Q1: Is $2^{n+1} = O(2^n)$?

A1: $\lim_{n \to \infty} \frac{2^{n+1}}{2^n} = \lim_{n \to \infty} 2^1 = 2$

$\because \lim_{n \to \infty} \frac{f(n)}{g(n)} = c = 2$

$\therefore 2^{n+1} = O(2^n)$

Q2: Is $2^{2n} = O(2^n)$?

A2: $\lim_{n \to \infty} \frac{2^{2n}}{2^n} = \lim_{n \to \infty} 2^n = 2^\infty = \infty$

$\because \lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$

$\therefore 2^{2n} \neq O(2^n)$

Q3: Is $(0.25)^n = O(n)$?

A3: $\lim_{n \to \infty} \frac{(0.25)^n}{n} = \frac{0}{0}$

$= \lim_{n \to \infty} \frac{((0.25)^n)'}{(n)'} = \lim_{n \to \infty} \frac{(0.25)^n \times ln(0.25)}{1} = 0$

$\because \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$

$\therefore (0.25)^n = O(n)$