x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.84	1.96	2.21	2.45	2.94	3.18

1) $y = a + bx$

	x_i	y_i	$x_i y_i$	x^2	$x^3 y$	x^5	x^6
	1	1.84	1.84	1	1.84	1	1
	1.1	1.96	2.156	1.21	2.6088	1.611	1.77156
	1.3	2.21	2.873	1.69	4.8554	3.713	4.82681
	1.5	2.45	3.675	2.25	8.2688	7.594	11.3906
	1.9	2.94	5.586	3.61	20.165	24.76	47.0459
	2.1	3.18	6.678	4.41	29.45	40.84	85.766
Sum	8.9	14.58	22.808	14.17	67.19	79.52	181.80

$14.58 = 6a + 8.9b$

$22.808 = 8.9a + 14.17b$

deg 3

$y = dx^3 + cx^2 + bx + a$

$a = .6208950 \quad b = 1.219621$

$y = .620895 + 1.219621x$

$y = a + bx + cx^2$

$14.58 = 6a + 8.9b + 14.17c$

$22.808 = 8.9a + 14.17b + 24.023c$

$38.0962 = 14.17a + 24.023b + 42.8629c$

	$x^2 y$	x^3	x^4
	1.84	1	1
	2.3716	1.331	1.4641
	3.7349	2.197	2.8561
	5.5125	3.375	5.0625
	10.6134	6.859	13.0321
Sum	38.0962	24.023	42.8629

$S = \{1, x+1, x^2\}$ on $[0,1]$

$c_0\Phi_0 + c_1\Phi_1 + c_2\Phi_2 = 0$

2 Gram-Schmidt for orthonormal basis

$c_0\Phi_0 + c_1\Phi_1 + c_2\Phi_2 = 0$

$c_0 + c_1(x+1) + c_2(x^2) = 0$

$c_2x^2 + c_1x + c_1 + c_0 = 0$

$c_2 = 0$

$c_1 = 0$

b) $\int_0^1 w(x)\,\Phi_k(x)\,\Phi_j(x)\,dx = \begin{cases} 0 & \text{when } j \neq k \\ \alpha_i & \text{when } j = k \end{cases}$

$\Phi_0 = 1 \quad \Phi_1 = x+1 \quad \Phi_2 = x^2$ is orthogonal wrt $w(x) = 1$ on $[0,1]$

$\int_0^1 \Phi_0\Phi_1\,dx$

$\int_0^1 x+1\,dx = 3/2$

$\int_0^1 \Phi_0\Phi_2\,dx = 1/3$

$\int_0^1 \Phi_1\Phi_2\,dx = 7/12$

$\psi_0 = \dfrac{\Phi_0}{\left(\int_0^1 \Phi_0^2\,dx\right)^{1/2}} \quad \psi_1 = \dfrac{\Phi_1}{\left(\int_0^1 \Phi_1^2\,dx\right)^{1/2}} \quad \psi_2 = \dfrac{\Phi_2}{\left(\int_0^1 \Phi_2^2\,dx\right)^{1/2}}$

$\dfrac{1}{\sqrt{1}} \qquad \dfrac{x+1}{\sqrt{7/3}} \qquad \dfrac{x^2}{\sqrt{\frac{1}{5}}}$

$$f(x) = a_0 P_0(x) + a_1 P_1(x) + a_2 P_2(x) + a_3 P_3(x)$$

3) $f(x) = 2x^3 - 3x + 1$

$$P_n(x) = \frac{(-1)^n}{2^n n!} \frac{d^n}{dx^n} (1-x^2)^n$$

$P_0(x) = 1$ $\quad P_1(x) = x$ $\quad P_2(x) = \frac{-1+3x^2}{2}$ $\quad P_3(x) = \frac{5x^3 - 3x}{2}$

$$2x^3 - 3x + 1 = a_0(1) + a_1(x) + a_2\left(\frac{-1+3x^2}{2}\right) + a_3\left(\frac{5x^3-3x}{2}\right)$$

$a_0 - \frac{a_2}{2} = 1$ $\quad a_0 = 1$

$-\frac{3\left(\frac{4}{5}\right)}{2} = -3$

$a_1 - \frac{3a_3}{2} = -3$ $\quad a_1 = -9/5$

$\frac{3a_2}{2} = 0$ $\quad a_2 = 0$

$\frac{5a_3}{2} = 2$ $\quad a_3 = 4/5$

$$T_n(x) = \frac{(-2)^n n!}{(2n)!}\sqrt{1-x^2}\frac{d^n}{dx^n}(1-x^2)^{n-1/2}$$

Chebyshev Poly $T_n(x)$

Verify $n=1,2,3$

$(1-x^2)y'' - xy' + n^2y = 0$

$T_1(x) = x$

$-1\sqrt{1-x^2}(-1/2)(1-x^2)^{\frac{1}{2}-1}(-2x) \Rightarrow x$

$n=1$

$(1-x^2)0 - x + x = 0$

$y = x \quad y' = 1 \quad y'' = 0$

$\frac{1}{3}\sqrt{1-x^2}\ \frac{6x^2-3}{\sqrt{1-x^2}} \Rightarrow 2x^2-1 = T_2(x)$

$n=2$

$(1-x^2)4 - x(4x) + 4(2x^2-1) = 0$

$y = 2x^2-1 \quad y' = 4x \quad y'' = 4$

$\frac{-1}{15}\sqrt{1-x^2}\ \frac{45x-60x^3}{\sqrt{1-x^2}} \Rightarrow 4x^3-3x = T_3(x)$

$n=3$

$24x - 24x^3 - 12x^3 + 3x + 36x^3 - 27x = 0$

$y = 4x^3-3x \quad y' = 12x^2-3$

$y'' = 24x$

$T_3(x) = 4x^3 - 3x$

fourier series

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

5 $f(x) = x^2 \quad (-\pi, \pi)$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\frac{x^3}{3}\Big|_{-\pi}^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\left(\frac{x^2}{n}\sin(nx) + \frac{2x}{n^2}\cos(nx) - \frac{2}{n^3}\sin(nx)\right.$$

$$b_n = \frac{1}{n}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\left(\frac{-x^2}{n}\cos(nx) + \frac{2x}{n^2}\sin(nx) + \frac{2}{n^3}\cos(nx)\right)$$

$$\frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{1}{\pi}\cos(nx)\left(\frac{x^2}{n}\sin(nx) + \frac{2x}{n^2}\cos(nx) - \frac{2}{n^3}\sin(nx)\right) +$$

$$\sum_{n=1}^{\infty}\frac{1}{\pi}\sin(nx)\left(\frac{-x^2}{n}\cos(nx) + \frac{2x}{n^2}\sin(nx) + \frac{2}{n^3}\cos(nx)\right)$$