122b HW1

2.3 a) $T(n) = 3T(n/2) + O(n)$, what is k^{th} term, what value k?

$$T(n) = 3\left(T\left(\tfrac{n}{2}\right) + O(n)\right) \leq 3\left(3\left(T\left(\tfrac{n}{4}\right) + c\left(\tfrac{n}{2}\right)\right)\right) + cn$$

$$T(n) \leq 3^k T\left(\tfrac{n}{2^k}\right) + kcn$$

$k = \log_2 n$

b) $T(n) = T(n-1) + O(1)$

$$T(n) = T(n-1) + O(1) \cdot \leq T(n-1) + cn$$

$$T(n-2) + c(n-1) + cn$$

$$c \sum_{i=0}^{n} (n-i)$$

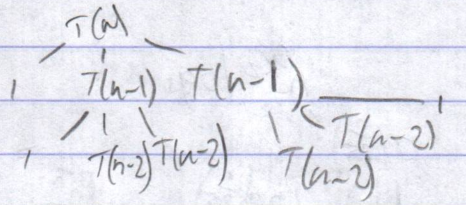
2.4 Algo A $= 5T\left(\tfrac{n}{2}\right) + O(n)$

Algo B $= 2T(n-1) + O(1)$

Algo C $= 9T\left(\tfrac{n}{3}\right) + O(n^2)$

A = if $O(n^{\log_2(5)-\varepsilon}) \rightarrow \Theta(n^{\log_2(5)})$ for any $\varepsilon > 0$

B = $O(2^n)$



C = $O(n^2 \log n)$

$$O(n^2) = \Theta\left(n^{\log_3(9)}\right) = \Theta\left(n^2 \log(n)\right)$$

Choose Algo C as it is the fastest runtime

2.5 a) $T(n) = 2T(\frac{n}{3}) + 1$

$1 = O(n^{\log_3 2 - \varepsilon})$ $\varepsilon > 0$

a) $\boxed{\Theta(n^{\log_3(2)})}$

$n^{3^{1} - \varepsilon} = 2$

b) $T(n) = 5T(\frac{n}{4}) + n$

$n = O(n^{\log_4 5 - \varepsilon})$ $\varepsilon > 0$

b) $\boxed{\Theta(n^{\log_4(5)})}$

c) $T(n) = 7T(\frac{n}{7}) + n$

$n = n^{\log_7(7)}$

c) $\boxed{\Theta(n \log n)}$

d) $T(n) = 9T(\frac{n}{3}) + n^2$

$n^2 = n^{\log_3(9) = 2}$

d) $\boxed{\Theta(n^2 \log n)}$

e) $T(n) = 8T(\frac{n}{2}) + n^3$

$n^3 = n^3$

e) $\boxed{\Theta(n^3 \log n)}$

f) $T(n) = 49T(\frac{n}{25}) + n^{3/2} \log n$

$n^{3/2} \log n = n^{\log_{25}(49)}$

f) $\boxed{\Theta(n^{\log(49)})}$

$T(\frac{n}{25})$, $n^{3/2} \log n$

$(\frac{n}{25})^{3/2} \log(\frac{n}{25})$ $\sqrt{\frac{n^3}{25^3}} \log\frac{n}{25}$

g) $T(n) = T(n-1) + 2$

g) $\boxed{\Theta(n)}$

$T(n-1)$, 2

$T(n-2)$, 4 $T(n-K) + 2K$

h) $T(n) = T(n-1) + n^c$, where $c \geq 1$ is a constant

$T(n-1)$, n^c

$(n-1)^c$

$T(n) = T(n-K) + n^c + (n-1)^c$

$\Theta(n^{c+1})$

let $c = 1$

$\boxed{\Theta(n^2)}$

i) $T(n) = T(n-1) + c^n$, where $c > 1$ is some constant

$T(n-1) = T(n-2) + c^{(n-1)}$

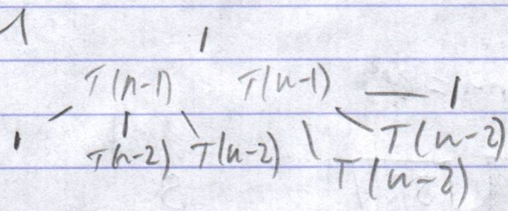
$T(n) = T(n-2) + c^n + c^{(n-1)}$

$T(n-2) = T(n-3) + c^{(n-2)}$

$T(n) = T(n-3) + c^n + c^{(n-1)} +$

i) $\boxed{\Theta(c^n)}$

j $T(n) = 2T(n-1) + 1$



$2^{k+1} - 1$

J) $\boxed{\Theta(2^n)}$

k) $T(n) = T(\sqrt{n}) + 1$

$T(n) = T(n^{1/2}) + 1$

$T(n^{1/2}) = T(n^{1/2^2}) + 1$

$T(n) = T(n^{1/2^2}) + 2$

$T(n) = (n^{1/2^k}) + k$

$n^{\left(\frac{1}{2^k}\right)} = 2$

$\log_2 n = 2^k$

$\boxed{\log_2(\log_2 n) = k}$

4 a) input elements in groups of 5. Linear time in group of 7? 3?

7 groups $\Rightarrow O(n)$ time

Selection $(\bar{5}, (n/4)/2)$

$n \log n$ cost time

each group is $7 \log 7$ time

$n/7 \times (7 \log 7) \approx O(n)$ time

$a < c/7$

$T(n) = T(n/7) + T\left(\frac{5n}{7}\right) + O(n)$

$T(n) \leq T(n/3) + \left(\frac{4n}{6}\right) + O(n)$

$T(n) \leq T(n/3) + T\left(\frac{2n}{3}\right) + cn$

$\log_3 3$

$T(n) > O(n) \rightarrow T(n) = \Theta(n \log n)$

4b) 25 RNG first level of recursion & recursive calls for the 6th smallest #

Selection (S, k)

1) Choose Pivot v

2) sort into arrays

$\sum_{i=0}^{n-1} n-i = dn^2$

$\boxed{SL}$ $\boxed{\ }$ $\boxed{SR}$

v

divide S in groups of size 5

$S_1 = 2, 36, 5, 21, 8$

$S_2 = 13, 11, 20, 5, 4$

$S_3 = 12, 15, 7, 9, 24$

$S_4 = 3, 29, 42, 69, 30$

$S_5 = 1, 79, 22, 23, 42$

Let pivot = Selection($\{\bar{S}_1, \bar{S}_2, \ldots \bar{S}_5\}$

$\left\lceil \frac{25}{6} \right\rceil$))

5 $v =$ Selection($\{\bar{S}_1, \bar{S}_2, \ldots \bar{S}_n\}$), $\left\lceil \frac{n}{5/2} \right\rceil$)

Recursive Step if (|SL| ≥ k)

Selection(SL , k)

elseif (|SL| < k ≤ |SL| + |Sv|)

return v;

else

Selection(SR, k − |SL| − |Sv|);

b) the pivot gives the median of each set of groups. By giving median of median

$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + O(n)$

select pivot

worst case for recursing

$T(n) = n \log n$