Search to find

Q1 $\sum_{i=1}^{n} 2^i = ?$ let $n=1$ $\sum_{i=1}^{1} 2^i = 2$ $i=3$ $2^3=8$

$a_n = a \cdot r^n$ $2 \cdot 2 \cdot 2$

$\sum_{i=1}^{n} 2^i = 2^{n+1}-2$ $a_0 = a$ $2(4)$ $n=2 \to \Sigma=6$

$ar=2$ $a_0=2$ $2i^2$

$a_1 = a_0 \times r$ if $n=3$ if $n=4$

2) to Prove $\sum_{i=1}^{3} 2^i = 14$ $2(2n+1)$ $\Sigma=30$

$\sum_{i=1}^{n} 2^i = (2^{n+1}-2)$ $\forall n \geq 1$ by induction $2+4+8+16$

2^4+2^3

Base case let $n=1$ $2^5-2=30$

$\sum_{i=1}^{1} 2^i = 2^2-2=2$ $P(1)$ is true

$LHS=2$ $RHS=2$

let $k \geq 1$ be an arbitrary int to Prove $P(k)$ assume $P(k)$ is true to prove $P(k+1)$

$P(k)$ $\sum_{i=1}^{k} 2^i = 2^{k+1}-2$

$P(k+1)$ $\sum_{i=1}^{k+1} 2^i = 2^{k+2}-2$

by inductive Hypothesis

$(2^{k+1}-2) + k+1 = 2^{k+1+2} - 1 + k$

$(2)2^k - 2 +$ $2^k + 2 + k - 1$

Q2 $i=n$

Loop $i>1$ $\quad i/2 \quad j{+}{+} \quad K\times 2$

$i *= 1/2$

Loop2 $j<n$

$j=i$

$j{+}{+}$

Loop3 $K<n$

$K *= 2$

$O(n)$ because $i=n$ & $j=i$ then $j=n$ thus never satisfying the conditional statement $j<n$ $\therefore$ the 2^{nd} & 3^{rd} nested loop is not stepped in.

There exist some constant C for $O(n)$

Q3 $T(n) = 2n^4 + 5n^3 + 3n^3/\log n + 2n + 5$ is $O(n^4)$

for all $n \geq K$ $\exists$ a constant C

Such that $C \cdot n^4 \geq 2n^4 + 5n^3 + 3n^3 \log n + 2n + 5$

Ex let $n=1$

$$C \cdot 1^4 \geq 2 + 5 + 0 + 2 + 5$$

$$12$$

let $C \geq 12$

then $T(n)$ is asymptotically bounded by $O(n^4)$

Q4 $T(n) = 5n^6 + n^2 + 3$ is $O(\log n + n^6 + n)$

let $g(n) = \log n + n^6 + n$

by def of $O(g(n))$ there exists positive constants c & K such that $0 \leq T(n) \leq c\,g(n)$ for all $n \geq K$

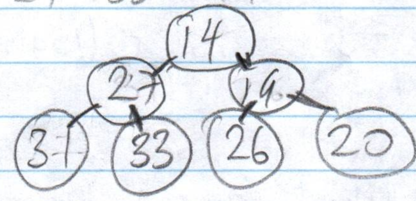
Let $L = \lim_{n \to \infty} \dfrac{T(n)}{g(n)}$ $\dfrac{5n^6 + \ldots}{n^6 + \ldots} = 5$

$L = 5$ is a positive constant then $T(n)$ is $\Theta(g(n))$ $\therefore$

$T(n)$ is $O(\log n + n^6 + n)$

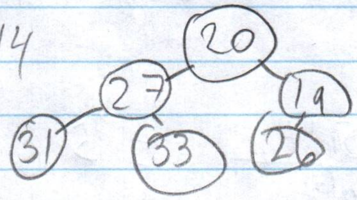
31 14 26 27 33 19 20

Q5

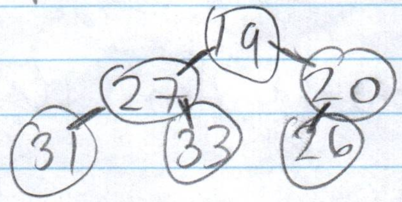


2)

remove 14



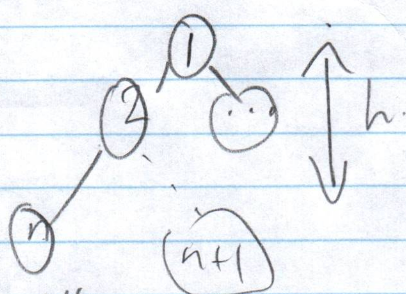
Reheap



3) insert new item in next position at bottom of heap $O(1)$

Reheap up/down $O(h)$ h = Tree height

Minheap (A)



$2^h \leq n \leq 2^{h+1}-1$

$O(\log n)$