

# ECS 122A B01-B03 FQ 2021 Homework 07

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TOTAL POINTS

**100 / 100**

QUESTION 1

1 Q1 30 / 30

✓ + 30 pts Correct

+ 0 pts Invalid/Wrong submission

+ 25 pts No steps shown in Part 2

+ 25 pts No steps shown in Part 1

+ 20 pts Incomplete

QUESTION 2

2 Q2 20 / 20

✓ + 20 pts Correct

+ 0 pts [Click here to replace this description.](#)

QUESTION 3

3 Q3 30 / 30

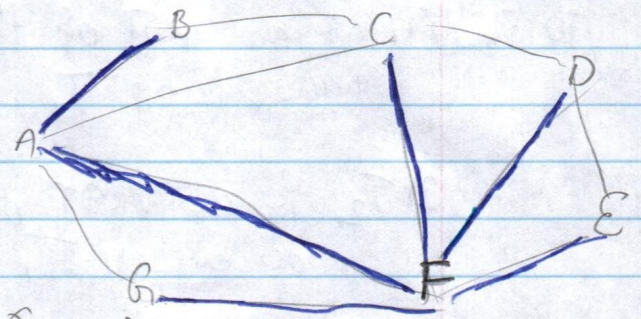
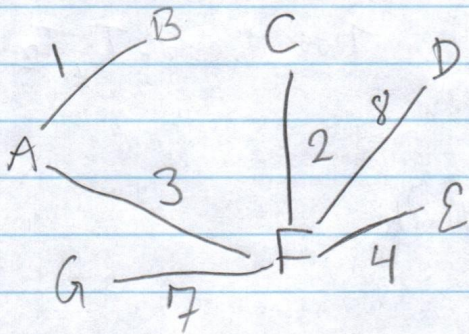
✓ + 30 pts Correct

QUESTION 4

4 Q4 20 / 20

✓ + 20 pts Correct

1) vertex	A	B	C	D	E	F	G	MST
key	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\{A\}$
→	0	1	6	<del><math>\infty</math></del>	$\infty$	3	10	$\{A, B\}$
→	0	1	5	$\infty$	$\infty$	3	10	$\{A, B, F\}$
→	0	1	2	8	4	3	7	$\{A, B, F, C\}$
→	0	1	2	8	4	3	7	$\{A, B, F, C, E\}$
→	0	1	2	8	4	3	7	$\{A, B, F, C, E, G\}$
→	0	1	2	8	4	3	7	$\{A, B, F, C, E, G, D\}$



Edge	AB	C,F	A,F	F,E	B,C	A,C	G,F	F,D	C,D	A,G	D,E
Wt.	1	2	3	4	5	6	7	8	9	10	11

	A	B	C	D	E	F	G
A							
B							
C							
D							
E							
F							
G							

A, B  
 C, F  
 A, F  
 F, E  
 G, F  
 F, D



1 Q1 30 / 30

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2) Suboptimality property:

Problem:  $G=(V, E, w)$

Subset problem

Graph induced by  $T_1$  &

Solution a mst  $T$  of  $G$

The graph induced by  $T_2$

The graph induced by  
a Spanning Tree =  
the original graph

Subset Solution

Remove an edge  $e \in T$   $(u,v)$   
from  $T$   $\therefore$

To show  $T_1$  is a mst of the graph  
induced by  $T_1$

Two trees:  $T_1, T_2 \subseteq T$

2)  $T_2$  is a mst of the graph  
induced by  $T_1$

$$w(T) = w(T_1) + w(T_2) + \sum_{(u,v) \in E} w(u,v)$$

Proof by Contradiction

Assume  $T_1$  is not a mst of the graph  
induced by  $T_1$ , Then  $\exists T_1'$  that is a mst  
of the graph induced by  $T_1$

$$w(T_1') < w(T_1)$$

then  $T_1' \cup T_2 \cup \{(u,v)\}$  is a Spanning tree of  
 $G$  & its weight  $< w(T)$

$$w(T_1') + w(T_2) + w(u,v) < w(T_1) + w(T_2) + w(u,v)$$

Conclusion:  $T_1$  is a mst of the graph  
induced by  $T_1$

2 Q2 20 / 20

✓ + 20 pts *Correct*

+ 0 pts [Click here to replace this description.](#)



3) Prim's Algorithm for max Spanning Tree  
vertex A B C  
weight 0 ~~1~~ -∞ init min Value

if  $\text{Weight} > \text{max weight}$

$\text{Max weight} = \text{Weight}$

4) All weights are the same @1 then  
any Spanning Tree is a minimum  
Spanning Tree.

Breadth First Search (Breadth First Traversal  
of a tree)

Then a node is only  
visited once  $\therefore O(E)$

3 Q3 30 / 30

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4 Q4 20 / 20

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