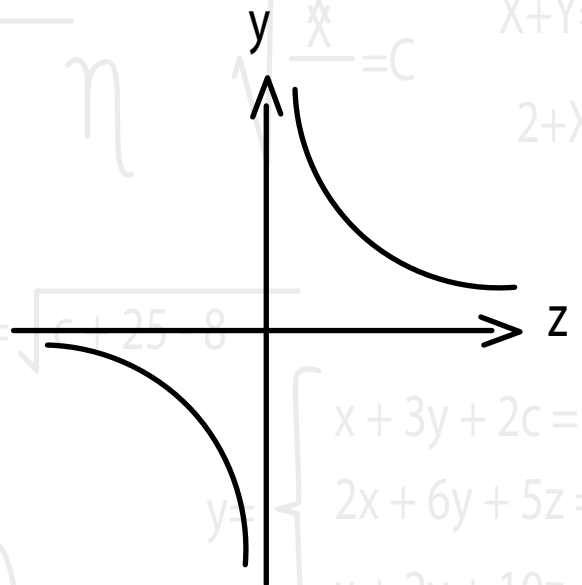


# MATHEMATICS



4

Introduction to Calculus

Pg.  
52

3

Geometry

Pg.  
29

2

Algebraic Process

Pg.  
23

1

Numbers and Numeration

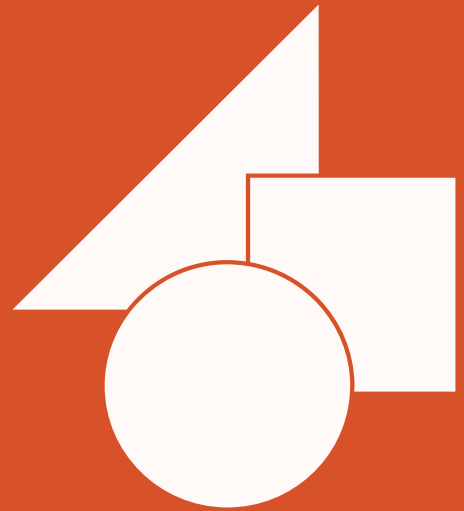
Pg.  
3

# Contents



THEME

01



---

**Numbers and Numeration.**

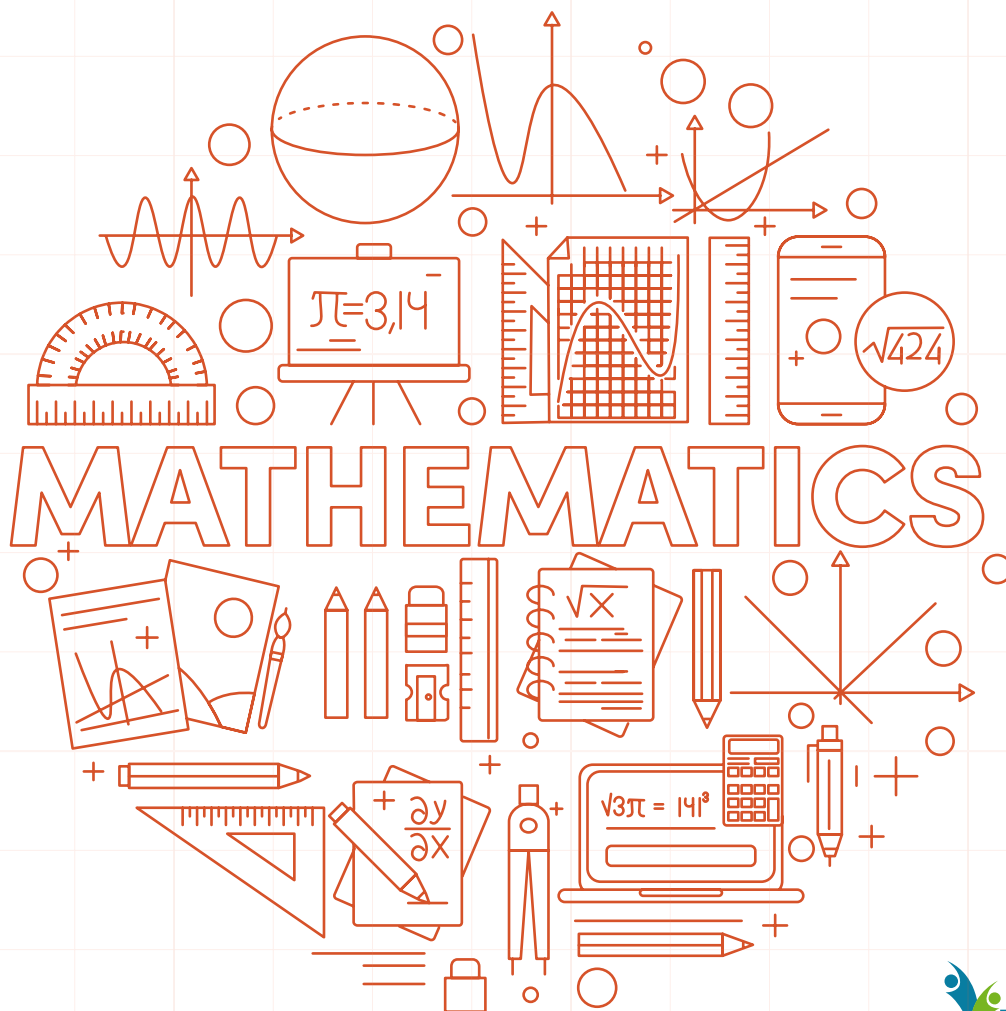
**Algebraic Process.**

**Geometry.**

**Introductory Calculus.**

# SURDS

## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS



1. Classify the expressions below into “Surds” and “not surds”

- A  $\sqrt{9}$
- B  $\sqrt{15}$
- C  $2\sqrt{3}$
- D  $\frac{1}{5}$
- E 0

**Correct Answer**

Surds = B, C: Not surds = A, D, E

**Explanation**

Options “B” and “C” are surds because they are irrational numbers and do not have a definite square root. However, A, D and E are not surds because A has a definite square root, D is a rational number, and E is a whole number

2. Match each question to its correct answer

- A  $4\sqrt{10}$
- B  $\frac{2}{3}\sqrt{3}$
- C  $6\sqrt{6}$
- D  $6\sqrt{2}$
- E  $2\sqrt{2}$



$2\sqrt{3} + 4\sqrt{3}$

Simplify  $3\sqrt{8}$

$17\sqrt{2} - 15\sqrt{3}$

$5\sqrt{10} + 2\sqrt{10} - 3\sqrt{3}$

Simplify  $\frac{2}{\sqrt{3}}$

### Correct Answer

PAIRS

A, D

B, E

C, A

D, B

E, C

### Explanation

$$\begin{aligned} \text{A. } 2\sqrt{3} + 4\sqrt{3} &= (2 + 4)\sqrt{3} \\ &= 6\sqrt{3} \end{aligned}$$

B. Simplify  $3\sqrt{8}$

$$\begin{aligned} \text{C. } 3\sqrt{8} &= 3\sqrt{(4 \times 2)} = 3 \times \sqrt{4} \times \sqrt{2} \\ &= 3 \times 2 \times \sqrt{2} \\ &= 6\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{D. } 17\sqrt{2} - 15\sqrt{2} &= (17 - 15)\sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{E. } 5\sqrt{10} + 2\sqrt{10} - 3\sqrt{10} &= (5 + 2 - 3)\sqrt{10} \\ &= 4\sqrt{10} \end{aligned}$$

E. Simplify  $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

3. Choose whether the following is true or false

A (True)  $2 + \sqrt{5}$  is the conjugate of  $5 + \sqrt{2}$

B (False)  $2\sqrt{2} - 3\sqrt{7}$  is a binomial surd.

### Correct Answer

Pairs:

A, B

B, A

### Explanation

$2 + \sqrt{5}$  is not the conjugate of  $5 + \sqrt{2}$ . This is because when you multiply both expressions, you cannot get a whole number.

$2\sqrt{2} - 3\sqrt{7}$  is a binomial surd because the expression has only two terms.

4. Evaluate  $\cos 30^\circ + \sin 45^\circ$  in surd form.

### Correct Answer

$$\frac{(\sqrt{3}+\sqrt{2})}{2}$$

### Explanation

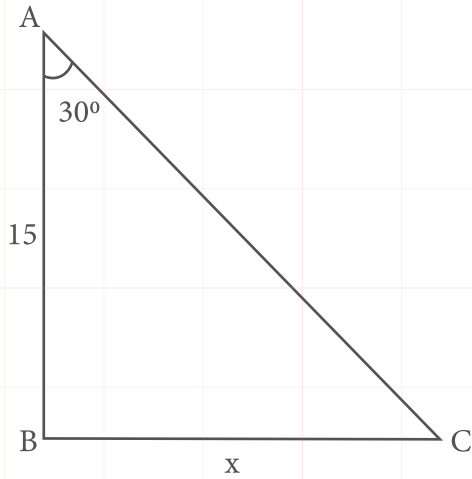
Since

$$\bullet \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \bullet \sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\text{Then, } \cos 30^\circ + \sin 45^\circ = \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}$$

$$\text{Giving us the answer - } \frac{(\sqrt{3}+\sqrt{2})}{2}$$

5. Find the value of  $x$  in the diagram below



### Correct Answer

The correct answers is  $x = 5\sqrt{3}$

### Explanation

$$\begin{aligned} \bullet \quad \tan 30^\circ &= \frac{x}{15} \\ \frac{\sqrt{3}}{3} &= \frac{x}{15} \end{aligned}$$

Cross multiply

$$3 \times x = 15 \times \sqrt{3}$$

$$3x = 15\sqrt{3}$$

$$x = \frac{15\sqrt{3}}{3}$$

$$x = 5\sqrt{3}$$



## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS



1. Which of the following statements are true?

- A A matrix is a circular arrangement of numbers, symbols and expression
- B The arrangement from left to right is called ROWS
- C The arrangement from left to right is called COLUMNS
- D A matrix is a rectangular arrangement of numbers, symbols and letters.
- E The arrangement from up to down is called COLUMNS

**Correct Answer**

The correct answers are B, D and E.

**Explanation**

Matrix is the rectangular arrangement of numbers, symbols and expressions into rows and columns. The arrangement from left to right is called ROWS while the arrangement from up to down is called COLUMNS

2. The order of matrix C is \_ by \_

$$C = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$$

**Correct Answer**

The correct answer is 2 by 2

**Explanation**

Correct! Matrix C has two rows and two columns

Matrix "C" has two rows and two columns. So the order is 2 by 2 or 2 X 2.

3. What type of Matrix is this?

$$\mathbf{C} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- A          Diagonal Matrix
- B          Triangular Matrix
- C          Middle Matrix
- D          Zero Matrix

**Correct Answer**

The correct answer is A.

**Explanation**

A diagonal matrix is a square matrix whose diagonal contains elements that is neither 0 nor 1

4. Solve the matrix below.

$$\begin{bmatrix} 3 & 12 & 5 \\ 13 & 16 & 7 \\ 8 & -4 & -9 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 18 \\ 2 & -5 & -10 \\ 4 & 3 & 0 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & -7 & -9 \end{bmatrix}$$

- A          4
- B          17
- C          2
- D          21
- E          9
- F          11

### Correct Answer

A, C

D, F

G, A

B, E

E, D

C, G

F, B

### Explanation

$$a = 3 - 1 = 2$$

$$d = 13 - 2 = 11$$

$$g = 8 - 4 = 4$$

$$b = 12 - 3 = 9$$

$$e = 16 - (-5) = 21$$

$$c = 5 - 18 = -13$$

$$f = 7 - (-10) = 17$$

Thats Correct!

Each element in A will subtract its corresponding element in B. Try again...

5. Then match the options to their corresponding correct answer.  
Choose the correct option.

$$\text{If, } \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 9 & -8 & 3 \\ 7 & 2 & 1 \end{bmatrix}$$

Then,  $\mathbf{B}^T =$

$$\mathbf{A} = \begin{bmatrix} 1 & 9 & 7 \\ 0 & -8 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 7 & 2 & 1 \\ 9 & -8 & 3 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & -8 & 2 \\ 1 & 9 & 7 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 1 & 9 & 7 \\ 1 & 3 & 1 \\ 0 & -8 & 2 \end{bmatrix}$$

### Correct Answer

A, C	D, F	G, A
B, E	E, D	
C, G	F, B	

### Explanation

$a = 3 - 1 = 2$	$d = 13 - 2 = 11$	$g = 8 - 4 = 4$
$b = 12 - 3 = 9$	$e = 16 - (-5) = 21$	
$c = 5 - 18 = -13$	$f = 7 - (-10) = 17$	

Thats Correct!

Each element in A will subtract its corresponding element in B. Try again...

If  $\mathbf{M} = \begin{bmatrix} -1 & -1 \\ 8 & 9 \end{bmatrix}$  Then the inverse of  $\mathbf{M}$ ,  $\mathbf{M}^{-1}$  is,

$$\mathbf{A} = \begin{bmatrix} -1 & -1 \\ 8 & 9 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 9 & -1 \\ 8 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} -9 & -1 \\ 8 & 1 \end{bmatrix}$$

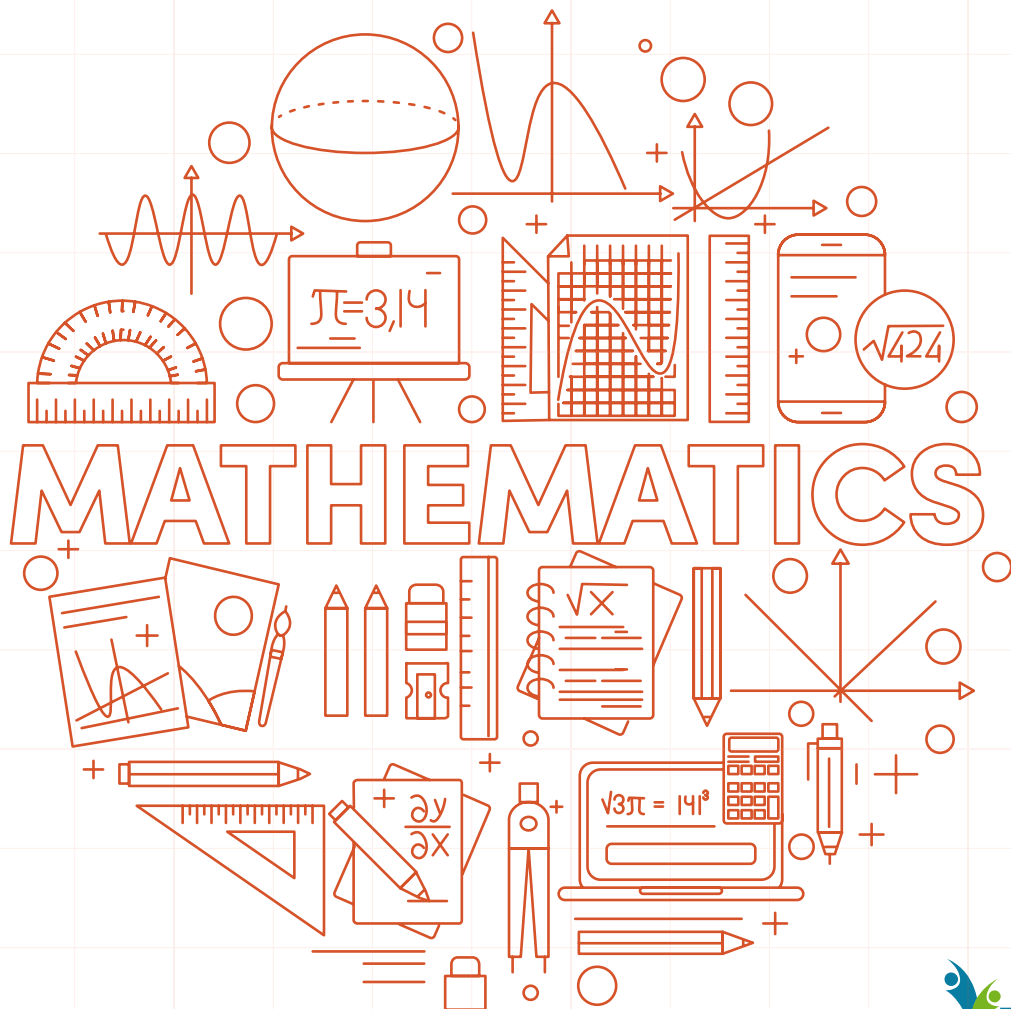
Solve the following simultaneous equation using matrix and find Det  $\mathbf{A}$ ,  $\mathbf{x}$  and  $\mathbf{y}$

$$x - 3y = 7, \quad x + y = 11$$

Det  $\mathbf{A} = \_$ ,  $\mathbf{x} = \_$  and  $\mathbf{y} = \_$

# LOGARITHM

## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS



1. Simplify  $\log_{10} 3 + \log_{10} 5$

- A  $\log_{10} 15$
- B  $\log_{10} 8$
- C  $\log_{10} 35$
- D  $\log_{10} 3.5$
- E  $\log_{10} 2$

**Correct Answer**

The correct answer is A.

**Explanation**

Since  $\log x + \log y = \log xy$ , then

$$\begin{aligned}\log 3 + \log 5 &= \log (3 \times 5) \\ &= \log 15\end{aligned}$$

Yes, that's correct! Addition becomes multiplication

2. Simplify  $\log 3y - \log y$

- A  $\log 3y^2$
- B  $\log 2y$
- C  $\log 3$
- D  $\log 4y$

**Correct Answer**

The correct answer is C.

**Explanation**

Since  $\log x - \log y = \log (x/y)$ , then

$$\begin{aligned}\log 3y - \log y &= \log (3y/y) \\ &= \log 3\end{aligned}$$

Subtraction becomes division

3.  $\log_5 5^2$  can be written as

- A  $\log 10$
- B 2
- C 1
- D 25

**Correct Answer**

The correct answer is B.

**Explanation**

Since  $\log xy = y \log x$ , this implies that

$$\log_5 5^2 = 2 \log 5$$

Also since  $\log 5^5 = 1$

$$\text{Then } 2 \log 5^5 = 2 \times 1 = 2$$

That's correct!

The power becomes the coefficient.

Non-Objective Question Assessments

- Simplify,  $8 \log m - \log m^3$

**Correct Answer**

$5 \log m$  or  $\log m^5$

**Explanation**

$$8 \log m - \log m^3$$

$$= \log m^8 - \log m^3$$

Correct answer:  $5 \log m$  or  $\log m^5$

From second law, we will have:

$$\log \frac{m^8}{m^3} = \log m^5 = 5 \log m$$



4. If  $\log x = 1.1257$ , then the value of  $x$  is,

- A 1.336
- B 13.36
- C 12.96
- D 1.296
- E 11.257

### Correct Answer

The correct answer is B.

### Explanation

Using antilog table, from the mantissa, look at the intersection of row 12 column 5. Add your answer to the intersection of column 7 in the mean difference section. Since you have the number 1 in the characteristic, add 1 to it, it becomes 2. Then you move your decimal 2 places to the right. The answer will be 13.36 which is B.

5. Use log tables to solve 
$$x = \frac{12.56}{4.097}$$

### Correct Answer

The correct answer is B.

### Explanation

$$\begin{aligned}\log x &= \log \frac{12.56}{4.097} \\ &= \log 12.56 - \log 4.097 \\ &= 1.0990 - 0.6125\end{aligned}$$

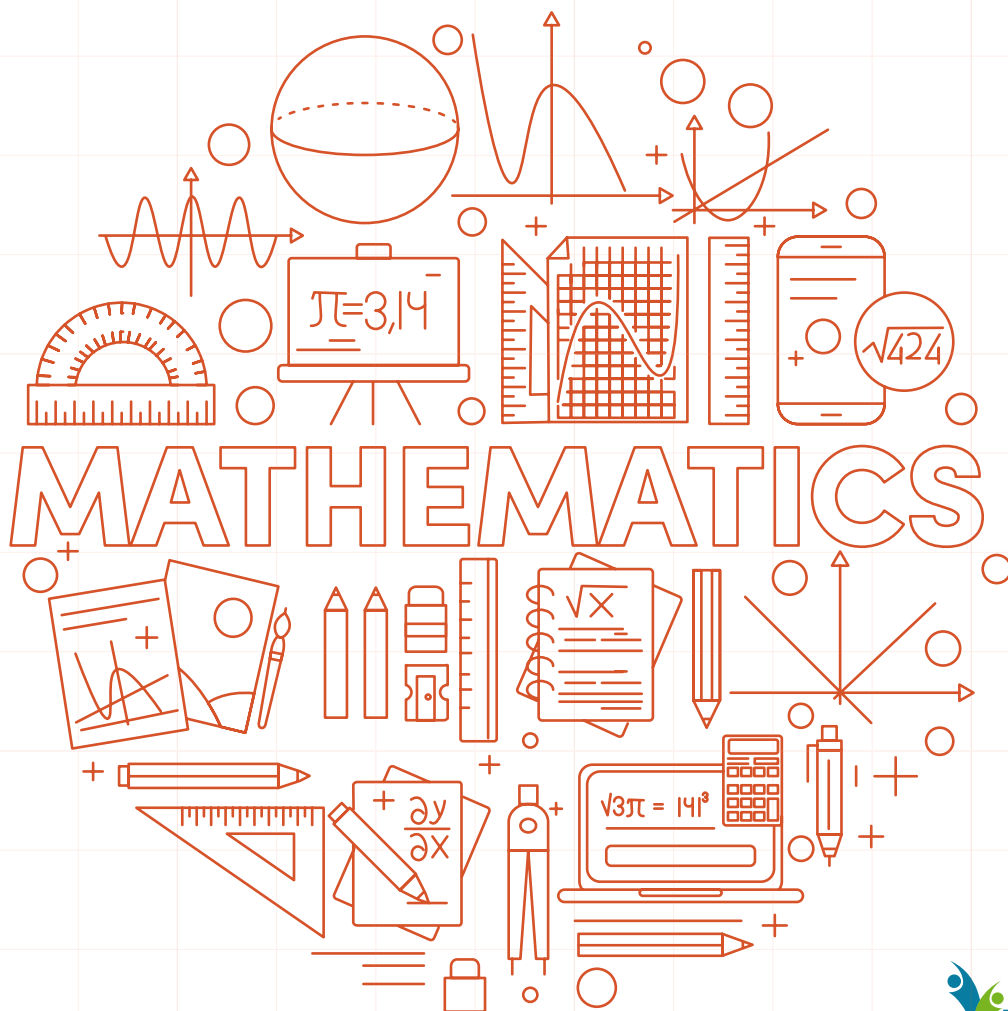
$$\log x = 0.4865$$

$$x = \text{antilog of } 0.4865 = 3.066$$

The final answer is 3.066

# ARITHMETIC OF FINANCE

INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS



1. The simple interest on #5,000 for 3 years at 2% interest is

- A      #500
- B      #750
- C      #90,000
- D      #150
- E      #300

**Correct Answer**

The correct answer is E.

**Explanation**

Simple Interest =  $P \times R \times T$

$$5,000 \times \frac{2}{100} \times 3$$
$$= \text{\#}300$$

2. What is the compound interest of #12,000 compounded biannually at the rate of 5% per year for three years?

- A      #1,916.32
- B      #13,916.32
- C      #12,000
- D      #1,000

### Correct Answer

The correct answer is A.

### Explanation

$$A = P \left( 1 + \frac{r}{n} \right)^{nt} \quad A = 12\,000 \left( 1 + \frac{0.05}{2} \right)^{5 \times 2}$$

$$\begin{aligned} A &= 12\,000(1 + 0.05)^6 \\ &= 12\,000(1.05)^6 \end{aligned}$$

$$A = 13,916.32$$

$$\begin{aligned} \text{Compound interest} &= \text{Amount} - \text{Principal} \\ &= \text{\#}13,916.32 - \text{\#}12,000 \\ &= \text{\#}1,916.32 \end{aligned}$$

3. Mrs. Martins bought a new pair of shoes at #15,000. Three years later she sells the shoe for #6,000, Calculate the depreciation expense of the shoe.

- A      #6,000
- B      #3,000
- C      #5,000
- D      #9,000

### Correct Answer

The correct answer is B.

### Explanation

$$\begin{aligned} \text{Depreciation} &= \frac{\text{\#}15,000 - \text{\#}6,000}{3} \times \frac{\text{\#}15,000 - \text{\#}6,000}{3} \\ &= \text{\#}3,000 \end{aligned}$$

4. How much money must be deposited now at 6% interest compounded quarterly in order to be able to withdraw #3 000 at the end of each quarter year for two years? (Approximate your answer to the nearest ten)

- A      #22460.259
- B      #22460
- C      #22450
- D      #22450.77

**Correct Answer**

The correct answer is B.

**Explanation**

$$\log x = \log \frac{12.56}{4.097}$$

$$R = 3,000$$

$$i = 0.06/4 \text{ (since it is quarterly)} = 0.015$$

$$n = 12$$

$$\text{Present value (P.V)} = \frac{1 - (1 + 0.015)^{-8}}{0.015} \times 3,000$$

$$= \frac{1 - 0.8877111238}{0.015} \times 10,000$$

$$= \frac{0.1122888762}{0.015} \times 10,000$$

$$= 7.48592508 \times 3000$$

$$= \text{\# } 22,457.77$$

$$= \text{\#22460 to the nearest ten}$$

This is our final answer

5. Halima borrows #200,000 at 10% annual interest on a 5 year loan. What is her monthly payment? Approximate your answer to two decimal places.

- A      4249.03
- B      2025814.35
- C      121.59

**Correct Answer**

The correct answer is A.

**Explanation**

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

$$P = \text{\#}200,000$$

$$i = \frac{10}{12}\% = 0.00833 \text{ (since we are looking for monthly payment, we have to divide the annual rate by 12)}$$

$$n = 5 \text{ yrs} \times 12 = 60 \text{ months (since it is monthly)}$$

$$A = 20,000 \left( \frac{0.00833 + (1+0.00833)^{60}}{(1+0.00833)^{60}-1} \right)$$

$$A = \frac{200000 \times 0.00833 \times 1.00833^{60}}{1.00833^{60}-1}$$

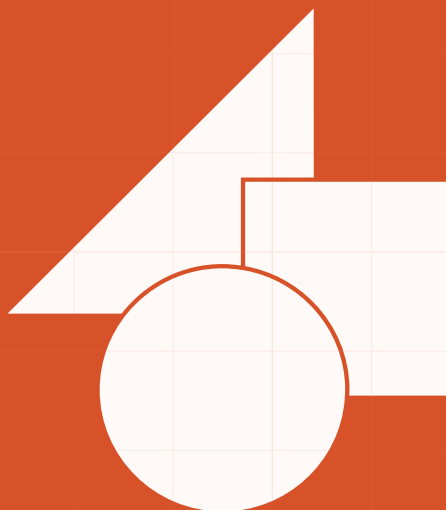
$$A = \frac{1666 \times 1.64498}{0.64498} = \frac{2740.53668}{0.64498}$$

$$A = \text{\#}4249.03$$



THEME

02



Numbers and Numeration.

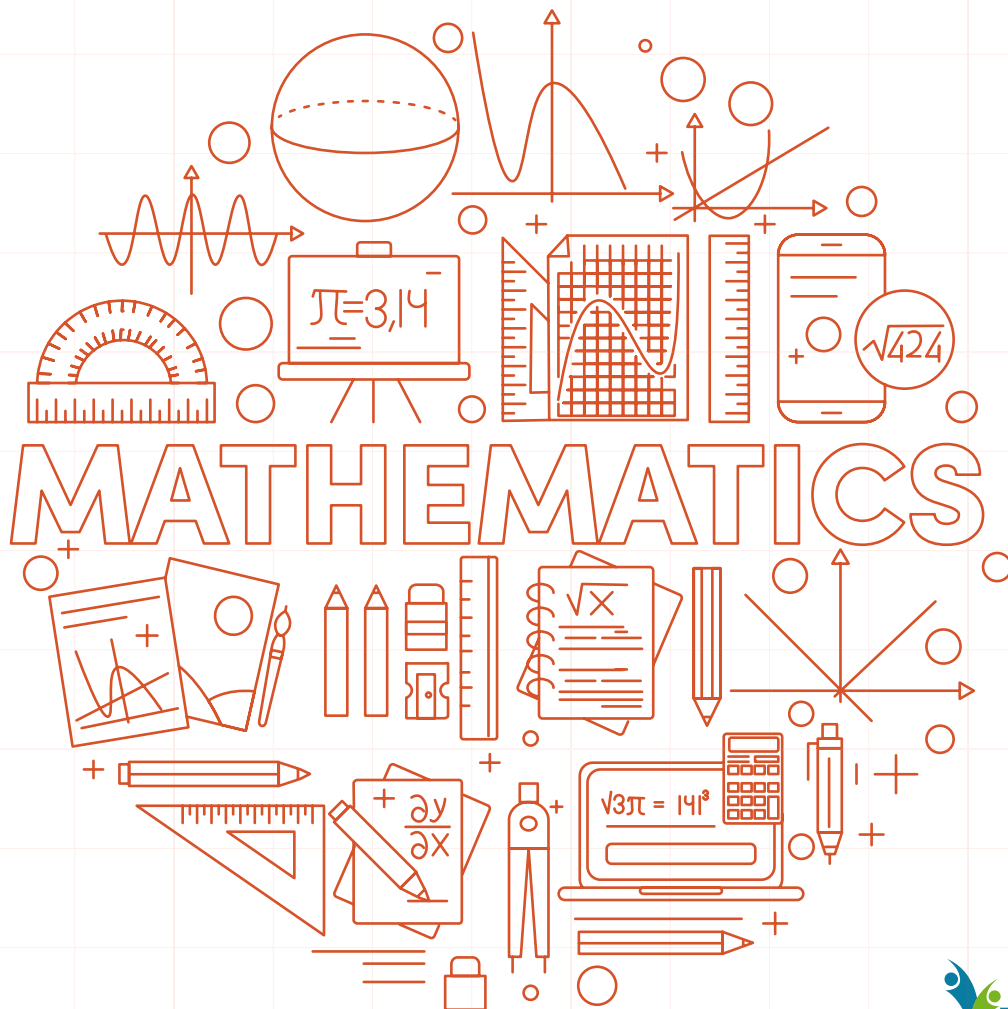
---

**Algebraic Process.**

Geometry.

Introductory Calculus.

## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS





1. Solve the simultaneous equations

$$y = 2x^2$$

$$y = x + 10$$

- A  $(-2, 8)$  and  $(5/2, 25/2)$
- B  $(2, 10)$  and  $(8, 200)$
- C  $(2, 5)$  and  $(8, 50)$
- D  $(-2, 5/2)$  and  $(8, 25/2)$

### Correct Answer

The correct answer is A.

### Explanation

$$Y = 2x^2$$

$$Y = x + 10$$

$$2x^2 = x + 10$$

$$2x^2 - x - 10 = 0$$

Factorising, we will have

$$x = -2 \text{ or } x = \frac{5}{2}$$

From,  $y = 2x^2$

When  $x = -2$   $y = 2(-2)^2$

$$Y = 8$$

Therefore  $x = -2$  and  $y = 8$

$$= 2x \frac{25}{4} = \frac{25}{2} = 12.5$$

Therefore  $x = \frac{5}{2}$   $y = \frac{25}{2} = 12.5$

The pairs are  $(-2, 8)$  and  $(\frac{5}{2}, \frac{25}{2})$

2. Solve  $y = x^2 - 3x + 4$  and  $y - x = 1$

- A (1,3) and (2,4)
- B (-1,-2) and (-3,-4)
- C (1,2) and (3,4)
- D 20

### Correct Answer

The correct answer is C.

### Explanation

$$y = x^2 - 3x + 4$$

$$y = 1 + x$$

We will have

$$x^2 - 3x - x + 4 - 1 = 1 + x$$

Collecting like terms, we will have

$$x^2 - 3x - x + 4 - 1 = 0$$

$$x^2 - 4x + 3 = 0$$

Factorizing we will have

$$X = 1 \text{ or } x = 3$$

Using equation 2,

$$\text{When } x = 1, y = 1 + 1$$

$$Y = 2$$

$$\text{When } x = 3, y = 1 + 3 = 4$$

The pairs are (1,2) and (3,4)

3. The mean of two numbers is 6 and the product is 35. Find the numbers.

- A 2, 3
- B -5, -7
- C 4, 6
- D 5, 7

### Correct Answer

The correct answer is option D

### Explanation

Let the numbers be x and y

$$\frac{x+2}{2} = 6 \quad (1)$$

$$x \times y = 35 \quad (2)$$

From (1)

$$x + y = 12, \text{ implying that } y = 12 - x \quad (3)$$

From (2)

$$y = \frac{35}{x} \quad (4)$$

Equating (3) and (4), we will have

$$12 - x = \frac{35}{x}$$

Cross multiplying, we will have

$$x(12-x) = 35$$

$$12x - x^2 = 35$$

Collecting like terms

$$-x^2 + 12x - 35 = 0$$

Multiplying each term by -1 we will get

$$x^2 - 12x + 35 = 0$$

Factorizing we will have

$$x = 5 \text{ or } 7$$

4. Mrs. Esther invested #5,900 is invested in two accounts. One account earns 3.5% and another earns 4.5%. If the interest for 1 year is #229.50, the much is invested in each account?

- A      A = #3,600 B = #2,300
- B      A = #360 B = #230
- C      A = #36,000 B = #23,000
- D      A = #3,000 B = #2,900

### Correct Answer

The correct answer is option A

### Explanation

Account A earns 3.5% interest

Account B earns 4.5% interest

Interest for A + Interest for B = #229.50

I for A =  $x \times 0.035 \times 1$

=  $0.035x$

I for B =  $(5900 - x) \times 0.045 \times 1$

=  $265.5 - 0.045x$

This means

$0.035x + 265.5 - 0.045x = 229.50$

$265.5 - 0.01x = 229.5$

Collecting like terms

$-0.01x = 229.5 - 265.5$

$-0.01x = -36$

$0.01x = 36$

$x = \frac{36}{0.01}$

$x = 3600$

This means that

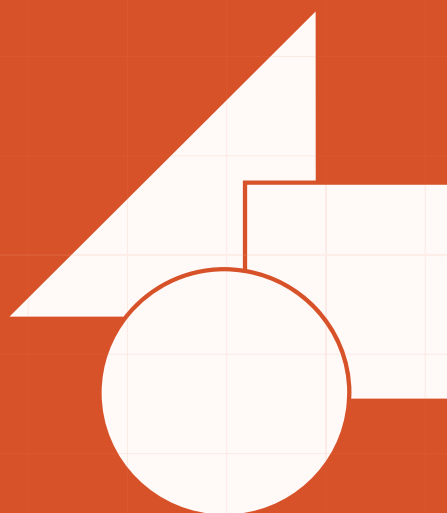
Money invested in Account A = #3,600

Money invested in Account B = #5,900 - #3,600 = #2,300



THEME

03



Numbers and Numeration.

Algebraic Process.

---

Geometry.

Introductory Calculus.

## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS

# MATHEMATICS

## INTERACTIVE ASSESSMENT QUESTIONS

1. (a) Copy and complete the table of values for the function  $y = 2 \cos 2x + \sin x$

x	$-120^\circ$	$-90^\circ$	$-60^\circ$	$-30^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$0^\circ$
y	-1.87	-3.00	-1.87	0.50	1.50	-0.13	-1.00	-0.13	

### Correct Answer

(a)  $y = 2 \cos 2x + \sin x$

### Explanation

For  $x = -120^\circ$

$$-120^\circ \equiv 240^\circ$$

$$\begin{aligned}
 y &= 2 \cos 2 \times 240 + \sin 240 \\
 &= 2 \cos 480 + \sin 240 \\
 &= 2(-0.5) + (-0.8660) \\
 &= -1 - 0.8660 \\
 &= -1.87
 \end{aligned}$$

For  $x = -30^\circ$

$$-30^\circ \equiv 330^\circ$$

$$\begin{aligned}
 y &= 2 \cos 2 \times 330^\circ + \sin 330^\circ \\
 &= 2 \cos 660^\circ + \sin 330^\circ \\
 &= 2(0.5) + (-0.5) \\
 &= 1 - 0.5 \\
 &= 0.5
 \end{aligned}$$

- (b) Using a scale of 2cm to  $30^\circ$  on x-axis and 2cm to 1 unit on y-axis draw the graph of  $y = 2 \cos 2x + \sin x$  for  $-120^\circ < x < 120^\circ$

(c) Using the same scale and axes draw the graph of

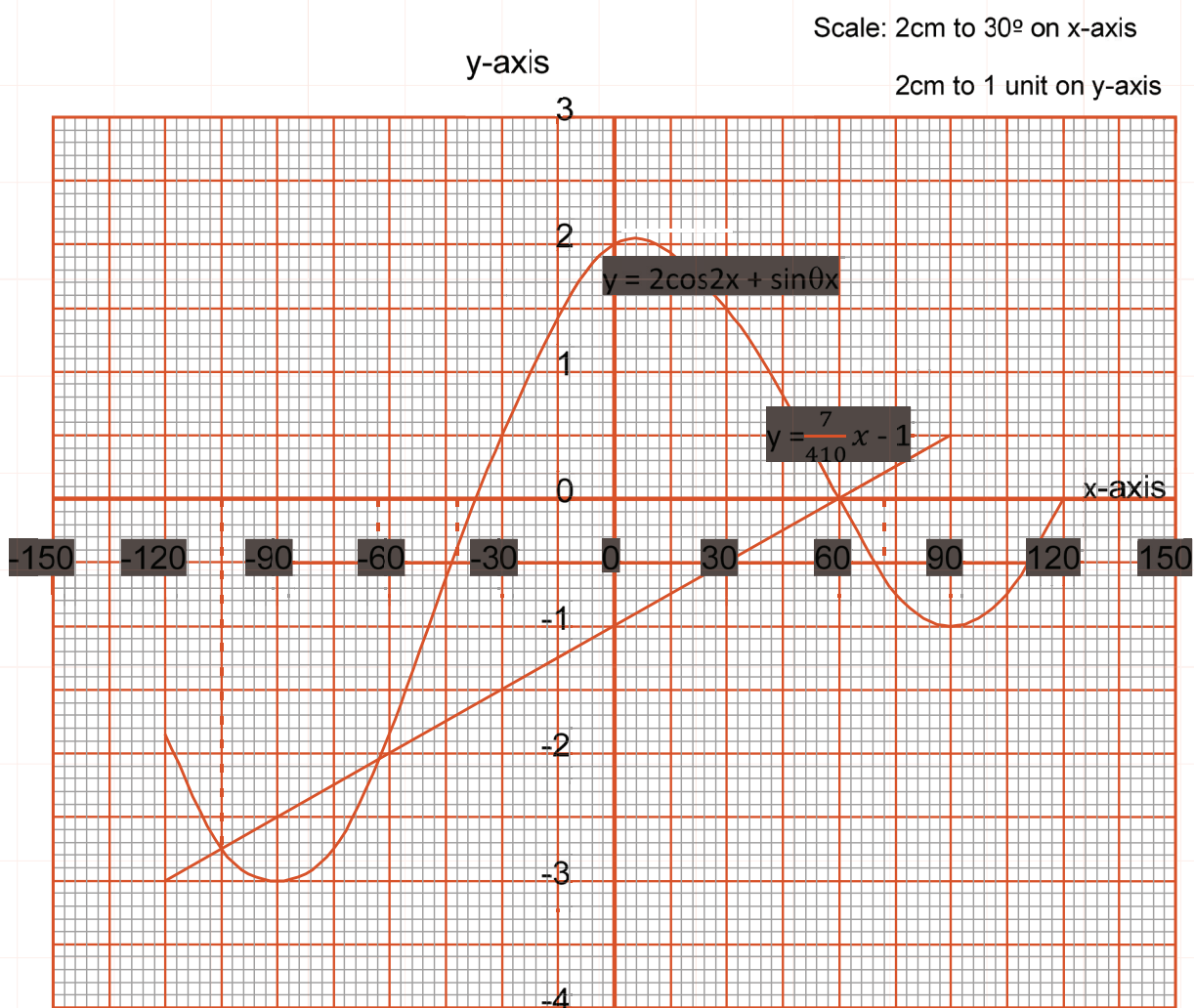
$$y = \frac{7x}{410} + 1$$

(d) From your graph, find the roots of the following equations

i.  $\sin x + 2 \cos 2x = 0$

ii.  $\sin x + \frac{1}{2} + 2 \cos 2x = 0$

iii.  $2 \cos 2x + \sin x = \frac{7x}{410} - 1$





### Explanation

For  $x = -90$

$$-90^\circ \equiv 270^\circ$$

$$\begin{aligned}y &= 2 \cos 2 \times 2700 + \sin 2700 \\&= 2 \cos 540 + \sin 270 \\&= 2(-1) + (-1) \\&= -2 - 1 \\&= -3\end{aligned}$$

For  $x = 30$

$$\begin{aligned}y &= 2 \cos 2 \times 30 + \sin 300 \\&= 2 \cos 60^\circ + \sin 30^\circ \\&= 2(0.5) + (0.5) \\&= 1 + 0.5 \\&= 1.5\end{aligned}$$

x	$-120^\circ$	$0^\circ$	$9^\circ$
y	-3.05	-1	0.54

### Explanation

For  $x = -120$

$$y = \frac{-7 \times 120}{410} - 1$$

$$y = -2.05 - 1$$

$$y = -3.05$$

For  $x = 0$

$$y = \frac{7 \times 120}{410} - 1$$

$$y = -1$$

$$\text{For } x = 90^\circ$$

$$y = \frac{7 \times 90}{410} - 1$$

$$y = \frac{630}{410} - 1$$

$$y = 1.54 - 1$$

$$y = 0.54$$

(d) (i) The roots of  $2 \cos x + \sin x = 0$  is at the points where  $y = 0$  i.e. where the graph crosses the x-axis.

$$x = -35^\circ \text{ or } x = 59^\circ$$

$$(ii) 2 \cos 2x + \sin x + \frac{1}{2} = 0$$

$$2 \cos 2x + \sin x = -\frac{1}{2}$$

The roots are the values of  $x$  where  $y = -\frac{1}{2}$  i.e.  $x = -42^\circ$  or  $x = 69^\circ$  or  $x = 112^\circ$

$$(iii) 2 \cos 2x + \sin x = \frac{7x}{410} - 1$$

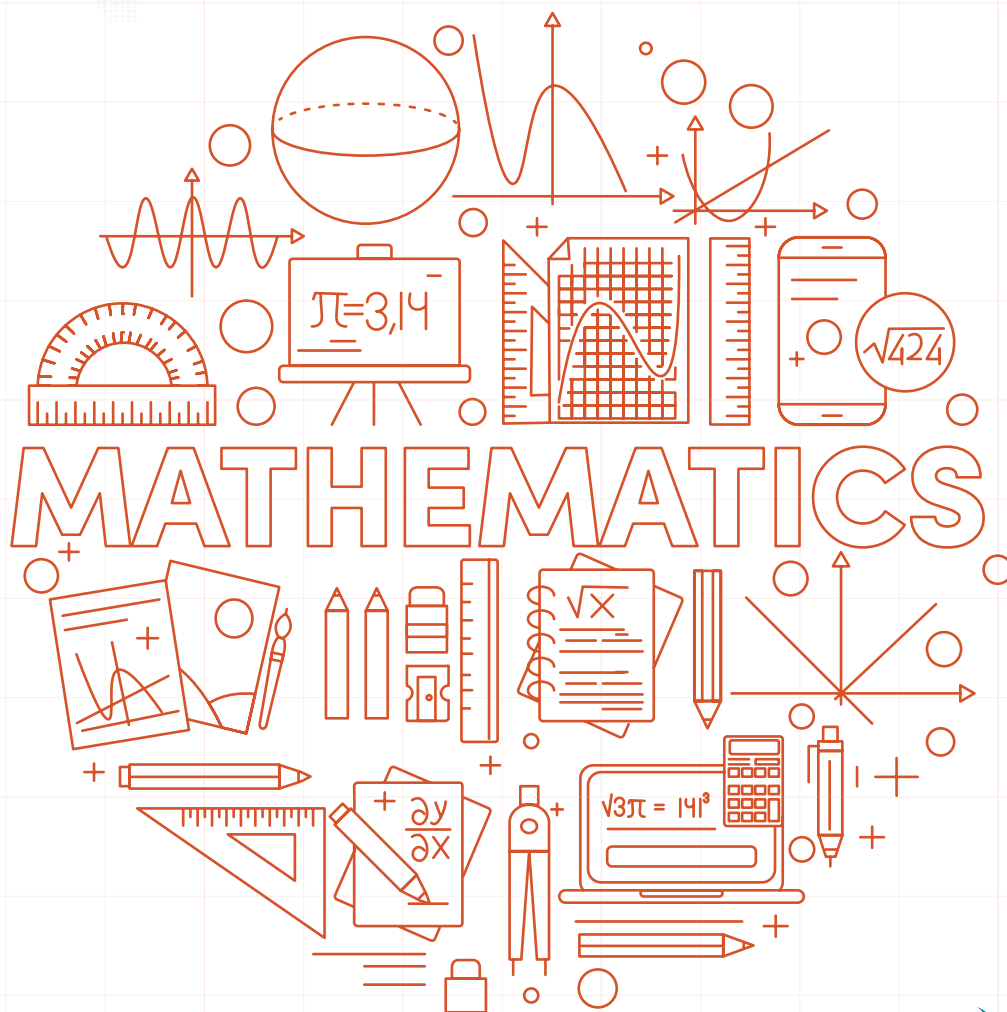
The roots are at the points where the two graphs  $y = 2 \cos 2x + \sin x$  and

$$y = \frac{7x}{410} - 1$$

Meet. i.e.  $x = -1030$  or  $x = -640$  or  $x = 600$

# SURFACE AREA AND VOLUME OF SPHERE

INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS



1. The formula for the total surface area of a sphere is written as

A  $\frac{4}{3} \pi r^3$

B  $4\pi r^2$

C  $2\pi r^2$

D  $\frac{1}{3} \pi r^2$

**Correct Answer**

The correct answer is option E.

**Explanation**

$$\text{T.S.A} = 4\pi r^2$$

2. What is the total surface area of a sphere of diameter 10cm? (to the nearest two decimal places)

A  $4190.48\text{cm}^2$

B  $4190.47\text{cm}^3$

C  $1257.14\text{cm}^3$

D  $1257.14\text{cm}^2$

E None of the above

**Correct Answer**

The correct answer is option E.

**Explanation**

$$\text{Total surface area T.S.A} = 4\pi r^2$$

$$\pi = \frac{22}{7}, r = 10\text{cm}$$

$$\text{T.S.A} = 4 \times \frac{22}{7} \times 10 \times 10 = 1257.14 \text{ cm}^2$$

3. The total surface area of a sphere is measured in \_\_\_ while the volume of a sphere is measured in \_\_\_

- A Square units, cubic units
- B Cubic units, square units
- C Square units, square units
- D Cubic units, cubic units
- E One unit

**Correct Answer**

The correct answer is option A. Square units, cubic units

4. Which of the following statements is true?

- A The volume of a sphere = 2 x volume of a cube
- B The volume of a sphere = 2 x volume of a cylinder
- C Volume of a sphere = 2 + volume of a cone
- D The volume of a sphere = 2 x volume of a circle
- E The volume of a sphere = 2 x volume of a cone

**Correct Answer**

The correct answer is option E.

5. Kingsley wants to know how much air can fill a spherical hot air balloon. How can he go about this? He should calculate....

- A The surface area of the sphere
- B The volume of the sphere
- C The perimeter of the sphere
- D How much the balloon is sold in the market
- E The color of air to fill the balloon

**Correct Answer**

The correct answer is option B.

**Explanation**

Right, the volume of a solid is the amount of substance the solid can hold, in this question, the amount of air.

6. Fatima noticed air leaking from a spherical advertising balloon at the rate of 30 cubic feet per minute. If the radius of the ball is 7 feet, how many minutes will it take for the balloon to lose its air completely? Round your answer to the nearest whole number (Take  $\pi=3.14$ )

- A 47 minutes
- B 47.8 minutes
- C 48 minutes
- D 1436 minutes

**Correct Answer**

The correct answer is option D.

### Solution

To know how long it will take for the balloon to lose its air, we need to calculate the volume of the spherical balloon.

We know, the volume of sphere =  $\frac{4}{3}\pi r^3$ ,

$$\pi = 3.14 \quad r = 7 \text{ inches}$$

$$\text{Volume} = \frac{4}{3} \times 3.14 \times 7 \times 7 \times 7 = \frac{4308.08}{3}$$

$$= 1436.0267 \text{ cubic inches}$$

Since air is leaking from the balloon at the rate of 30 cubic feet per minute, to know how long it will take for the balloon to become empty, we divide the volume by 30,

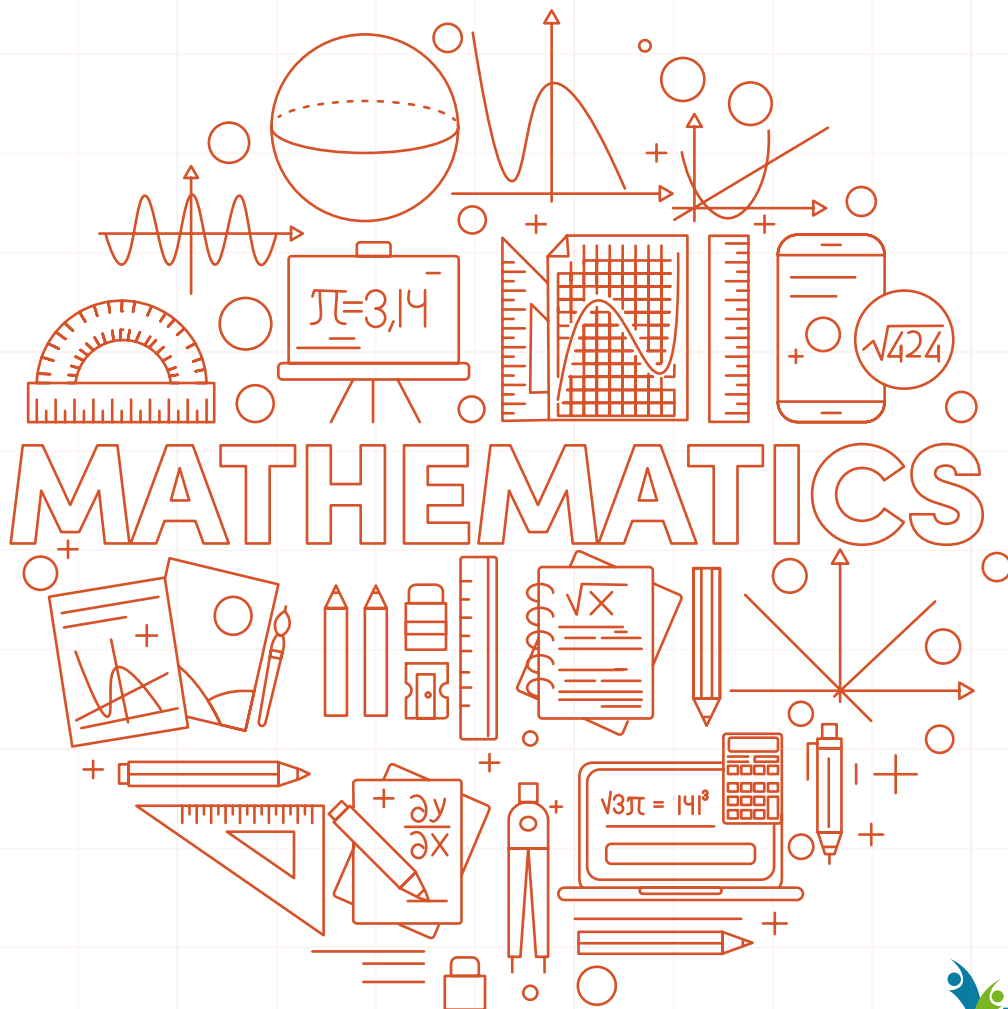
$$= \frac{1436.0267}{30}$$

$$= 47.8676 \text{ minutes}$$

$$\cong 48 \text{ minutes.}$$

# LATITUDE AND LONGITUDE

## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS





1. The  $0^\circ$  line of latitude is the
- A Prime Meridian
  - B Equator
  - C Contour line
  - D International dateline

**Correct Answer**

The correct answer is option B.

2. How many parts does the equator divide the earth into?
- A 6
  - B 8
  - C 4
  - D 2

**Correct Answer**

The correct answer is option D.

**Explanation**

The equator divides the earth into the Northern and Southern Hemispheres. So the correct answer is option [D]

3. The  $0^\circ$  mark of longitude is the
- A Prime meridian
  - B Contour line
  - C Equator
  - D International dateline

### Correct Answer

The correct answer is option A.

### Explanation

00 lies at the center of the sphere, which is the Prime meridian. So, the correct answer is option [A]

4. Lines of longitude

- A Never meet
- B Are called parallels
- C Are real lines painted on the earth
- D Meet at north and south poles

### Correct Answer

The correct answer is option D.

### Explanation

Lines of longitude go from north to south. They converge at the North and South poles. Therefore, the correct answer is option [D]

5. What is the reason why latitude lines never intersect?

- A They converge at the poles
- B They are cool
- C They are parallel
- D They are vertical lines

### Correct Answer

The correct answer is option C.

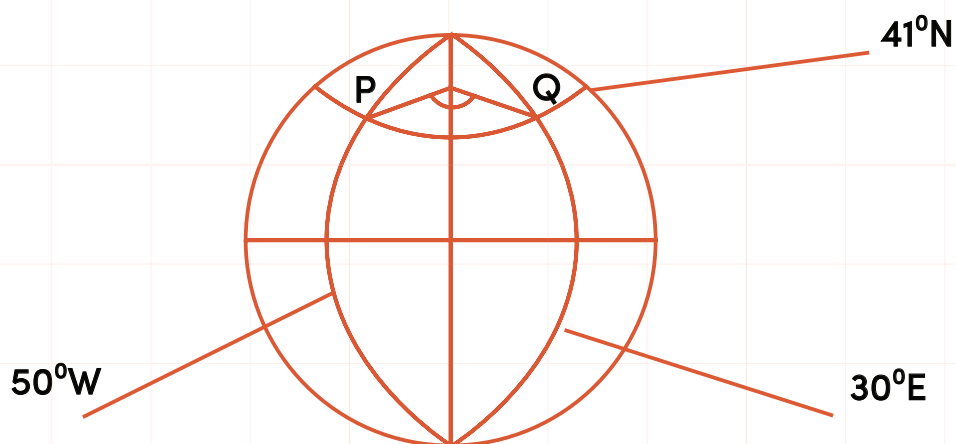
6. Two points P( $41^{\circ}\text{N}$ ,  $50^{\circ}\text{W}$ ) and Q( $41^{\circ}\text{N}$ ,  $30^{\circ}\text{E}$ ) are on the earth's surface. Given that  $\pi = 3.142$ ,  $R = 6400\text{km}$ , the distance between P and Q to the nearest km is

- A 6745km
- B 6740km
- C 1686km
- D 4830.3km

### Correct Answer

The correct answer is option A.

### Explanation



$$|PQ| = \frac{\theta}{360} \times 2\pi r \text{ (since the points lie on the small circle)}$$

$$\theta = 50^{\circ} + 30^{\circ} = 80^{\circ}$$

$$r = R \cos \alpha \text{ } (\alpha = \text{common latitude} = 41^{\circ}\text{N})$$

$$r = 6400 \times \cos 41^{\circ}$$

$$r = 4830.14\text{km}$$

$$\approx 4830\text{km}$$

$$\text{Therefore, } |PQ| = \frac{80}{360} \times 2 \times 3.142 \times 4830$$

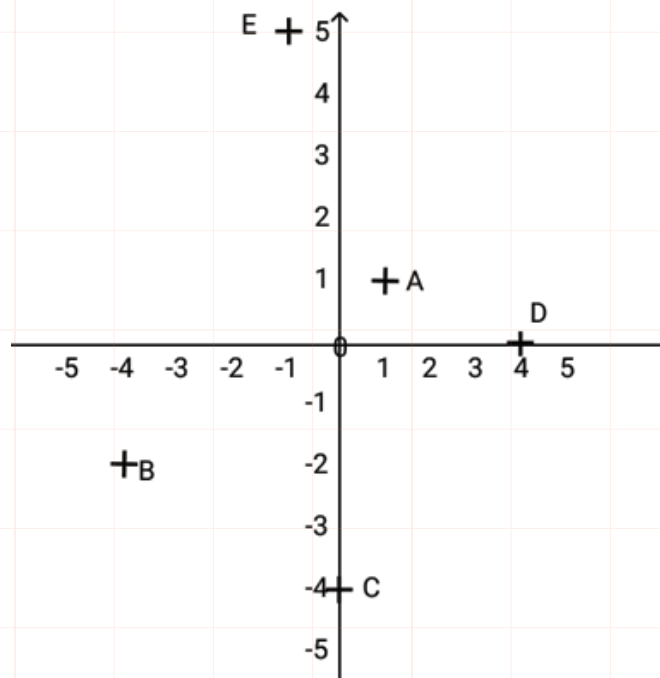
$$= 6744.8$$

$$\cong 6745 \text{ km}$$

## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS



1. Match the points to their corresponding cartesian coordinates using the graph below:



- A (0,4)
- B (1,1)
- C (4,0)
- D (-1,5)
- E (-4,-2)

### Correct Answer

The correct pairs are as follows: Match

[A] to [B],

[B] to [E],

[C] to [A],

[D] to [C] and

[E] to [D]

### Explanation

The first entry is the x-coordinate while the second entry is the y-coordinate

2. Set of points that form a straight line is called a

- A Parabola
- B Set graph
- C Pointed graph
- D Linear graph

**Correct Answer**

The correct answer is Linear graph.

**Explanation**

The set of points that form a straight line is called a linear graph.

3. The distance between point R(2,-6) and S(-4,-3) is

- A  $\sqrt{45}$  units
- B  $\sqrt{17}$  units
- C  $\sqrt{13}$  units
- D  $\sqrt{65}$  units

**Correct Answer**

The correct answer is  $\sqrt{45}$  units

**Solution**

$$|RS| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For R (2, -6),  $x_1 = 2$ ,  $y_1 = -6$

For S (-4, -3)  $x_2 = -4$ ,  $y_2 = -3$

$$|RS| = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$$

$$|RS| = \sqrt{((-4 - 2)^2 + (-3 - (-6))^2)} = \sqrt{(36 + 9)} = \sqrt{45} \text{ units.}$$

3. The midpoint of the line joining  $(-1,4)$  and  $(2,-3)$  is

- A  $(2, -3)$
- B  $(8, 5)$
- C  $(-1/2, -1/2)$
- D  $(1/2, 1/2)$

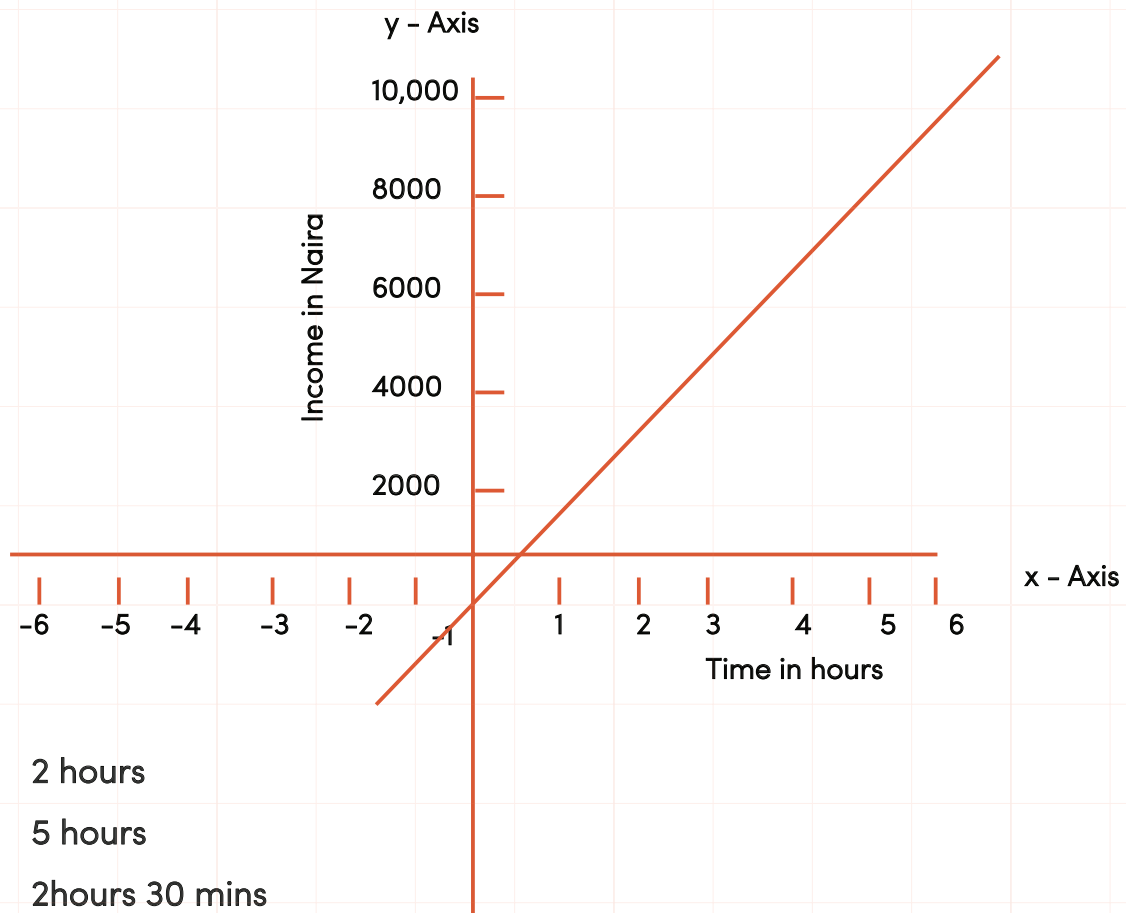
**Correct Answer**

The correct answer is option D.

**Explanation**

$$\begin{aligned}\text{Midpoint} &= \left( \frac{x_1 + x_2}{2} \right), \left( \frac{y_2 + y_1}{2} \right) = \left( \quad \quad \right), \left( \frac{4 + (-3)}{2} \right) \\ &= \left( \frac{1}{2}, \frac{1}{2} \right)\end{aligned}$$

4. A cleaning job pays #2,000 per hour. A graph of income per hour is drawn below. With the help of the graph shown below, determine how many hour(s) is/are needed to earn #5,000.



- A 2 hours
- B 5 hours
- C 2 hours 30 mins
- D 1 hour

### Correct Answer

The correct answer is C.

### Solution

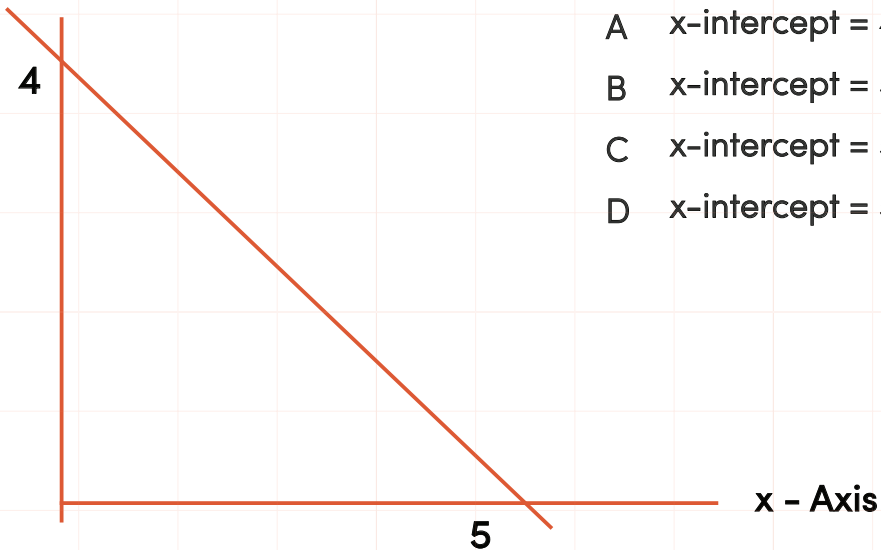
#5,000 is in between #4,000 and #6,000

By determining the #5,000 amount on the vertical axis, you can follow a horizontal line through the value until it meets the graph line. Follow a vertical line straight down from there till it meets the horizontal axis. The value in hours should be in between 2 and 3 hours which is 2.5 hours or 2 hours 30 minutes.



5. In the graph below, the x-intercept is \_\_\_\_\_, the y-intercept is \_\_\_\_ and the gradient is \_\_\_\_\_.

**y - Axis**



- A x-intercept = 4, y-intercept= 5 gradient = 0
- B x-intercept = 5, y-intercept= 4 gradient = 0
- C x-intercept = 5, y-intercept= 4 gradient =  $-4/5$
- D x-intercept = 5, y-intercept= 4 gradient =  $4/5$

**Correct Answer**

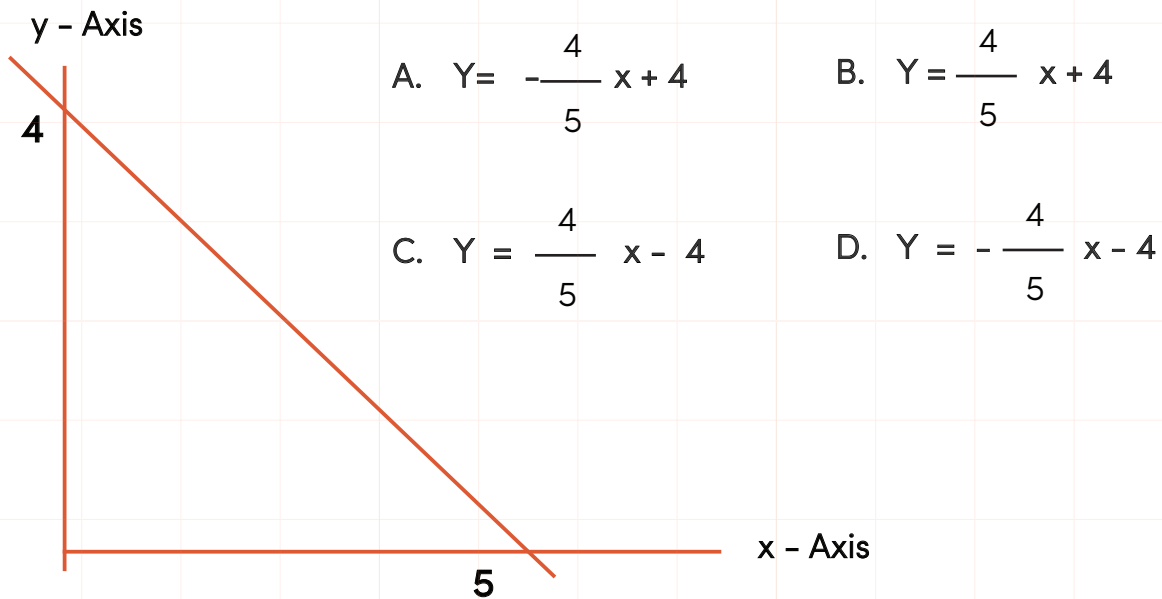
The correct answer is C.

**Solution**

x-intercept = 5, y-intercept= 4 gradient =  $-4/5$ .

The x-intercept is the point where the line intercepts the x-axis while the y-intercept is the point where the line intercepts the y axis. The gradient is the slope which is the rate of change y to change in x.

6. What is the equation formed by the graph below is...



**Correct Answer**

The correct answer is A.

**Solution**

Equation of a line is  $y = mx + c$ .  $m = -\frac{4}{5}$ ,  $c = 4$ . So the correct answer is option A

7. The angle between the lines with equations  $3x + 4y - 12 = 0$  and  $5x + 12y + 13 = 0$  is

- A       $13.24^\circ$
- B       $71.83^\circ$
- C       $20.76^\circ$
- D       $14.25^\circ$

**Correct Answer**

The correct answer is D.

From  $\tan \theta =$

### Solution

$$M_1 = - \left( \frac{3}{4} \right)$$

$M_2$  = slope of equation of second line

From equation 2,  $5x + 12y + 13 = 0$

$$M_1 = - \left( \frac{3}{4} \right)$$

From the formula  $\tan \theta = \left( \frac{m_1 - m_2}{1 + m_1 m_2} \right)$

$$\tan \theta = \left( \frac{16}{63} \right)$$

$$\theta = \tan^{-1} \frac{16}{63} = 14.250$$

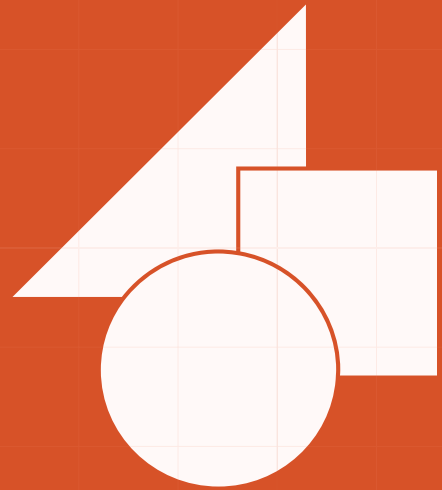
Recall, the formula for calculating the angle between two intersecting straight lines

is  $\tan \theta = \left( \frac{m_1 - m_2}{1 + m_1 m_2} \right)$



THEME

# 04



Numbers and Numeration.

Algebraic Process.

Geometry.

---

**Introductory Calculus.**

## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS



1. Differentiate  $y=7x-2x^{-3}$  concerning  $x$

A.  $\frac{dy}{dx} = 7 + 6x^{-4}$

B.  $\frac{dy}{dx} = 7 - 6x^{-4}$

C.  $\frac{dy}{dx} = 7 + 5x^{-4}$

D.  $\frac{dy}{dx} = 7 - 1x^{-4}$

### Correct Answer

The correct answer is A.

### Explanation

$$\frac{dy}{dx} = 7 + 6x^{-4}$$

$$\frac{dy}{dx} = 7 - (-3)2x^{-4}$$

$$\frac{dy}{dx} = 7 - 6x^{-4}$$

2. The derivative of  $\cos x$  is

A  $-\cos x$

B  $\sin x$

C  $-\sin x$

D  $\cos x$

E  $\tan x$

### Correct Answer

The correct answer is C.

### Solution

The derivative of  $\cos x$  is  $-\sin x$

3. Differentiate  $Y = 6x^3 - x^2 - 4x + 1$  concerning  $x$ .

A.  $\frac{dy}{dx} = 18x^2 - 2x - 4 + 1$

B.  $\frac{dy}{dx} = 18x^2 - 2x - 4x$

C.  $\frac{dy}{dx} = 18x^2 - 2x + 4$

D.  $\frac{dy}{dx} = 18x^2 - 2x - 4$

### Correct Answer

The correct answer is D.

### Solution

$$y = 6x^3 - x^2 - 4x + 1$$

$$\frac{dy}{dx} = 18x^2 - 2x - 4$$

$$\therefore \frac{dy}{dx} = 18x^2 - 2x - 4$$

4. Differentiate  $(2x^2 + 1)^4$  to  $x$ .

A.  $\frac{dy}{dx} = 8x(2x^2 + 1)^3$

B.  $\frac{dy}{dx} = 16x(2x^2 + 1)^3$

C.  $\frac{dy}{dx} = 16x(2x^2 - 1)^3$

D.  $\frac{dy}{dx} = 16x(2x^2 + 1)^2$

### Correct Answer

The correct answer is B.

### Solution

$$\text{Let } y = (2x^2 + 1)^4$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dv}{dx}$$

$$y = (2x^2 + 1)^4$$

$$\text{let } u = 2x^2 + 1$$

$$y = u^4$$

$$\frac{dy}{dx} = 4u^3$$

$$\frac{dy}{dx} = 4x$$

$$\frac{dy}{dx} = 4u^3 \times 4x$$

$$= 4(2x^2+1)^3 \times 4x$$

$$\frac{dy}{dx} = 16x(2x^2+1)^3$$

5. Differentiate the following for x;  $y = (3+2x)(1-x)$ .

A.  $\frac{dy}{dx} = -4x + 1$

B.  $\frac{dy}{dx} = 4x - 1$

C.  $\frac{dy}{dx} = 4x - 1$

D.  $\frac{dy}{dx} = 4x + 1$

### Correct Answer

The correct answer is C.

### Solution

$$y = (3+2x)(1-x)$$

$$\text{Let } u = 3+2x \quad v = 1-x$$

$$\frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = -1$$



$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\frac{dy}{dx} = (1-x)^2 + (3+2x)(-1)$$

$$\frac{dy}{dx} = 2 - 2x - 3 - 2x$$

$$\frac{dy}{dx} = -4x - 1$$

6. Find the  $\frac{d^3y}{dx^3}$  of  $y = x^4 - 6x^3 + 5$

A.  $\frac{d^3y}{dx^3} = 4x^3 - 18x^2$

B.  $\frac{d^3y}{dx^3} = 24x - 36$

C.  $\frac{d^3y}{dx^3} = 12x^2 - 36x$

D.  $\frac{d^3y}{dx^3} = 24x + 36$

### Correct Answer

The correct answer is C.

### Solution

$$\frac{dy}{dx} = 4x^3 - 18x^2$$

$$\frac{dy}{dx} = 12x^2 - 36x$$

$$\frac{dy}{dx} = 24x - 36$$

7. If  $f(x) = (x^2+3)^3$ , find the gradient of  $f(x)$  at  $x = \frac{1}{2}$

A.  $\frac{dy}{dx} = 6.5$

B.  $\frac{dy}{dx} = 0.5$

C.  $\frac{dy}{dx} = -6.5$

D.  $\frac{dy}{dx} = 6$

### Correct Answer

The correct answer is C.

### Solution

Let  $y = (x^2 + 3)^2$

$$\frac{dy}{dx} = 2 \times 2 \times (x^2 + 3)$$

$$\frac{dy}{dx} = 4x(x^2 + 3)$$

Therefore, the gradient at  $x = \frac{1}{2}$  is

$$\frac{dy}{dx} = 4 \times \frac{1}{2} \left( \frac{1^2}{2} + 3 \right)$$

$$\frac{dy}{dx} = 2 \left( \frac{1}{4} + 3 \right)$$

$$\frac{dy}{dx} = 0.5 + 6 = 6.5$$

8. A particle moves along a straight line in such a way that after  $t$  seconds it has gone  $s$  meters, where  $s = t^2 + 2t$ . Find the velocity of the particle after 3 seconds.

A 6m/s

B 7m/s

C 5m/s

D 8m/s

### Correct Answer

The correct answer is D.

### Solution

$$\text{Velocity} = ds/dt$$

$$S = t^2 + 2t$$

$$ds/dt = 2t + 2$$

After 3 seconds, velocity will be

$$= 2(3) + 2$$

$$= 6 + 2$$

$$= 8\text{m/s}$$

### Solution

Remember, velocity is a change of distance with time, so velocity is  $ds/dt$ . When you get the value of the velocity, substitute the value of time for  $t$  in the new equation and you will get your final answer. See the solution for more details.

9. Find the range of values of  $x$  for which  $x^2 - x$  is decreasing?

A  $X > -1$

B  $X > 1$

C  $X < -1$

D  $X < 1$

### Correct Answer

The correct answer is D.

### Solution

$$\text{Let } y = x^2 - 2x$$

$$\frac{dy}{dx} < 0 \quad \frac{dy}{dx} = 2x - 2 < 0$$

$$2x < 2 \Rightarrow X < 1$$

Recall,  $x^2 - x$  is decreasing if  $\frac{dy}{dx} < 0$ . Check the solution for a detailed explanation

10. Find the maximum or minimum value of the curve  $y=x^2-6x+5$

- A minimum point is (3,4)
- B maximum point is (3,-4)
- C minimum point is (3,-4)
- D maximum point is (3,-4)

**Correct Answer**

The correct answer is C.

**Solution**

$$\text{Let } y = x^2 - 2x$$

$$\frac{dy}{dx} = 2x - 6 = 0$$

$$2x - 6 = 0$$

$$2x = 6$$

$$x = 3$$

$$\frac{d^2y}{dx^2} = 2, 2 > 0 \text{ therefore, it is minimum}$$

To obtain y,

$$y = 3^2 - 6(3) + 5$$

$$y = 9 - 18 + 5$$

$$y = -4$$

minimum point is (3,-4)

## INTERACTIVE ASSESSMENT QUESTIONS AND ANSWERS



1. Solve  $\int 3x^2 \, dx$

- A  $x^3 + C$
- B  $3x^2 + C$
- C  $3x^3 + C$
- D  $2x^3 + C$
- E  $x^2 + C$

**Correct Answer**

The correct answer is A.

**Explanation**

$$\begin{aligned}\int 3x^2 \, dx &= 3 \int x^2 \, dx \\ &= 3 \left( \frac{x^{2+1}}{2+1} \right) + c \\ &= \frac{3x^3}{3} + C\end{aligned}$$

Recall, to integrate, add one to the power and divide by the power.

2. The integral of  $2x(x^2 - 1)$  for  $x$

- A  $2x^4 - x^2 + C$
- B  $0.5x^2 - x^2 + C$
- C  $0.5x^4 - x^2 + C$
- D  $0.5x^4 - x + C$

**Correct Answer**

The correct answer is C.

### Solution

$$\begin{aligned}\int 2x(x^2 - 1) dx &= \int (2x^3 - 2x) dx \\&= \int 2x^3 dx - \int 2x dx \\&= 0.5x^4 - x^2 + C\end{aligned}$$

### Explanation

First, open the brackets before finding the integral. Check the solution for a detailed explanation.

3. Solve the integral below  $\int (x+1)^3 dx$   
(Hint: Use integration by substitution)

A.  $\frac{(x+1)^3}{4}$

B.  $\frac{(x+1)^4}{4}$

C.  $\frac{(x+1)^3}{3}$

D.  $\frac{(x+1)^4}{1}$

### Correct Answer

The correct answer is B.

### Solution

$$\begin{aligned}\int (x+1)^3 dx &\text{ can be written as } \int (x+1)^3 - 1 du \\&\int (x+1)^3 - 1 dx\end{aligned}$$

$$\text{Let } (x+1) \text{ be } u \quad \frac{du}{dx} = 1 \quad du = 1 dx$$

$$\begin{aligned}\int u^3 du &= \frac{u^4}{4} + c \\&= \frac{(x+1)^4}{4}\end{aligned}$$

### Explanation

Use integration by substitution to solve the question. Check the solution for a detailed explanation

4. Solve the integral below:  $\int e^x x dx$

- A  $e^x (x + 1)$
- B  $e (x - 1)$
- C  $xe^x (x - 1)$
- D  $e^x (x - 1)$

### Correct Answer

The correct answer is D.

### Solution

$$\int e^x x dx$$

$$\text{Let } u = x, v = e^x$$

$$\text{Differentiate } u = 1$$

$$\text{Integrate } v = \int e^x dx = e^x$$

Putting it together

$$u \int v dx - \int u' (\int v dx) dx$$

$$x (e^x) - \int 1.(e^x) dx$$

$$xe^x - \int e^x dx$$

$$xe^x - e^x + c$$

$$e^x (x-1)$$

### Explanation

Use integration by part to solve the question. Note that when choosing  $u$  and  $v$ , choose  $u$  for the term you can easily differentiate while you should choose  $v$  for the term you can easily integrate. Check the solution for a detailed explanation.



5. A particle moves in a straight line in such a way that its velocity after  $t$  seconds is  $(3t + 4)$  m/s. The distance traveled in the first 3 seconds is

- A  $S = 25.5\text{m}$
- B  $S = 26.5\text{m}$
- C  $S = 19.75\text{m}$
- D  $S = 19.5\text{m}$

**Correct Answer**

The correct answer is A.

**Explanation**

$$V = 3t + 4$$

$$ds/dt = 3t + 4$$

$$ds = (3t + 4)dt$$

$$\int_0^3 ds = \int (3t+4)dt$$

$$S = \left( \frac{3t^3}{2} + 4t \right)$$

$$S = 27/2 + 12$$

$$S = 25.5\text{m}$$

6. The velocity,  $V$  m/s of a body after time  $t$  seconds is given by  $V=3t^2 - 2t - 3$ . Find the distance covered during the 4th second. Using Simpson's rule with 4 strips, evaluate

- A  $S = 18\text{m}$
- B  $S = 45\text{m}$
- C  $S = 27\text{m}$
- D  $S = 23\text{m}$

### Correct Answer

The correct answer is C.

### Explanation

Let  $S$  be the distance covered

$$S = \int (3t^2 - 2t - 3) dt$$

$$S = \left[ t^3 - t^2 - 3t \right]_3^4$$

$$S = (64 - 16 - 12) - (27 - 9 - 9)$$

$$S = 27\text{m}$$

### Explanation

Recall that the integral of velocity is distance. Check the solution for the explanation.

7.  $\int_1^6 2^x dx$  Correct to 2 decimal places.

A      86.67

B      86.7

C      86.6

D      86.66

### Correct Answer

The correct answers are B, D and E.

### Explanation

$$\int_2^6 2^x dx$$

$$b - a = 6 - 2 = 4 \quad n = 4$$

$$h = \frac{b - a}{n} = \frac{4}{4} = 1$$

X	Y	First last ordinates	Odd ordinates	Remaining ordinates
2	$Y_0$	4		
3	$Y_1$		8	
4	$Y_2$			16
5	$Y_3$		32	
6	$Y_4$	64		
Totals		68	40	16

$$\int_2^6 2^x dx = \frac{1}{3} \times 1 [(y_0 + y_4) + 4(y_1 + y_3) + 2y_2]$$

$$= \frac{1}{3} (68 + 4(40) + 2(16))$$

$$= \frac{1}{3} (68 + 4(40) + 2(16))$$

$$= 86.67 \text{ (2 d.p.)}$$

Copyright © E-Malezi LLP. All rights reserved.

No part of this publication, either image, layout or text may be copied, distributed or posted online without the prior written permission of the publisher.

For permission on usage or any other requests, write to the publisher, at [hello@malezi.ng](mailto:hello@malezi.ng)



#### **BUSINESS DETAILS**

Website : [Malezi.co.ke](http://Malezi.co.ke) / [learnathome.ng](http://learnathome.ng)

Business : Education

Concern : E- Learning