1. $T(n) = T(n-1) + n$

Sub Problem 1 — root

Size $= (n-1)$

$$n$$
$$n-1$$
$$n-2$$

cost $T(n-1) = T(n-2) + n - 1$

$n \to n-1 \to n-2 \to \dots \to 1$

$T(n-2) = T(n-3) + n - 2$

$$T(n) = n + (n-1) + (n-2) + \dots 1$$

$$T(n) = \frac{n(n+1)}{2}$$

$$T(n) = O(n^2)$$

2. $T(n) = T\left(\frac{n}{2}\right) + 1$

$$1 \downarrow \frac{n}{4} \downarrow n/8$$

$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + 1$

$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + 1$

$1 \to \frac{n}{4} \to \frac{n}{8} \to \dots$

$$1 + \frac{n}{4} + \frac{n}{8} + \frac{n}{16} + \dots$$

$$n\left(\frac{1}{2^h}\right) + 1 =$$

$$n \cdot \left(\frac{1}{2}\right)^h = \quad n = 2^h$$

$$h = \log n$$

3. $T(n) = T\left(\frac{n}{2}\right) + n^2$

$$n^2$$

$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2$

$$\left(\frac{n}{2}\right)^2$$

$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2$

$$\left(\frac{n}{4}\right)^2$$

$$n^2 \to \left(\frac{n}{2}\right)^2 \to \left(\frac{n}{4}\right)^2 \to \dots \to$$

$n^2\left(\sum_{k=1}^{h} \frac{1}{2^h}\right) = 1$

$$n^2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right)$$

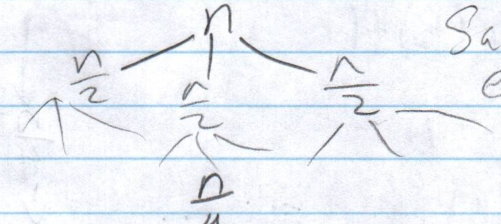
$$n^2\left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^h}\right)$$

$O(n \log n)$

4. $T(n) = 3T\left(\frac{n}{2}\right) + n$

$T\left(\frac{n}{2}\right) = 3T\left(\frac{n}{4}\right) + \frac{n}{2}$

$T\left(\frac{n}{4}\right) = 3T\left(\frac{n}{8}\right) + \frac{n}{4}$



Same cost of each sub Problem

$$n \to n/2 \to \frac{n}{2^2} \to \frac{n}{2^3} \to \dots$$

$n \cdot \frac{1}{2^h} = 1$

$n = 2^h \to h = \log n$

3 Times $\left(\frac{1}{2}\right)^h$

$\log n$

$\log n$

$O(n^{\log 3})$

$n\left(\frac{1}{2^h}\right)^3 = 1$

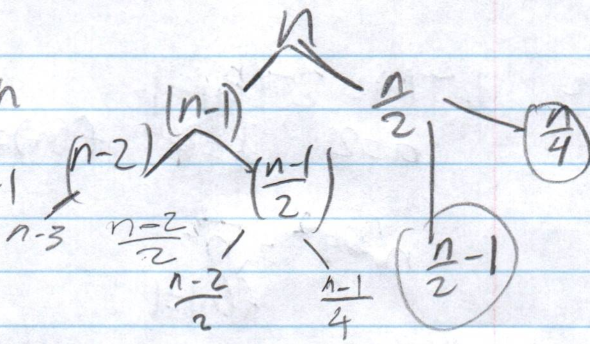
$\log.$

$n = \frac{1}{\,} \frac{2}{3}$

$h = \log_{\frac{2}{3}} h$

to Prove $O(n2^n)$

5. $T(n) = T(n-1) + T\left(\frac{n}{2}\right) + n$



$T(n-1) = T(n-2) + \left(\frac{n-1}{2}\right) + n-1$

$T\left(\frac{n}{2}\right) = T\left(\frac{n}{2}-1\right) + T\left(\frac{n}{4}\right) + \frac{n}{2}$

Sub Problem 1

$n-1 \to n-2 \to n-3 \to \ldots$

$n-1 \to \frac{n-1}{2} \to \frac{n-3}{2}$

$\left(\frac{n}{2}-1\right)-1 + \left(\frac{\frac{n}{2}-1}{2}\right)$

$\frac{n}{2}-2 \qquad \frac{n}{4}-\frac{1}{2}$

$\left(\frac{n-1}{4}\right)-1 + \left(\frac{\frac{n-1}{4}}{2}\right)$

$n-1 \to n-2 \to n-3 \to \ldots \qquad n-h=1$

$n-1 \to \frac{n-1}{2} \to \frac{n-1}{4} \to \frac{n-1}{8} \to \ldots \qquad \frac{n-1}{4} \qquad \frac{n-1}{2^h}$

Sub problem 2

$\frac{n}{2} \to \frac{n}{2}-1 \to \frac{n}{2}-2 \to \frac{n}{2}-3 \to \ldots$

$\frac{n}{2} \to \frac{n}{4} \to \frac{n}{8} \to \frac{n}{16} \to \ldots$

$n-h=1$

$\frac{n-1}{2^h}=1$

$\frac{n}{2}-h=1$

$\frac{n}{2^h}=1$

$n=2^h$

$n=\log h$

Qc 1. $T(n) = 2T(\frac{n}{4}) + 1$

$a=2 \quad b=4 \quad f(n)=1$

$n^{\log_4 2} = \sqrt{n}$

$f(n)=1$

Case 1 $f(n)$ is smaller

Then

$f(n) = O(n^{.5-\varepsilon})$

$T(n) = \Theta(n^{\log_4 2})$

2. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

$a=2 \quad b=4$

$n^{\log_4 2} = \sqrt{n}$

$f(n) = \sqrt{n}$

Case 2 $\quad T(n) = \Theta(\sqrt{n} \log n)$

3. $T(n) = 2T(\frac{n}{4}) + n$

$n^{\log_4 2} = \sqrt{n}$

$f(n) = n$

$f(n) > \sqrt{n}$

Case 3 must satisfy regularity condition

$af(\frac{n}{b}) \leq cf(n)$ where $c > 1$

$2(\frac{n}{4}) \leq c(n)$

$\frac{n}{2} \leq c(n) \rightarrow T(n) = \Theta(f(n)) = n$

4. $T(n) = 2T(\frac{n}{4}) + n^2$

$f(n) = n^2$ $a=2$ $b=4$

$f(n) > \sqrt{n}$ $\sqrt{n}$

Case 3 must satisfy regularity condition

$2\left(\frac{n^2}{4}\right)$

$\frac{n^2}{2} \leq C n^2$ $C > 1$ then

$T(n) = \Theta(n^2)$