

# ECS 122b Midterm 1

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TOTAL POINTS

**91 / 120**

## QUESTION 1

### 1 Problem 1 32 / 40

- 0 pts Correct
- 1 pts Runtime for line 1 incorrect
- 3 pts Runtime for lines 2,3,4 incorrect
- 3 pts Runtime for line 5 incorrect
- ✓ - 4 pts *Runtime for lines 6,7,8 incorrect*
- ✓ - 2 pts *End Recurrence Relation Incorrect*
  - 5 pts Minor mistake in solving recurrence relation
- ✓ - 2 pts *Incorrect summation end solution*
  - 5 pts Not used recurrence tree method
  - 2 pts Wrong recursive relation for lines 6,7,8
- 💬 Your recurrence relation should've been  $5T(n/5) + O(n)$ . The end solution for (b) should've been  $O(n \log n)$

## QUESTION 2

### 2 Problem 2 20 / 30

- 0 pts Correct
- 5 pts Issues with recursive formula
- ✓ - 10 pts *Major issues with recursive formula*
  - 5 pts Small error in part c
  - 10 pts Major error in (b) proof
  - 5 pts Small error in c
  - 10 pts no accounting for  $y/14$  or  $y+2/14$
  - 5 pts not understanding the order of

operations i.e. pivot is found before creation of SL and  $S_r$  i.e.  $T(n/7)$

- 5 pts Wrong guess for (b)

## QUESTION 3

### 3 Problem 3 27 / 30

- 6 pts Part (a) major mistake
- 6 pts Part (b) major mistake
- 6 pts Part (c) major mistake
- 0 pts All correct
- ✓ - 3 pts *Minor mistake in (a)*
  - 3 pts Minor mistake in (b)
  - 3 pts Minor mistake in (c)
  - 2 pts Not given the character for  $S[26]$  but given correct Z score.
- 💬 Good job but (a) should be first letter

## QUESTION 4

### 4 Problem 4 12 / 20

- 0 pts Correct
- 1 pts Minor mistake in suffix tree
- 2 pts Minor mistake in suffix tree runtime
- 2 pts Minor mistake in z-algo runtime
- ✓ - 4 pts *Major mistake in suffix tree runtime*
- ✓ - 4 pts *Major mistake in z-algo runtime.*
  - 10 pts No Suffix Tree drawn
  - 4 pts Major mistake in suffix tree drawn
  - 20 pts Not attempted



Runtime for suffix tree should be  $O(m^6)$   
and runtime for z-algo should be  $O(m^8)$ .

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### INSTRUCTIONS

You will use your  $y$  variable which is calculated by taking the last digit of your student id mod 3 then add 6 to that. Everything in the test is self explanatory if you are confused state your assumption and answer the question. We will grade with your assumption in mind. We cannot answer any questions related to your interpretation. For example my last digit is 3. Then  $3 \% 3 = 0$  Therefore my final  $y$  is  $y = 0 + 6 = 6$  I will use the number 3 in place of  $y$  everywhere on the test.

What is your  $\$y\$$  ? 5

### PROBLEM 1: (40 POINTS)

(a) Use your  $y$ ! void foofoo(int n)::

```

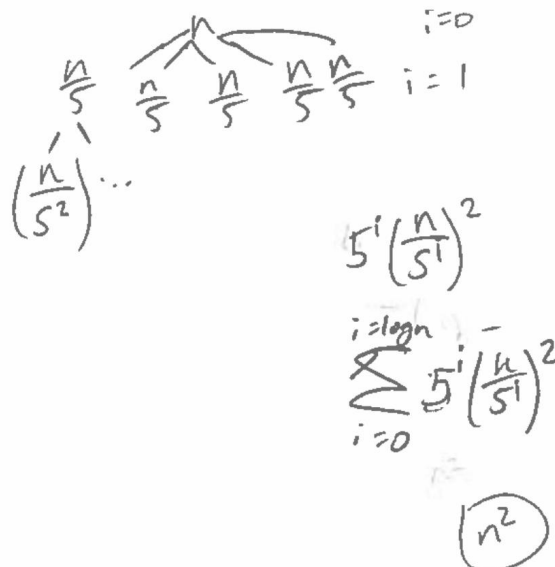
O(1) 1. int A[n];
O(n) 2. for(i=1 to n){
O(1) 3.     A[i] = a random number btw. 1 and n^8 //O(1) operation
O(1) 4. } //endfor
O(n) 5. int x = deterministic_selection(A,y);
O(1) 6. for(x=1 to y) {
O(n) 7.     foofoo(n/y);
O(1) 8. } //endfor

```

Provide the recurrence of the code above. Do not just state the recurrence but explain the run-time line by line. In the blanks above.

$$T(n) = 5T\left(\frac{n}{5}\right) + O(n^2)$$

(b) Solve recurrence relation from above using the recurrence tree method.



## PROBLEM 2: (30 POINTS)

**Problem Description:** Suppose someone invented a new way to select the pivot for the deterministic selection algorithm. You are guaranteed that:

- the pivot is at least equal to larger than  $\frac{y}{14}n$  of the data in S  $\frac{y}{14}n$
- the pivot is at least equal to or smaller than  $\frac{y+2}{14}n$  of the data in S
- the pivot is found by recursively with input size  $\frac{1}{7}$ th of the original size of S.  $(\frac{1}{7})$

(a) State the recurrence of the algorithm based on this new way of selecting the pivot. Show your work. (Hint What is the  $\max(S_L, S_R)$ )

$$\begin{aligned}
 |S_R| &\geq \frac{5}{14}n \\
 |S_L| &\leq \frac{1}{2}n \\
 |S| &= T\left(\frac{1}{7}\right)
 \end{aligned}
 \quad \max\left(\frac{5n}{14}, \frac{1}{2}n\right)$$

$$T(n) = T\left(\frac{n}{7}\right) + O(n)$$

(b) Find the asymptotic run-time of the above algorithm through substitution. ( If you couldn't figure out (a) guess what the  $T(n)$  is and solve via substitution)

$$\begin{aligned}
 T(n) &= T\left(\frac{n}{7}\right) + O(n) \\
 T(n) &\leq T\left(\frac{n}{7}\right) + Cn \\
 T(n) &\leq T\left(\frac{n}{7^2}\right) + C\left(\frac{n}{7}\right) \\
 &\leq T\left(\frac{n}{7^k}\right) + C\left(\frac{n}{7^{k-1}}\right) \\
 &\quad \text{1st term} \\
 &\quad \text{term} \leq T\left(\frac{n}{7^n}\right) + C\left(\frac{n}{7^{n-1}}\right) \\
 &\quad n = n \\
 T(n) &\leq T\left(\frac{n}{7^n}\right) + C\left(\frac{n}{7^{n-1}}\right)
 \end{aligned}$$

$y=5$

15.

- (c) For the original deterministic selection covered in class. Please replace the  $y$  variable with your  $y$ . Given  $S = \{18, 18, 4, 18, 18, 61, 61, 48, 61, 61, y, y, -1y, -2y, y\}$  What would be the initial pivot (first level recursion), and what would  $S_L$  set contain?

groups of 5

$(-10, -5)(4, 5)(5, 5)(18, 18)(18, 18)(61, 61)(61, 61)(61)$

$-10, -5, 4$

$V_L = 18$

Initial Pivot = 18

$\{-10, -5, 4, 5, 5, 5, 18\}$

$S_L$

### PROBLEM 3: 30 POINTS

Geeg

- (a) Create a random string  $T$  length 4 made up of the first 2 characters of your first name. For example my name is yelena and hence my first two characters are  $y$  and  $e$ . I will make up the string eyye. Rember you can make up any string length 4.

What is your string:  $T = \text{Geeg}$

Given  $W$  is some string such that  $|W| = 30$ . You do not need to know the characters of  $W$  to answer the questions below.

Let  $S$  be the concatenation of  $T$  and  $W$  i.e.  $S = T.W$  // Clearly,  $|S| = 34$

Given  $Z_7 = 2$ ,  $Z_{20} = 10$ , What is  $S[26] = e$

$$K' = 26 - 20 + 1 = 7$$

$$Z_{K'} = 2$$

$Z_{K'}(P)$  20 21 22 23 24 25 26

$$|P| = r - k + 1$$

$$20 + 10 = 30$$

$$1 = 20 = 10$$

- (b) Now assume you ran Z-algorithm on a string of size 100. Given  $Z_k = y$ ,  $Z_3 = 15$

What is  $Z_{k+2} = 3$

$$Z_k(P) \quad \beta = 4 \quad \beta = k + 4 - k \quad Z_k = 5$$

$$K \quad K+1 \quad K+2$$

- (c) Now assume you ran Z-algorithm on a string of size 100. Given  $Z_k = y$ ,  $Z_3 = 2$

What is  $Z_{k+4} = 2$

$$Z_{k+2} = 2$$

$$Z_k = 5$$

$$Z_{k+4} =$$

$$Z_3 = 2$$

3

$$\beta = k + 4 - k$$

$$Z_k = 2$$

$$\beta = 4$$

$$Z_k = 1 \beta$$

# PROBLEM 4: (20 POINTS)

- (a) You are deciding which algorithm to implement for you pattern P search in text T. <sup>string</sup>  
 Your given that some requirements:

**Pattern:** the the size of the pattern is always  $m$  i.e.  $|P| = m$  <sup>substring</sup>

**Text:**  $|T| = m^3$ .

**Number of Searches performed:** You will search for  $X$  different patterns inside the same text T.  $X = m^y$

What is the run-time for searching for  $X$  patterns length  $P$  in text  $T$  given you decide to use Suffix Trees. <sup>suffix tree  $O(|S| \cdot |m|)$ ?</sup>

$(S_c \text{ of Pattern})^3$



$\times$

$O(m^3 \cdot m)$

What about if you use the Z-algorithm?  $O(n)$

$O(m^3)$

- (b) Makeup a random string length 4 made up of the first two characters in your first name. Create a suffix Tree for that string.

My string is: Greeg

The tree is:

