For every given $f(n)$ and $g(n)$ prove that $f(n)$ = $\Theta(g(n))$:-

Q1: $g(n) = n^3$, $f(n) = 3n^3 + n^2 + n$

A1: $\lim\limits_{n \to \infty} \frac{3n^3+n^2+n}{n^3} = \lim\limits_{n \to \infty} 3 + \frac{1}{n} + \frac{1}{n^2} = 3 + \frac{1}{\infty} + \frac{1}{\infty}$

$= 3 + 0 + 0 = 3$

$\because \lim\limits_{n \to \infty} \frac{f(n)}{g(n)} = c = 3$

$\therefore f(n) = \Theta(g(n))$ #

Q2: $g(n) = 2^n$, $f(n) = 2^{n+1}$

A2: $\lim\limits_{n \to \infty} \frac{2^{n+1}}{2^n} = \lim\limits_{n \to \infty} 2^1 = 2$

$\because \lim\limits_{n \to \infty} \frac{f(n)}{g(n)} = c = 2$

$\therefore f(n) = \Theta(g(n))$ #

Q3: $g(n) = ln(n),\ f(n) = log(n) + log(log(n))$

A3: $\lim_{n \to \infty} \frac{log(n)+log(log(n))}{ln(n)} = \lim_{n \to \infty} \frac{\frac{1}{nlog(n)\times ln(2)}\times(n\times\frac{1}{nln(2)}+log(n)\times 1)}{\frac{1}{n}}$

$= \lim_{n \to \infty} \left(\frac{\frac{n}{nln(2)}}{nlog(n)\times ln(2)} + \frac{log(n)\times 1}{nlog(n)\times ln(2)}\right) \times n$

$= \lim_{n \to \infty} \left(\frac{1}{ln(2)} \times \frac{1}{nlog(n)\times ln(2)} + \frac{1}{nln(2)}\right) \times n$

$= \lim_{n \to \infty} \frac{1}{log(n)\times(ln(2))^2} + \frac{1}{ln(2)} = \frac{1}{log(\infty)\times(ln(2))^2} + \frac{1}{ln(2)}$

$= \frac{1}{\infty} + \frac{1}{ln(2)} = \frac{1}{ln(2)}$

$\because \lim_{n \to \infty} \frac{f(n)}{g(n)} = c = \frac{1}{ln(2)}$

$\therefore f(n) = \Theta(g(n))$ #
