midterm #1

4 $\sum_{i=1}^{n}(2i-1)=n^2$ weak induction

IH let $k\geq 1$ assume $\sum_{i=1}^{n}(2i-1)=n^2 \ \forall \ n\geq 1$

Base holds when $n=1$ IH when $n=k$

$$\sum_{i=1}^{1} 2i-1 = 1^2$$

$$2(1)=1^2 \checkmark$$

When $n=k+1$

$$\sum_{i=1}^{k+1}(2i-1)=(k+1)^2 = (k+1)(k+1)$$

$k^2+k+1 = k^2+2k+1$

$$k^2+2k+1 =$$

When $i=k+1$ then

the Proposition for
$n=k+1$ & $n+1=k+2$
$\forall$ integers $n$

Q5 $T(n) = 4n^3 + 2n + n\log n$ is $O(n^3)$

let $g(n) = n^3 + n\log n + 2n$ — add lower order Terms — by degree

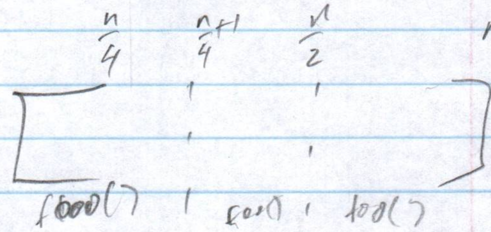
by squeeze $\frac{T(n)}{g(n)} \lim_{n \to \infty} \frac{4n^3 + n\log n + 2n}{n^3 + n\log n + 2n}$ $L = 4$

$L$ is a positive constant then $T(n)$ is $g(n)$ ∴ $T(n)$ is $O(n^3)$ by definition

Q6 Let A[0...n-1] arr length n  i: 0:n-1  if i>j

if n<2  # base case arr len =1
  return 1
i=1
i ≤ n
  boo(arr)  # $O(m)$
  i *=2

m = size of input arr for boo()




a = $\frac{n}{4}$
b = $\frac{n}{4}+1 : \frac{n}{2}$

c = $\frac{n}{2}+1 : n-1$

2 comparisons 5 assignments

1) $T(n) = 2T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n$  $T(0) = O(1)$

3 sub Problems

2) $2T\left(\frac{n}{4}\right) == T\left(\frac{n}{2}\right)$

Tree ends at base case 1

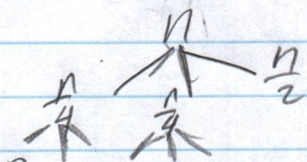
$T(n) = 2T\left(\frac{n}{2}\right) + n$

Sub Problem 1 & 2

$T\left(\frac{n}{4}\right)$ size = $\frac{n}{4}$

$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{16}\right) + T\left(\frac{n}{8}\right) + \frac{n}{4}$

$T\left(\frac{n}{16}\right) = 2T\left(\frac{n}{16^2}\right) + T\left(\frac{n}{8(16)}\right)$




Sub Problem 3
$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{8}\right) + T\left(\frac{n}{4}\right) + \frac{n}{2}$

$\frac{n}{2}$

$n$  base $n=1$

$O(m \cdot \log_3 n)$?

3) $T(1) = 1$
$T(n) = 2T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + cO(2^n)^2$?

4) $T(n) = 2T\left(\frac{n}{4}\right) + T\left(\frac{2n}{4}\right) + n$

$T(n) = 2T\left(\frac{n}{2}\right) + n$

a=2  b=2  f(n)=n
$\log_2 2 = 1$