A Bayesian model for data flow: BikeMi

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What we are doing

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The BikeMi stations net in Milan

Two prospective of the problem

We followed two distinct paths:

Global model: the total volume of bikes travels in a specific day Y_t without considering the graph structure. This results in a single time series.

Network model: dividing in the different nodes and analysing the flow of bikes in the net. The dimensionality is much higher.

Poisson model

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Day by day Poisson:

$$\begin{cases} Y_t \sim \mathrm{Po}(Y_t|\lambda_t) \\ \lambda_t = \exp\{\alpha + \boldsymbol{\beta} \cdot \mathbf{x}_t\} \\ \alpha \sim \mathcal{N}(0, \sigma_\alpha^2) \\ \beta_i \overset{iid}{\sim} \mathcal{N}(0, \sigma_\beta^2) \end{cases} \tag{1}$$

Poisson model

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With covariates $\mathbf{x}_t$:

- Y_{t-1} volume on the previous day
- Y_{t-7} volume on the same weekday of the previous week
- W_t dummy for weekday / weekend
- R_t, R_{t-1} dummies for rain in the current and previous day
- T_t mean temperature for the day
- S_t, M_t dummies for Saturday and Monday

Predictive distribution of the poisson model

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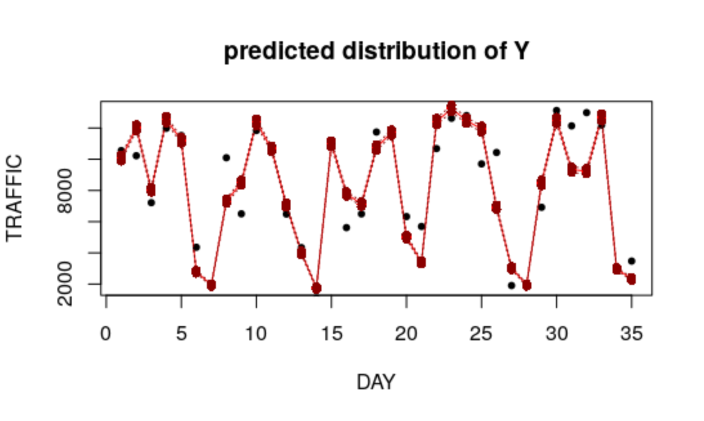
Only 3 of 35 in the 90% credible interval



Time series model: BSTS

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$$\begin{cases} \mathbf{X}(t) = \mathbf{f}(\mathbf{X}(1:(t-1))) + \boldsymbol{\epsilon}_1(t) \\ \mathbf{Y}(t) = \mathbf{g}(\mathbf{X}(t)) + \boldsymbol{\epsilon}_2(t) \end{cases} \tag{2}$$

with suitable initial conditions and priors

Time series model: BSTS

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Elementary model with precisions

$$\begin{cases} Y_t = \mu_t + \gamma_t + \frac{1}{\sqrt{\tau_\epsilon}}\tilde{\epsilon}_t \\ \\ \mu_t = \mu_{t-1} + \delta_{t-1} + \frac{1}{\sqrt{\tau_\eta}}\tilde{\eta}_t \\ \delta_t = \delta_{t-1} + \frac{1}{\sqrt{\tau_v}}\tilde{v}_t \\ \gamma_t = \sum_{i=1}^{S-1} \gamma_{t+i-S} + \frac{1}{\sqrt{\tau_w}}\tilde{w}_t \\ \\ \tilde{\epsilon}_t, \tilde{\eta}_t, \tilde{v}_t, \tilde{w}_t, \overset{iid}{\sim} \mathcal{N}(0,1) \end{cases} \tag{3}$$

Time series model: BSTS

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Elementary model priors

$$\begin{cases} \mu_0 \sim \mathcal{N}(m, \tau_m) \\ \delta_0 \sim \mathcal{N}(d, \tau_d) \\ \boldsymbol{\gamma}_{0:(2-S)} \sim \mathcal{N}_{S-1}(\mathbf{g}, \tau_g \mathbf{I}) \\ \\ \tau_* \sim Gamma(a_*, b_*),\ with \ast = \{\epsilon, \eta, v, w\} \\ \\ \{\tilde{\epsilon}_t, \tilde{\eta}_t, \tilde{v}_t, \tilde{w}_t, \mu_0, \delta_0, \boldsymbol{\gamma}_{0:(-S+2)}, \tau_\epsilon, \tau_\eta, \tau_v, \tau_w\} \\ independent. \end{cases} \tag{4}$$

Time series model: BSTS

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Complete model with precisions

$$\begin{cases} Y_t = \mu_t + \gamma_t + \rho_t + \boldsymbol{\beta}^T \mathbf{z}_t + \frac{1}{\sqrt{\tau_\epsilon}} \tilde{\epsilon}_t \\ \\ \mu_t = \mu_{t-1} + \delta_{t-1} + \frac{1}{\sqrt{\tau_\eta}} \tilde{\eta}_t \\ \delta_t = \delta_{t-1} + \frac{1}{\sqrt{\tau_v}} \tilde{v}_t \\ \gamma_t = \sum_{i=1}^{S-1} \gamma_{t+i-S} + \frac{1}{\sqrt{\tau_w}} \tilde{w}_t \\ \rho_t = \alpha \rho_{t-1} + \frac{1}{\sqrt{\tau_u}} \tilde{u}_t \\ \\ \tilde{\epsilon}_t, \tilde{\eta}_t, \tilde{v}_t, \tilde{w}_t, \tilde{u}_t \overset{iid}{\sim} \mathcal{N}(0,1) \end{cases} \tag{5}$$

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Complete model priors

$$
\begin{cases}
\mu_0 \sim \mathcal{N}(m, \tau_m) \\
\delta_0 \sim \mathcal{N}(d, \tau_d) \\
\boldsymbol{\gamma}_{0:(2-S)} \sim \mathcal{N}_{S-1}(\mathbf{g}, \tau_g \mathbf{I}) \\
\rho_0 \sim \mathcal{N}(r, \tau_r) \\
\boldsymbol{\beta} \sim \mathcal{N}_p(\mathbf{0}, \tau_b \mathbf{I}) \\
\alpha \sim \mathcal{N}(a, \tau_a) \\
\tau_* \sim Gamma(a_*, b_*), \ with * = \{\epsilon, \eta, v, w, u\} \\
\{\tilde{\epsilon}_t, \tilde{\eta}_t, \tilde{u}_t, \tilde{w}_t, \tilde{v}_t, \mu_0, \delta_0, \boldsymbol{\gamma}_{0:(-S+2)}, \rho_0 \, \boldsymbol{\beta}, \alpha, \tau_\epsilon, \tau_\eta, \tau_v, \tau_w, \tau_u\} \\
independent.
\end{cases} \tag{6}
$$

Time series model: BSTS

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Robust model with precisions

$$
\begin{cases}
Y_t = \mu_t + \gamma_t + \rho_t + \boldsymbol{\beta}^T \mathbf{z}_t + \frac{1}{\sqrt{\tau_\epsilon}}\tilde{\epsilon}_t \\
\\
\mu_t = avgpred(\mu_t) + avgpred(\delta_t) + \frac{1}{\sqrt{\tau_\eta}}\tilde{\eta}_t \\
\delta_t = avgpred(\delta_t) + \frac{1}{\sqrt{\tau_v}}\tilde{v}_t \\
\gamma_t = \sum_{i=1}^{S-1} \gamma_{t+i-S} + \frac{1}{\sqrt{\tau_w}}\tilde{w}_t \\
\rho_t = \alpha\rho_{t-1} + \frac{1}{\sqrt{\tau_u}}\tilde{u}_t \\
\\
\tilde{\epsilon}_t, \tilde{\eta}_t, \tilde{v}_t, \tilde{w}_t, \tilde{u}_t \overset{iid}{\sim} \mathcal{N}(0,1)
\end{cases}
\tag{7}
$$

where $avgpred(\phi_t) = \frac{1}{2}\phi_{t-1} + \frac{1}{3}\phi_{t-S} + \frac{1}{6}\phi_{t-2S}$
and same priors as before

Time series model: BSTS

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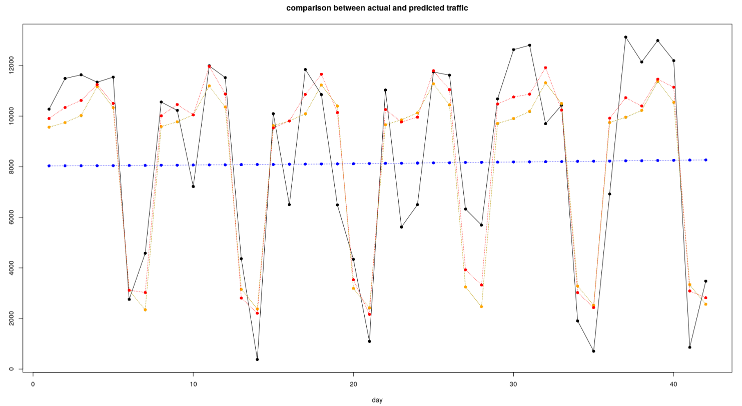
Instability of the errors, τ_ϵ explodes, bad autocorrelation



Time series model: BSTS

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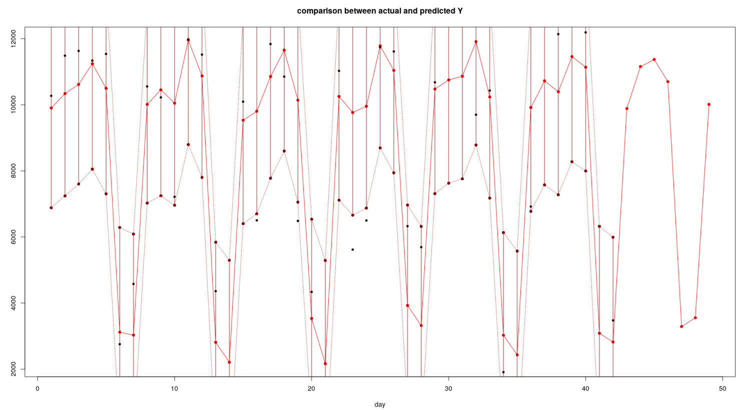
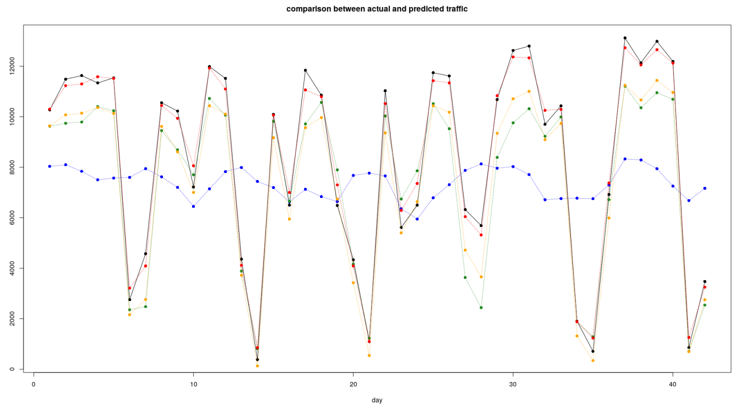
Instability of the errors, τ_ϵ explodes, bad autocorrelation



Time series model: BSTS

Blocking the maximum variance, τ_* under control, partial mixing of state variables



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Time series model: BSTS

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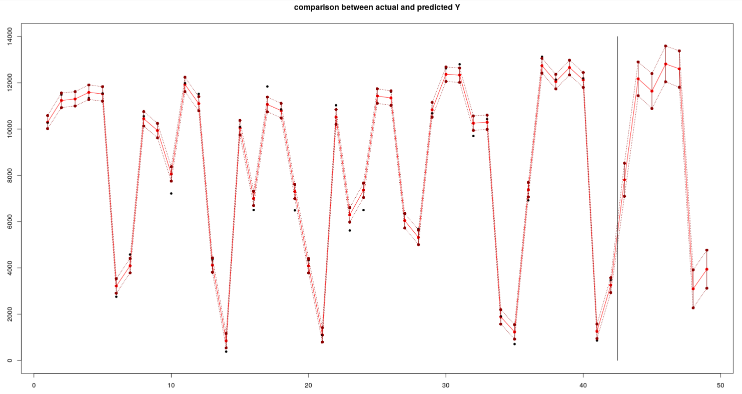
Smaller induced variability, attempt of prediction with real weather.



Time series model: BSTS

Comparison truncated-variance robust BSTS vs Poisson.

Observations inside credibility intervals:

- Poisson 3/35
- BSTS 24/42

WAIC:

- Poisson -0.0171
- BSTS -0.0160

Time series model: BSTS

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Robust model for time zones with precisions

$$
\begin{cases}
Y_{th} = \mu_t + \gamma_t + \rho_t + \chi_k + \boldsymbol{\beta}^T \mathbf{z}_t + \frac{1}{\sqrt{\tau_\epsilon}} \tilde{\epsilon}_{th} \\
\\
\mu_t = avgpred(\mu_t) + avgpred(\delta_t) + \frac{1}{\sqrt{\tau_\eta}} \tilde{\eta}_t \\
\delta_t = avgpred(\delta_t) + \frac{1}{\sqrt{\tau_v}} \tilde{v}_t \\
\gamma_t = \sum_{i=1}^{S-1} \gamma_{t+i-S} + \frac{1}{\sqrt{\tau_w}} \tilde{w}_t \\
\rho_t = \alpha \rho_{t-1} + \frac{1}{\sqrt{\tau_u}} \tilde{u}_t \\
\chi_k = \sum_{i=1}^{F-1} \chi_{k+i-F} + \xi \delta_{t(k)=6,7} + \frac{1}{\sqrt{\tau_\zeta}} \tilde{\zeta} \\
\\
\tilde{\epsilon}_t, \tilde{\eta}_t, \tilde{v}_t, \tilde{w}_t, \tilde{u}_t, \tilde{\zeta} \overset{iid}{\sim} \mathcal{N}(0,1)
\end{cases}
\tag{8}
$$

where $avgpred(\phi_t) = \frac{1}{2}\phi_{t-1} + \frac{1}{3}\phi_{t-S} + \frac{1}{6}\phi_{t-2S}$, $h = mod(k,4)+1$ and priors of the same class as before

Time series model: BSTS

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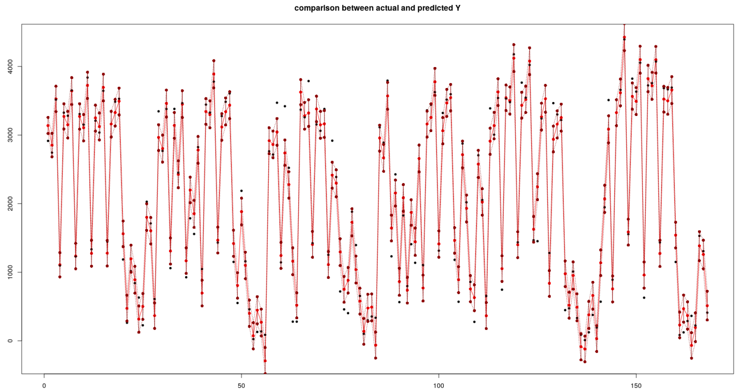
Robust truncated variance model for time zones



Two prospective of the problem

We followed two distinct paths:

Global model: the total volume of bikes travels in a specific day Y_t without considering the graph structure. This results in a single time series.

Network model: dividing in the different nodes and analysing the flow of bikes in the net. The dimensionality is much higher.

Network model

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For every (i, j) edge of the graph and $t \in 1 : 42$ we have $Y_{ij}(t)$ the number of travels from node i to j at day t.

Problem:

more than 4 million variables $\Rightarrow$

Computationally untreatable

Solutions:

- clusterization through DBSCAN
- simplification of the variables

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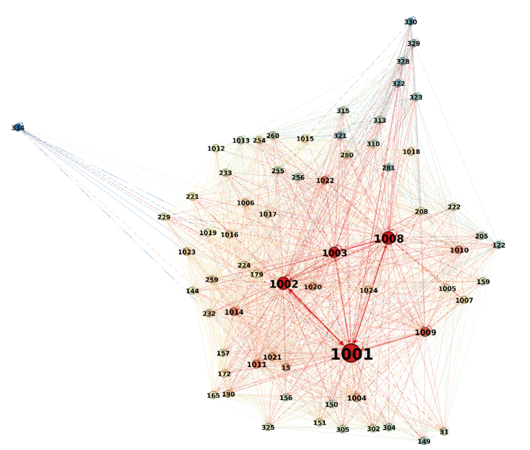
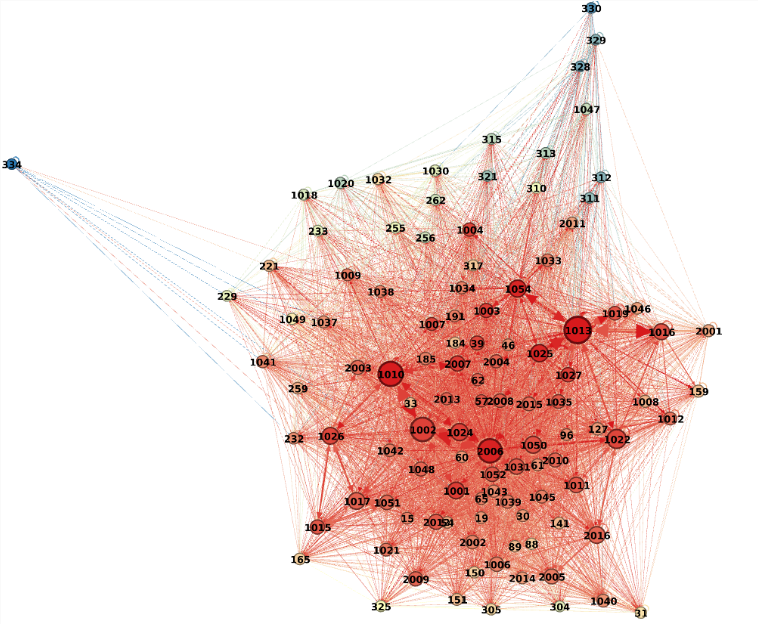
Algorithm to divide the nodes into initial clusters. We introduced a modified version with a weight to break up the bigger clusters, minimizing the autorings presence.



From 334 nodes to 140.



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Modeling the flux

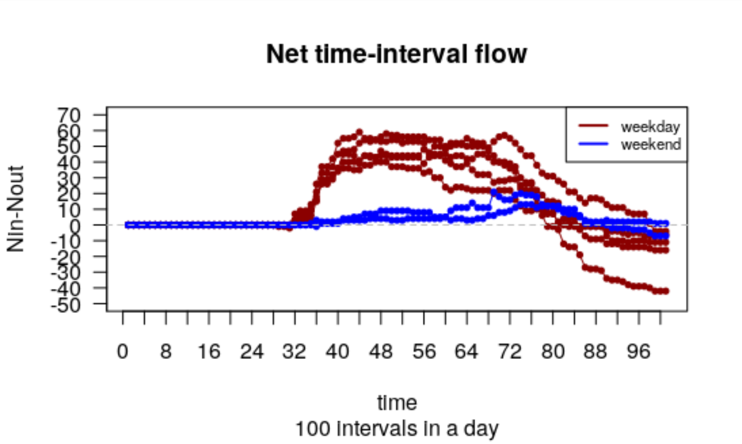
To further simplify the analysis we focus on the bikes arriving and departing from each node $N_i^{IN}(\Delta t)$ and $N_i^{OUT}(\Delta t)$ in the time interval Δt.

We are interested in the smallest Δt as possible but this would increase the number of variables $\Rightarrow$ consider them as functional data.

$$V_i(t) = \lim_{\Delta t \to 0} \frac{N_i^{IN}(\Delta t) - N_i^{OUT}(\Delta t)}{\Delta t}$$

$$\Phi_i(t) = \int_0^t V_i(u)\,\mathrm{d}u$$

We can analyse when new bikes should be brought to which station.



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