ECS 122b HW1 - gradescope fix

Geoffrey Mohn

TOTAL POINTS

3 / 3

QUESTION 1

1 Questions 1-2 **1 / 1**

✓ **- 0 pts** *Correct*

QUESTION 2

2 Question 3 **1 / 1**

✓ **- 0 pts** *Correct*

QUESTION 3

3 Question 4-5 **1 / 1**

✓ **- 0 pts** *Correct*

122b HW1

2.3 a) $T(n) = 3T(n/2) + O(n)$. What is k^{th} term, what value k?

$T(n) = 3\left(T\left(\frac{n}{2}\right) + O(n)\right) \leq 3\left(3\left(T\left(\frac{n}{4}\right) + c\left(\frac{n}{2}\right)\right) + cn\right)$

$T(n) \leq 3^k T\left(\frac{n}{2^k}\right) + kcn$

$k = \log_2 n$

b) $T(n) = T(n-1) + O(1)$

$T(n) = T(n-1) + O(1) \cdot \leq T(n-1) + cn$

$T(n-2) + c(n-1) + cn$

$c \sum_{i=0}^{n} (n-i)$

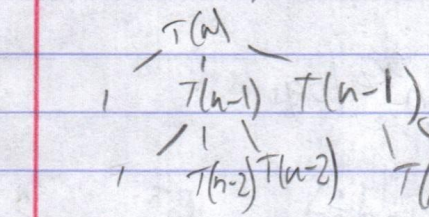
2.4 Algo A = $5T\left(\frac{n}{2}\right) + O(n)$

Algo B = $2T(n-1) + O(1)$

Algo C = $9T\left(\frac{n}{3}\right) + O(n^2)$

A = if $O\left(n^{\log_2(5)-\varepsilon}\right) \rightarrow \Theta\left(n^{\log_2(5)}\right)$ for any $\varepsilon > 0$

B = $O(2^n)$



C = $O(n^2 \log n)$

$O(n^2) = \Theta\left(n^{\log_3(9)}\right) = \Theta\left(n^2 \log(n)\right)$

Choose Algo C as it is the fastest runtime

1 Questions 1-2 1 / 1

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2.5 a) $T(n) = 2T(\frac{n}{3}) + 1$

$1 = O(n^{\log_3 2 - \varepsilon})$ $\varepsilon > 0$

$n^{3^{-1}-\varepsilon} = 2$

a) $\boxed{\Theta(n^{\log_3(2)})}$

b) $T(n) = 5T(\frac{n}{4}) + n$

$n = O(n^{\log_4 5 - \varepsilon})$ $\varepsilon > 0$

b) $\boxed{\Theta(n^{\log_4(5)})}$

c) $T(n) = 7T(\frac{n}{7}) + n$

$n = n^{(\log_7(7))}$

c) $\boxed{\Theta(n \log n)}$

d) $T(n) = 9T(\frac{n}{3}) + n^2$

$n^2 = n^{\log_3(9)=2}$

d) $\boxed{\Theta(n^2 \log n)}$

e) $T(n) = 8T(\frac{n}{2}) + n^3$

$n^3 = n^3$

e) $\boxed{\Theta(n^3 \log n)}$

f) $T(n) = 49T(\frac{n}{25}) + n^{3/2}\log n$

$n^{3/2}\log n = n^{\log_{25}(49)}$

f) $\boxed{\Theta(n^{\log(49)})}$

$n^{3/2}\log n$

$T(\frac{n}{25})$,

$(\frac{n}{25})^{3/2}\log(\frac{n}{25})$ $T\frac{n^3}{25^3}\log\frac{n}{25}$

g) $T(n) = T(n-1) + 2$

$T(n-1)$, 2

$T(n-2)$, 4 $T(n-k) + 2k$

g) $\boxed{\Theta(n)}$

h) $T(n) = T(n-1) + n^c$, where $c \geq 1$ is a constant

$T(n-1)$, n^c

$\frac{1}{(n+1)^c}$

$T(n) = T(n-k) + n^c + (n-1)^c$

$O(n^{c+1})$

$\boxed{O(n^2)}$ let $c=1$

i) $T(n) = T(n-1) + c^n$, where $c > 1$ is some constant

$T(n-1) = T(n-2) + c^{(n-1)}$

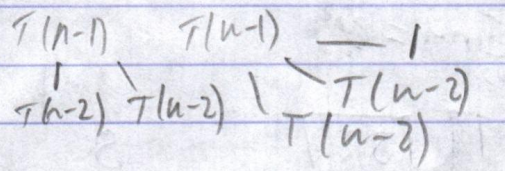
$T(n) = T(n-2) + c^n + c^{(n-1)}$

$T(n-2) = T(n-3) + c^{(n-2)}$

$T(n) = T(n-3) + c^n + c^{(n-1)} +$

i) $\boxed{\Theta(c^n)}$

j) $T(n) = 2T(n-1) + 1$



$2^{k+1} - 1$

j) $\boxed{\Theta(2^n)}$

k) $T(n) = T(\sqrt{n}) + 1$

$T(n) = T(n^{1/2}) + 1$

$T(n^{1/2}) = T(n^{1/2^2}) + 1$

$T(n) = T(n^{1/2^2}) + 2$

$T(n) = (n^{\frac{1}{2^k}}) + k$

$n^{\left(\frac{1}{2^k}\right)} = 2$

$\log_2 n = 2^k$

$\boxed{\log_2(\log_2 n) = k}$

4 a) input elements in groups of 5. Linear time in group of 7? 3?

7 groups $\Rightarrow O(n)$ time

$n \log n$ Cost time

each group is $7 \log 7$ time

$n/7 \times (7 \log 7) \approx O(n)$ time

Selection $(5, (n/4)/2)$

$a < c/7$

$T(n) = T(n/7) + T(5n/7) + O(n)$

$T(n) \leq T(n/3) + \left(\frac{4n}{6}\right) + O(n)$

$T(n) \leq T(n/3) + T\left(\frac{2n}{3}\right) + cn$

$T(n) > O(n) \rightarrow T(n) = O(n \log n)$

2 Question 3 **1 / 1**

✓ **- 0 pts** *Correct*

i) $T(n) = T(n-1) + c^n$, where $c > 1$ is some constant

$T(n-1) = T(n-2) + c^{(n-1)}$

$T(n) = T(n-2) + c^n + c^{(n-1)}$

$T(n-2) = T(n-3) + c^{(n-2)}$

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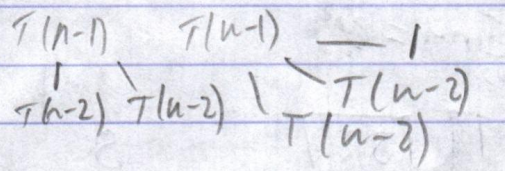

$2^{k+1} - 1$

j) $\boxed{\Theta(2^n)}$

k) $T(n) = T(\sqrt{n}) + 1$

$T(n) = T(n^{1/2}) + 1$

$T(n^{1/2}) = T(n^{1/2^2}) + 1$

$T(n) = T(n^{1/2^2}) + 2$

$T(n) = (n^{\frac{1}{2^k}}) + k$

$n^{\left(\frac{1}{2^k}\right)} = 2$

$\log_2 n = 2^k$

$\boxed{\log_2(\log_2 n) = k}$

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$T(n) \leq T(n/3) + T\left(\frac{2n}{3}\right) + cn$

$T(n) > O(n) \rightarrow T(n) = O(n \log n)$

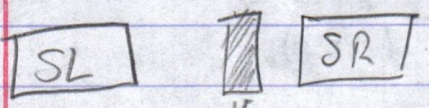
4b) 25 RNG, first level of recursion & recursive calls for the 6th smallest #

Selection (S, K)

1) Choose Pivot v

2) Sort into arrays

$\sum_{i=0}^{n-1} n-i = 2n$



divide S in groups of size 5

$S_1 = 2, 36, 5, 21, 8$

$S_2 = 13, 11, 20, 5, 4$

$S_3 = 12, 15, 7, 9, 24$

$S_4 = 3, 29, 42, 69, 30$

$S_5 = 1, 79, 22, 23, 42$

Let Pivot = Selection($\{\bar{S}_1, \bar{S}_2, \ldots \bar{S}_5\}$, $\lceil \frac{25}{6} \rceil$)

5 $v = \text{Selection}(\{\bar{S}_1, \bar{S}_2, \ldots \bar{S}_n\}, \lceil \frac{n}{5/2} \rceil)$

Recursive Step if ($|SL| \geq K$)

Selection (SL, K)

elseif ($|SL| < K \leq |SL| + |Sv|$)

return v;

else

Selection (SR, K - |SL| - |Sv|);

6) the Pivot gives the median of each set of groups. By giving median of median

$T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + O(n)$

select Pivot — worst case for recursion

$T(n) = n \log n$

3 Question 4-5 1 / 1

✓ **- 0 pts** *Correct*