Telescope sensitivity analysis due to M1 AHU vibration forces

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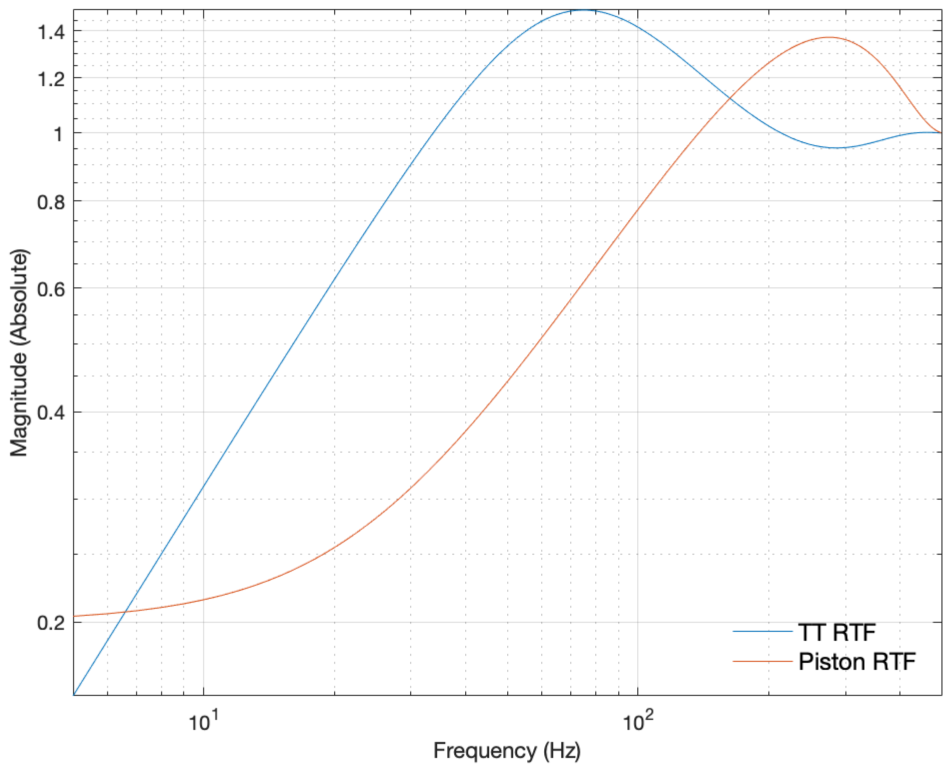
Preamble

- Clear workspace
- Radians to mas (10^{-3} arc seconds) conversion constant

Load PSD of ATA vibration data

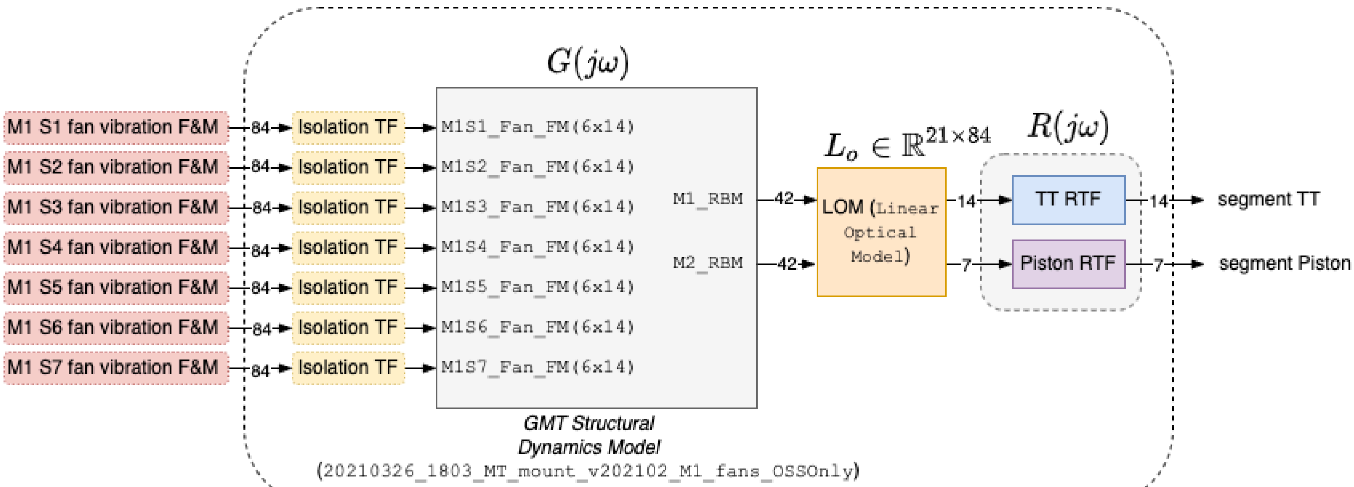
Load NS or LTAO RTFs

The variable `rtf` is a 21×21 matrix transfer function. The NS or LTAO (depending on the argument state of `NS_MODE`) tip-tilt rejection transfer function (RTF) fills the first 14 main diagonal entries. The differential piston RTF fills the last 7 diagonal elements if the checkbox is unchecked, meaning that one assumes the LTAO observing mode. Otherwise, those diagonals are filled with 1s. The rejection transfer function matrix aims at reproducing the effect of the wavefront control loops.



Vibration Analysis Model

Let $G(\jmath\omega)$ the frequency response matrix represent the GMT structural dynamics whose inputs are the forces and moments applied to nodes at the M1 fan locations and the outputs are the M1 and M2 rigid-body motions. As illustrated in the model diagram below, using a linear approximation (LOM), one gets the segment tip-tilt (TT) and the piston at the exit pupil.



AHU to seg TTP Transfer Function

$H(j\omega)$

The full telescope model reads as

$$H(j\omega) = R(j\omega)L_oG(j\omega)\left(I_{n_u} \otimes H_{\rm iso}(j\omega)\right),$$

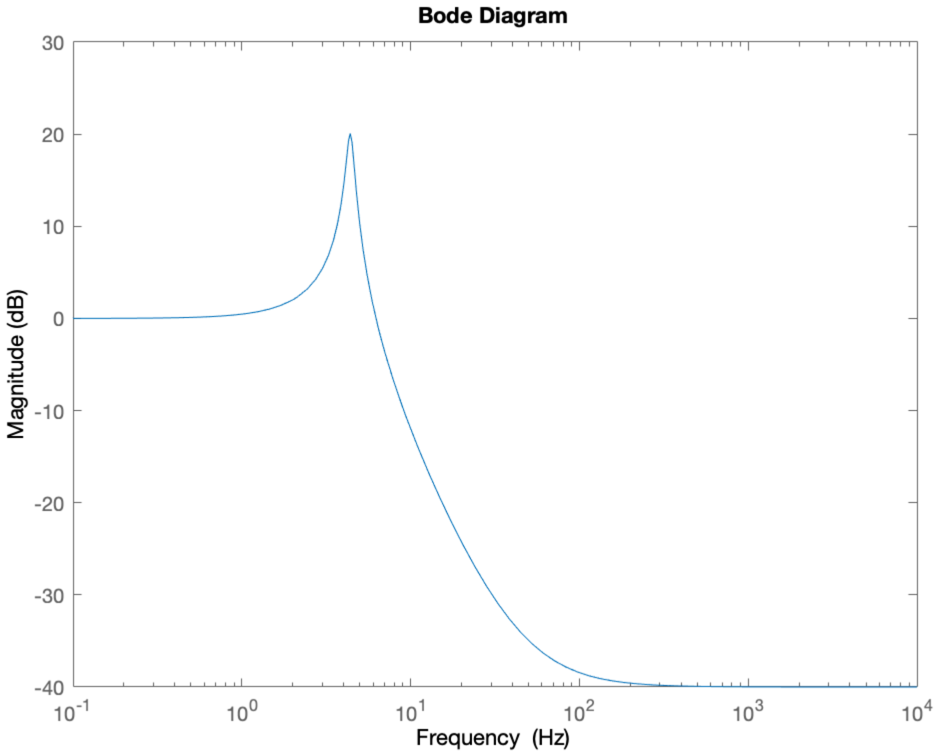
where $H_{\rm iso}(j\omega)$, $R(j\omega)$ and L_o represent the AHU isolation model, the rejection transfer function frequency response matrix and the linear optical model transformation, respectively. The n_u-dimensional matrix is denoted by I_{n_u} ($n_u = 588$) and the symbol $\otimes$ represents the Kronecker product.

Isolation transfer function

The baseline isolation transfer function (refer to Section 6.3 of GMT-DOC-04861 for the parameters sensitivity study) reads as

$$H_{\rm iso}(s) = \frac{\omega_0^2}{\omega_1^2}\frac{s^2 + 2\omega_1 s + \omega_1^2}{s^2 + 2\zeta_{\rm iso}\omega_0 s + \omega_0^2},$$

where $\omega_0 = 2\pi 4.4$ rad/s, $\omega_1 = 10\omega_0$, and $\zeta_{\rm iso} = 0.05$.



White noise response

We compute the TT and piston PSD ($\mathrm{mas}^2/\mathrm{Hz}$ and $\mathrm{nm}^2/\mathrm{Hz}$, respectively) using

$$Y_{\rm TTP}(j\omega) = |H(j\omega)|^2(\mathbf{1}_{98} \otimes U_{\rm fan}(j\omega))$$

The symbol $\otimes$ stands for the Kronecker product and $\mathbf{1}_{n_1}$ is an n_1-dimensional column vector filled with ones, such that

$$\mathbf{1}_3 \otimes U_{\text{fan}}(j\omega) = \begin{bmatrix} U_{\text{fan}}(j\omega) \\ U_{\text{fan}}(j\omega) \\ U_{\text{fan}}(j\omega) \end{bmatrix}.$$

The white noise response assumes that a $1\,\mathrm{N}^2/\mathrm{Hz}$ (or $1\,(\mathrm{Nm})^2/\mathrm{Hz}$) power density is applied to each fan input force (or moment) degree of freedom. The power spectrum density Y_{TTP} is 21-dimensional. According to the telescope frequency response model, the first 14 elements of Y_{TTP} are the tip-tilt PSD and the last 7 provide the segment piston.

In terms of implementation, the telescope frequency response representing the effect of the load applied on each M1 segment is available from a different data file (`m1s`***X***`fanTF.mat`) computed beforehand. For this reason, the code below performs two `for` loops: an outer one, which considers the frequency response model due to the excitation on each segment, and an inner iteration, to compute the tip-tilt and piston PSD of each segment.

```
Computing WFE response to white noise vibration...
```

```
Done!
```

Overall tip-tilt and piston effects

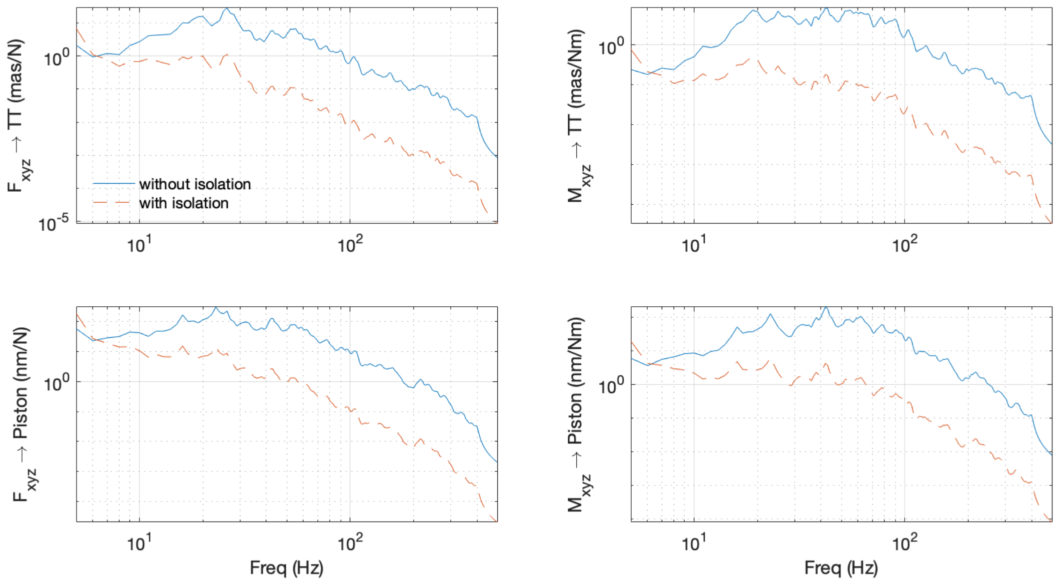
Let the $Y_i(j\omega)$ the i^{th} element of $Y_{\text{TTP}}(j\omega)$, we compute the overall TT (in mas) as

$$\phi = \sqrt{\int_0^\infty \sum_{i=1}^{14} Y_i(j\omega)d\omega}.$$

Similarly, the overall differential piston (in meters) reads as

$$\rho = \sqrt{\int_0^\infty \sum_{i=15}^{21} Y_i(j\omega)d\omega}.$$

The overall tip-tilt and piston response to white-noise are as follows.

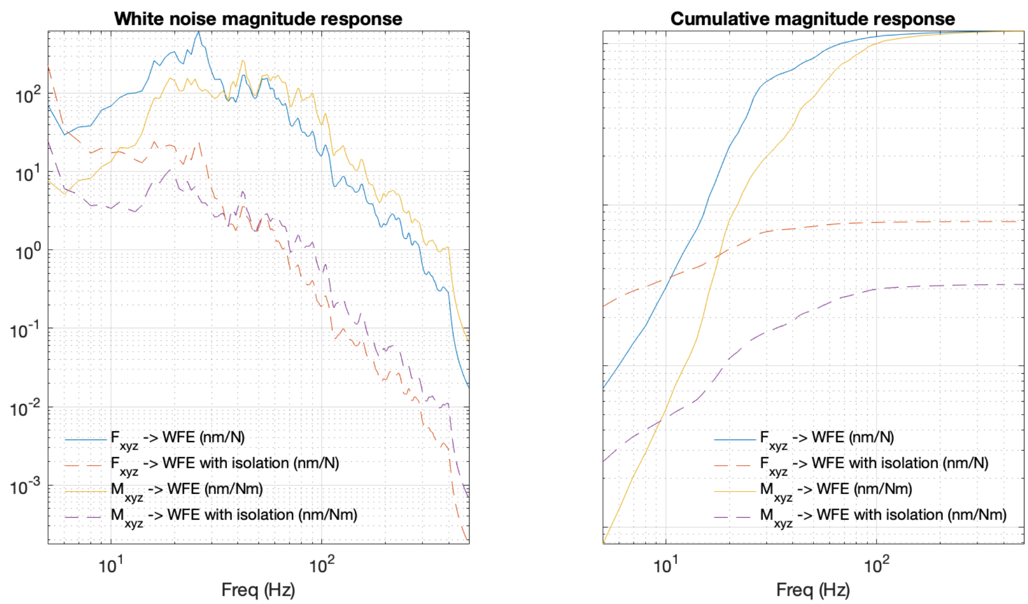


Overall wavefront error (WFE) effect

The expression of the overall M1 AHU vibration effect on the wavefront error (WFE) in (in meters) is

$$\mathrm{WFE} = \sqrt{\frac{R^2}{4}\left(\frac{1}{3.6\times 10^6}\frac{\pi}{180}\phi\right)^2 + \rho^2},$$

where $R = 8.4$ m (see Appendix E of GMT-DOC-04861).



Vibration effect analysis based on ATA's data

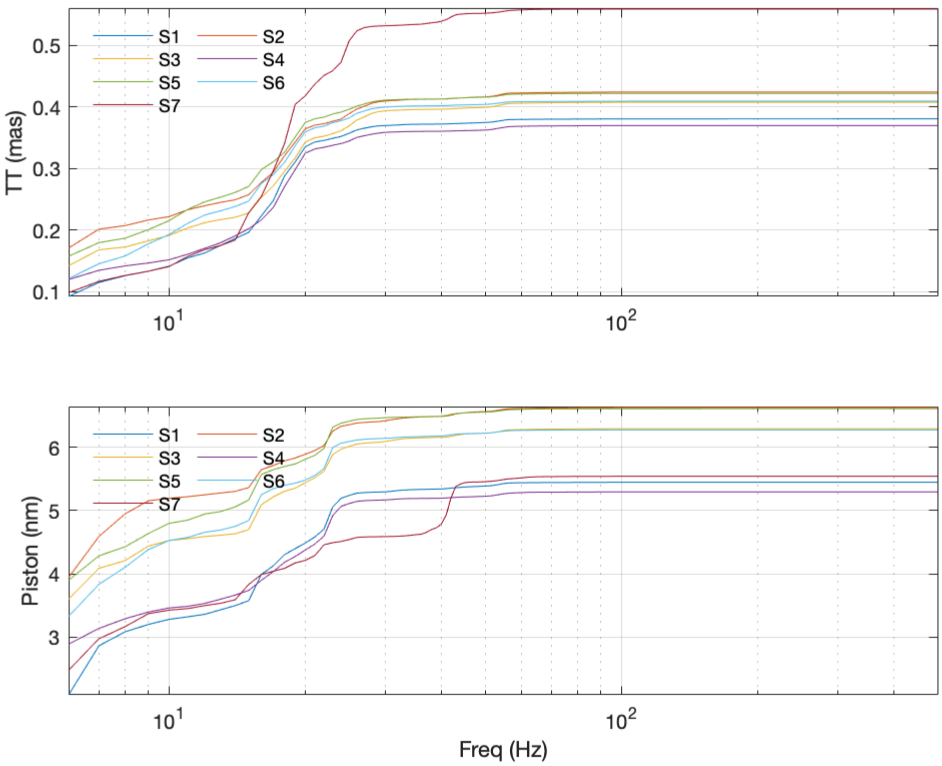
Here we compute $Y_{\mathrm{TTP}}(j\omega)$ considering the vibration data PSD provided by ATA.

```
TT and Piston PSD computed for S1.
```

```
TT and Piston PSD computed for S2.
TT and Piston PSD computed for S3.
TT and Piston PSD computed for S4.
TT and Piston PSD computed for S5.
TT and Piston PSD computed for S6.
TT and Piston PSD computed for S7.
```

Cumulative TT and Piston Power Spectrum Densities

Below, we present the cumulative power spectrum (CPS). Each line represents the effect of the load applied on a particular M1 segment.




```
Overall optical metrics:

TT=0.417mas   Piston=6.04nm

 Total vibration induced WFE:10.4nm
```

Rejection transfer functions (RTFs)

Function to compute the NS RTF (according to REQ-L3-OAD-35337)

Function to compute the GLAO RTF (according to REQ-L3-OAD-35398)

Function to compute the differential piston RTF