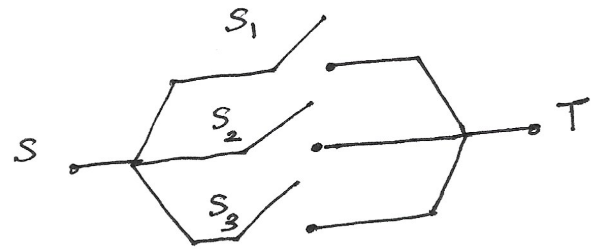
Course name: **Probability and Statistical modeling for Computer Science** Course: ECS 132
Instructor: **Dr. Vidhyacharan Bhaskar** Time duration of Quiz: **2 hours**

Instructions: a) Answers can be handwritten or typed (diagrams required for Problems 1 and 5)
b) Show complete working on all questions

Total: **50 points**

1. **[4 + 4 = 8 points]** Consider Figure 1 containing a parallel and independently operating network of switches: S_1, S_2, S_3. The switches can be open or closed, with the probability that the switch is closed is p. When one or more switches are closed, the input signal from S arrives at the output T.
(a) Compute the probability of receiving an input signal at the output T.
(b) If it is given to you that the input signal arrives at T, what is the probability that switch S_1 is open.

Figure 1



$S_1, S_2, S_3 \rightarrow$ Switches
$S \rightarrow$ Source
$T \rightarrow$ Target

2. **[2 + 2 = 4 points]**
(a) Let X be a uniformly distributed continuous random variable in the open interval (0, 1). Let x_u be specific values of X for u = 0.1, 0.2, ..., 0.9. Find x_u as a function of u.
(b) Let X be an exponentially distributed continuous random variable with probability density function $f(x) = e^{-2x}U(x)$.

3. **[5 + 2 + 2 = 9 points]**
Let X be a random variable following Erlang density function, $f(x) = c^4 x^3 \exp(-cx)U(x)$. Find the maximum likelihood (ML) estimate, $\hat{c}$ of c. You also need to use the second derivative to prove that this estimate indeed produces a maximum. Express the Log-Likelihood function in terms of this ML estimate.

4. **[2 + 2 + 2 = 6 points]** Suppose a group of 12 sales price records has been sorted as follows:
5, 10, 11, 13, 15, 35, 50, 55, 72, 92, 204, 215.
Partition them into three bins by each of the following methods:
(a) equal-frequency (equal-depth) partitioning
(b) equal-width partitioning
Plot the bar charts for (a) and (b).

5. **[2 + 3 + 2 + 1 = 8 points]** The following table (Figure 2) shows midterm and final exam scores obtained for students in a course.
(a) Plot the data points and infer if x and y have a linear relationship.

Figure 2

MIDTERM	FINAL
72	84
50	63
81	77
74	78
94	90
86	75
59	49
83	79
65	77
33	52
88	74
81	90

(b) Use linear regression to predict the student's final exam score based on the student's midterm score in the course (or in other words, construct the linear regression equation relating y and x). Plot the linear model and the data points on the same graph. Provide legends.
(c) Predict the final exam score of a student who received 86 points on the midterm exam.

6. [2 + 2 + 2 = 6 points] Multiple linear regression problem
Consider the following table which presents the relation between three variables: $Y,\ X_{1,}X_2$. Arrive at a regression equation, $Y = \beta_1 X_1 + \beta_2 X_2 + \beta_0$ by calculating the estimates of $\beta_0,\ \beta_{1,},\beta_2$.

Y	X_1	X_2
0	1	1
4	2	1
7	4	1

7. [3 + 6 = 9 points] Let the random variable X denote the number of heads generated when a fair coin is tossed 10 times.
(a) Derive an expression for the cumulative distribution function $F_X(x)$.
(b) If $Y = (X-3)^2$, derive an expression for the cumulative distribution function, $F_Y(y)$.