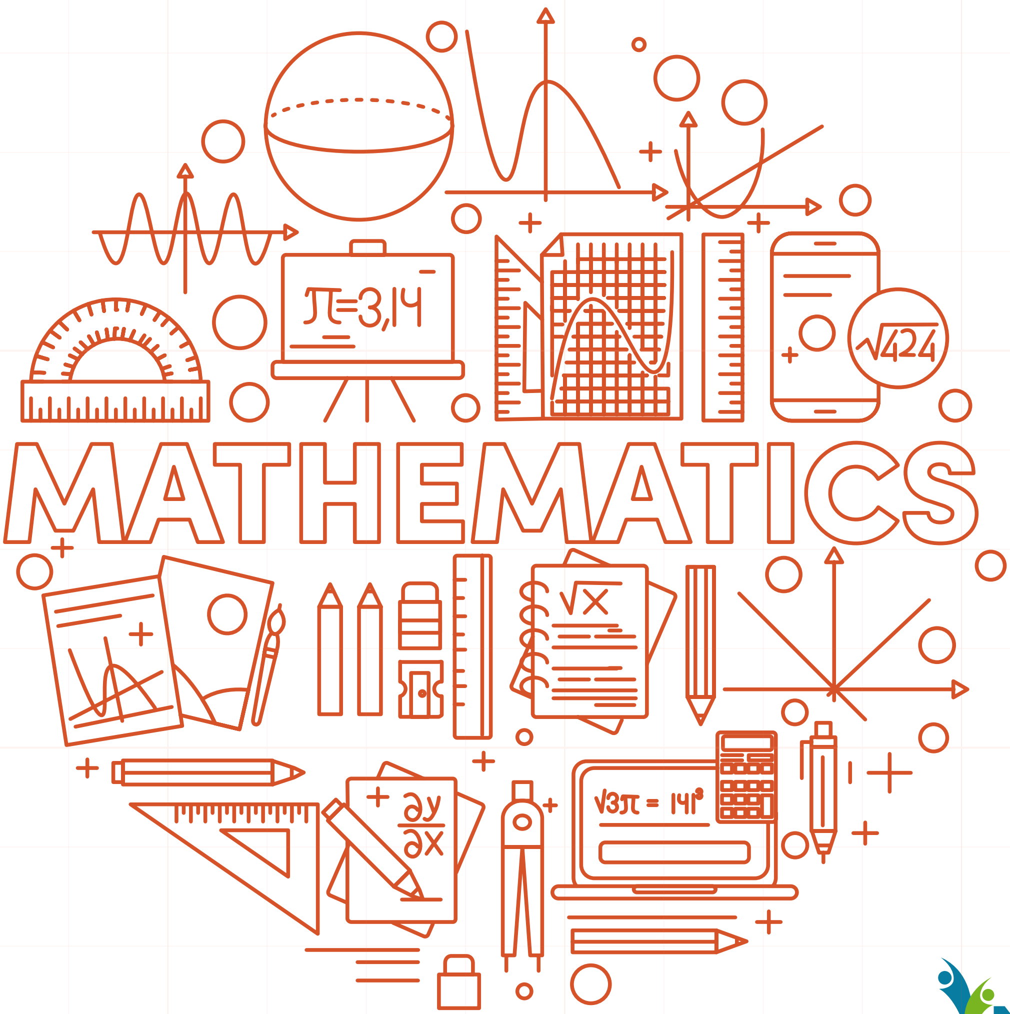
Geometry.

Introductory Calculus.

APPLICATION OF LINEAR AND QUADRATIC EQUATIONS TO CAPITAL MARKET

PERFORMANCE OBJECTIVES

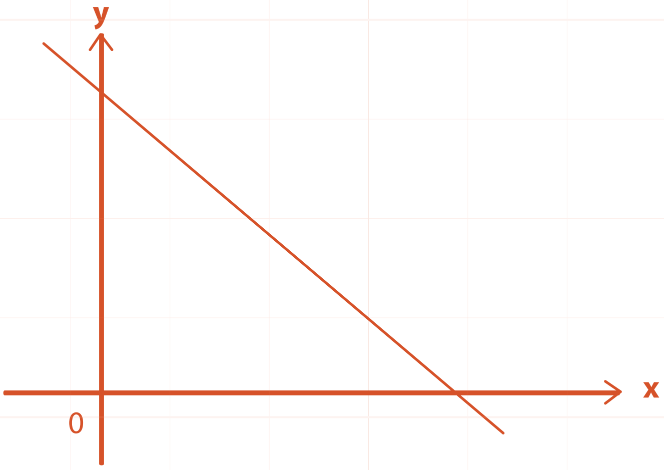
1. Solve Simultaneous Linear And Quadratic Equations.
2. Solve Word Problems On Linear, Quadratic, And Simultaneous Linear And Quadratic Equation
3. Solve Problems On A Linear Equation Involving Capital Markets.

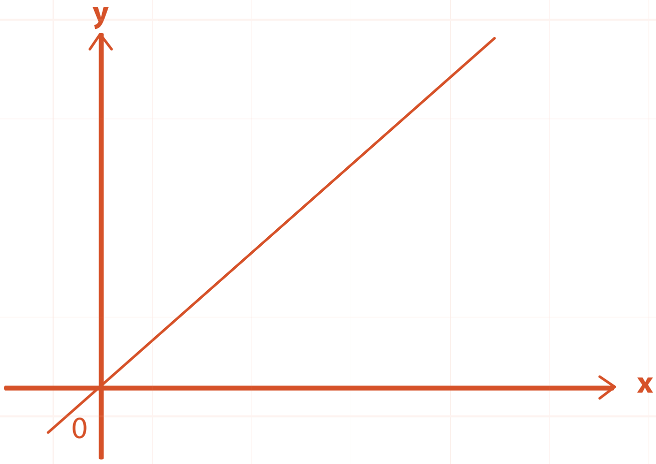


LESSON

GRAPH OF LINEAR EQUATIONS

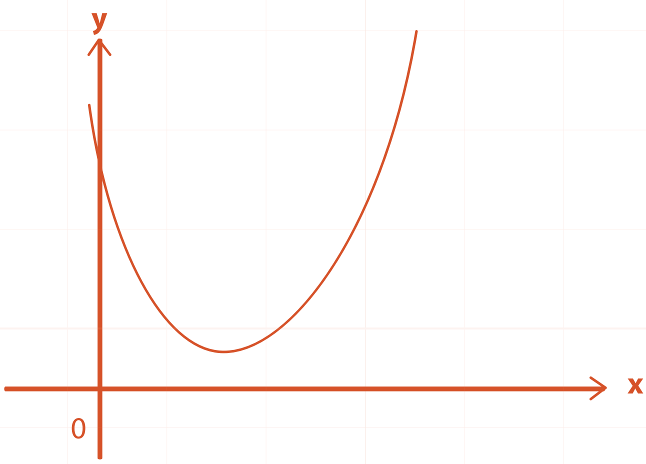
An equation is a complete mathematical statement, like $x+2 = 5$. A linear equation is an equation whose highest degree is 1, like $2x+5=7$. A linear equation forms a straight line on a graph.

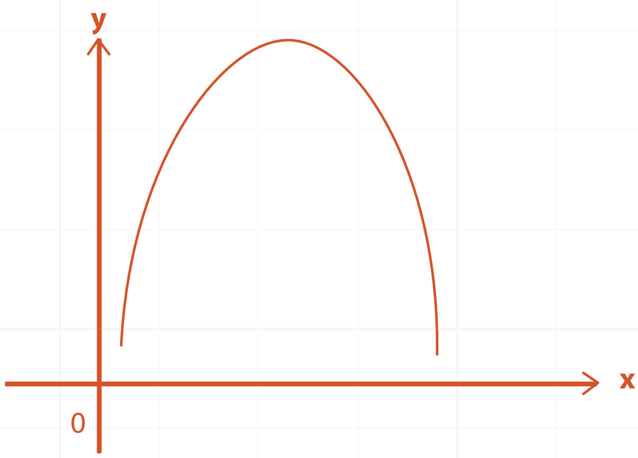




GRAPH OF QUADRATIC EQUATIONS

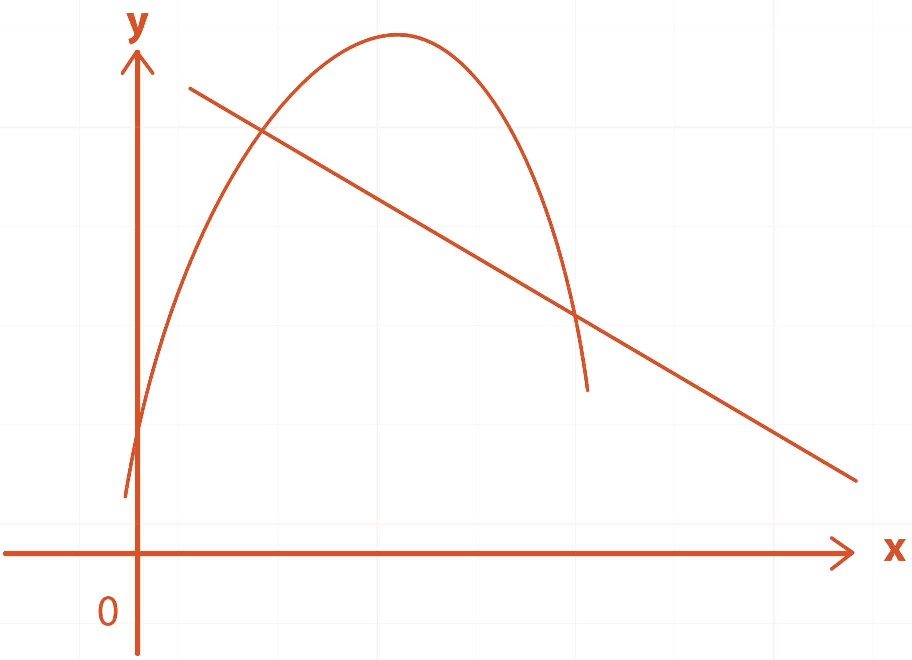
A quadratic equation is an equation whose highest power is 2, like $x^2 - 3x + 5 = 13$. A quadratic equation forms a curve like a parabola on a graph.





GRAPH OF LINEAR/QUADRATIC EQUATION

Putting a linear and quadratic equation together, they will look like this:



Example

Solve these two equations:

$y = x^2 + 2x - 6$ ----(1)

$y = 3x - 8$ -------(2)

We can equate (1) and (2) since $y = x^2 + 2x + 6$ and the same $y - 3x + 8$.

This means that $x^2 + 2x + 6 = 3x + 8$

Collect like terms. Move each term to one side. Leave the right-hand side to be 0.

$x^2 + 2x - 3x + 6 - 8 = 0$, Hence $x^2 - x - 2 = 0$

Factorizing the equation , we get

$(x -2) (x - 1) = 0$

$x = 2$ or $x = -1$

From equation (2),

$Y = 3x + 8$

When $x = 2$ $y = 3(2) + 8$ and $y = 6 + 8 = 14$

Therefore, the values of x and y are 2 and 14 respectively

When $x = -1$,

$y = 3(-1) + 8 = -3 + 8 = 5$ Therefore, the values of x and y are –1 and 5 respectively

So, the solution is **(2,14)** and **(-1,5)**

WORD PROBLEMS ON SIMULTANEOUS LINEAR AND QUADRATIC EQUATION

Example:

The product of two numbers is 24 and their sum is 10.

Find the numbers

Solution

Let the numbers be x and y

$X + y = 10$ (1)

$Xy = 24$ (2)

From (1),

$Y = 10 - x$ (3)

From (2)

$Y = 24/x$ (4) {The idea here is to make y the subject of the formula in both equations (1) and (2) to equate them later on}

Equating (3) and (4),

$10 - x = 24/x$

Cross multiply

X(10 – x) = 24

10x – x2 = 24

Bringing all terms to one side,

10x – x2 – 24 = 0

Which can also be written as

X2 – 10x + 24 = 0

Factorizing we will have

(x – 6) (x – 4) = 0

X = 6. x = 4

The two numbers are 6 and 4. To check, the sum of the numbers is 6 + 4 which is 10, while to product is 6 times 4 which is 24. So we are correct.

2. The sum of two numbers is **4**. The sum of their squares is 26. Find the numbers.

Solution

Let the numbers be x and y

$X + y = 4 \qquad (1)$

$X^2 + y^2 = 26 \qquad (2)$

From (1)

$Y = 4 – x \quad (3)$

From (2)

$Y^2 = 26 – x^2$

$Y = \sqrt{(26-x^2)}$

Equating (3) and (4),

$4 – x = \sqrt{(26-x^2)}$

Square both sides to eliminate the square root

$(4 – x)^2 = (\sqrt{(26-x^2)})^2$

X(10 – x) = 24

10x – x2 = 24

Bringing all terms to one side,

10x – x2 – 24 = 0

Which can also be written as

X2 – 10x + 24 = 0

Factorizing we will have

(x – 6) (x – 4) = 0

X = 6. x = 4

The two numbers are 6 and 4. To check, the sum of the numbers is 6 + 4

LINEAR EQUATIONS INVOLVING CAPITAL MARKETS

Example: Joshua invests his savings of #8000 in two accounts. The first account yielded 5% last year and the second 7.5%. If his total interest for the year was #450, How much was in each account?

Solution

Let the first account be called A

Let the second account be called B

Recall, Simple interest = P X R X T

Let the money invested in A be called x.

Since the sum of money invested in both accounts = #8,000, then the money invested in B will be #8,000 – x

Now, Interest for A = P X R X T = x X 0.05 X 1 = 0.05x

Interest for B = (8000 – x) X 0.075 X 1 = 600 – 0.075x

Since the total interest = #450, this means that

Interest for A + Interest for B = #450

$0.05x + 600 - 0.075x = 450$

Hence, $0.05 - 0.075x = 450 - 600$

$-0.025x = -150$,

$0.025x = 150$

Dividing both sides by 0.025, we will have

X = # 6000

This means that Joshua invested **#6000** in Account A and **#2000** (#8000-#6000) in account B

SUMMARY

So far, we have learnt how to

1. Solve Simultaneous Linear And Quadratic Equations
2. Solve Word Problems On Linear, Quadratic, And Simultaneous Linear And Quadratic Equation
3. Solve Problems On A Linear Equation Involving Capital Markets.

INTERACTIVE ASSESSMENT QUESTIONS

1. Solve the simultaneous equations

$$y = 2x^2$$
$$y = x + 10$$

A (-2, 8) and (5/2, 25/2)

B (2,10) and (8,200)

C (2, 5) and (8, 50)

D (-2, 5/2) and (8, 25/2)

2. Solve $y = x^2 - 3x + 4$ and $y - x = 1$

A (1,3) and (2,4)
B (-1,-2) and (-3,-4)
C (1,2) and (3,4)
D 20

3. The mean of two numbers is 6 and the product is 35. Find the numbers.

A 2, 3
B -5, -7
C 4, 6
D 5, 7

4. Mrs. Esther invested #5,900 is invested in two accounts. One account earns 3.5% and another earns 4.5%. If the interest for 1 year is #229.50, the much is invested in each account?

A A = #3,600 B = #2,300
B A = #360 B = #230
C A = #36,000 B = #23,000
D A = #3,000 B = #2,900

THEME 03

Numbers and Numeration.

Algebraic Process.

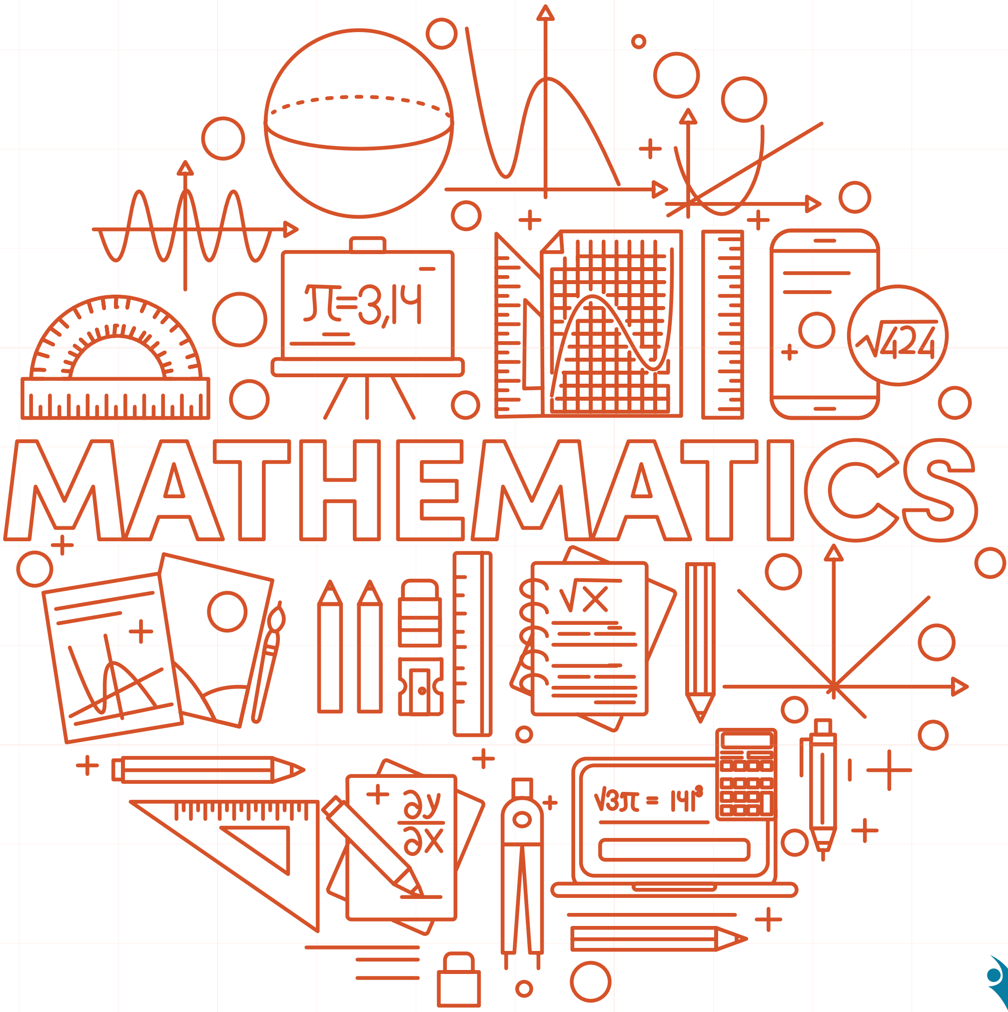
Geometry.

Introductory Calculus.

TRIGONOMETRIC GRAPHS OF TRIGONOMETRIC RATIOS

PERFORMANCE OBJECTIVES

1. Plot graphs of sine and cosine for angles $0^0 < x < 360^0$
2. Interpret/read and plot graphs of trigonometric ratio.
3. Solve major problems involving trigonometric ratios.
4. Plot graphs of tan for angles $0^0 < x < 360^0$
5. Carry out graphical solutions of simultaneous linear equations and trigonometric equations



LESSON

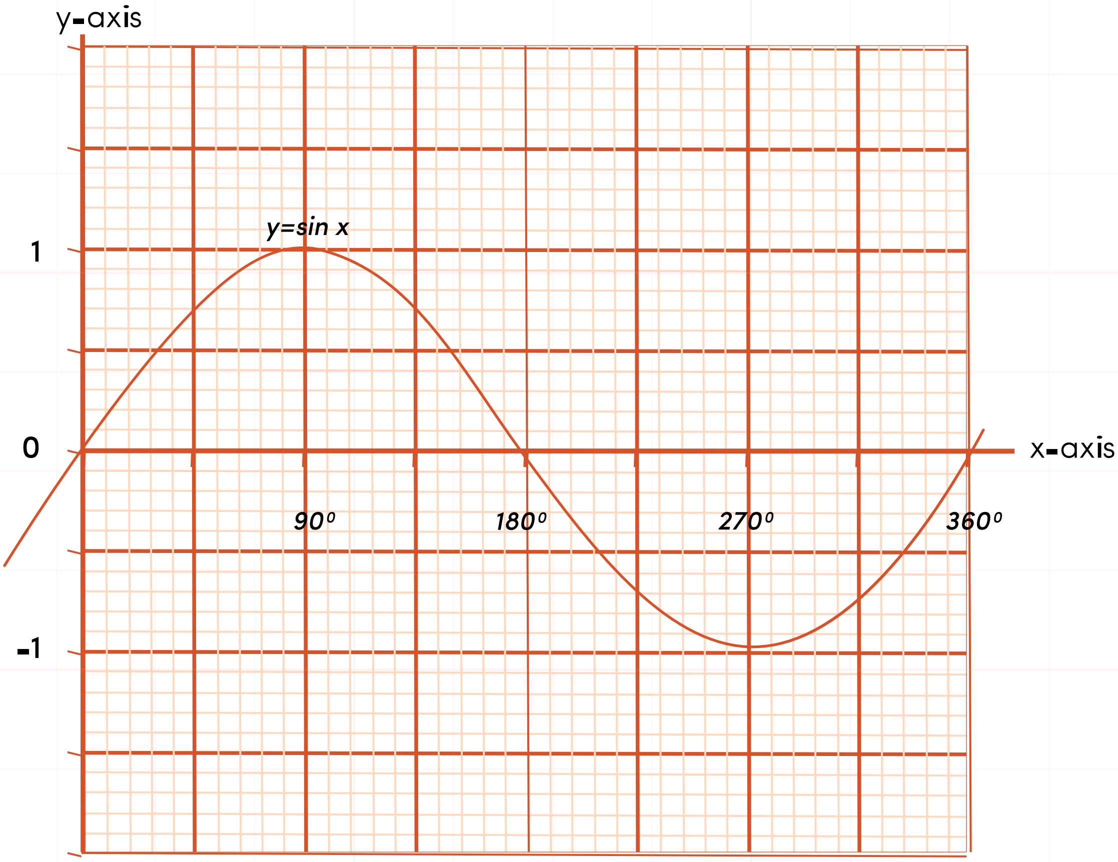
GRAPH OF TRIGONOMETRIC RATIOS

The graph of basic trigonometric ratios is drawn by considering the table of values for sin x, cos x and tan x from $x = 0^0$ to $x = 360^0$ at intervals of 90^0 as shown in the table below.

For $Y = \sin x$ for $0^0 < \mathbf{x} < 360^0$, the table of values and the graph will look like this:

x	0^0	90^0	180^0	270^0	360^0
y = sin x	0	1	0	-1	0

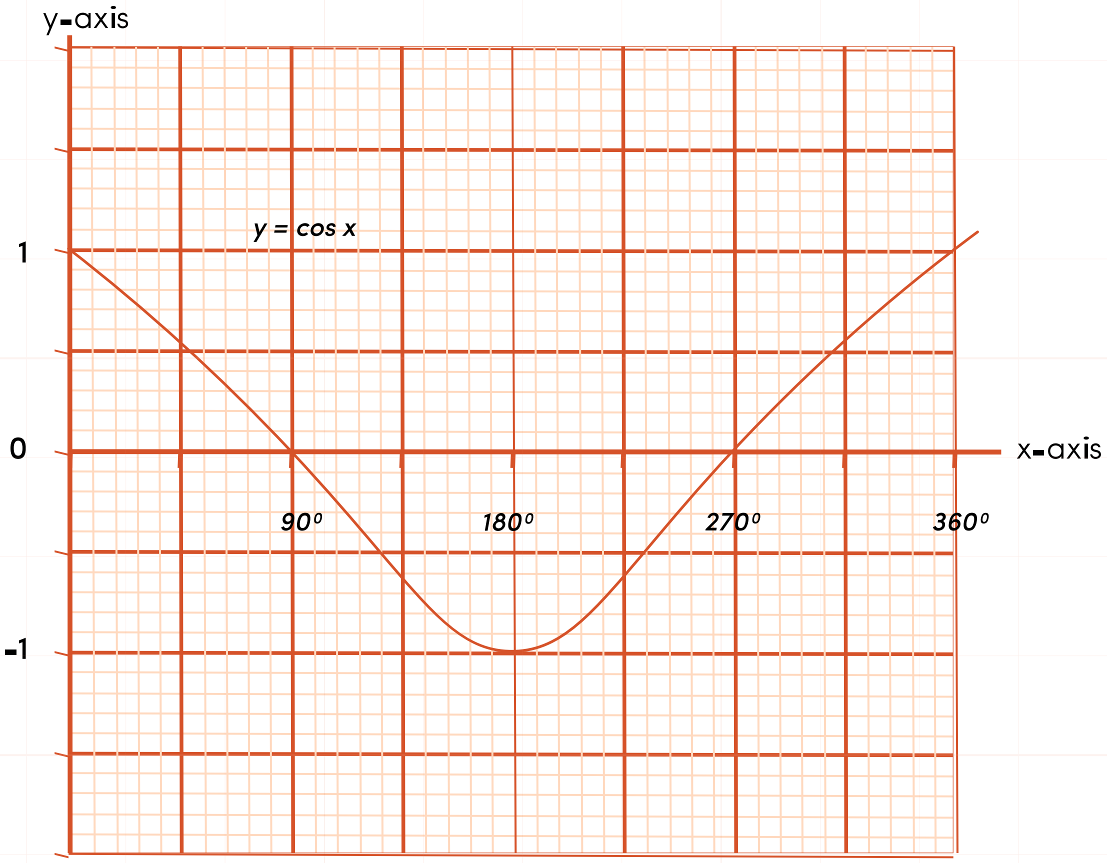
The graph of **y = sin x** for $0^0 < \mathbf{x} < 360^0$



For $y = \cos x$ for $0^0 < \mathbf{x} < 360^0$, the table of values and the graph will look like this:

x	0^0	90^0	180^0	270^0	360^0
y = cos x	1	0	-1	0	1

The graph of **y = cos x** for $0^0 < \mathbf{x} < 360^0$

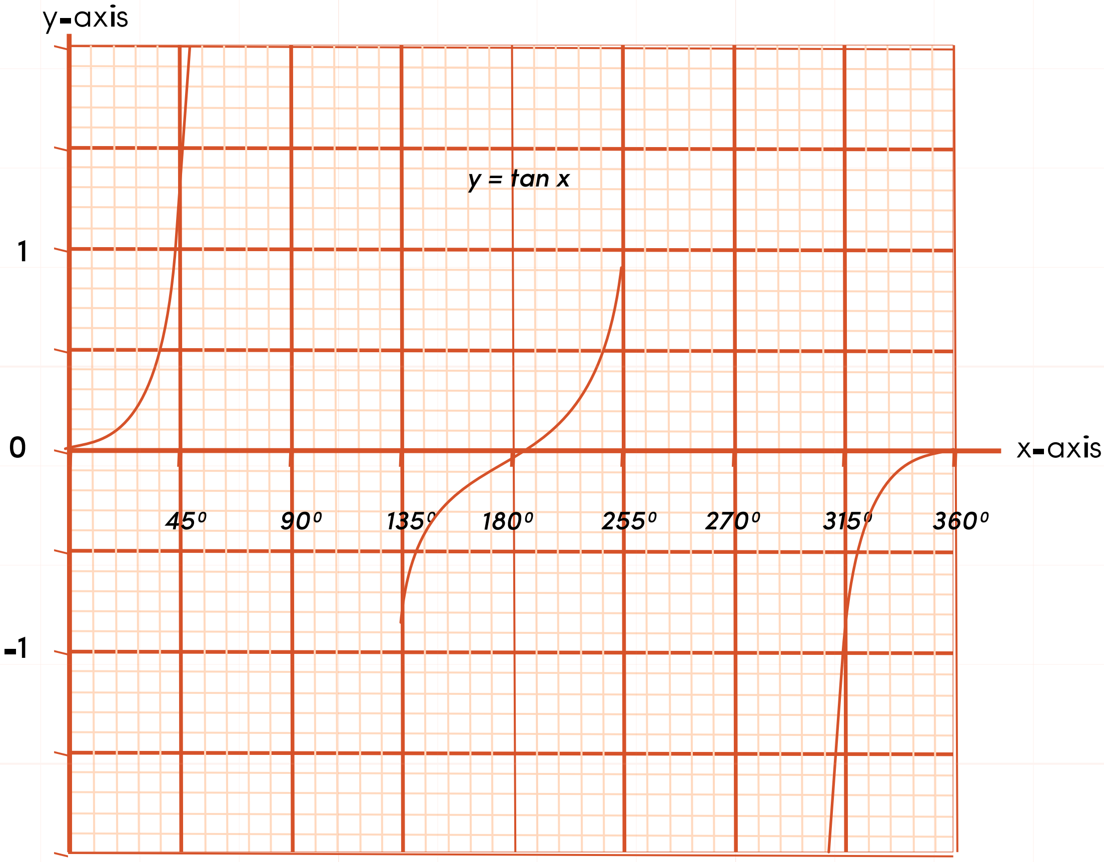


The graph of **y = tan x** for $0^0 < \mathbf{x} < 360^0$

The graph of y = tan x is drawn by considering the table of values for tan x from $x = 0^0$ to $x = 360^0$ at intervals of 45^0 as shown below.

x	0^0	45^0	90^0	135^0	180^0	255^0	270^0	315^0	360^0
y = tan x	0	1	udf	-1	0	1	udf	0	1

udf is Undefined



GRAPHICAL SOLUTION OF SIMULTANEOUS LINEAR AND TRIGONOMETRIC GRAPH

Example 1:

(a) Copy and complete the table of values for the function
$y = 2 \cos 2x - 1$

x	0^0	30^0	60^0	90^0	120^0	150^0	180^0
$y = 2 \cos 2x - 1$	1.0	0.0					1.0

(b) Using a scale of 2cm to 30^0 on the x-axis and 2cm to 1 unit on the y-axis draw the graph of $y = 2 \cos 2x - 1$ for $0^0 < x < 180^0$

(c) Draw the graph of

$y = \frac{1}{180}\ (x - 360^0)$

(d) Use your graph to find the

(i) Values of x for which 2 cos 2x + ½ = 0

(ii) Roots of the equation

$2\cos 2x - \frac{1}{180} + 1 = 0$

Solution

y = 2cos2x – 1

x	0^0	30^0	60^0	90^0	120^0	150^0	180^0
y = 2 cos2x - 1	1.0	0.0	**-2.0**	**-3.0**	**-2.0**	**0.0**	1.0

For x = 600

$y = 2\cos 2 \times 60^0 - 1$

= 2 cos 120 – 1

= -2 cos (180 – 120) – 1

$= -2\cos 60^0 - 1$

= -2 x 0.5 – 1

= -1 – 1

= -2

For x = 90

$y = 2\cos 2 \times 90^0 - 1$

= 2 cos 180 – 1

= -2 – 1

= -3

For x = 120

$y = 2\cos 2 \times 120^0 - 1$

= 2 cos 240 – 1

= -2 cos (240 – 180) – 1

= -2 cos 60 – 1

= -2 x 0.5 – 1 = -1 – 1

= - 2

For x = 150^0

y = 2cos 2x – 1

= 2cos 2 X 150 – 1 = 2cos 300 – 1

= 2 cos (360 – 300) – 1

= 2 cos 60 – 1

= 2 X 0.5 – 1 = 1 – 1

= 0

(b) Turn to the next page for graph

(c) To draw the graph of $y = \frac{1}{180}(x - 360^0)$

select any three values from the x-axis of the table above

x	0^0	90^0	180^0
y	-2.0	-1.5	-1.0

(d) (i)

$2\cos 2x + 1/2 = 0$

$2\cos 2x = 1/2$

$2\cos 2x - 1 = -1/2 - 1$

$2\cos 2x - 1 = -1½$

The values of x for which $2\cos 2x + ½ = 0$ can be obtained at the point where $y = -1½$ (point **A** and **B** on the graph)

i.e. $x = 52^0$ or $x = 129^0$

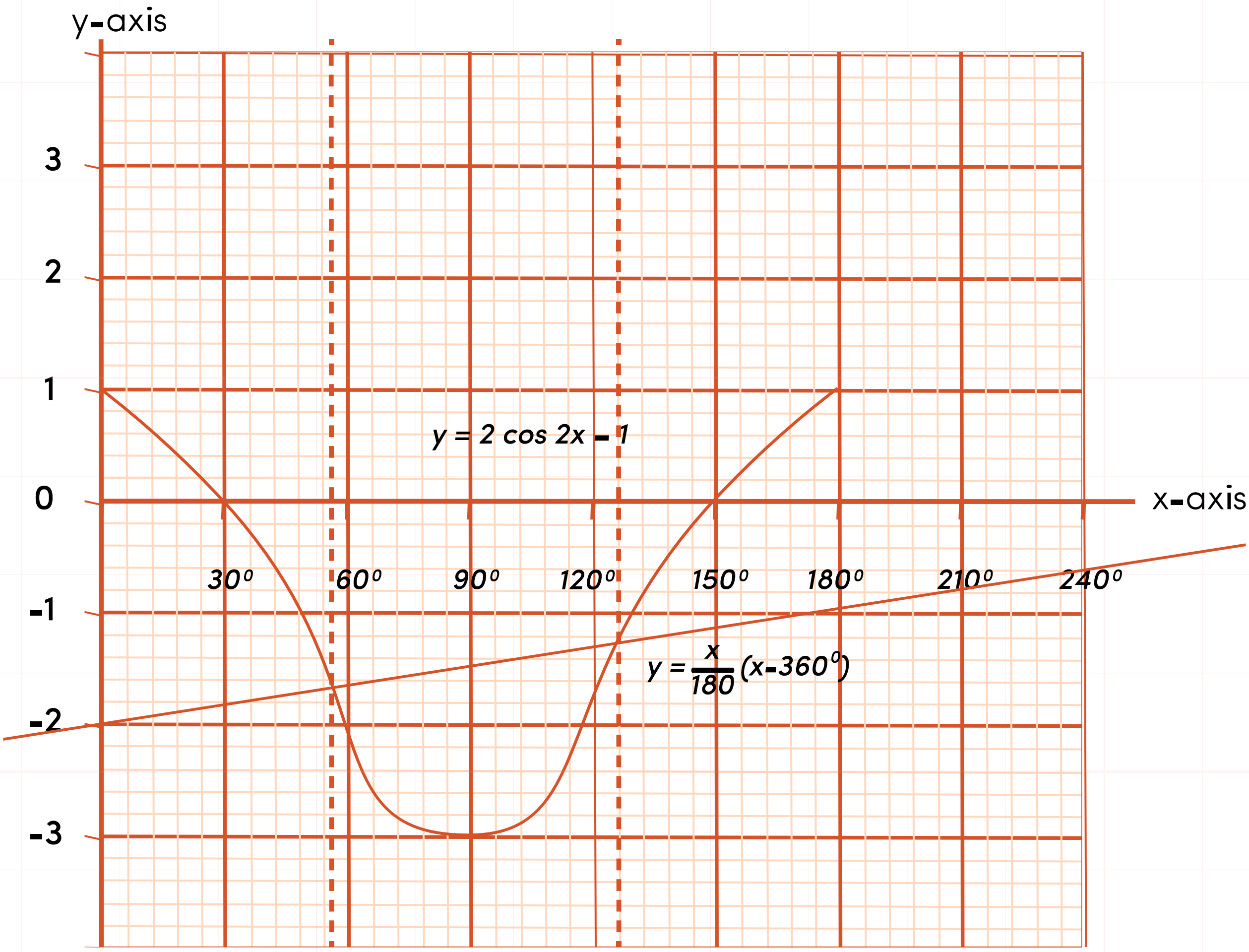
$$2\cos 2x - \frac{x}{180} + 1 = 0$$

$$2\cos 2x = \frac{x}{180} - 1$$

$$\text{Where, } \frac{x}{180} - 2 = \frac{x}{180}(x - 3600)$$

The roots are found at the point where the two graphs $y = 2\cos 2x - 1$ and $y = 1/180\,(x - 3600)$ meet (points C and D on the graph)

i.e. **x = 550** or **x = 1320**



Scale: 2cm to 30 º on x-axis

2cm to 1unit on y-axis

SUMMARY

So far, we have learnt how to

1. Plot graphs of sine and cosine for angles $0^0 < x < 360^0$.
2. Interpret/read and plot graphs of trigonometric ratio.
3. Solve major problems involving trigonometric ratios.
4. Plot graphs of tan for angles $0^0 < x < 360^0$
5. Carry out graphical solutions of simultaneous linear equations and trigonometric equations.

INTERACTIVE ASSESSMENT QUESTIONS

1. (a) Copy and complete the table of values for the function $y = 2 \cos 2x + \sin x$

x	-120^0	-90^0	-60^0	-30^0	30^0	60^0	90^0	120^0	0^0
y			-1.87			-0.13	-1	0.13	2

(b) Using a scale of 2cm to 30^0 on x-axis and 2cm to 1 unit on y-axis draw the graph of $y = 2 \cos 2x + \sin x$ for $-120^0 \leq \mathbf{x} \leq 120^0$

(c) Using the same scale and axes draw the graph of

$$y = \frac{7x}{410} + 1$$

(d) From your graph, find the roots of the following equations

i. $\sin x + 2 \cos 2x = 0$

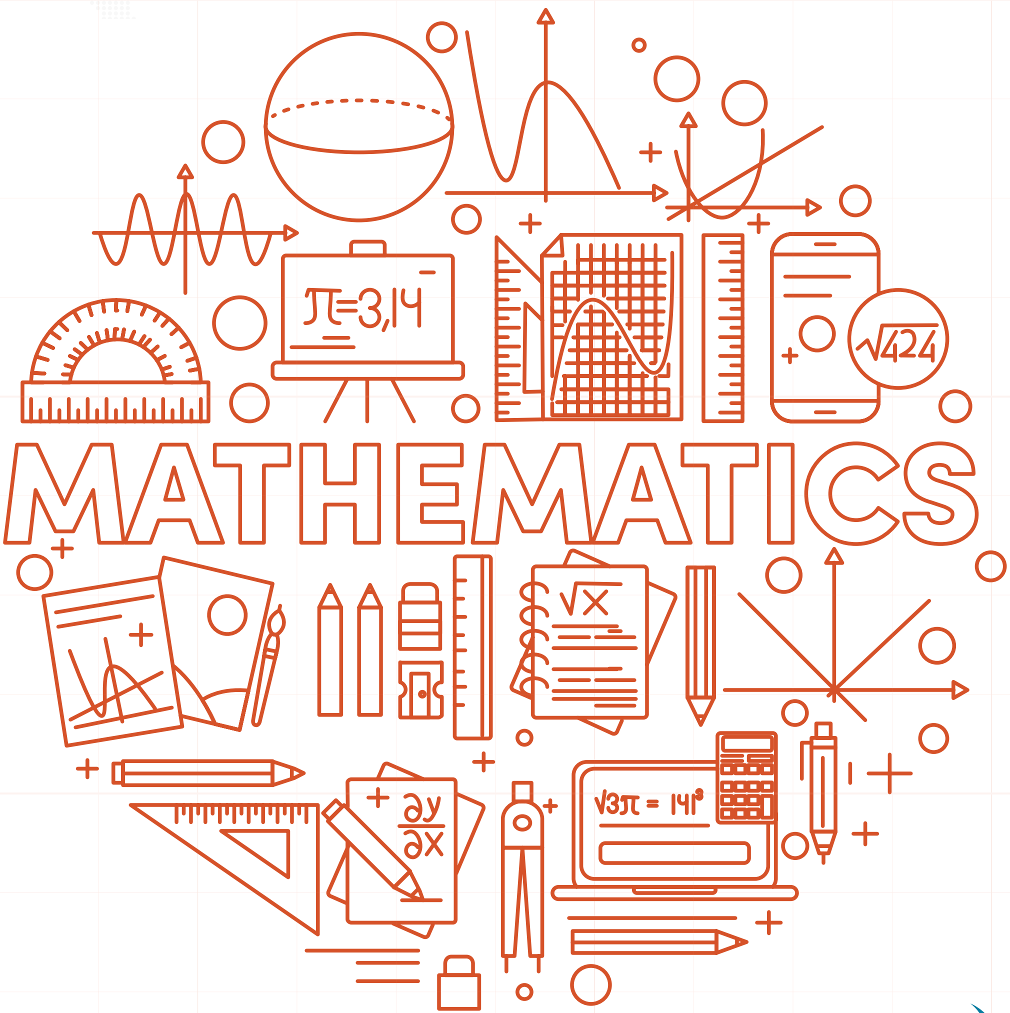
ii. $\sin x + ½ + 2 \cos 2x = 0$

iii. $2 \cos 2x + \sin x = \frac{7x}{410} - 1$

SURFACE AREA AND VOLUME OF SPHERE

PERFORMANCE OBJECTIVES

1. Find the surface area of a sphere.
2. Find the volume of a sphere.



LESSON

SURFACE AREA OF A SPHERE

What is the surface area?

The surface area of any solid is the area that describes the material that can be used to cover the solid completely. Look at this spherical tennis ball. If we wrap this paper completely around the ball, the area of material that covers the ball is called the area.

The total surface area of a sphere is four times as large as the area of a circle with the same radius. This means that

Total surface area = $4\,\pi r^2$

Where r = radius

π = pi as we know it.

Example: Find the total surface area of a sphere of radius 7cm

Solution

Total surface area (T.S.A) = $4\pi r^2$

$\pi = 22/7$, r = 7cm

There fore T.S.A = $4 \times 22/7 \times 7 \times 7$

= $616cm^2$

Example: If the surface area of a sphere is $154cm^2$, what is the size of the radius?

Solution

T.S.A. = $4\pi r^2$

$\pi = 22/7$, T.S.A. = $154cm^2$, r = ?

$154cm^2 = 4 \times 22/7 \times r^2$

Cross multiply

$R^2 = \frac{154 \text{ X } 7}{4 \text{ X } 22} = \frac{49}{4}$ Hence, $R = \sqrt{\frac{49}{4}} = \frac{7}{2}$

$R = 3.5cm$

Remember that areas are measured in square units.

Next, we move on to finding the volume of a sphere.

VOLUME OF A SPHERE

The volume of a solid is defined as how much substance the solid can hold. It is measured in cubic units.

Now, how do we derive the formula for the volume of a cube?

Consider this sphere and these two cones. The cones have the same base radius and height such that twice the radius of the sphere is equal to the height of the cones. This means that if we fill these cones with water and pour the water into the sphere, we will notice that the water from the two cones fills up the sphere completely. Thus, we can say that The volume of sphere = 2 x volume of a cone

Volume of a cone = $\frac{1}{3}\pi r^2 h$. So,

Volume of sphere = $2 \times \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

Remember, the height of the sphere is the diameter which is equal to twice the radius (i.e, diameter = height = 2r). We will have

The volume of sphere $= \frac{2}{3}\pi r^2 (2r)$

Therefore, the volume of sphere = $\frac{4}{3}\pi r^3$

Thus we have derived the volume of a sphere.

Let's solve some questions on the volume of a sphere

Example:

1. Find the volume of a sphere whose radius is 4.2cm.

   (Take $\pi = \frac{22}{7}$)

Solution

We know, the volume of sphere $= \frac{4}{3}\pi r^3$, $(\pi = \frac{22}{7})$, r = 4.2cm

Volume (V) = $\frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2$ = 295.68cm3

Example: Faith measures the diameter of a ball as 7 inches. How many cubic inches of air can the ball, hold, to the nearest hundredth? (Take $\pi = \frac{22}{7}$)

Solution

To know how much air the ball can hold, we need to find the volume of the ball, which is a spherical shape.

We know, the volume of sphere $= \frac{4}{3}\pi r^3$,

π = 3.14 r = 7 inches

Volume = $\frac{4}{3}$ x 3.14 x 7 x 7 x 7 = $\frac{4308.08}{3}$

= 1436.0267 cubic inches

To the nearest hundredth, the volume will be 1436.03cubic inches.

INTERACTIVE ASSESSMENT QUESTIONS

1. The formula for the total surface area of a sphere is written as

A $\frac{4}{3}\pi r^3$

B $4\pi r^2$

C $2\pi r^2$

D $\frac{1}{3}\pi r^2$

2. What is the total surface area of a sphere of diameter 10cm? (to the nearest two decimal places)

A $4190.48cm^2$
B $4190.47cm^3$
C $1257.14cm^3$
D $1257.14cm^2$
E None of the above

3. The total surface area of a sphere is measured in ___ while the volume of a sphere is measured in ___

A Square units, cubic units
B Cubic units, square units
C Square units, square units
D Cubic units, cubic units
E One unit

Which of the following statements is true?

A The volume of a sphere = 2 x volume of a cube
B The volume of a sphere = 2 x volume of a cylinder
C Volume of a sphere = 2 + volume of a cone
D The volume of a sphere = 2 x volume of a circle
E The volume of a sphere = 2 x volume of a cone

Kingsley wants to know how much air can fill a spherical hot air balloon. How can he go about this? He should calculate....

A The surface area of the sphere
B The volume of the sphere
C The perimeter of the sphere
D How much the balloon is sold in the market
E The color of air to fill the balloon

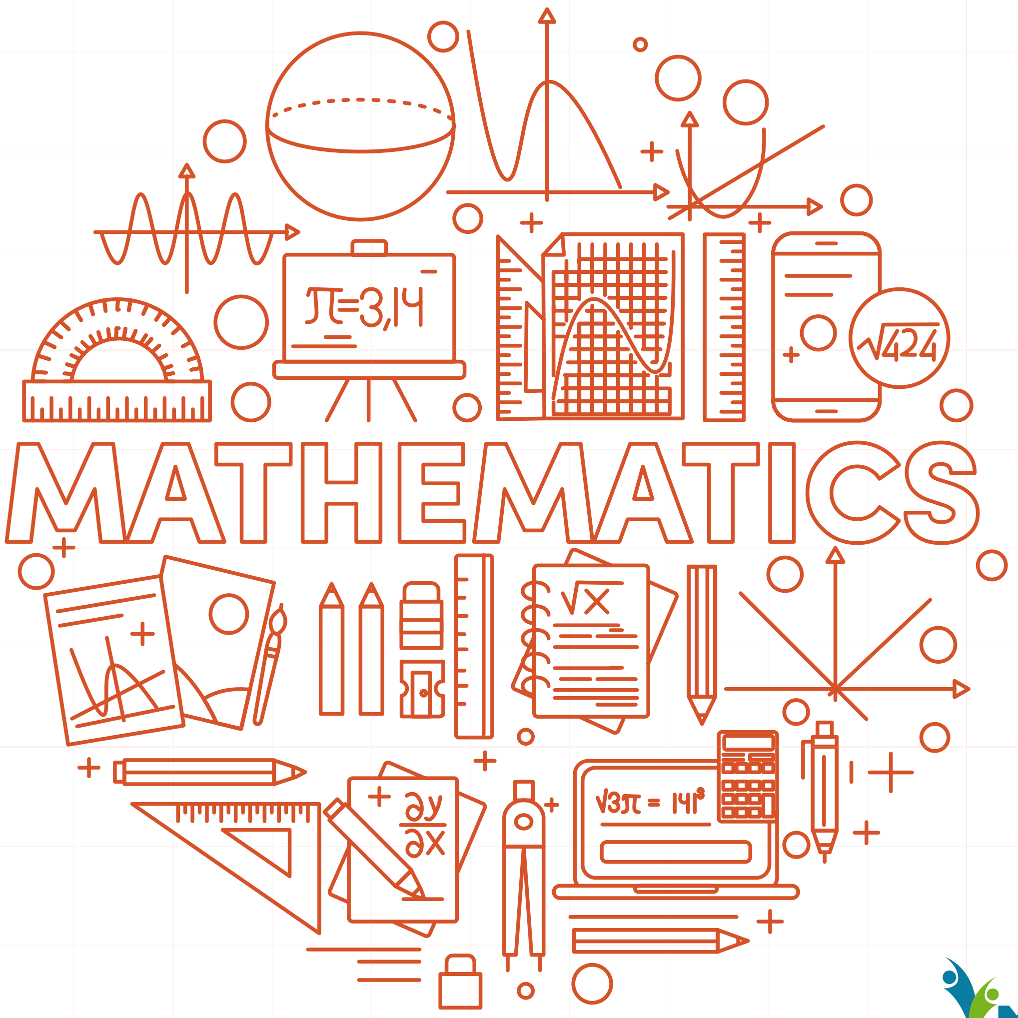
Fatima noticed air leaking from a spherical advertising balloon at the rate of 30 cubic feet per minute. If the radius of the ball is 7 feet, how many minutes will it take for the balloon to lose its air completely? Round your answer to the nearest whole number (Take π=3.14)

A 47 minutes
B 47.8 minutes
C 48 minutes
D 1436 minutes

LATITUDE AND LONGITUDE

PERFORMANCE OBJECTIVES

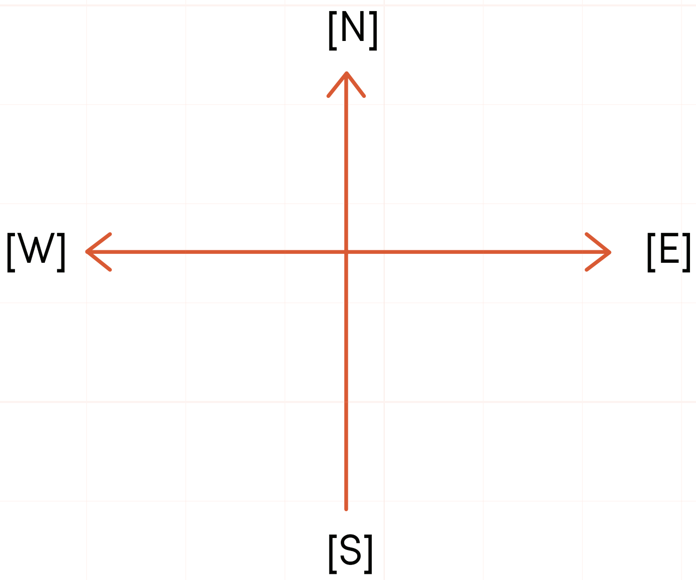
1. Describe the earth as a sphere
2. Identify (using skeletal globe) and locate (using real globe) and appreciate the following: North and South poles, Longitude, Latitude, Meridian and equator, parallel of latitude, the radius of a parallel of latitude etc
3. Recall, state formula, and solve problems on arc length of a curve and on longitude and latitude.

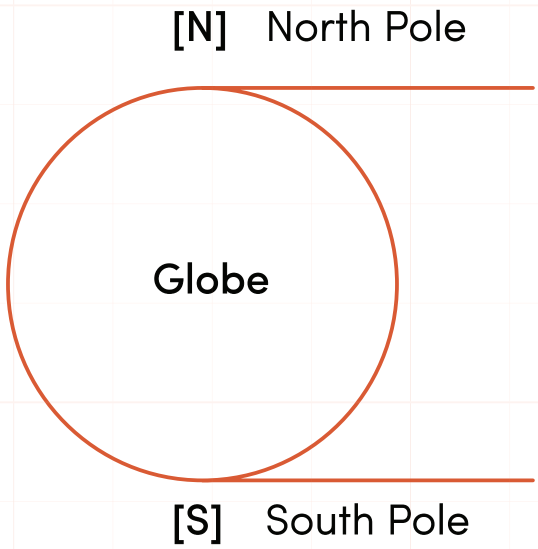


LESSON

THE EARTH AS A SPHERE

A sphere is a round solid figure. The earth is spherical.
The four basic cardinal points are north, south, east, and west

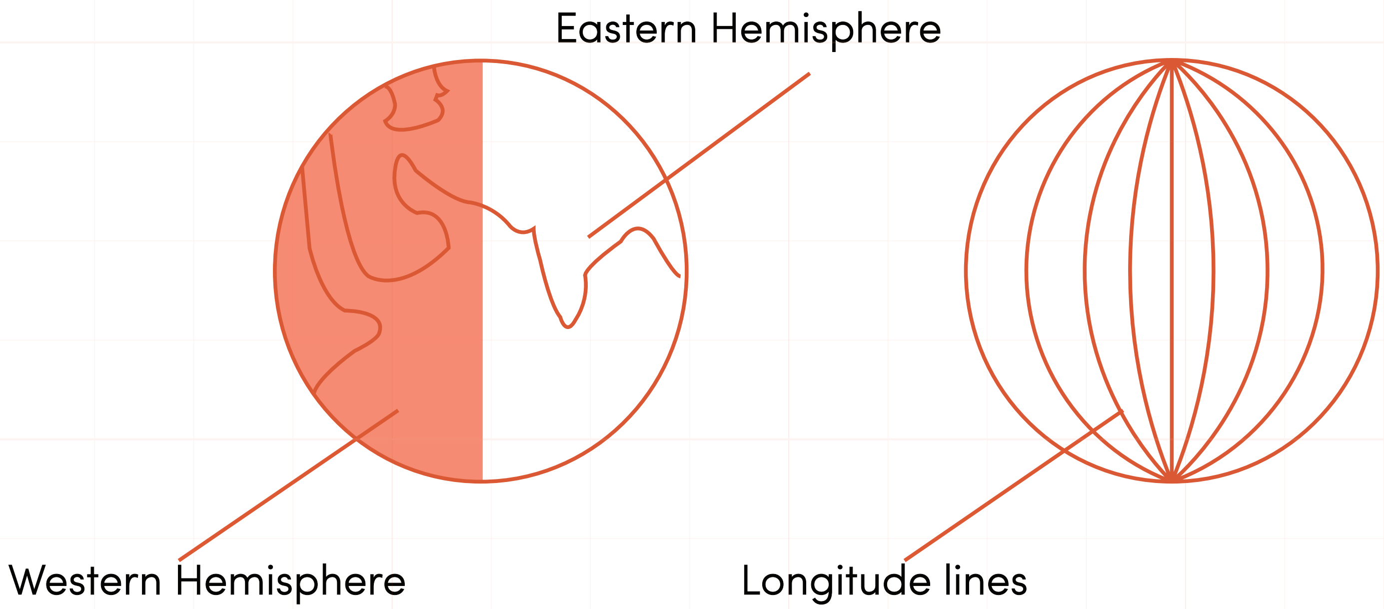




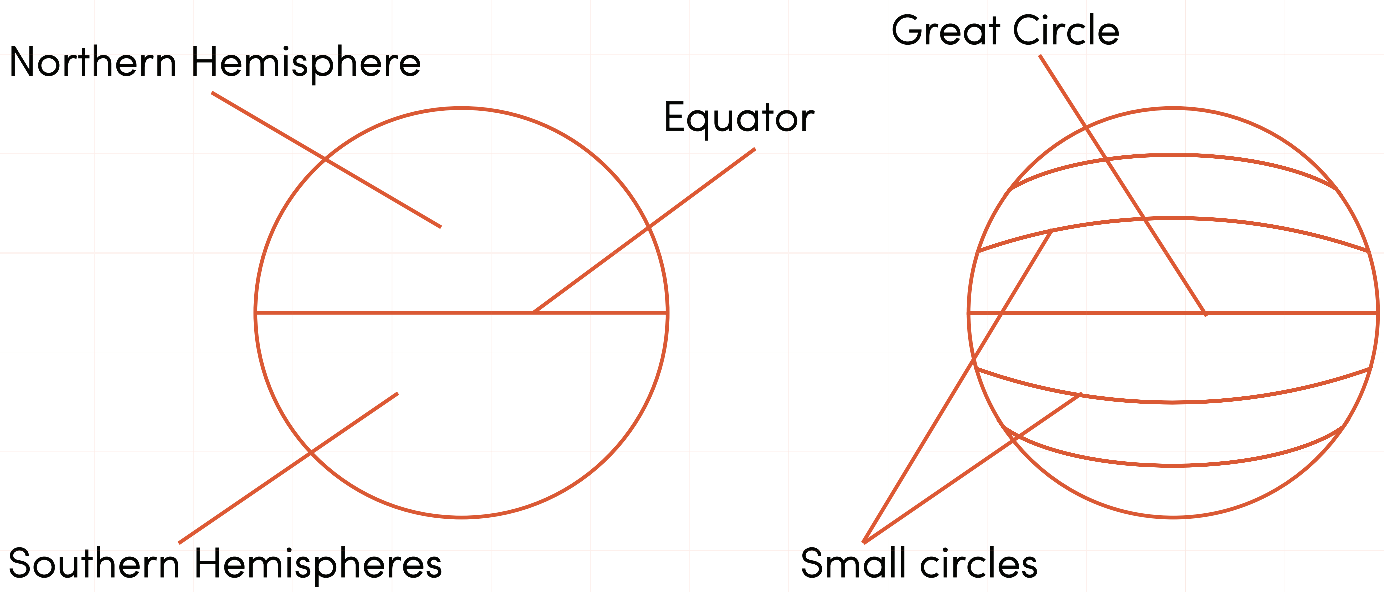
IDENTIFICATION OF DIFFERENT TERMS

LONGITUDE AND LATITUDE

Longitude lines are made by circles that intersect with both North and South Poles. The lines of longitude run through North and South, and they are called MERIDIANS OF LONGITUDE. The meridian at the center is called the Prime Meridian. These lines of longitude divide the earth into Eastern and Western Hemisphere.



Latitudes, on the other hand, divide the earth into Northern and Southern Hemispheres. These lines run from east to west on the globe. The line at the center is called the Equator.



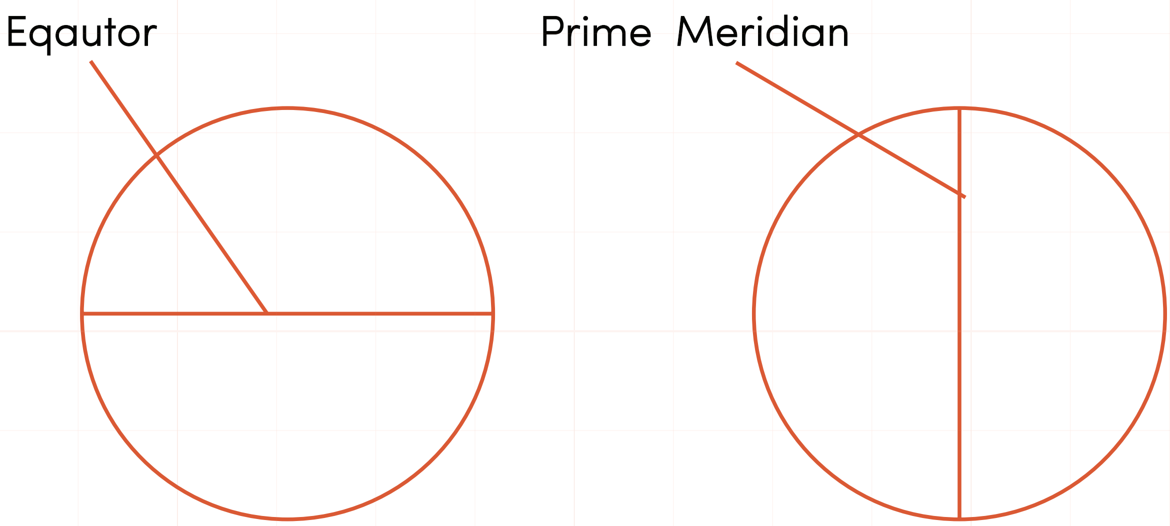
SMALL AND GREAT CIRCLES

These are two types of circles that can be drawn on the surface of a sphere. Great circle: It is the largest circle that can be drawn on the

surface of a sphere. It is found at the centre of a circle and it is called the "Radius of the Earth Sphere" which is calculated as 6400km. It is denoted as the letter R (That is, R = 6400km)
Small circle: It is anywhere but the centre of a sphere. Here the parallels of latitude apart from the equator are all small circles. It is denoted by r ($r = R\cos\alpha$)

MERIDIAN AND EQUATOR

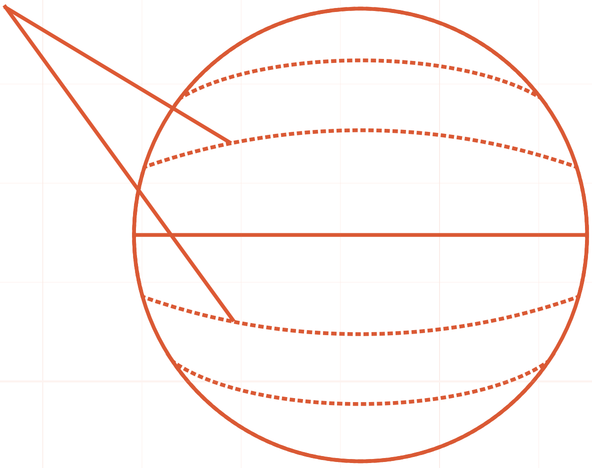
A sphere can either be divided horizontally (from East to West) or vertically (from North to South). The line which cut along the sphere from North to South is called the Meridian while line that cuts across the sphere from East to West is called the Equator.



PARALLELS OF LATITUDE

These are also called lines of latitude. They are called parallels of latitude because one latitude line is parallel to another. They are the imaginary lines that run through East and West of the globe.

Parallels of latitude



RADIUS OF THE PARALLEL OF LATITUDE

Recall, the radius of the earth has a constant value (6400km), while the radii of the parallel of latitude have varying values. So how do we calculate the radius of the parallel of latitude? We use the formula

$r = R\cos\theta$

where R = Radius of the Earth

θ = latitude (in degrees)

Now let's look at an example:

Example: Find the radius of the parallel of latitude 600 S.

(Given that R = 6400km)

Recall that parallel of latitude are the small circles and to find the radius, we use the formula

$r = R\cos\alpha$

r =?

R = 6400km

$\alpha = 600$

r = 6400 X cos 600

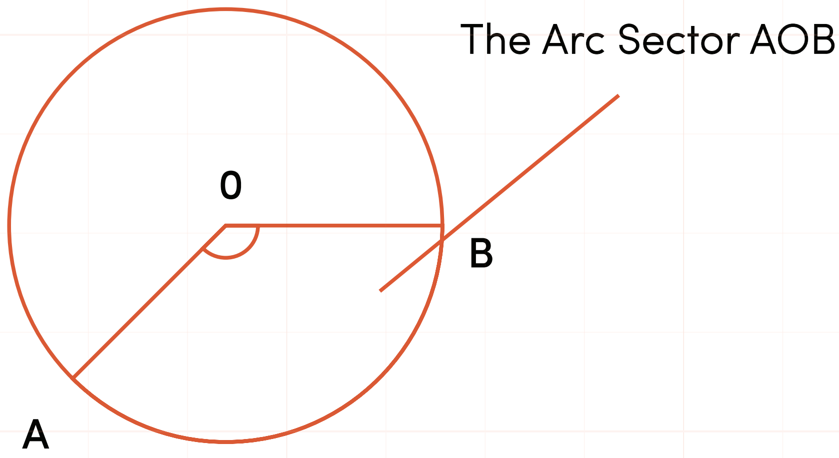
r = 6400 X 1/2 = **3200km**

Therefore, the raidus of the parallel of latitude 600S is 3200km

PROBLEMS ON LONGITUDE AND LATITUDE

REVISION OF ARC LENGTH OF A CURVE

Recall, that the formula for finding the length of an arc is $\theta/360 \times 2\pi r$. We will be using this formula subsequently.



CALCULATIONS OF DISTANCE BETWEEN THE POINTS ON THE EARTH

To calculate the distance between two points in a circle, we use the formula for the length of an arc. However, since we have two types of circles (great circle and small circle), then

The distance on small circles = $\theta/360 \times 2\pi r$, while

The distance on great circles = $\theta/360 \times 2\pi R$

The difference between the two formulas is the letter r and R.

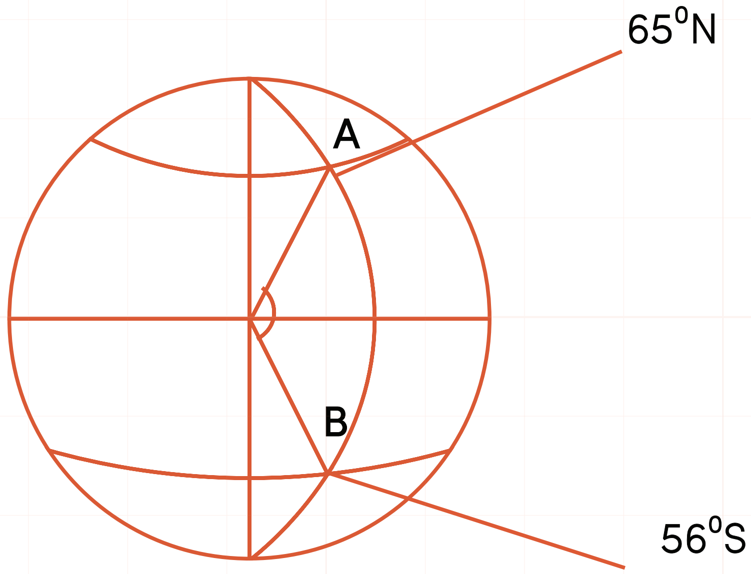
"θ"="difference in longitude/latitude"

r= radius of the parallel of latitude (small circle)

R = radius of the earth (great circle)

Now, let's consider an example.

Example: Find the distance between points A (560S, 800E) and B(650N, 800E), take π= 3.142 and R = 6400km



In this diagram, locations A and B lie along the same longitude, therefore they qualify to be great circles.

The distance on great circles = $\theta/360 \times 2\pi R$

"θ"="difference in longitude or latitude" . You will notice that there are two locations on the great circle (65^0N and 56^0S. So, to get the longitude difference, we do either of two things:

Add up the longitudes if they are on different cardinal points (such as N and S).

Subtract the longitude with a lower value from the longitude with a higher value if they are on the same cardinal point (such as N and N, S and S, E and E, W and W)

Therefore, difference in latitude = $65^0 + 56^0 = 121^0$

R = radius of the earth (great circle) = 6,400km

Distance AB = $\theta/360 \times 2\pi R$ (since it is on the great circle)

$= 121/360 \times 2 \times 3.142 \times 6400$

$= 13{,}517.6\text{km} \approx 13{,}518\text{km}$

Example 2:

Two points P(320N, 47^{0}W) and Q(32^{0}N, 28^{0}E) are on the earth's surface. Given that π=3.142, R = 6400km, calculate:

a. The radius of the parallel of latitude

b. Distance between P and Q.

Solution

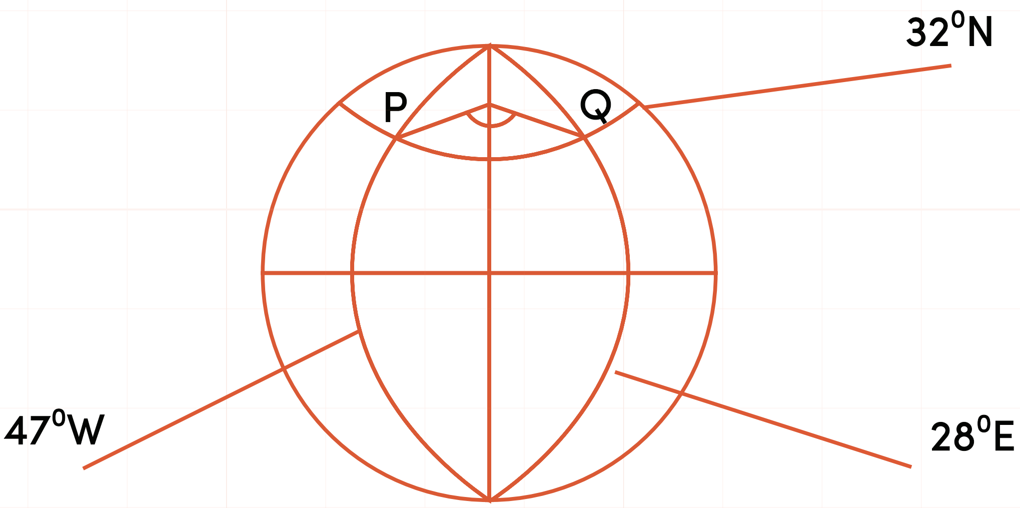
Example 2:

Two points P(32^{0}N, 47^{0}W) and Q(32^{0}N, 28^{0}E) are on the earth's surface. Given that π=3.142, R = 6400km, calculate:

a. The radius of the parallel of latitude

b. Distance between P and Q.

Solution



Difference in longitude = (47 + 28)^0 = 75^0

Common latitude = 32^{0}N

a. Radius of the parallel of latitude

$r = R \cos \alpha$

$R = 6400$km

α = common latitude = 32^0N

$r = 6400 \text{ X } \cos 32^0$

$r = 6400 \text{ X } 0.8480$

$r = 5{,}428$km.

b. Distance between P and Q

$|PQ| = \theta/360 \times 2\pi r$

$|PQ| = 75/360 \times 2 \times 3.142 \times 5428$

? $|PQ|$ □ 7105km

SUMMARY

So far, we have learnt how to

1. Describe the earth as a sphere
2. Identify (using skeletal globe) and locate (using real globe) and appreciate the following: North and South poles, Longitude, Latitude, Meridian and equator, Parallel of latitude, The radius of a parallel of latitude and The radius of the earth
3. Recall, state formula, and solve problems on arc length of a curve
4. Solve problems on longitude and latitude

INTERACTIVE ASSESSMENT QUESTIONS

1. The θ^{0} line of latitude is the

A Prime Meridian
B Equator
C Contour line
D International dateline

2. How many parts does the equator divide the earth into?

A 6
B 8
C 4
D 2

3. The θ^{0} mark of longitude is the

A Prime meridian
B Contour line
C Equator
D International dateline

4. Lines of longitude

A Never meet
B Are called parallels
C Are real lines painted on the earth
D Meet at north and south poles

5. What is the reason why latitude lines never intersect?

A They converge at the poles
B They are cool
C They are parallel
D They are vertical lines

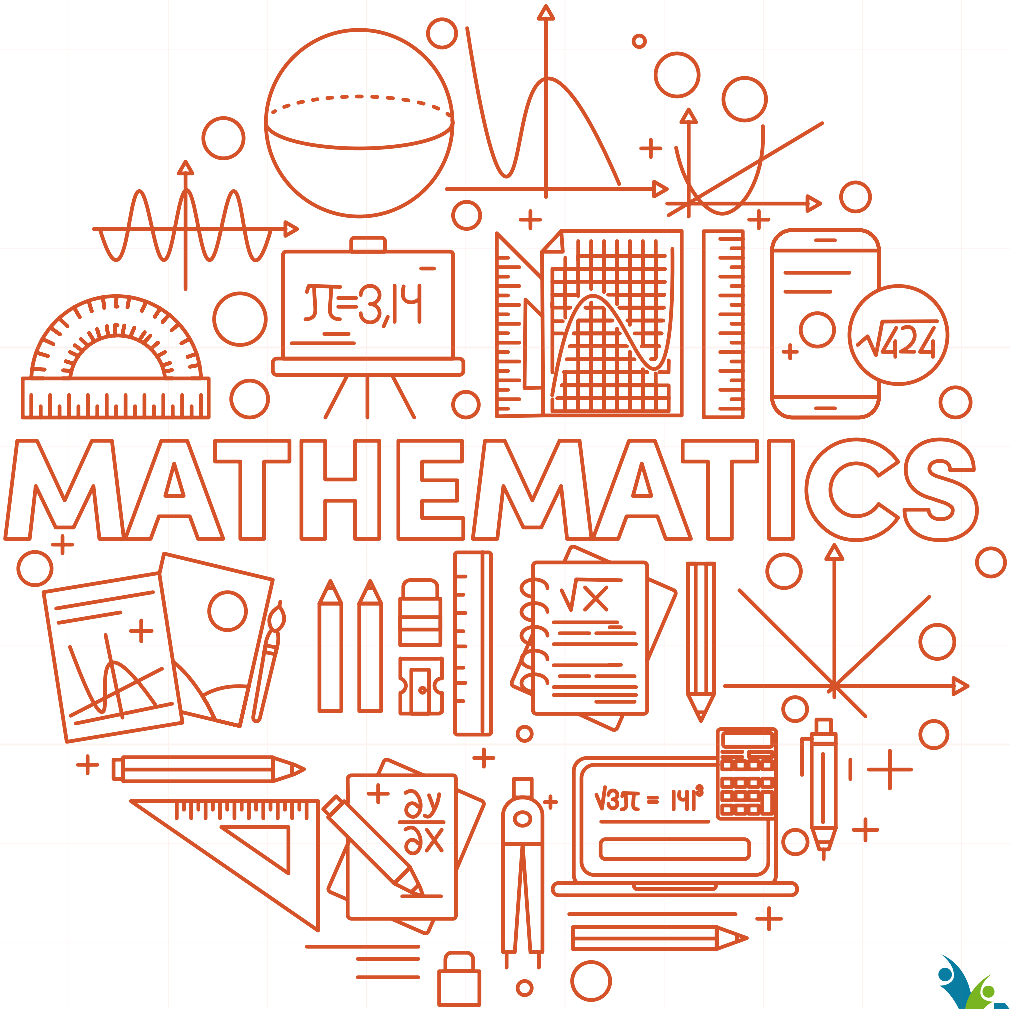
6. Two points P(41^0N, 50^0W) and Q(41^0N, 30^0E) are on the earth's surface. Given that π= 3.142, R =6400km, the distance between P and Q to the nearest km is

A 6745km
B 6740km
C 1686km
D 4830.3km

COORDINATE GEOMETRY OF STRAIGHT LINES

PERFORMANCE OBJECTIVES

1. Identify the Cartesian rectangular coordinate
2. Draw and interpret linear graphs and apply to real-life situations.
3. Determine the distance between two coordinate points
4. Find the midpoint of the line joining two points
5. Apply the concept to a real-life situation
6. Define and determine the gradient and intercept of a line

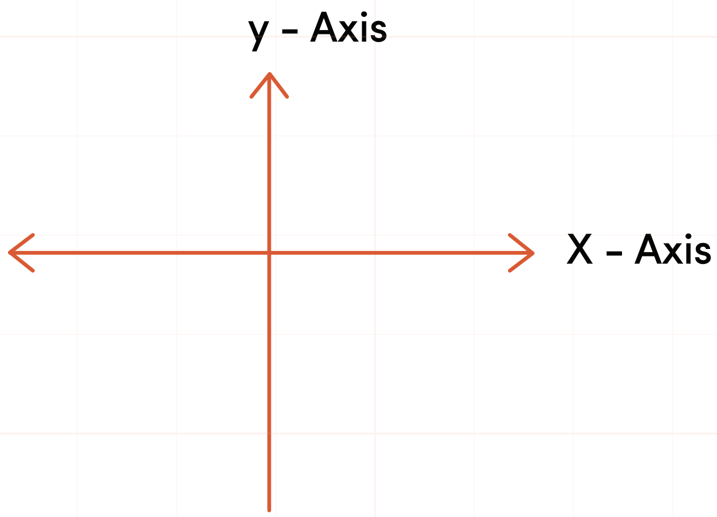


MALEZI
EMPOWERING THOSE WHO CARE

LESSON

RECTANGULAR CARTESIAN COORDINATE

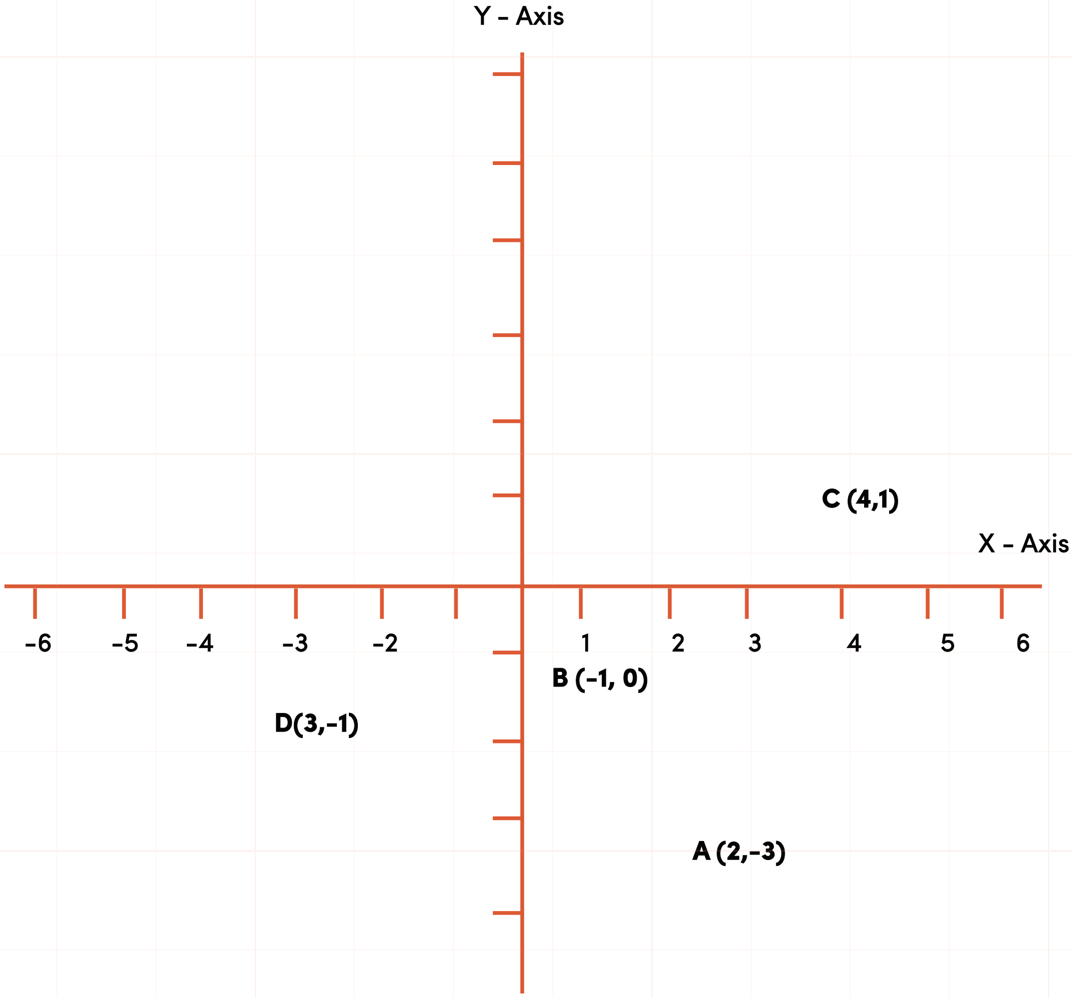
The graph consists of an intersection of a vertical and horizontal number line at a right angle. The intersected number lines form a flat surface called a plane. The horizontal number line(from left to right) is called the x-axis, while the vertical number line(from up to down) is called the y-axis. Each point on the plane consists of two coordinates, the first being the x-coordinate and the second being the y-coordinate. For instance, if we have a point (a, b) on the plane, a is the x-coordinate while b is the y-coordinate.



The intersection (0) between the x and y-axis is called the origin (the point where all calibrations start from)

Example: Plot the set of ordered pairs A(2,-3), B(-1,0), C(4,1), D(-3,-1)

To plot to point a, locate the intersection of +2 on the x-axis and -3 on the y-axis. Do the same for the other points. An illustration is shown below:

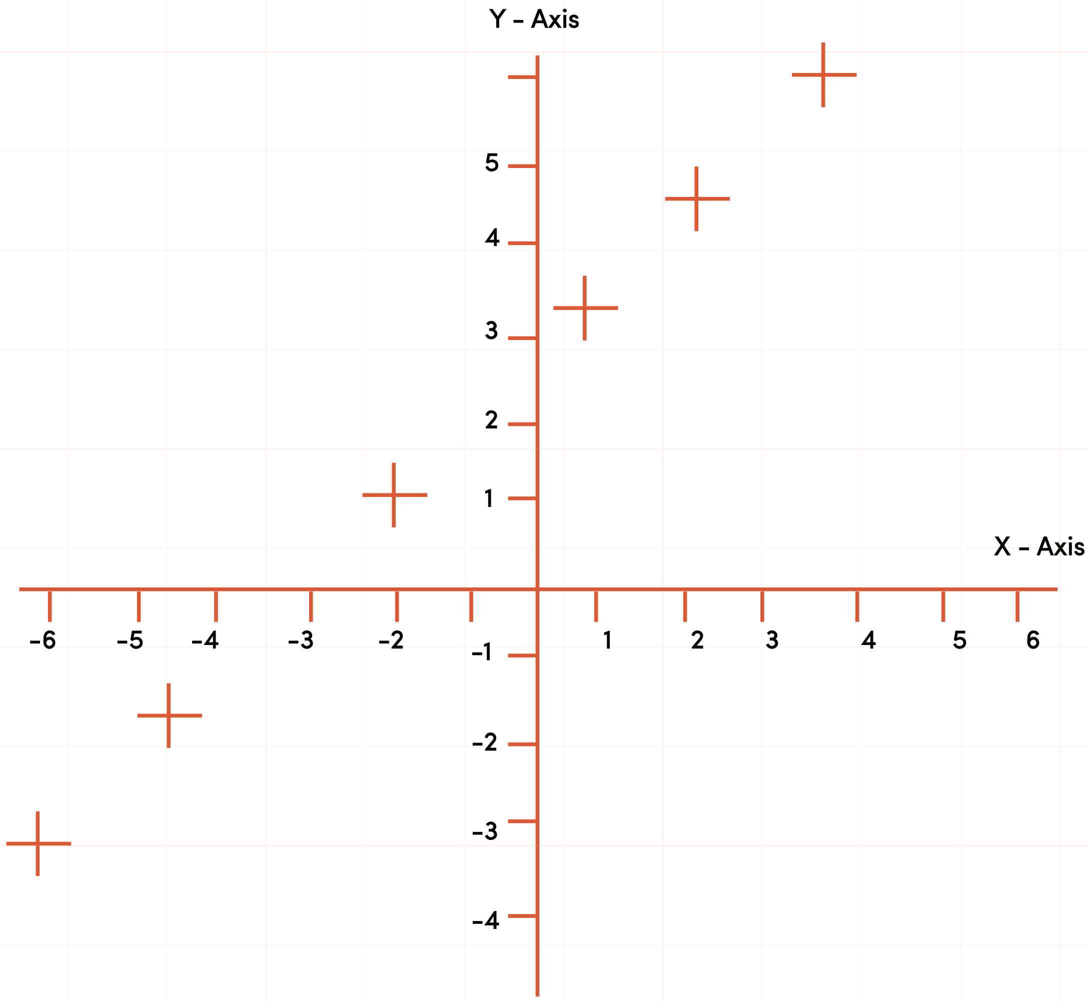


LINEAR GRAPHS

Now that we have learnt to plot points on the graph, let's plot some more!

Example:

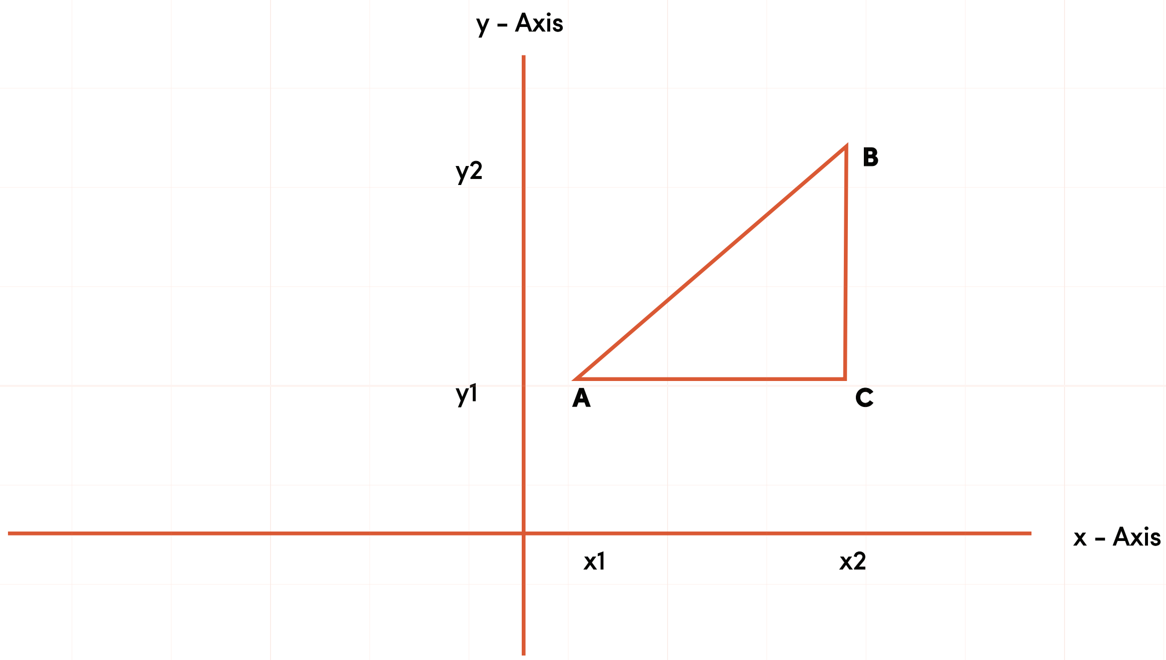
Plot the set of ordered pairs: (–5, –3), (–3, –1), (–1, 1), (1, 3), (3, 5)



In this graph, you will notice that the points appear to lie along a straight path. Joining the points will form a line. Thus, we say that the graph above is linear.

DISTANCE BETWEEN TWO POINTS

How do we calculate the distance between two points?
Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$.



After joining the points, form a right angle triangle. |AB| becomes the hypotenuse, which is also the distance between A and B.

Recall, in Pythagoras' theorem,

$Hyp^2 = adj^2 + opp^2$. So,

$|AB|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$. Therefore,

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

This is the formula used to calculate the distance between two points on a line.

Example: Find the distance between points M(1, 0) and N(-1, 5)

Solution

For M(1, 0), $x_1 = 1, y_1 = 0$

For N(-1, 5) $x^2 = -1, y^2 = 5$

$|\mathbf{MN}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|\mathbf{MN}| = \sqrt{(-1-1)^2 + (5-0)^2} = \sqrt{(4+25)} = \sqrt{\mathbf{29}}$ units.

MIDPOINT OF A LINE JOINING TWO POINTS

The formula for finding the midpoint of a line joining two points is:

$$\left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_2 + y_1}{2}\right)$$

Example:

Find the midpoint of the line joining F(-1,-2) and G(7,4)

Solution

For F(1,2), $x_1 = 1$ and $y_1 = 2$

For G(7,4) $x_1 = 7$ and $y_2 = 4$

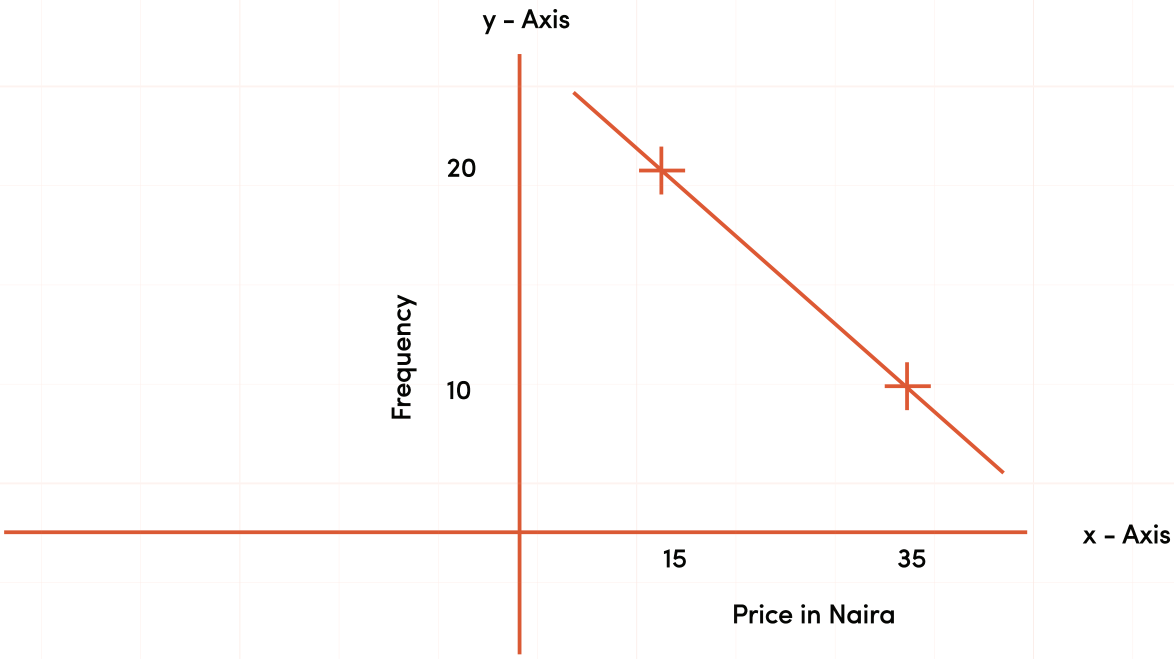
Midpoint between F(1,2) and G(7,4) is

$$\left(\frac{1 + 7}{2}\right), \left(\frac{2 + 4}{2}\right) = (4,3)$$

PRACTICAL APPLICATION OF COORDINATE GEOMETRY

Let's consider a practical question

Example: The linear graph below shows the number of pencils sold with the price of pencils in the year 2018.



What are the average price of the pencil and the average number of pencils sold in the year 2018?

Solution

To find the average, we find the midpoint of the line joining the two points (15,20) and (35,10)

For (15,20), x1 = 15, y1 = 20

For (35,10), x2 = 35, y2 = 10

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_2 + y_1}{2}\right) = \left(\frac{15 - 54}{2}\right), \left(\frac{20 - 10}{2}\right)$$

= (8,8)

Therefore, the average price is #25 while the average number of pencils sold is 15.

GRADIENT AND INTERCEPT OF A LINE

The intercept of a line is a point where the line crosses the x-axis or y-axis.

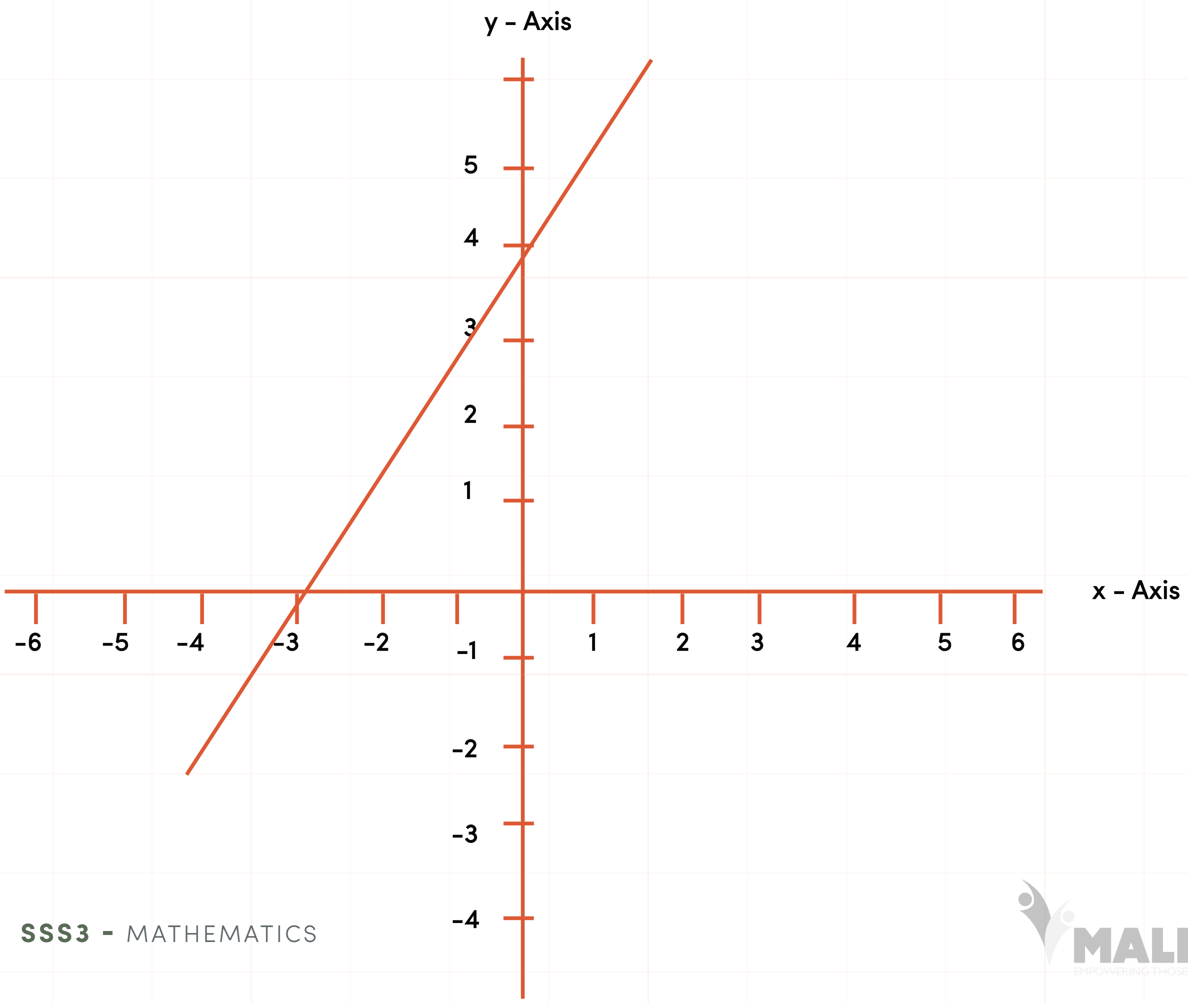
So, we have x-intercept and y-intercept.

The gradient on the other hand is the rate of change in y to change in x. It is also called the slope. It is denoted as the letter "m". The formula for the gradient is:

$$\mathbf{M} = \left(\frac{\text{Change in y}}{\text{Change in x}}\right), \text{ So } \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Let's consider an example:

Find the (i) gradient (ii) y-intercept (iii) x-intercept of the linear graph below:



Solution

x-intercept = -3

y-intercept = 4

To find the gradient, pick any two points on the line. Let's pick the two intercept points (-3,0) and (0,4).

$$\mathbf{M} = \left(\frac{\text{Change in y}}{\text{Change in x}}\right) \quad \text{, So} \quad \mathbf{M} = \left(\frac{y_2-y_1}{x_2-x_1}\right)$$

$$\mathbf{M} = \left(\frac{4 - 0}{0-(-3)}\right) = \left(\frac{4}{3}\right)$$

Note that the gradient of a straight line is constant at every point of the line. So, if you picked another two points on the same line and calculated the gradient, you will get the same answer.

The y-intercept is denoted as "c" in the equation of a straight line.

The gradient is 0 if parallel to x and 1 if parallel to y.

EQUATION OF A STRAIGHT LINE

The equation of a straight line is defined as

$Y = mx + c$

Where m= slope, c= y-intercept

So, if we know the slope and y-intercept, then we can write the equation of the line.

Let's consider this example:

Example: If the slope of a line si7 and the y-intercept is -1, write the

The equation of a straight line is defined as

$Y = mx + c$

Where m= slope, c= y-intercept

So, if we know the slope and y-intercept, then we can write the equation of the line.

Let's consider this example:

Example: If the slope of a line si7 and the y-intercept is -1, write the equation of the line.

Solution

Equation of a line is $y = mx + c$

m= slope = 7, c= y-intercept = -1

Equation of the line will be

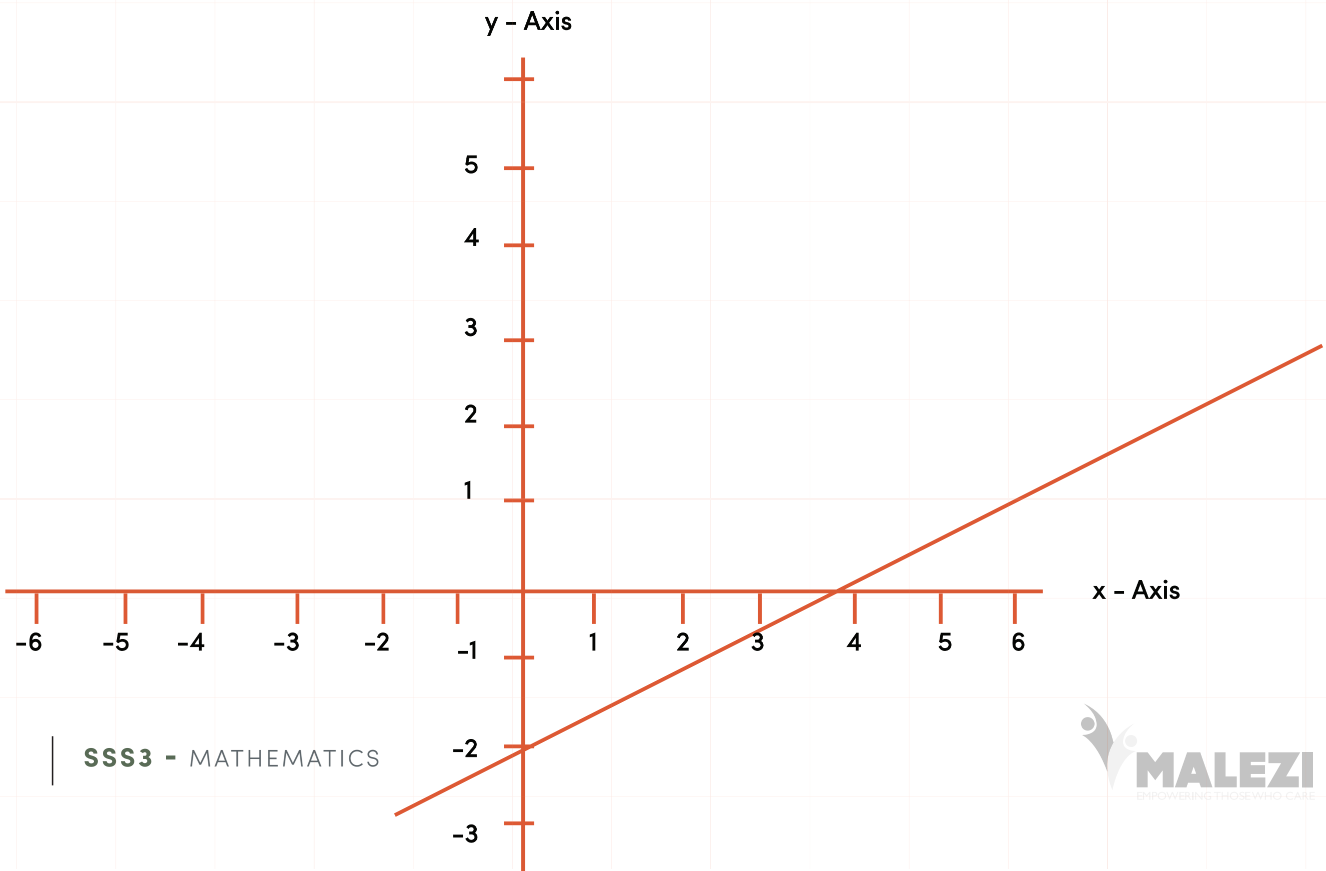
$y = 7x + (-1)$

Therefore, $y = 7x - 1$

To plot this on a graph, you need to find the x-intercept and the y-intercept. To get the x-intercept, put y=o in the equation and find x. To get the y-intercept, put x=0 in the equation and find y. For the example above,

x-intercept = 1/7 and y-intercept = -1. Then the two points will be (x-intercept, 0) and (0, y-intercept) which in this case, are (1/7, 0) and (0, -1). Plot the points and join the line. Voila!! You've got a linear graph!!

Example 2: What is the equation of the line in the graph below?



Solution

We need two things: (i) the y-intercept (ii) the slope

y-intercept = -3

To find the slope, let's choose points (4,0) and (0,-3). The slope will be

$$\mathbf{M} = \left(\frac{\text{Change in y}}{\text{Change in x}}\right), \text{ So } \left(\frac{y_2-y_1}{x_2-x_1}\right)$$

$$\mathbf{M} = \left(\frac{-3-0}{0-4}\right) = \left(\frac{-3}{-4}\right) = \left(\frac{3}{4}\right)$$

Therefore, the equation of the line, y= mx +c will be

$$y = \frac{3x}{4} - 1$$

ANGLE BETWEEN TWO INTERSECTING STRAIGHT LINES

Consider two intersecting straight lines AB and CD (draws the lines on the board)

Let the slope of AB be m1

Let the slope of CD be m2

To find the angle between them, we use this formula:

$$\textbf{Tan } \theta = \left(\frac{m_1-m_2}{1+m_1 m_2}\right)$$

The two straight lines bordering the right-hand side means absolute value, i.e., positive value.

The angle between two intersecting straight lines can also be called the measure of angles between the lines.

If the two lines are perpendicular, then $\theta = 90^0$

If the two lines are parallel, then $\theta = 0^0$, that is, there is no angle between them.

Let's consider these examples:

1. Find the angle between the lines $3x - 2y = 4$ and $x + 4y = 1$

Solution

$$\textbf{Tan } \theta = \left(\frac{m_1 - m_2}{1 + m_1 m_2} \right)$$

We need to find m1 and m2

m1= slope of equation of first line

$y = mx + c$ Hence, $3x - 2y = 4$

Make y the subject

$3x - 4 = 2y$, $2y = 3x - 4$

$y = 3x/2 - 4/2 = 3x/2 - 2$

Recall, $y = mx + c$

$$M_1 = \frac{3}{2}$$

M_2 = slope of equation of second lilne

$X + 4y = 1$ Hence, $M2 = (-1)/4$

From $\textbf{tan } \theta = \left(\frac{m_1 - m_2}{1 + m_1 m_2} \right)$, $\textbf{tan } \theta = \left(\frac{3/2 - (-1/4)}{1 + 3/2 \times (-1)/4} \right)$

From **tan** θ = $\left(\dfrac{14}{5}\right)$ θ = **tan**$^{-1}\left(\dfrac{14}{5}\right)$

$\theta = 70.35^0$

Example: 2. Given two intersecting lines AB and BC, and A(-2,1), B(2,3), and C(-2,4) are the points, find the angle between the lines.

Solution

From **tan** θ = $\left(\dfrac{m_1 - m_2}{1 + m_1 m_2}\right)$

We need to find m1 and m2
M1 = slope of line AB
M2 = slope of line BC
Note: You can interchange them, you will still get the same answer
For line AB, A(-2,1) and B(2,3) are the points

$$M_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \left(\frac{3 - 1}{2 - (-2)}\right) = \left(\frac{2}{4}\right) = \left(\frac{1}{2}\right)$$

For line BC, B(2,3) and C(-2,-4) are the points

$$M_2 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \left(\frac{-4 - 3}{-2 - 2}\right) = \left(\frac{-7}{-4}\right) = \left(\frac{7}{4}\right)$$

$$M_1 = \left(\frac{1}{2}\right), \quad M_2 = \left(\frac{7}{4}\right)$$

Having gotten the value of m1 and m2 we can go ahead and find the angle

$$\text{From } \tan\theta = \left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) = \left(\frac{1/2 - 7/4}{1 + (1/2 \times 7/4)}\right) = \left(\frac{-5/4}{1 + 7/8}\right)$$

$$= \left(\frac{-5}{4}\right)\left(\frac{8}{15}\right) = \left(\frac{2}{3}\right)$$

Hence $\theta = \tan^{-1}\left(\frac{2}{3}\right)$ $\quad \theta = \mathbf{33.69^0}$

Therefore, the angle between the lines = $\mathbf{33.69^0}$

APPLICATION OF LINEAR GRAPHS TO REAL LIFE SITUATIONS

We can also use the knowledge gained here to apply it in real-life situations. Some of them are

- The research process, to prepare government budget
- To determine whether our weight is proportional to our height
- To know if we are making a profit in business (using demand/supply curve)

Let's consider an example below:

Example: A water company charges a flat fee of #50 and an additional #150 per gallon of water. Write a linear equation to approximate the cost (in naira) in terms of x, the gallons of water.

How much would 5 gallons of water cost?

Solution:

The equation of a line is $y = mx + c$

C = constant = flat fee= #50

The total cost is equal to the rate per gallon times the number of gallons ordered plus the cost of the flat fee.

That is, $y = mx + c$

$y = 150x + 50$

To calculate the cost of 5 gallons, substitute 5 for x into the equation.

$y = 150x + 50 = 150(5) + 50 = 750 + 50$

= #800

SUMMARY

So far, we have learnt how to

1. Identify the cartesian rectangular coordinate
2. Draw and interpret linear graphs
3. Determine the distance between two coordinate points
4. Find the midpoint of the line joining two points
5. Apply the concept to a real-life situation
6. Define and determine the gradient and intercept of a line
7. Determine the equation of a line
8. Find the angle between two intersecting straight line
9. Apply linear graphs to real-life situations.

INTERACTIVE ASSESSMENT QUESTIONS

1. Set of points that form a straight line is called a

A Parabola
B Set graph
C Pointed graph
D Linear graph

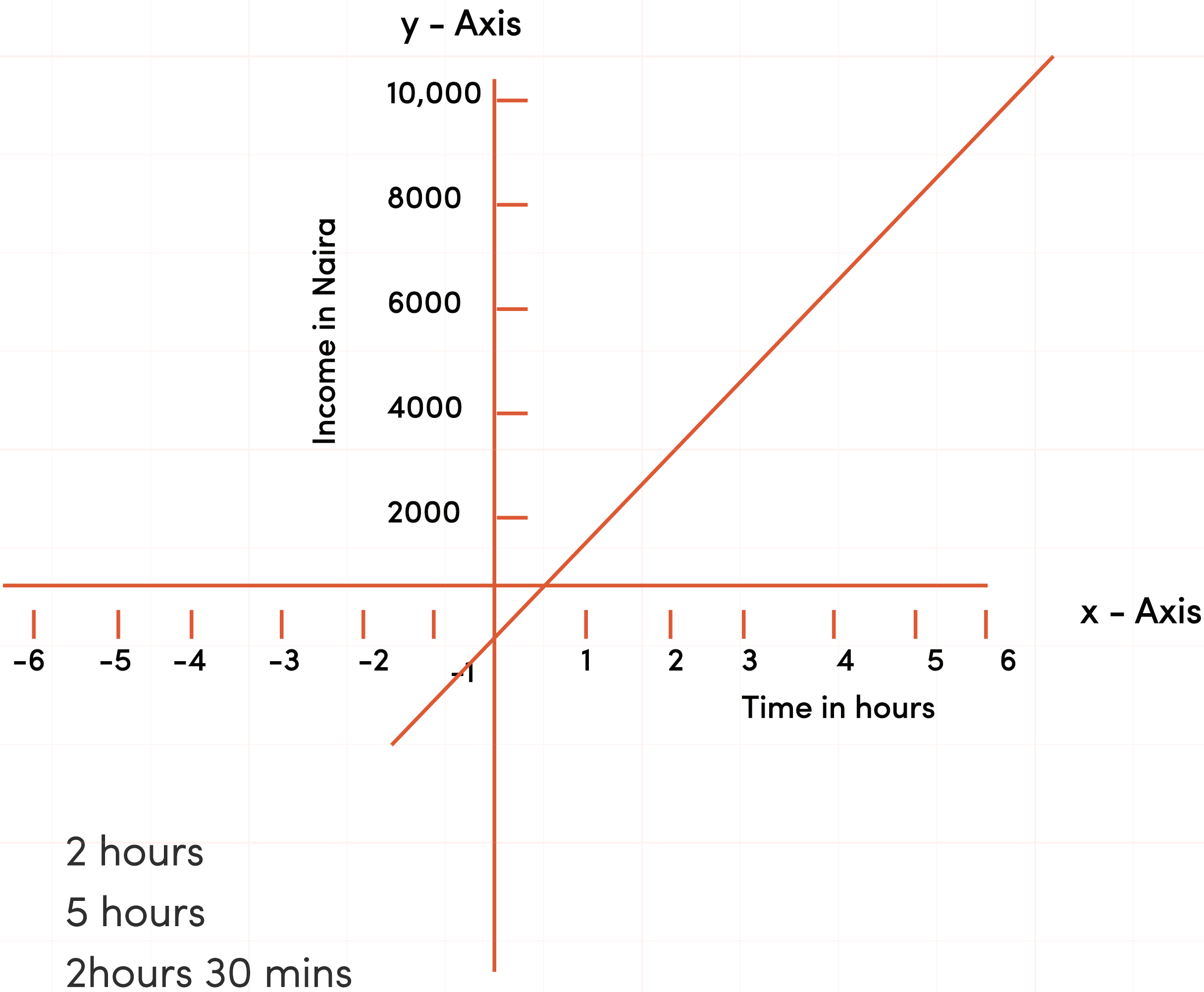
2. The distance between point R(2,-6) and S(-4,-3) is

A $\sqrt{45}$ units
B $\sqrt{17}$ units
C $\sqrt{13}$ units
D $\sqrt{65}$ units

3. The midpoint of the line joining (-1,4) and (2,-3) is

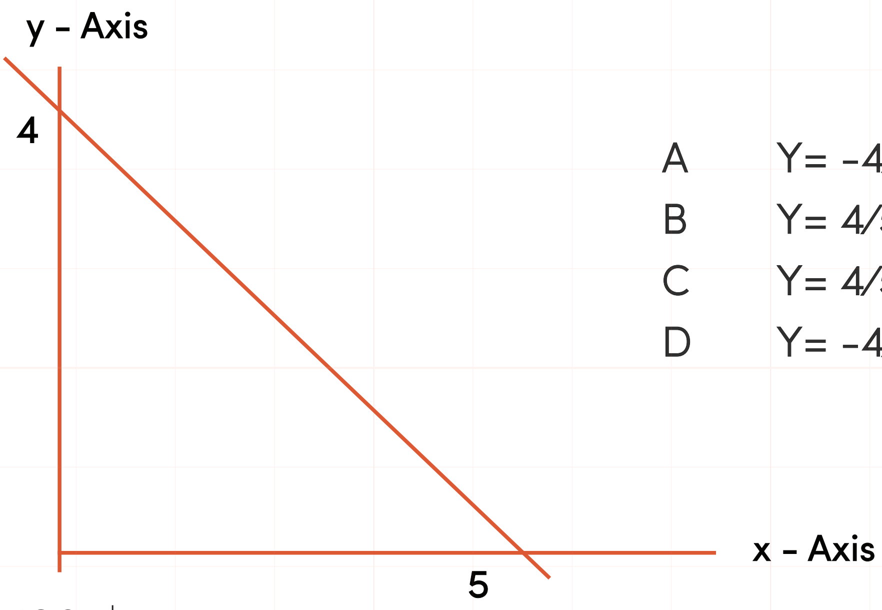
A (2, -3)
B (8, 5)
C (-1/2 , -1/2)
D (1/2 , 1/2)

4. A cleaning job pays #2,000 per hour. A graph of income per hour is drawn below. With the help of the graph shown below, determine how many hour(s) is/are needed to earn #5,000.



A 2 hours
B 5 hours
C 2hours 30 mins
D 1 hour

5. What is the equation formed by the graph below is...



A Y= -4/5 x+4
B Y= 4/5 x+4
C Y= 4/5 x-4
D Y= -4/5 x-4

THEME

04

Numbers and Numeration.

Algebraic Process.

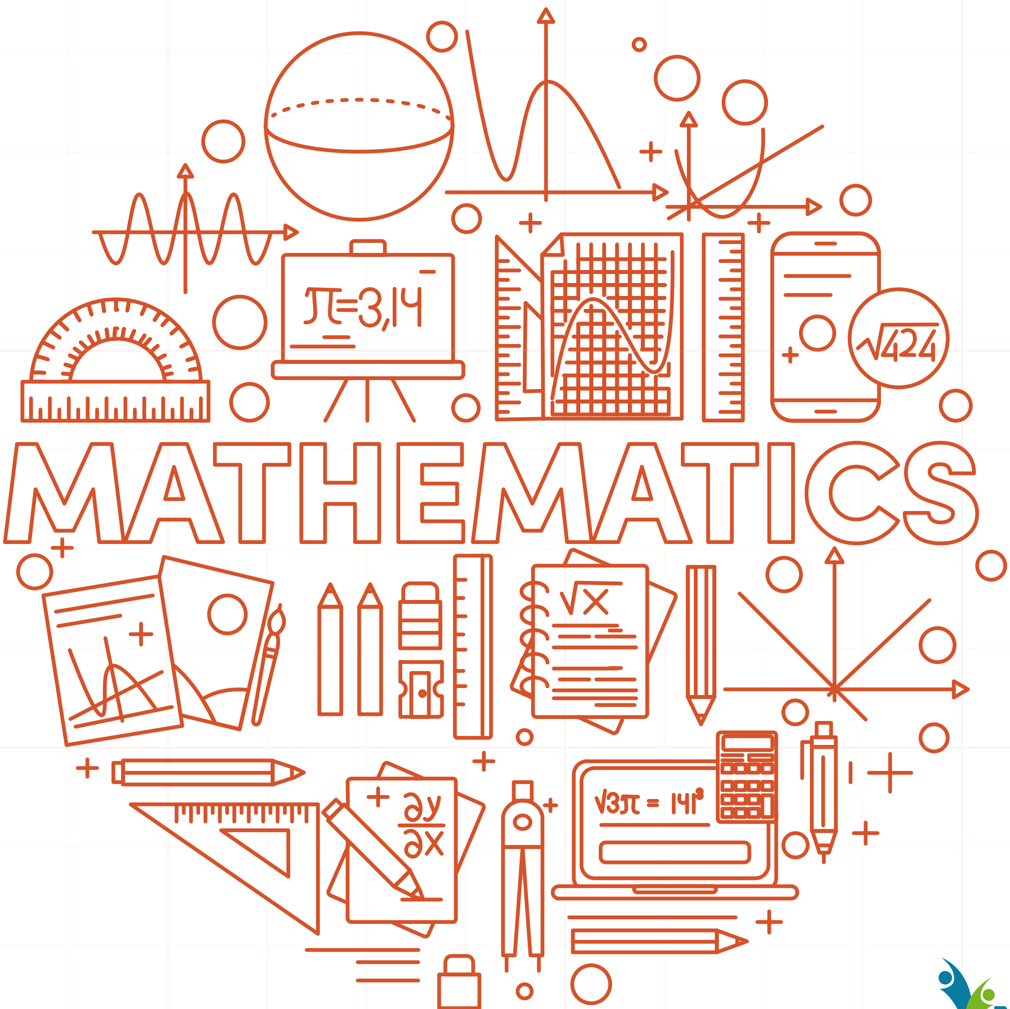
Geometry.

Introductory Calculus.

DIFFERENTIATION OF ALGEBRAIC FUNCTIONS

PERFORMANCE OBJECTIVES

1. Explain the meaning of differentiation
2. Differentiate from the first principle
3. Recognize the derivatives of some basic functions
4. Apply the rules of differentiation of functions
5. Apply differentiation of capital market and real-life situations.



MEANING OF DIFFERENTIATION/DERIVED FUNCTION

Differentiation is the process of finding the rate of change or derivative in a function when some quantities in the function are either decreased or increased.

For example, given the function $y = f(x)$, a change in x will produce a corresponding change in y. When y is increased, x is bound to increase in proportion and vice versa. Note: The reverse of differentiation is integration.

DIFFERENTIATION FROM THE FIRST PRINCIPLE

The method of finding the derivative of a function from the definition is called differentiation from the first principle. Note: A change in x to $x+\Delta x$ produces a corresponding change in y to $y+\Delta y$.

Example 1:
Differentiate the following from the first principle
$y=2x+5$
$y=x^2$

Solution

A. $y=2x+5$
Take increment in both x and y
$y+\Delta y=2(x+\Delta x)+5$
$\Delta y=2(x+\Delta x)+5-y$
$\Delta y=2(x+\Delta x)+5-(2x+5)$
$\Delta y=2x+2\Delta x+5-2x-5$
$\Delta y=2\Delta x$

Divide both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{2\Delta x}{\Delta x}$$ Take limits of both sides as Δx tends to zero

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} 2$$

$$\therefore \frac{dy}{dx} = \mathbf{2}$$

B. $y=x^2$

Take increment in both x and y

$y+\Delta y=(x+\Delta x)^2$

$\Delta y=(x+\Delta x)^2-y$

$\Delta y=(x+\Delta x)^2-x^2 \quad = \quad (x+\Delta x)(x+\Delta x)-x^2$

$\Delta y=x^2+x\Delta x+x\Delta x+(\Delta x)^2-x^2$

$\Delta y=2x\Delta x+(\Delta x)^2$

divide both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \frac{2x\Delta x}{\Delta x} + \frac{(\Delta x)^2}{\Delta x}$$

$$= 2x + \Delta x$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x)$$

$$\therefore \frac{dy}{dx} = 2x + 0 = \mathbf{2x}$$

EXAMPLE 2:

Differentiate the following from the first principle

$$y = \frac{1}{x}$$

Take increment in both x and y

$$y + \Delta y = \frac{1}{x + \Delta x}$$

$$\Delta y = \frac{1}{x + \Delta x} = \frac{1}{x + \Delta x} - \frac{1}{x} = \frac{x - (x + \Delta x)}{x(x + \Delta x)}$$

$$\Delta y = \frac{-\Delta x}{x(x + \Delta x)}$$

divide both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{-\Delta x}{x(x + \Delta x)} \div \Delta x = \frac{-\Delta x}{x(x + \Delta x)} \text{ x } \frac{1}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{-1}{x(x + \Delta x)}$$

take limits of both sides as $\Delta x \to 0$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{-1}{x(x + \Delta x)} \right)$$

$$\therefore \frac{dy}{dx} = \frac{-1}{x(x + 0)} = \frac{-1}{x \times x}$$

$$= \mathbf{\frac{-1}{x^2}}$$

STANDARD DERIVATIVES OF SOME BASIC FUNCTIONS

(1) if y = a, where 'a' is constant, then $\frac{dy}{dx} = 0$

(2) if $y = ax$, then $\frac{dy}{dx} = a$

(3) if $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

(4) if $y = ax^n$, then $\frac{dy}{dx} = nax^{n-1}$

(5) if $y = (ax+b)^n$, then $\frac{dy}{dx} = na(ax+b)^{n-1}$

(6) if $y = \sin ax$, then $\frac{dy}{dx} = a \cos ax$

(7) if $y = \cos ax$, then $\frac{dy}{dx} = -a \sin ax$

(8) if $y = e^{ax}$, then $\frac{dy}{dx} = ae^{ax}$

I would like to note that the one most commonly used here is Number (4), i.e, If $y = ax^n$ where a is the constant and n is the power, then

$$\frac{dy}{dx} = nax^{n-1}$$

EXAMPLE 1:

Use the standard derivatives given above to find $\frac{dy}{dx}$ of the following functions

A. $y=4$

$$\frac{dy}{dx} = 0$$

B. $y=7x$

$$\frac{dy}{dx} = 7$$

C. $y= x^3$

$$\frac{dy}{dx} = 3x^2$$

D. $y= 3x^5$

$$\frac{dy}{dx} = 3 \times 5 \times x^4$$

$$\frac{dy}{dx} = 15x^4$$

EXAMPLE 2:

Differentiate the following functions with respect to x

A. $y=x^{-4}$

$$\frac{dy}{dx} = -4x^{-5}$$

B. $y= \sin 2x$

$$\frac{dy}{dx} = 2\cos 2x$$

C. $y= e^{-4x}$

$$\frac{dy}{dx} = -4e^{-4x}$$

D. $y= (4x-1)^{\frac{-1}{2}}$

$$\frac{dy}{dx} = -\frac{1}{2} \times 4(4x-1)^{\frac{-3}{2}}$$

$$\frac{dy}{dx} = -2(4x-1)^{\frac{-3}{2}}$$

RULES OF DIFFERENTIATION

SUM AND DIFFERENCE RULE

a. If y = u + v (sum rule)

$$\frac{dy}{dx} = \frac{d\,(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

b. If y = u – v (difference rule)

$$\frac{d\,(u-v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

EXAMPLE1:

Differentiate $y = x^4 + 4x^3 - 2x + 1$ concerning x

$$y = x^4 + 4x^3 - 2x + 1$$

$$\frac{dy}{dx} = x^4 + 4x^3 - 2x + 1 \quad = 4x^3 + 12x^2 - 2$$

$$\therefore \frac{dy}{dx} = 4x^3 + 12x^2 - 2$$

EXAMPLE 2:

find $\frac{dy}{dx}$ of the equation of the curve $x^3 + 3x^2 - 9x + 5$

$$y = x^3 + 3x^2 - 9x + 5$$

$$\frac{dy}{dx} = 3x^2 + 6x - 9$$

$$\therefore \frac{dy}{dx} = 3x^2 + 6x - 9$$

CHAIN RULE

FUNCTION OF A FUNCTION

The chain rule is used to find the derivatives of functions that have powers. For example, $(x-3)^5$, $(2x-5)^3$ etc.

The chain rule formula is:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

EXAMPLE1:

Differentiate a. $y = (2x+8)^4$ and b. $y = \sqrt{(x^2-1)}$

Solutions

a. $y = (2x+8)^4$ and b. $y = \sqrt{(x^2-1)}$

let $u = 2x + 8$, $\rightarrow y = u^4$

$$\frac{dy}{dx} = 2 = 4u^3 = 4u^3 \times 2 = 8u^3$$

$$\therefore \frac{dy}{dx} = 8(2x+8)^3$$

b. $y = \sqrt{(x^2-1)}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

let $y = \sqrt{u}$ and $u = x^2 - 1$

$y = u^{\frac{1}{2}}$

$$\frac{dy}{du} = \frac{1}{2}u^{\frac{-1}{2}} \qquad \therefore \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{1}{2}u^{\frac{-1}{2}} \times 2x = \frac{2x}{u^{\frac{1}{2}}} = \frac{x}{(x^2-1)^{\frac{1}{2}}} = \frac{x}{\sqrt{(x^2-1)}}$$

PRODUCT AND QUOTIENT RULES

SUM AND DIFFERENCE RULE

If y=uv, where u and v are separate functions of x, then the product rule states that:

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Similarly,if $y=\frac{v}{u}$, then the quotient rule states that:

EXAMPLE 1:

Find the derivative of the function: $(2x-1)^3 \quad (x^2-1)^2$

Let $u = (2x-1)^3$ and $v = (x^2-1)^2$

$$\frac{dy}{dx} = 3 \times 2(2x-1)^2 \qquad \frac{dy}{dx} = 2 \times 2x(x^2-1)^1$$

$$\frac{dy}{dx} = 6(2x-1)^2 \qquad \frac{dy}{dx} = 4x(x^2-1)$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = (x^2-1)^2 \times 6(2x-1)^2 + (2x-1)^3 \times 4x(x^2-1)$$

$$\frac{dy}{dx} = 2(x^2-1)(2x-1)^2\,[3(x^2-1)+2x(2x-1)]$$

$$\frac{dy}{dx} = 2(x^2-1)(2x-1)2\,[3x^2-3+4x^2-2x]$$

$$\frac{dy}{dx} = \mathbf{2(x^2-1)(2x-1)^2(7x^2-2x-3)}$$

EXAMPLE 2:

Find the derivative of the function: $\frac{1 - x^2}{1 + x^2}$

Let $u = 1 - x^2$ and $v = 1 + x^2$

$$\frac{dy}{dx} = -2x^2 \qquad \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(1 + x^2)(-2x)-(1-x^2)2x}{(1 + x2)^2} = \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{-4x}{(1 + x2)^2}$$

HIGHER DERIVATIVE

If $y=x^n$

Then $\frac{dy}{dx} = nx^{n-1}$

Also, $\frac{d^2x}{dy^2} = n(n-1)x^{n-2}$ (second derivative)

$\frac{dx^3}{dy^3} = n(n-1)(n-2)x^{n-3}$ (third derivative) etc.

EXAMPLE:

Find the second derivative of $y=3x^3-5x^2$

Solution

$Y=3x^3-5x^2$

Then $\frac{dy}{dx}=9x^2-10x$

Also, $\frac{d^2y}{dx^2}=18x-10$

APPLICATION OF DIFFERENTIATION TO REAL LIFE SITUATIONS

GRADIENT

If y or f is a function of x , then the first derivative $\frac{dy}{dx}$ or f '(x) is called the gradient function. The gradient of a curve at any point $P(x_1,y_1)$ is obtained by substituting the values of x_1 and y_1 into the expression for $\frac{dy}{dx}$. This is the same as the gradient of the tangent at that point.

EXAMPLE 1:

Find the gradient of the curve $y= x^2 + 7x - 2$ at the point (2,16)

Solution

$y = x^2 + 7x - 2$

$\frac{dy}{dx} = 2x+7$, Thus at (2,16)

$\frac{dy}{dx} = 2(2) + 7 = 11$

VELOCITY AND ACCELERATION

Suppose that a particle's distance, s meters, after t seconds is given by s=t2+3t+5. The velocity is the rate of change of s compared with t, i.e. $\frac{ds}{dt}$ Since $s= t^2+3t+5$, then $\frac{ds}{dt}=2t+3$

Hence the velocity after t seconds is given by 2t+3.

Acceleration is the rate of change of velocity in comparison with time. If velocity is $\frac{vm}{s}$ then the acceleration is given by $\frac{dv}{dt}$

EXAMPLE 1:

A particle moves in a straight line specified by the equation $x = 3t^2- 4t^3$. Find the velocity and acceleration after 2 seconds.

Solution

$$x = 3t^2- 4t^3$$

$$v = \frac{dx}{dt} \quad 6t - 12t^2 \quad , \text{at } t = 2, \text{ we have } 6(2) - 12(2)^2 = 12- 48$$

$$= 36m/s$$

$$a = \frac{d^2x}{dt^2} \; 6-24t \quad t = 2, \quad a = 6-24(2) = 6 - 48$$

$$a=\mathbf{-42m/s^2}$$

MALEZI

EXAMPLE 1:

An object projected vertically upwards satisfies the relation
$h = 27t - 3t^2$, where hm is the height after t seconds.

a. Find the time the object will take to reach its highest point.

b. How high does it go?

Solution

a. $h = 27t - 3t^2$

$$v = \frac{dx}{dt} = 27-6t$$

when the object reaches the highest point,the velocity,v = 0
i.e. $27-6t=0$
$6t=27$
$t = 27/6 = 4.5$seconds

b. to find the highest point, substitute t=4.5s into the expression for h.

$h = 27(4.5) - 3(4.5)^2$

h = **60.75m**

INCREASING AND DECREASING FUNCTIONS

EXAMPLE 1:

An object projected vertically upwards satisfies the relation
$h = 27t - 3t^2$, where hm is the height after t seconds.

A function y is increasing if $\frac{dy}{dx} > 0$ while a function is decreasing if, $\frac{dy}{dx} < 0$

Solution

Let $y=x^2-x$

x^2-x is increasing if $\frac{dy}{dx} > 0$

$$\frac{dy}{dx} = 2x - 1 > 0 \quad 2x > 1$$

$$x > \frac{1}{2}$$

RATE OF CHANGE

EXAMPLE 1:

Find the approximate increase in the area of a circle if the radius increases from 2cm to 2.02cm.

Solution

Let A denote the area of the circle of radius r.

Then,

$$A = \pi r^2$$

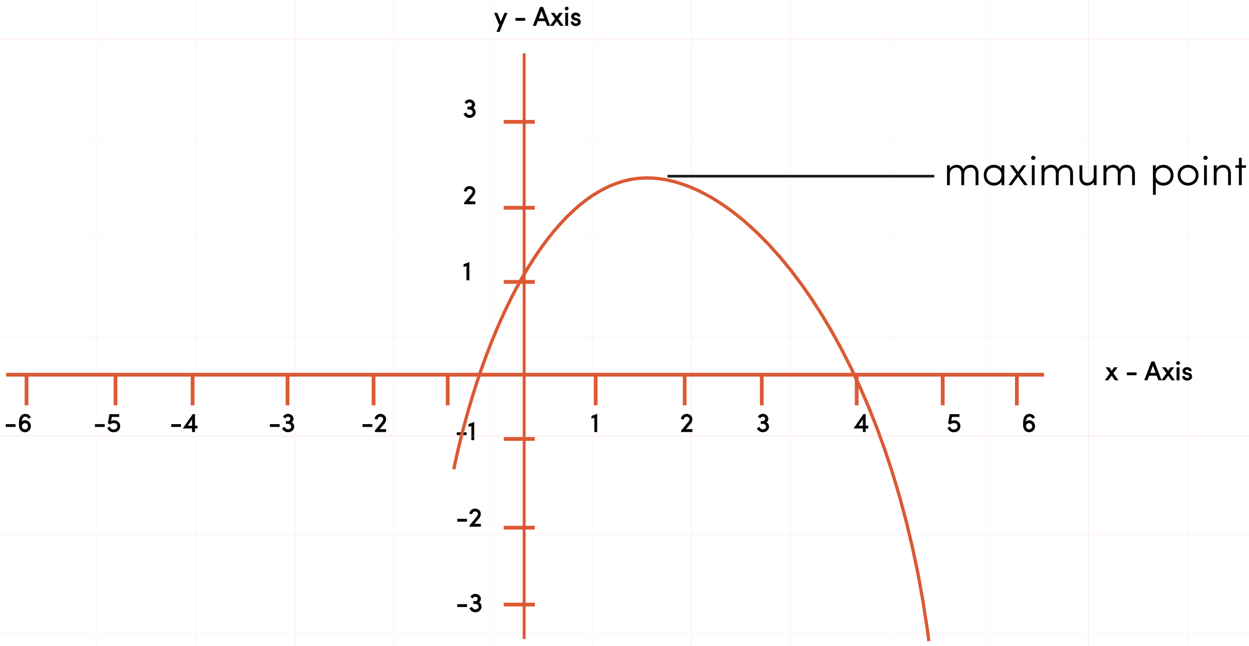
$$\frac{dA}{dr} = 2\pi r \qquad now, \delta A = \frac{dA}{dr}\delta r$$

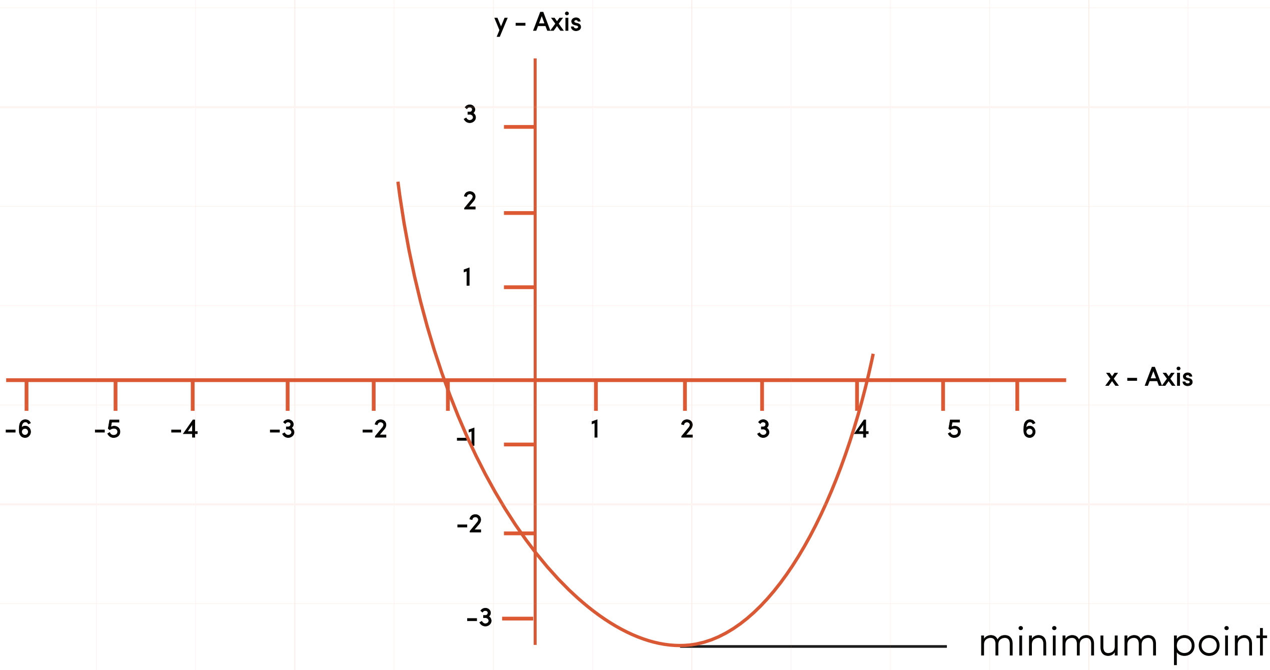
$$2\pi r(0.02) = 0.04\pi r = 0.04 \times \frac{22}{7} \times 2$$

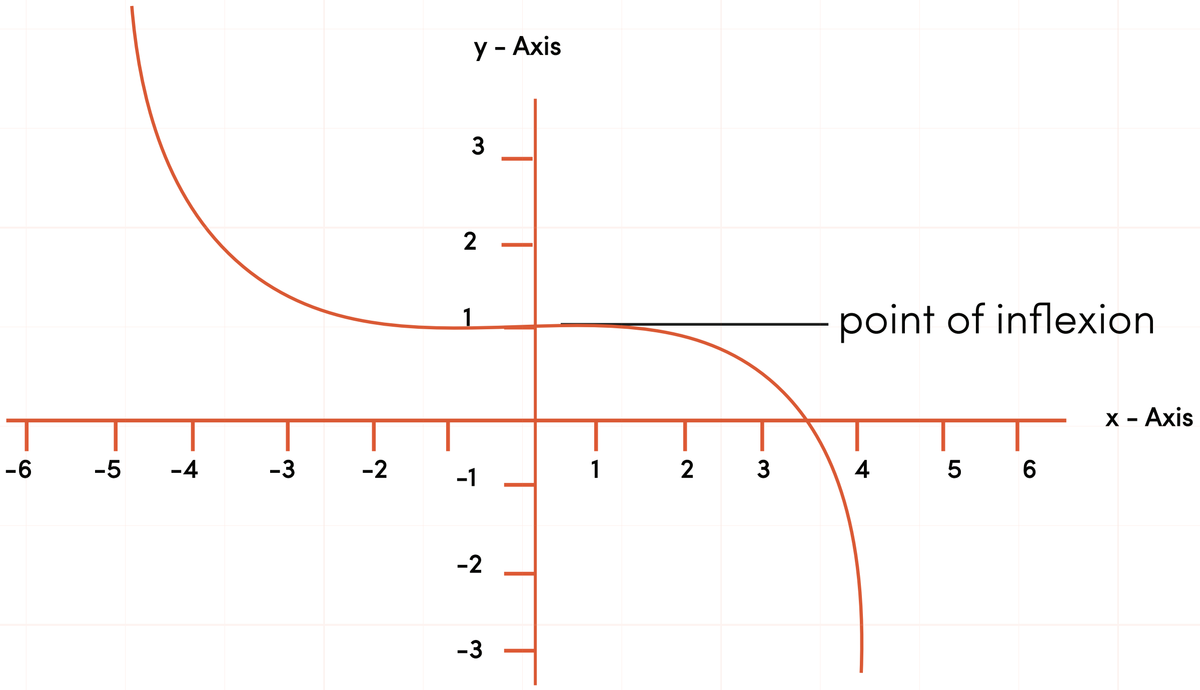
$$= \mathbf{0.2514cm^2}$$

MAXIMA AND MINIMA

A turning point/stationary point of a curve is a point at which the gradient is zero. The turning point is either maximum point (highest point) or the minimum point (lowest point) or the point of inflexion.







PROCEDURE FOR TESTING AND DISTINGUISHING BETWEEN STATIONARY POINTS

Given $y = f(x)$, determine $\frac{dy}{dx}$

Put $\frac{dy}{dx} = 0$ and solve for x, Substitute x into the equation to obtain the y, i.e. (x, y) of the turning point.

NATURE OF TURNING POINT

Using $\frac{d^2y}{dx^2}$

A. If $\frac{d^2y}{dx^2} < 0$ (i.e. negative) – the point is maximum

D. If $\frac{d^2y}{dx^2} > 0$ (i.e positive) – the point is minimum

C. If $\frac{d^2y}{dx^2} = 0$ – point of inflection.

EXAMPLE 1:

A curve is defined by the function $y=x^3-6x^2-15x-1$, find the maximum and minimum point.

Solution

First, we find $\frac{dy}{dx}$ and equate to zero.

$$\frac{dy}{dx} \quad 3x^2 - 12x - 15 = 0$$

$$x^2 - 4x - 5 = 0$$

$$x^2-5x+x-5 = 0$$

$$x(x-5)+1(x-5) = 0$$

$$(x-5)(x+1) = 0 \qquad x = 5 \text{ or } -1$$

To test for maximum or minimum, we differentiate the second time

$$\frac{dy}{dx} = 3x^2 - 12x - 15$$

$$\frac{d^2y}{dx^2} = 6x - 12$$

Put $x=5$, $6(5)-12=18>0$.............................minimum point at $\mathbf{x=5}$

Put $x=-1$, $6(-1)-12=-6-12=-18<0$..............maximum point at $\mathbf{x=-1}$

To find the corresponding y put x=5 to the first equation

i.e. $y = x^3-6x^2-15x-1$

$y = 125 - 150 - 75 - 1 = -101$, hence the minimum point $(5, -101)$

Put $x = -1$

$y = -1-6+15-1 = 7$

Maximum point $(-1,7)$

Maximum point$(-1,7)$, minimum point$(5,-101)$

NOTE: The maximum value = **7** and the minimum value = **-101**

SUMMARY

So far, we have learnt how to

1. Explain the meaning of differentiation
2. Differentiate from the first principle
3. Recognize the derivatives of some basic functions
4. Apply the rules of differentiation of functions
5. Apply differentiation of capital market and real-life situations.

INTERACTIVE ASSESSMENT QUESTIONS

1. Differentiate $y=7x-2x^{-3}$ concerning x

A. $\frac{dy}{dx} = 7 + 6x^{-4}$ **B.** $\frac{dy}{dx} = 7 - 6x^{-4}$

C. $\frac{dy}{dx} = 7 + 5x^{-4}$ **D.** $\frac{dy}{dx} = 7 - 1x^{-4}$

2. The derivative of cos x is

1. [A] - Cos x
2. [B] Sin x
3. [C] - Sin x
4. [D] Cos x
5. [E] Tan x

3. Differentiate $Y = 6x^3 - x^2 - 4x + 1$ concerning x.

A. $\frac{dy}{dx} = 18x^2 - 2x - 4 + 1$ **B.** $\frac{dy}{dx} = 18x^2 - 2x - 4x$

C. $\frac{dy}{dx} = 18x^2 - 2x + 4$ **D.** $\frac{dy}{dx} = 18x^2 - 2x - 4$

4. Differentiate $(2x^2 + 1)^4$ to x.

A. $\frac{dy}{dx} = 8x(2x^2 + 1)^3$ **B.** $\frac{dy}{dx} = 16x(2x^2 + 1)^3$

C. $\frac{dy}{dx} = 16x(2x^2 - 1)^3$ **D.** $\frac{dy}{dx} = 16x(2x^2 + 1)^2$

5. Differentiate the following for x; $y = (3+2x)(1-x)$.

A. $\frac{dy}{dx} = -4x + 1$ **B.** $\frac{dy}{dx} = 4x - 1$

C. $\frac{dy}{dx} = 4x - 1$ **D.** $\frac{dy}{dx} = 4x + 1$

6. Find the $\frac{d^3y}{dx^3}$ of $y = x^4 - 6x^3 + 5$

A. $\frac{d^3y}{dx^3} = 4x^3 - 18x^2$ **B.** $\frac{d^3y}{dx^3} = 24x - 36$

C. $\frac{d^3y}{dx^3} = 12x^2 - 36x$ **D.** $\frac{d^3y}{dx^3} = 24x + 36$

7. If $f(x) = (x^2+3)^3$, find the gradient of $f(x)$ at $x= ½$

A. $\frac{dy}{dx} = 6.5$

B. $\frac{dy}{dx} = 0.5$

C. $\frac{dy}{dx} = -6.5$

D. $\frac{dy}{dx} = 6$

8. A particle moves along a straight line in such a way that after t seconds it has gone s meters, where $s=t^2+2t$. Find the velocity of the particle after 3 seconds.

A 6m/s
B 7m/s
C 5m/s
D 8m/s

9. Find the range of values of x for which x^2-x is decreasing?

A $X > -1$
B $X > 1$
C $X < -1$
D $X < 1$

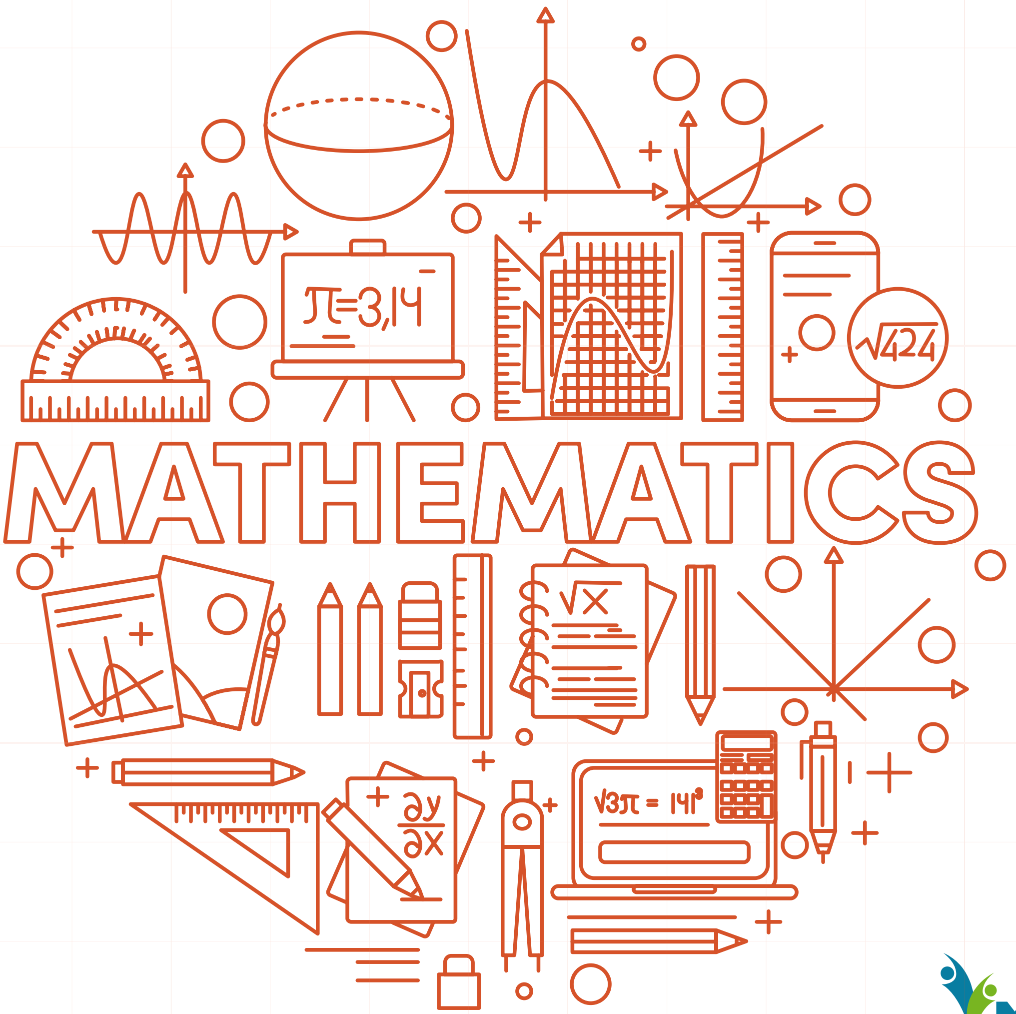
10. Find the maximum or minimum value of the curve $y=x^2-6x+5$

A minimum point is (3,4)
B maximum point is (3,-4)
C minimum point is (3,-4)
D maximum point is (3,-4)

INTEGRATION OF SIMPLE ALGEBRAIC FUNCTIONS

PERFORMANCE OBJECTIVES

1. Recognize integration as the reverse of differentiation
2. Recognize some standard integrals of polynomials and algebraic functions
3. Apply some techniques of integration such as (a) integration by substitution (b) integration by parts (c) integration by partial fractions.
4. Apply integration to real-life situations.



INTEGRATION AND EVALUATION OF DEFINITE SIMPLE ALGEBRAIC FUNCTIONS

Integration is the opposite of Differentiation. Since differentiation is the process of finding the derivative in a function, then integration is the process of obtaining a function from its derivative. A function F (x) is an anti-derivative of a given function F (x) if d $\frac{f(x)}{dx}$ = f (x).

In general, if F (x) is any antiderivative of f (x), then the most general antiderivative of f(x) is specified by f (x) + c and we write: ∫f(x)dx + c. The symbol ∫ is called an integral sign and ∫f(x)dx is called the indefinite integral. The arbitrary constant c is called the constant of integration, and the function f (x) is called the integral.

For example, f′(x) = x4 + c is an anti-derivative of $f(x) = 4x^3$ because $f'(x) = \frac{dx^4}{dx} = 4x^3 = f(x)$.

In general, if n ≠ -1, then an anti derivative of $f(x) = x^n$ is

F (x) = + C

To integrate a power of x (apart from power n = - 1, increase the power of x by 1 (one) and divide by the new power.

EXAMPLE 1:

a. $\int x5\, dx = \frac{x^{5+1}}{5+1} + c$

$$= \frac{x^6}{6} + c$$

To get the original value, we must differentiate:

Let c=1, -2 0r 3 (c can be any constant)

$$\frac{d}{dx}\left(\frac{x^6}{6} + 1\right) = \frac{6x^5}{6} = x^5$$

$$\frac{d}{dx}\left(\frac{x^6}{6} - 2\right) = x^5$$

$$\frac{d}{dx}\left(\frac{x^6}{6} + 3\right) = x^5$$

Similarly, $\frac{d}{dx}\left(\frac{ax^{n+1}}{n+1} + C\right) = ax^n \qquad (n \neq -1)$

$$\int a\ x^n\, dx = \frac{ax^{n+1}}{n+1} + C \qquad (n \neq -1)$$

b. Integrate the following:

(i). $\int 2x\, dx$ (ii). $\int (x^2+x-10)dx$ (iii). $\int x^{-0.5}\, dx$

(iv). If $\frac{dy}{dx} = 4$ and $y = 2$ when $x = -1$, find y in terms of x.

Solution

(i). $\int 2x\, dx$

$$\int 2x\, dx = 2\int x\, dx = \left(2\ \frac{x^{1+1}}{1+1}\right) + C = \left(\frac{2x}{2}\right) + C$$

$$= x^2 + C$$

(ii). $\int(x2+x-10)dx$ This can be done term by term.

$$\int(x^2+x-10)dx = \int x^2\, dx + \int x dx - \int 10 dx$$

$$= \frac{x^{2+1}}{2+1} + \frac{x^{2+1}}{2+1} \int 10x^0\, dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} - 10\frac{x^{0+1}}{0+1} + C$$

$$= \frac{1}{3}x^3 + \frac{x^2}{2}x^2 - 10x + C$$

Notice that instead of giving three different constants of integration, the three can be combined and written as one.

(iii). $\int x^{-0.5}\, dx = \frac{x^{-0.5+1}}{-0.5+1} + C$

$$= \frac{x^{0.5}}{-0.5} + C$$

$$= 2\sqrt{x} + C$$

(iv). $\frac{dy}{dx} = 4$, so $dy = 4dx$

$\int dy = \int 4dx$ ie $y = 4x + C$. When $y = 2$, $x = -1$ $2 = 4(-1) + C$

$C = 6$ Hence, $y = 4x + 6$

The integral $\int f(ax+b)dx$

Let$u = ax + b$

$\frac{du}{dx} = a,$

$du = adx,$ so $dx = 1/a\, du$

$\int f(ax+b)dx = \int f(u).\frac{1}{a}\, du = 1/a \int f(u)du.$

EXAMPLE 1:

a. Integrate (i). $\int (3x+2)^4 \, dx$ (ii). $\int \frac{3dx}{(2x-1)^2}$

Solution :

For $\int (3x+2)^4 \, dx$, Let $u = 3x + 2$, $\frac{du}{dx} = 3$

So $3dx = du$, hence $dx = 1/3du$, Meaning $\int (3x+2)4 \, dx = \int u4 \frac{1}{3} du$

$$= \frac{1}{3} \int u^4 du = \frac{1}{3} \left(\frac{u^5}{5} \right) + C = \frac{1}{15} u^5 + C$$

For b. (ii). $\int \frac{3dx}{(2x-1)^2}$, Let $U = 2x - 1$

$$\frac{du}{dx} = 2$$

$$2dx = du$$

$$dx = 0.5 \, du$$

So, $\int \frac{3dx}{(2x-1)^2}$, $= \frac{3\left(\frac{1}{2}\right)du}{u^2}$

$$= \frac{3}{2} \int \frac{du}{u^2} = \frac{3}{2} \int u^{-2} du = \frac{3}{2} \left(\frac{u^{-2+1}}{-2+1} \right) + C$$

$$= \frac{3}{2} \left(\frac{u^{-1}}{-1} \right) + C = \frac{-3}{2} \left(\frac{1}{u} \right) + C$$

$$= \frac{-3}{2} \left(\frac{1}{2x - 1} \right) + C = \frac{-3}{4x - 2} + C$$

For b. Integrate (i).$\int\left(\frac{x^4 + 3x^3 - 4}{x2}\right)$

$$= \int\left(\frac{x^4 + 3x^3 - 4}{x2}\right)dx = \int\left(\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{4}{x^2}\right)dx$$

$$= \int x2\,dx + \int 3xdx - 4\int x(-2)\,dx$$

$$= \frac{x^3}{3} + \frac{3}{2}x^3 + \frac{4}{x} + C$$

INTEGRATION BY SUBSTITUTION

This is the counterpart to the chain rule for differentiation. It can also be thought of as using the chain rule "backward". (Reverse chain rule).

STEP 1:

We should be able to write our integral in this form

$\int f\,[g(x)]\ g1\ (x)dx$

Note that we all have g(x) and its derivative g1(x)

In this example

$\int(x^2+1)^2xdx$

Here, $f(g(x)) = x^2 + 1, g^1(x)\ dx = 2x$

This integral is good to go

When our integral is set up like that, we can do this substitution:

$\int f(g(x))\ g^1\ (x)dx$

Put g(x) as u and g1(x) dx as du

$\int f(u)du$

Then we can integrate f(u), and finish by putting g(x) back as u.

We will have:

$\int(x2+1).\ 2x\,dx$

Since 2x is the derivative of (x2 + 1), we are good to go.

Put x2 + 1 as g(x) and 2x dx as g1(x)

So, we will have:

$\int f(g(x)).\ g^1(x)dx = \int f(u)du = \int(u)du$

$$\frac{u^2}{2} + C = \frac{(x^2+1)^2}{2} + C$$

EXAMPLE 1:

a. $\int \cos\ (x^2)\ 2x\,dx$

Solution

Since 2x is the integral of x^2, then we are good to go. So, we will have:

$\int \cos(u)du$

Now integrate

$\int \cos\ (u)\ du = \sin\ (u)+c$

Finally, put $u = x^2$ back again $= \sin x^2 + c$

Therefore,

$\int\cos\ (x^2)\ 2x\,dx = \sin x^2 + c$

However, we might have examples like this:

$\int\cos(x^2)\ 6x\,dx$

Here it is 6x, which is not the derivative of x2. What do we do?

We can rearrange the integral like this:

$\int\cos(x^2)\ 6x\,dx = 3\int \cos\ (x^2)\ 2x\,dx$

Since 2x is the derivative of u2, we can go on

$3\int \cos\ (x^2)\ 2x\,dx = 3\int \cos\ (u)\ du = 3\ \mathrm{Sin}\ (u) + c = 3\ \mathrm{Sin}\ (x^2) + c$

Done!!

EXAMPLE 1:

solve $\int x \cos(x)\, dx$

Solution

Let u = x

V = cos x

$\int uv\, dx = u \int v\, dx - \int u^1 \; (\int v dx)\, dx$

Differentiate U:U = x^1 = 1

Integrate V: $\int v dx = \int \cos(x)\; dx = \sin(x)$

$u\int v dx - \int u^1 \;\; (\int v dx)\;\; dx$

$x\,(\sin(x)) - \int 1\,(\sin(x))\; dx = x \sin x - \int \sin x\, dx = \sin x + \cos(x) + c$

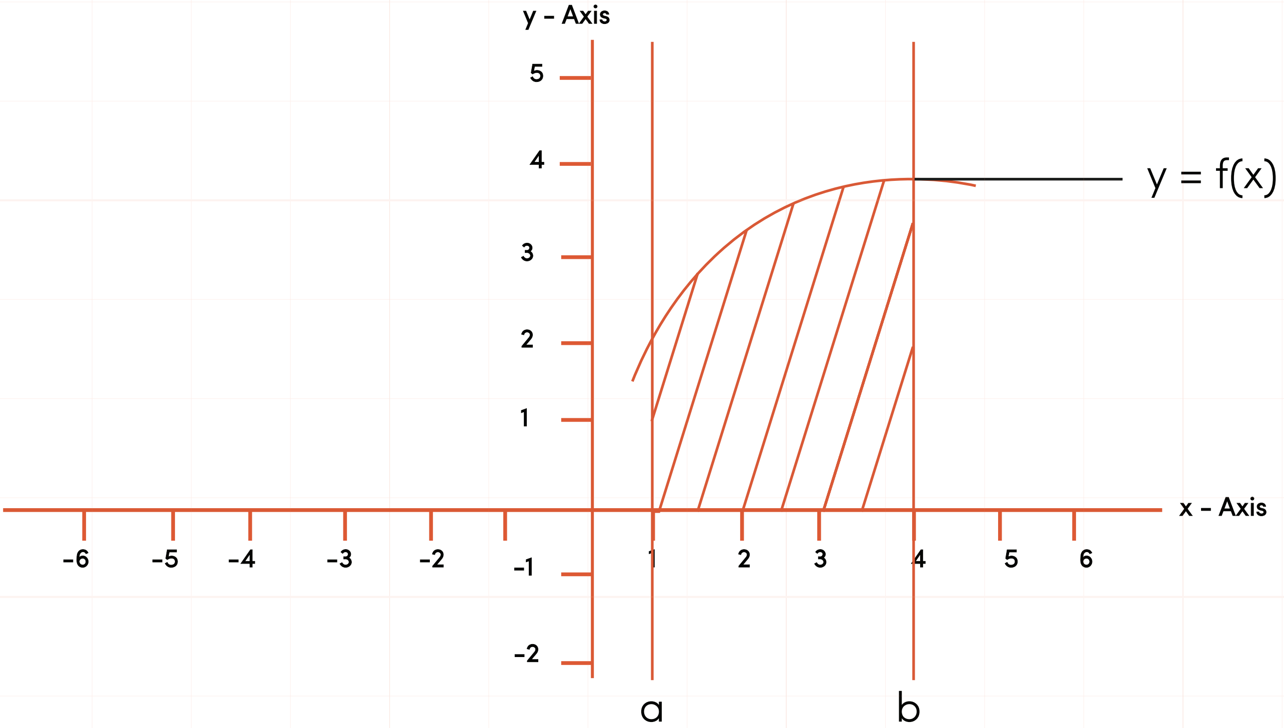
The Steps simply are

i. Choose u and v
ii. Differentiate u = u1
iii. Integrate v: $\int vdx$
iv. Put "u", u1, and $\int vd\,x$ in the expression $u\int vdx - \int u1 \;(\int vdx)\; dx$
v. Simplify and solve.

Note: Be careful when choosing u and u. Choose the "u" that is easy to differentiate and choose the v that is easy to differentiate.

APPLICATION OF INTEGRATION IN CALCULATING AREA UNDER THE CURVE

The integral $\int_b^a f(x)dx$ is called the definite integral of the function f(x) with 'a' and 'b' the lower and upper limits of the integral respectively. $\int_b^a f(x)\, dx$ geometrically represents the area bounded by the curve y = f(x), the lines x = a , x =b and the x-axis.



EXAMPLE 2:

Evaluate $\int_1^4 3x^2\,dx$

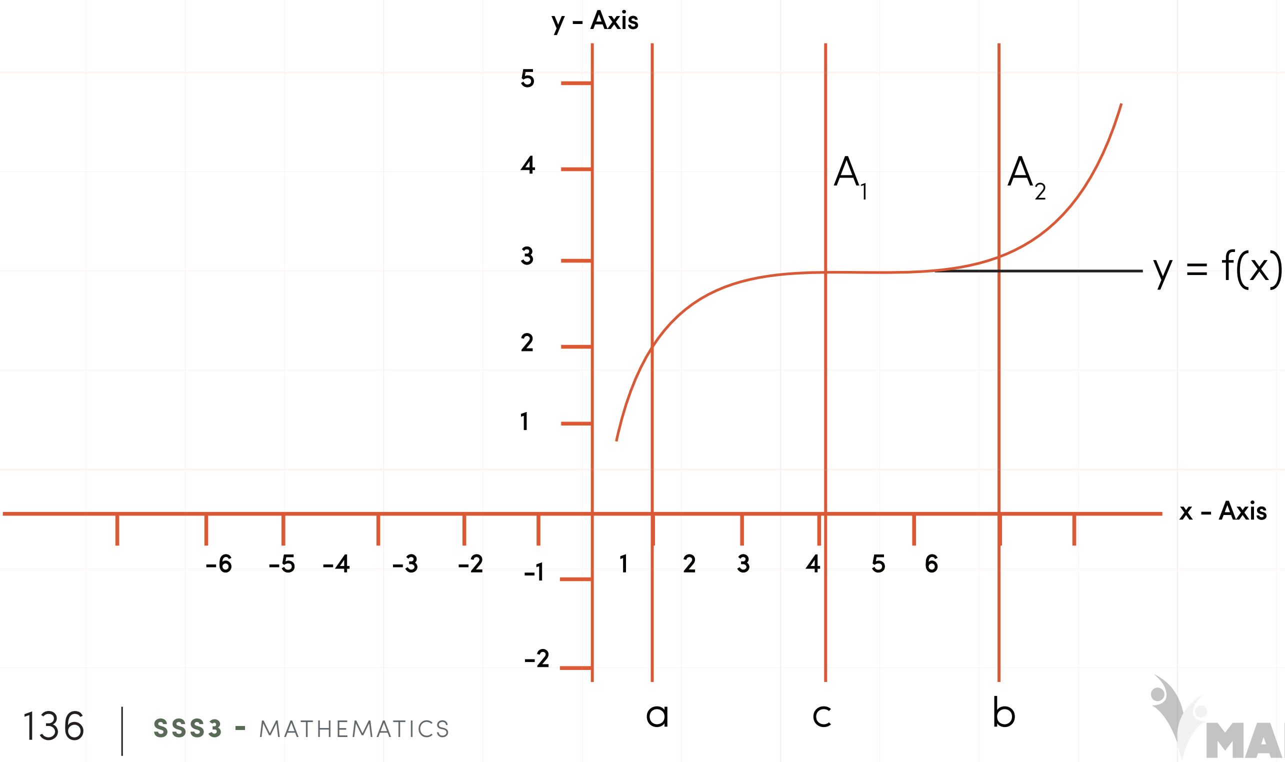
Solution

$\int_1^4 3x^2\,dx = [x^3 + c]_1^4$

Now substitute the value of the upper limit for x minus when you substitute the lower limit.

$$= (4^3 + c) - (1^3 + c) = 64 + c - 1 - c = 63$$

Now we shall examine some properties of the definite integral,



$$A_1 = \int_a^c f(x)dx$$

$$A_2 = \int_c^b f(x)dx$$

$$\int_a^c f(x)\,dx = A_1 + A_2 \quad = \int_c^b f(x)dx + \int f(x)dx$$

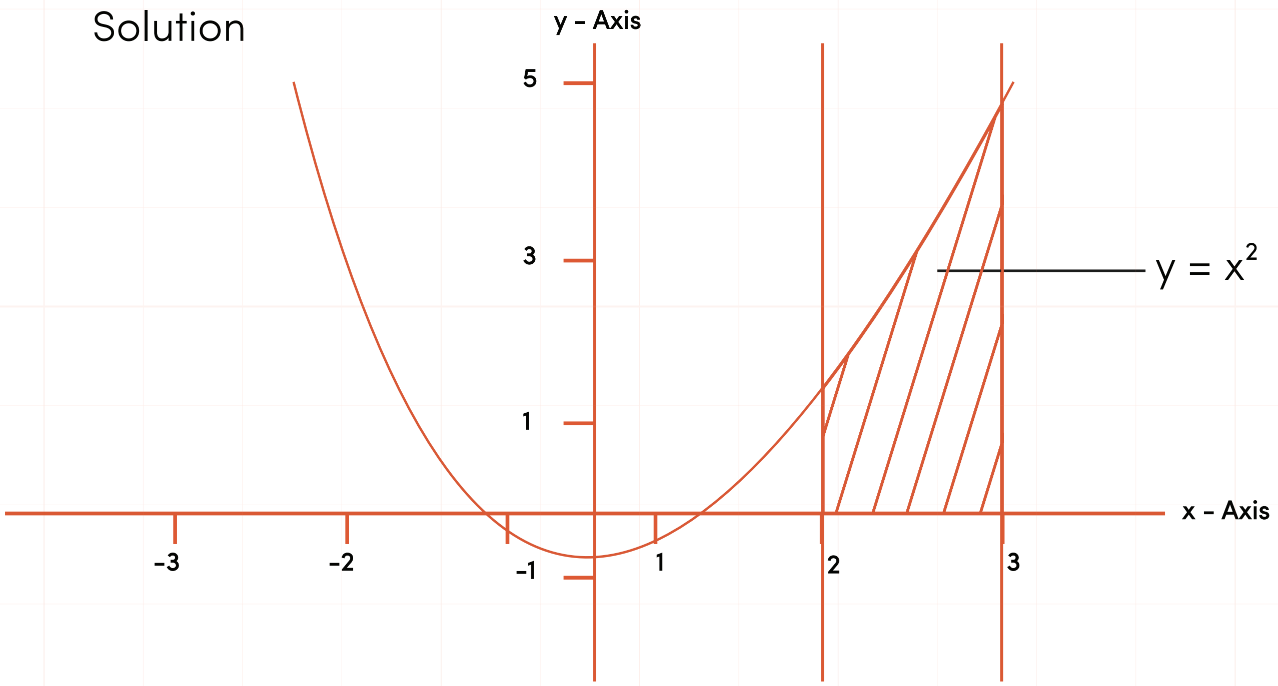
If the area is below the x-axis it will have a negative sign attached to it. Negating such an area will make it positive.

It is very essential to sketch the curve y = f(x) if the definite integral, $\int_a^b f(x)dx$ is to be used in finding the area bounded by the curve y= f(x), the x-axis, and the lines x=a and x=b.

EXAMPLE 2:

Find the area bounded by the curve y=x2 the lines x = 2, x = 3 and the x –axis.

Solution



Let the shaded area be the required area. i.e $\int_a^b ydx$

The area = $\int_2^3 x^2\,dx$

$$= \left(\frac{x^3}{3}\right)_2^3$$

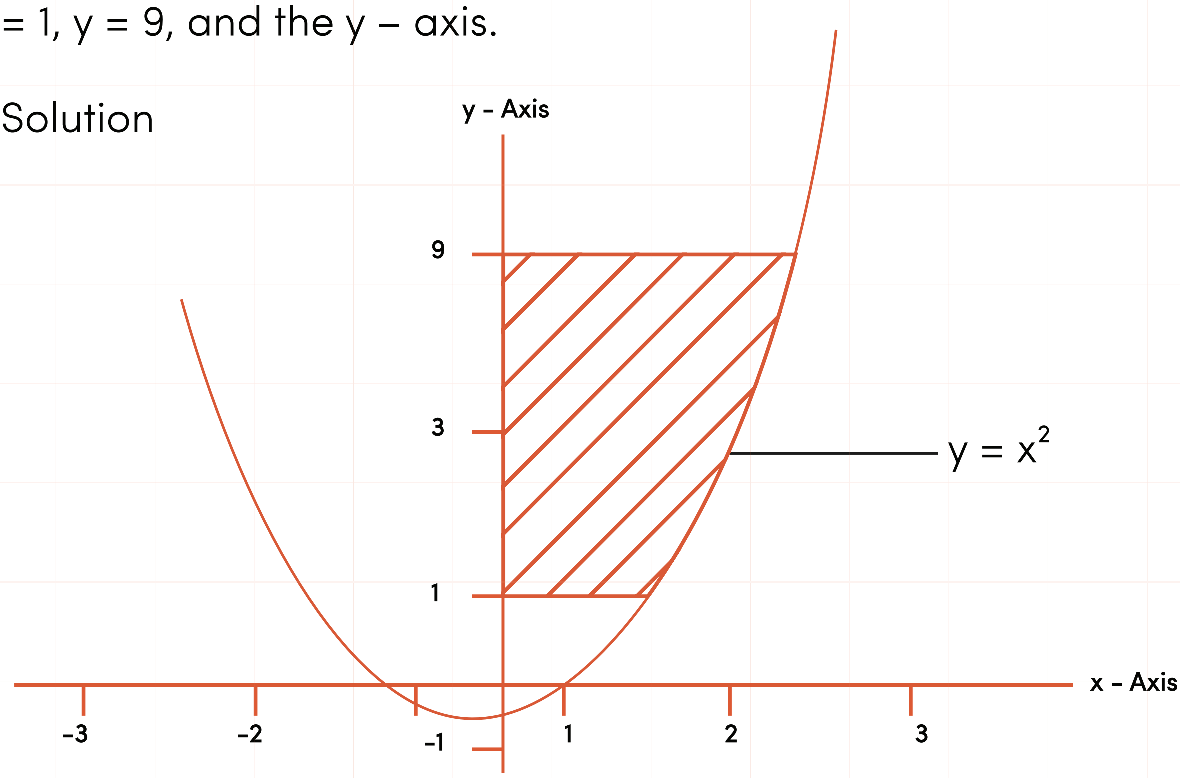
$$= \frac{3^3}{3} - \frac{2^3}{3} = \frac{27-8}{3} = \frac{19}{3}$$

$$= 6\frac{1}{3} \text{ sq. units}$$

EXAMPLE 3:

Find the area of the finite region bounded by the curve $y = x^2$ the line y = 1, y = 9, and the y – axis.

Solution



The area = $\int_1^9 x\,dy$ as y = x2 then x = √y, so the area = $\int_a^b \sqrt{y}\,dy$

$$= \left(\frac{y^{0.5+1}}{0.5+1}\right)_1^9$$

$$= \frac{2}{3}\left(9^{\frac{3}{2}} + 1^{\frac{3}{2}}\right) = \frac{2}{3}\left(3^3 - 1^3\right) = \frac{2}{3}\left(26\right)$$

$$= 17\frac{1}{3} \text{ sq. units}$$

APPLICATION OF INTEGRATION IN CALCULATING AREA UNDER THE CURVE

EXAMPLE 2:

A curve passes through the point (0,1) and its gradient at any point P(x,y) = $3x^2 - 5$. Find the equation of the curve.

Solution

Let $\frac{dy}{dx} = 3x^2 - 5$

$dy = (3x^2 - 5)dx$

$\int dy = \int (3x^2 - 5)dx$

$y = x^3 - 5x + c$

at (0,1), $1 = 0 - 5(0) + c$

$C = 1$

The equation is $y = x^3 - 5x + 1$

VELOCITY AND ACCELERATION

In differentiation, the derivative of distance is velocity. The derivative of velocity is acceleration. However, in integration, the integral of acceleration is velocity, the integral of velocity is distance.

Example: A particle moves in a straight line with a constant acceleration of $2cm/s^2$. If its velocity after t seconds is vcm/s, find u in terms of t, given that the velocity after 3 seconds is 12cm/s.

Solution

Let $\frac{dy}{dx} = 2$

$dv=\int 2dt$

$v = 2t + c$

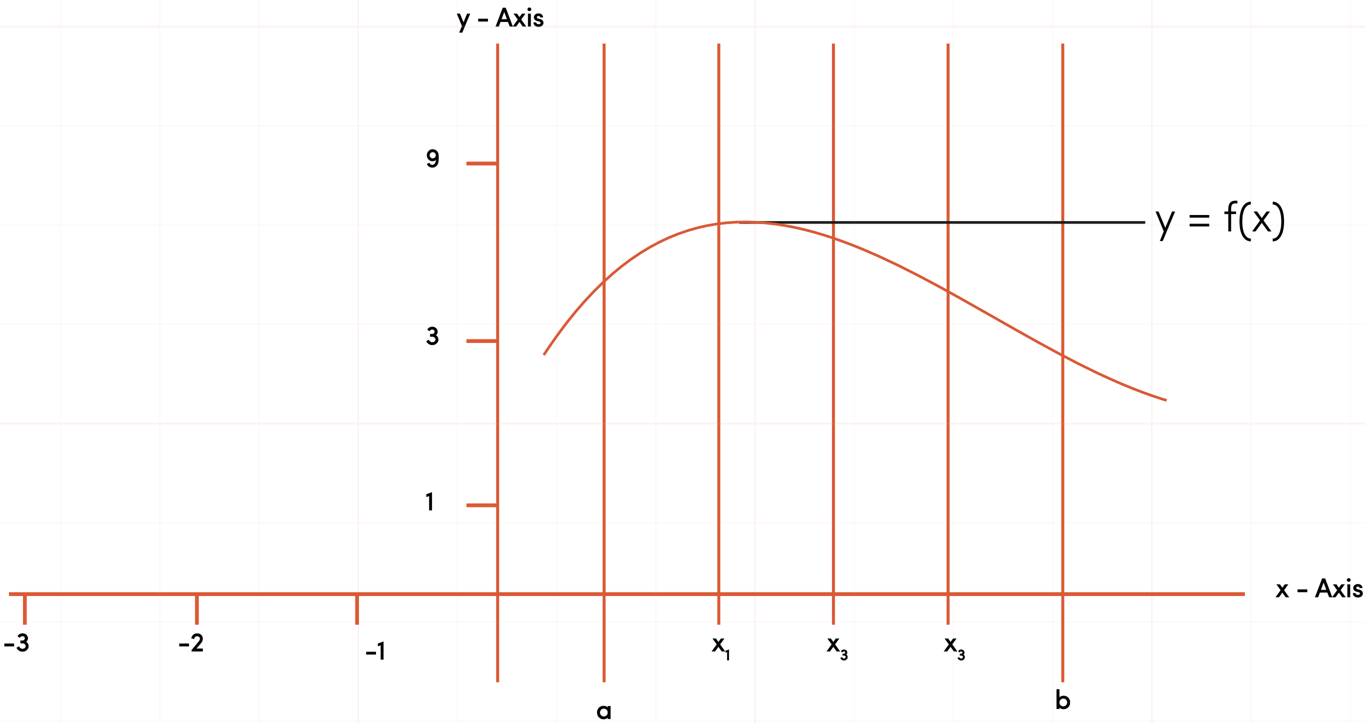
When $t = 3$ and $v = 12$

$12 = 2 \times 3 + c$

$12 = 6 + c$, $c = 6$ hence $v=2t+6$

SIMPSON'S RULE

Another rule for numerical integration is attributed to Thomas Simpson (1710-1761) an English Mathematician.



By Simpson's rule the interval an≤x≤b

is divided into an even number n of subintervals of length

$$h = \frac{b - a}{n}$$

with equally spaced points

$a = x_o, x_1, x_2, x_4, \ldots xn = b$

and their corresponding ordinates at y0,y1,y2...,yn, Simpson showed

that$\int_a^b f(x)dx \simeq \frac{1}{3} h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_{n-2} + 4y_{n-1} + y_n)$

this can also be written as

$$\int_a^b f(x)dx = \frac{1}{3} h[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1} + 2(y_2 + y_4 + y_{n-2})]$$

EXAMPLE 1:

Using Simpson's rule with **8** strips, evaluate $\int_1^5 \frac{1}{x} dx$

Correct to 2 decimal places.

Solution

The integration interval = 5 - 1 = 4 i.e. b - a = 4 n = 8

$h= \frac{b-a}{8} = \frac{4}{8} = 0.5$

x	y	First last ordinates	Odd ordinates	Remaining ordinates
1	Y0	1		
1.5	Y1		0.67	
2.0	Y2			0.50
2.5	Y3		0.40	
3.0	Y4			0.33
3.5	Y5		0.29	
4.0	Y6			0.25
4.5	Y7		0.22	
5.0	Y8	0.2	1.58	
totals		1.2		1.08

The working is set in a tabular form as follows:

$$\int_1^5 \frac{1}{x} dx \ \frac{1}{3} \times 0.5[1.2 + 4(1.48)+2(1.08)]$$

$$\int_1^5 \frac{1}{x} dx \ \simeq \frac{1}{3} \times 0.5[1.2 + 6.32 + 2.16]$$

$\simeq 1.613$

Hence, $\int_1^5 \frac{1}{3} dx \ \simeq 1.61$ (2d.p)

SUMMARY

So far, we have learnt how to

1. Recognize integration as the reverse of differentiation
2. Recognize some standard integrals of polynomials and algebraic functions
3. Apply some techniques of integration such as (a) integration by substitution (b) integration by parts
4. Apply integration to real-life situations

INTERACTIVE ASSESSMENT QUESTIONS

1. Solve $\int 3x^2 \ dx$

A $x^3 + C$

B $3x^2 + C$

C $3x^3 + C$

D $2x^3 + C$

E $x^2 + C$

2. The integral of $2x\,(x^2 - 1)$ for x

A $2x^4 - x^2 + C$
B $0.5\,x^2 - x^2 + C$
C $0.5\,x^4 - x^2 + C$
D $0.5\,x^4 - x + C$

3. Solve the integral below $\int (x+1)^3\,dx$
(Hint: Use integration by substitution)

A. $\dfrac{(x+1)^3}{4}$ **B.** $\dfrac{(x+1)^4}{4}$

C. $\dfrac{(x+1)^3}{3}$ **D.** $\dfrac{(x+1)^4}{1}$

4. Solve the integral below: $\int e^x\, x dx$

A $e^x\,(x + 1)$
B $e\,(x - 1)$
C $xe^x\,(x - 1)$
D $e^x\,(x - 1)$

5. A particle moves in a straight line in such a way that its velocity after t seconds is (3t + 4)m/s. The distance traveled in the first 3 seconds is

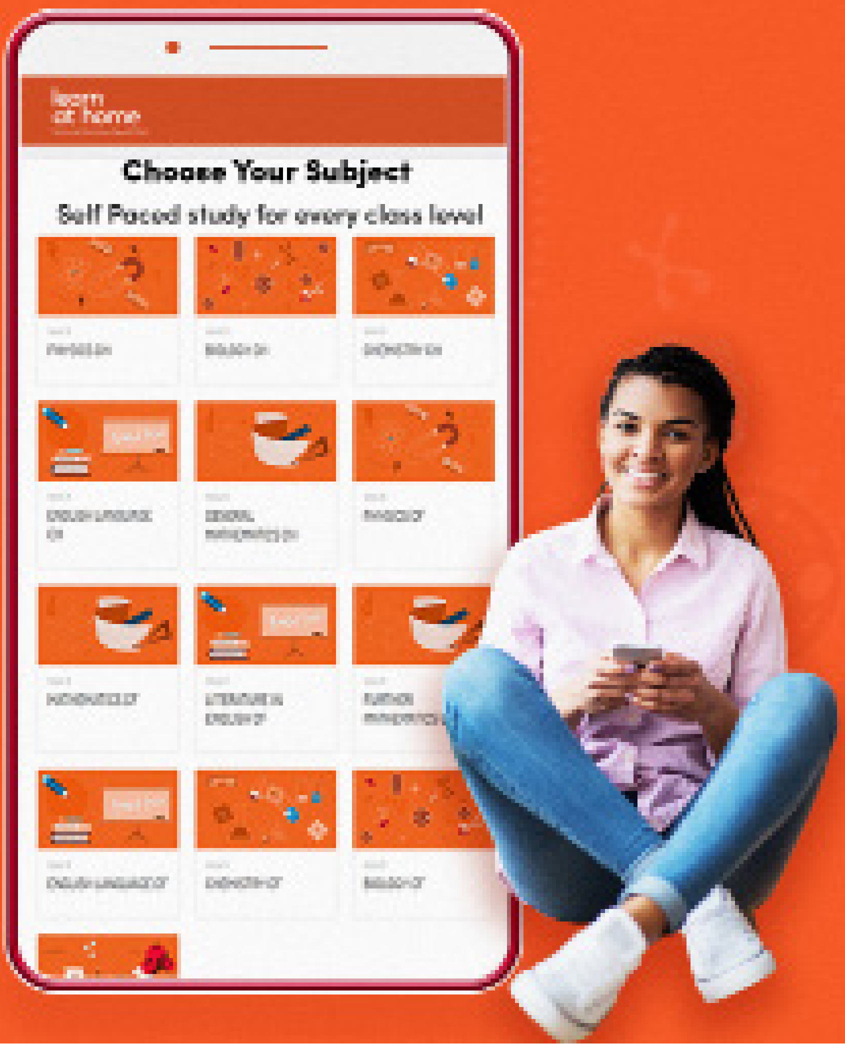
A S = 25.5m
B S = 26.5m
C S = 19.75m
D S = 19.5m

6. The velocity, Vms-1 of a body after time t seconds is given by $V=3t^2 - 2t - 3$. Find the distance covered during the 4th second. Using Simpson's rule with 4 strips, evaluate

A S = 18m
B S = 45m
C S = 27m
D S = 23m

7. $\int_1^6 2^x\,dx$ Correct to 2 decimal places.

A 86.67
B 86.7
C 86.6
D 86.66



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