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Method to Design Controller Gains for The Voltage Phase Angle Torque Feed-back Control System

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***Abstract*— In this paper, a method to design the control gains for the voltage phase angle feed-back control method by means of the linearization method, is proposed. The proposed controller designing method is verified by experimental test with a 1kW class PMSM.**

Keywords*— *IPMSM; Field weakening control; Torque feed-back;

I. Introduction

Permanent magnet synchronous motors (PMSMs) are featured by higher efficiency and higher torque density. Thus they are widely used for electric vehicles, elevators, consumer electronics and other applications. Especially, interior permanent magnet synchronous motors (IPMSMs) are expected to output higher torque by their reluctance torque due to the salient pole structure. In addition, IPMSMs are possible to reduce the size and weight. Therefore, by applying IPMSMs, mounting space of the main circuit equipment, e.g. on the electric vehicle, can be saved.

In the high-speed range operation, induced voltage of IPMSM exceeds the inverter's maximum voltage. Thus, IPMSMs require field-weakening control to reduce the terminal voltage. When output voltage of the inverter is limited in the field-weakening region, vector control cannot be applied; hence it is impossible to obtain high-performance torque response. Therefore, it is necessary to develop suitable control for field-weakening region.

There are various proposed control methods for field-weakening region of PMSMs [1]-[6]. As a method to control the torque, the authors proposed a method to determine the q-axis current command value by the feed-back signal of the calculated torque, to perform a field-weakening in the constant current control in [1]. This control method is called torque feed-back vector control. On the other hand, a method to determine terminal voltage phase angle by feed-back signal of the calculated torque has been proposed in [2]. This control method is called voltage phase angle torque feed-back control. This control method can highly utilize the voltage amplitude to control the motor current for the dynamic torque even under the limited voltage, compared with the other PMSM field weakening control methods. The control system configurations were shown also in [3]; however the analytical design method was not shown in neither [2] and [3]. In this paper, a method to design the control gains is proposed for the voltage phase angle feed-back control method by means of the linearization method. The proposed controller designing method is verified by experimental test with a 1kW class PMSM.

II. Voltage Phase Angle Torque Feed-back Control System

A. The torque control by voltage phase control

Voltage equations of the IPMSMs are expressed as follows:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} r_m + sL_d & -\omega L_q \\ \omega L_d & r_m + sL_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega\phi_f \end{bmatrix} \qquad (1)$$

Where r_m is stator coil resistance, L_d and L_q are d- and q-axis inductance, ω is electrical angular frequency, ϕ_f is permanent-magnet field, and s is differential operator. By assuming high speed operation and steady state ignoring armature resistance drop in the armature winding, hence (1) can be expressed as follows:

$$\begin{bmatrix} V_m \cos\theta \\ V_m \sin\theta \end{bmatrix} = \begin{bmatrix} 0 & -\omega L_q \\ \omega L_d & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega\phi_f \end{bmatrix} \qquad (2)$$

Voltages are represented in polar coordinate, whereas V_m and θ are the magnitude and phase angle of the voltage vector, respectively. Phase angle is based on the d-axis of which the counter-clockwise direction is positive.

Torque of IPMSMs can be expressed by (3), where p is pole pairs.

$$T_m = p\{\phi_f + (L_d - L_q)i_d\}i_q \qquad (3)$$

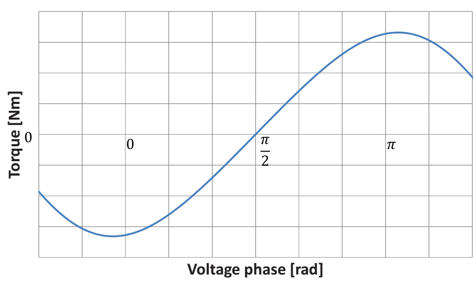


Fig. 1 Relationship of torque and voltage phase

By substituting i_d and i_q from (2) to (3), then T_m can be expressed as

$$T_m = \left(\frac{1}{L_d} - \frac{1}{L_q}\right)\frac{pV_m^2}{2\omega^2}\sin 2\theta - \frac{p\phi_f V_m}{\omega L_d}\cos\theta \tag{4}$$

Figure.Fig. 1 shows the relationship of torque and voltage phase in (4), and shows a fact that the motor torque T_m is controllable by changing the voltage phase angle θ even under the fixed voltage amplitude [2].

B. *Structure of the control system*

Figure.Fig. 2 shows voltage phase operation of the torque feed-back control system. Feed-forward unit calculates the q-axis current reference i_q^* by dividing torque reference T_m^* with $p\phi_f$. In addition, the d-axis current reference i_d^* is calculated from terminal voltage limitation formula as

$$V_{mmax}^* = \omega\sqrt{\left(L_q i_q^*\right)^2 + \left(\phi_f + L_d i_d^*\right)^2} \tag{5}$$

In the high speed range, the inverter voltage is maximal; hence stator resistance drop can be neglected. V_{mmax}^* is maximum voltage of inverter. If the inverter is operated in single-pulse mode, in which the rectangular pulse is output in a fundamental frequency, V_{mmax}^* can be expressed as follows:

$$V_{mmax}^* = \frac{\sqrt{6}}{\pi}V_{dc} \tag{6}$$

Where V_{dc} is input voltage of the inverter. i_d^* can be obtained from (5) and i_q^*, which can be represented as

$$i_d^* = -\frac{\phi_f}{L_d} + \sqrt{\frac{\phi_f}{L_d} - \frac{\omega^2\phi_f^2 + \omega^2 L_q^2 i_q^2 + V_m^2}{\omega^2 L_d^2}} \tag{7}$$

From (1), the d-axis voltage v_{dFF}^* and q-axis voltage v_{qFF}^* can be calculated from their current reference values as given below.

$$v_{dFF}^* = r_m i_d^* - \omega L_q i_q^* \tag{8}$$
$$v_{qFF}^* = r_m i_q^* + \omega\left(L_d i_d^* + \phi_f\right) \tag{9}$$

From the above results, voltage phase angle θ_{FF} can be expressed as follows:

$$\theta_{FF} = \tan^{-1}\frac{v_{qFF}^*}{v_{dFF}^*} \tag{10}$$

IN feed-back unit, the error between the torque reference T_m^* and the calculated torque T_{mcal} is compensated by a PID compensator, manipulating the feed-back component of the voltage phase angle θ_{FB} to follow T_{mcal} to T_m^*.

$$\theta_{FB} = G_{PID}(s)(T_m^* - T_{mcal}) \tag{11}$$

PID compensator $G_{PID}(s)$ is given as

$$G_{PID}(s) = K_p + \frac{K_i}{s} + K_d s \tag{12}$$

Where K_p is proportional gain, K_i is integration gain, and K_d is differential gain.

By using (10) and (11), the voltage phase angle θ can be expressed as in the following.

$$\theta = \theta_{FF} + \theta_{FB} \tag{13}$$

Afterwards, voltage vector v_d^* and v_q^* are calculated using voltage phase angle θ and V_{mmax}^* as given below.

$$v_d^* = V_{mmax}^*\cos\theta \tag{14}$$
$$v_q^* = V_{mmax}^*\sin\theta \tag{15}$$

Torque T_{mcal} is calculated by dividing the active power of motor input with electrical angular velocity [1].

$$T_{mcal} = \left(\eta p \textstyle\sum_{k=u,v,w} v_k i_k\right)/\omega \tag{16}$$

Where η is motor efficiency. In this case, accuracy of T_{mcal} depends on the η. Accordingly, (17) is introduced in [7].

$$T_{mcalnew} = p\sum_{k=u,v,w}\left\{v_k^* - \left(\frac{T_{dt}}{T_{sw}}V_{dc} + V_{fd}\right)\mathrm{sign}(i_k) - r_m i_k\right\} i_k/\omega \tag{17}$$

Where T_{dt} is dead time period, T_{sw} is switching period, V_{fd} is on-voltage drop of switching elements, and sign() is signum function. $T_{mcalnew}$ is the calculated torque that considers the output voltage error of inverter and the voltage drop caused by armature winding resistance. Equation (17) is confirmed to be able to calculate accurate torque at medium/high speed range.

III. Design Method of Feed-back Control System

A. *Linearization and gain determination method*

First, relation of d- and q-axis voltage to the voltage phase angle is linearized. Fig. 3 shows a vector diagram of the voltage phase angle control. When T_m^* is changed, $\boldsymbol{V_{mFF}^*}$ is determined by giving θ_{FF} as the feed forward signal. Then, the voltage phase angle θ is determined by adding the feed-back component θ_{FB} generated by the PID compensator to obtain

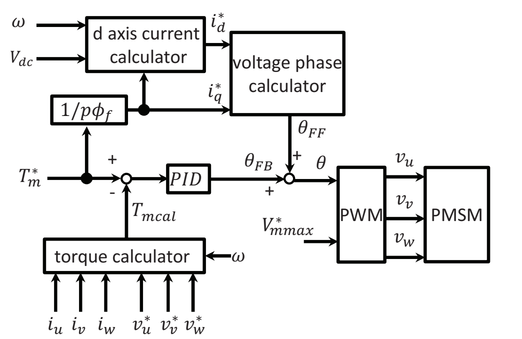


Fig. 2 Voltage phase operation torque feed-back control system

the voltage vector $\boldsymbol{V_m^*}$. $\boldsymbol{V_{mFF}^*}$ and $\boldsymbol{V_m^*}$ can be represented by $\boldsymbol{V_{mFF}^*} = (v_{dFF}^* \quad v_{qFF}^*)^T$ and $\boldsymbol{V_m^*} = (v_d^* \quad v_q^*)^T$ in d-q coordinate. The transformation from $\boldsymbol{V_{mFF}^*}$ to $\boldsymbol{V_m^*}$ is done using coordinate transformation θ_{FB}. Therefore, $\boldsymbol{V_m^*}$ can be expressed as

$$\begin{bmatrix} v_d^* \\ v_q^* \end{bmatrix} = \begin{bmatrix} \cos\theta_{FB} & -\sin\theta_{FB} \\ \sin\theta_{FB} & \cos\theta_{FB} \end{bmatrix} \begin{bmatrix} v_{dFF}^* \\ v_{qFF}^* \end{bmatrix} \tag{18}$$

Assuming that θ_{FB} given in the compensator is minimal, $\cos\theta_{FB}$ and $\sin\theta_{FB}$ can be approximated as 1 and θ_{FB}, respectively.

$$\begin{bmatrix} v_d^* \\ v_q^* \end{bmatrix} = \begin{bmatrix} 1 & -\theta_{FB} \\ \theta_{FB} & 1 \end{bmatrix} \begin{bmatrix} v_{dFF}^* \\ v_{qFF}^* \end{bmatrix} \tag{19}$$

In addition, assuming variation of $\boldsymbol{V_{mFF}^*}$ and $\boldsymbol{V_m^*}$ as $\boldsymbol{\delta V_m^*} = (\delta v_d^* \quad \delta v_q^*)^T$, then it can be expressed as follows [5]:

$$\begin{bmatrix} \delta v_d^* \\ \delta v_q^* \end{bmatrix} = \begin{bmatrix} 0 & -\theta_{FB} \\ \theta_{FB} & 0 \end{bmatrix} \begin{bmatrix} v_{dFF}^* \\ v_{qFF}^* \end{bmatrix} \tag{20}$$

The induced voltage by permanent magnet field in IPMSMs is high, hence $\boldsymbol{V_{mFF}^*}$ can be regarded as $v_{qFF}^* \gg v_{dFF}^*$. Thus, v_{dFF}^* can be omitted. In addition, v_{qFF}^* can be expressed by (21), if armature reaction drop and armature resistance drop can be regarded as much smaller than the PM flux induced voltage.

$$v_{qFF}^* = \omega_0 \phi_f \tag{21}$$

Then, torque calculation formula in (16) is linearized and represented on the d-q axis as in (22). However, the efficiency η is ignored.

$$T_{mcal} = p\left(v_d^* i_d + v_q^* i_q\right)/\omega \tag{22}$$

(23) is obtained by linearizing T_{mcal} in (22) by means of Tylor exploration.

$$T_{mcal} \approx T_{mcal0} + \left(v_{d0}\delta i_d + v_{q0}\delta i_q + i_{d0}\delta v_d^* + i_{q0}\delta v_q^*\right)/\omega_0 \tag{23}$$

Where T_{mcal0}, v_{d0}, v_{q0} are defined as follows:

$$T_{mcal0} = p\left(v_{d0} i_{d0} + v_{q0} i_{q0}\right)/\omega_0 \tag{24}$$

$$v_{d0} = r_m i_{d0} - \omega_0 L_q i_{q0} \tag{25}$$

$$v_{q0} = r_m i_{q0} + \omega_0 L_d i_{d0} + \omega_0 \phi_f \tag{26}$$

When the command value is slightly changed from the equilibrium point, it can be expressed as

$$T_m^* = T_{mcal0} + \delta T_m^* \tag{27}$$

The torque controller can be expressed by combining (10), (23) and (27) as

$$\theta_{FB} = G_{PID}(s) * \\ * \left\{\delta T_m^* - \left(v_{d0}\delta i_d + v_{q0}\delta i_q + i_{d0}\delta v_d^* + i_{q0}\delta v_q^*\right)/\omega_0\right\} \tag{28}$$

Equation (1) is assumed to be established for small value of v_d, v_q, i_d and i_q. Further, δv_d and δv_q are assumed to δv_d^* and δv_q^*. δv_d^*, δv_q^*, v_{d0} and v_{q0} are cancelled by (25) and (26).

Moreover, if the equilibrium point is on the terminal voltage limit ellipse, it is assumed to be in the vicinity of the maximum torque point. Therefore, torque controller can ignore the influence of the d-axis current; hence, the torque is considered to be proportional to the q-axis current. Consequently, the influence of δi_d to the torque controller can be neglected in (28), and rewritten as follows:

$$\theta_{FB} = G_{PID}(s)\left(\delta T_m^* - p\left\{\phi_f + \left(L_d - L_q\right)i_{d0}\right\}\delta i_q\right) \tag{29}$$

Block diagram of the linearized system is shown in Fig. 4 using (20), (21) and (29). Open-loop transfer function $G_{\theta T}(s)$ from θ_{FB} to T_{mcal} in Fig. 4 can be expressed as follows:

$$G_{\theta T}(s) = \frac{(\omega_0^2 \phi_f / L_q) \times p\{\phi_f + (L_d - L_q) i_{d0}\}}{s^2 + \{r_m(L_d + L_q)s / L_d L_q\} + \{(r_m^2 + \omega_0^2 L_d L_q)/L_d L_q\}} \tag{30}$$

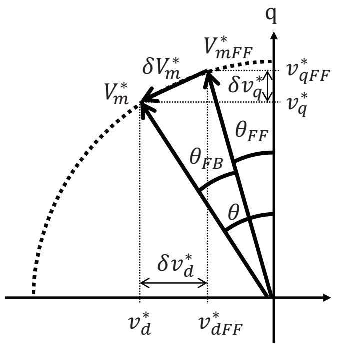


Fig. 3 Vector diagram of the voltage phase angle control

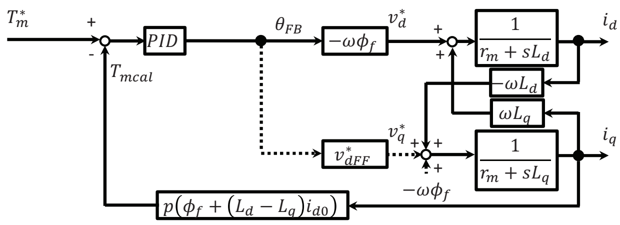


Fig. 4 Block diagram of the linearized system

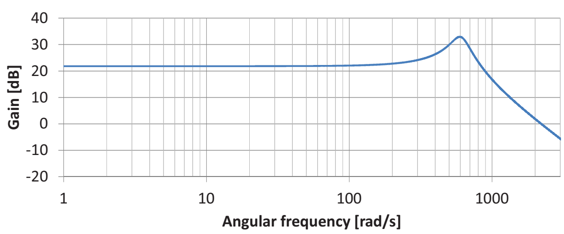


Fig. 5 Bode plot of the transfer function $G_{\theta T}(s)$

Figure.Fig. 5 shows bode plot of the transfer function $G_{\theta T}(s)$ using motor parameters shown in Table 1 and the rotational speed is 600 rad/s. $G_{\theta T}(s)$ has a resonance point due to interference by armature reaction induced voltage between the d-q axes, which causes oscillation. Hence, it is required to design the control system by determining the PID gains that cancel resonance point. Torque controller is often designed as minor loop of the upper controller, such as a speed control system. In that case, in order to simplify the upper controller design, one-order delay transfer function is seatible as a transfer function of acutual torque torque over torque reference. Gains in (31) are designed to make the transfer function one-order delay system, as shown in (32) to (34).

Therefore, (30) can be rewritten as follows:

$$G_{\theta T}(s) = \frac{b_0}{s^2+a_1 s+a_0} \tag{31}$$

$$a_0 = \left(r_m^2 + \omega_0^2 L_d L_q\right)/L_d L_q \tag{32}$$

$$a_1 = r_m\left(L_d + L_q\right)/L_d L_q \tag{33}$$

$$\mathrm{b}_0 = \left(\omega_0^2 \phi_f / L_q\right) \times p\{\phi_f + \left(L_d - L_q\right) i_{d0}\} \tag{34}$$

Open-loop transfer function of $G_{open}(s)$ from T_m^* to T_{mcal} in Fig. 4 can be expressed as follows:

$$G_{open}(s) = \frac{K_d s^2 + K_p s + K_i}{s} \times \frac{b_0}{s^2+a_1 s+a_0} \tag{35}$$

If T_t is assumed as time constant of the torque response, then the step response is the response of a first-order lag by determining the gains to match (35) to the integral element $1/T_t$, which are designed as

$$K_d = \frac{1}{T_t b_0} \tag{36}$$

$$K_p = \frac{a_1}{T_t b_0} \tag{37}$$

$$K_i = \frac{a_0}{T_t b_0} \tag{38}$$

According to the above results, we are able to design the gains of the PID compensator.

B. *Experimental verification of the controller design*

In order to verify of the proposed method, we did experiment using IPMSM with rating power of 1 kW. Figure.Fig. 6 shows the experimental system, whereas Table 1 shows specifications of the equipment used in the experiment. 2-level voltage source inverter is used in the main circuit to drive the IPMSM in the single-pulse mode operation. Disc-shaped inertial mass is used as a load. In this experimental system, we evaluated the torque step response.

Figure.Fig. 7 shows torque and q-axis current step response waveform when the rotational speed is 1800 rpm, where Fig. 7(a) shows the calculated torque response and Fig. 7(b) shows the q-axis current response. The results show that the torque and q-axis current correspond to 10 ms as designed. Therefore, the design method is effective and not affected by linearization.

Table 1 Parameters of the experimental system

● IPMSM rated power 1 kW
Stator coil resistance $r_m = 1.1$ [Ω]
Inductance $L_d = 12.0$ [mH] $L_q = 14.0$ [mH]
Constants of e.m.f $\phi_f = 0.21$ [V/rad/s]
Pole pairs $p = 4$
Rated speed 2000 [rpm]
DC link voltage $V_{dc} = 150$ [V]
Inertial load $J = 0.76$ [kgm²]
● Control parameters
Control period $T_s = 0.2$ [ms]
Torque control time constant $T_t = 10$ [ms]

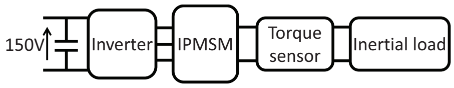
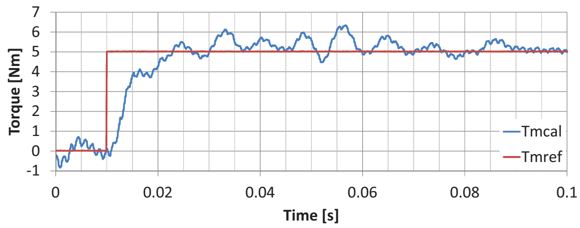
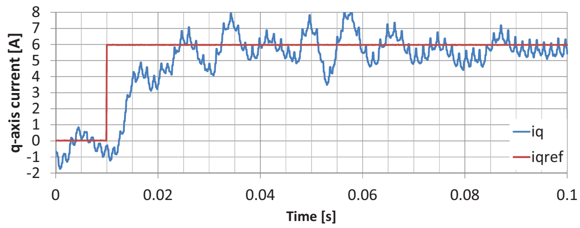


Fig. 6 Experimental system



(a) Calculated torque response



(b) q-axis current response

Fig. 7 Torque step response when the rotational speed is 1800 rpm

IV. Conclusions

In this paper, we proposed a design method of the voltage phase operation torque feed-back control by means of the linearized method. The proposed control method was verified by using experimental system. As a result, the proposed method was confirmed to be effective in the flux-weakening region.

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