ECS 122A B01-B03 FQ 2021 Homework 03

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TOTAL POINTS

100 / 100

QUESTION 1

1 Q1 **50 / 50**

- **- 3 pts** Q4. Not recursion tree

✓ **- 0 pts** *Q4. correct*

- **- 3 pts** Q4. Incorrect solution
- **- 1 pts** Q4. more work needed

QUESTION 2

2 Q2 **50 / 50**

✓ **- 0 pts** *Correct*

- **- 50 pts** Missing
- **- 3 pts** Q3 no regularity/regularity incorrect
- **- 7 pts** Q3 incorrect
- **- 3 pts** Did not choose epsilon

1. $T(n) = T(n-1) + n$

Sub Problem → Size = $(n-1)$; root

Cost $T(n-1) = T(n-2) + n-1$

n
$n-1$
$n-2$

$n \to n-1 \to n-2 \to \dots \to 1$

$T(n-2) = T(n-3) + n-2$

$T(n) = n + (n-1) + (n-2) + \dots 1$

$T(n) = \frac{n(n+1)}{2}$

$T(n) = O(n^2)$

2. $T(n) = T(\frac{n}{2}) + 1$

$T(\frac{n}{2}) = T(\frac{n}{4}) + 1$

$T(\frac{n}{4}) = T(\frac{n}{8}) + 1$

1
$\frac{n}{4}$
$n/8$

$1 \to \frac{n}{4} \to \frac{n}{8} \to \dots$

$1 + \frac{n}{4} + \frac{n}{8} + \frac{n}{16} + \dots$

$n(\frac{1}{2^h}) + 1 =$

$n \cdot (\frac{1}{2})^h = n = 2^h$

$h = \log n$

3. $T(n) = T(\frac{n}{2}) + n^2$

$T(\frac{n}{2}) = T(\frac{n}{4}) + (\frac{n}{2})^2$

$T(\frac{n}{4}) = T(\frac{n}{8}) + (\frac{n}{4})^2$

n^2

$(\frac{n}{2})^2$

$(\frac{n}{4})^2$

$n^2 \to (\frac{n}{2})^2 \to (\frac{n}{4})^2 \to \ldots \to$

$n^2\left(\sum_{k=1}^{h} \frac{1}{2^k}\right) = 1$

$n^2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right)$

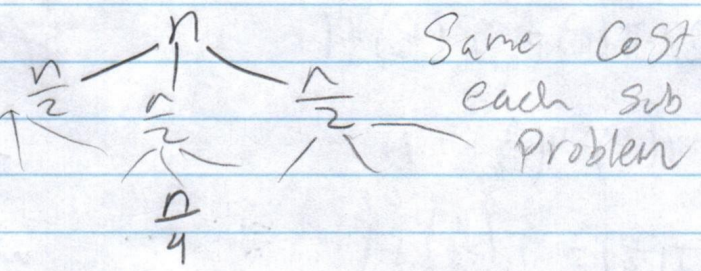
$n^2\left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^2}\right)$

$O(n \log n)$

4. $T(n) = 3T(\frac{n}{2}) + n$

$T(\frac{n}{2}) = 3T(\frac{n}{4}) + \frac{n}{2}$

$T(\frac{n}{4}) = 3T(\frac{n}{8}) + \frac{n}{4}$



$n \to n/2 \to \frac{n}{2^2} \to \frac{n}{2^3} \to \ldots$

$n \cdot \frac{1}{2^h} = 1$

$n = 2^h \to h = \log n$

3 times

$(\frac{1}{2})^h$

$n(\frac{1}{2^h})^3 = 1$

$\log n$

$\log n$

$\log$.

$n = \frac{1}{2^h}$

$h = \log_{\frac{1}{2}} n$

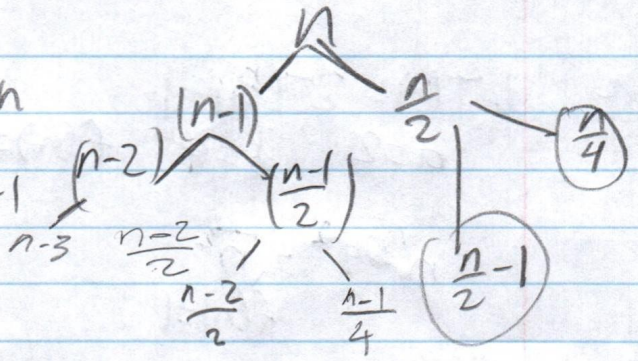
$O(n^{\log 3})$

to Prove $O(n2^n)$

5. $T(n) = T(n-1) + T\left(\frac{n}{2}\right) + n$

$T(n-1) = T(n-2) + \left(\frac{n-1}{2}\right) + n-1$

$T\left(\frac{n}{2}\right) = T\left(\frac{n}{2}-1\right) + T\left(\frac{n}{4}\right) + \frac{n}{2}$



Sub Problem 1

$n-1 \to n-2 \to n-3 \to \ldots$

$n-1 \to \frac{n-1}{2} \to \frac{n-3}{2}$

$\left(\frac{n}{2}-1\right)-1 + \left(\frac{\frac{n}{2}-1}{2}\right)$

$\frac{n}{2}-2 \qquad \frac{n}{4}-\frac{1}{2}$

$n-1 \to n-2 \to n-3 \to \ldots \qquad n-h=1$

$n-1 \to \frac{n-1}{2} \to \frac{n-1}{4} \to \frac{n-1}{8} \to \ldots$

$\left(\frac{n-1}{4}\right)-1 + \left(\frac{\frac{n-1}{4}}{2}\right)$

$\frac{n-1}{4} \qquad \frac{n-1}{2^h}$

Sub problem 2

$\frac{n}{2} \to \frac{n}{2}-1 \to \frac{n}{2}-2 \to \frac{n}{2}-3 \to \ldots$

$\frac{n}{2} \to \frac{n}{4} \to \frac{n}{8} \to \frac{n}{16} \to \ldots$

$n-h=1$

$\frac{n-1}{2^h}=1$

$\frac{n}{2}-h=1$

$\frac{n}{2^h}=1$

$n = 2^h$

$n = \log h$

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- **3 pts** Q4. Not recursion tree

✓ **- 0 pts** *Q4. correct*

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Qe 1. $T(n) = 2T(\frac{n}{4}) + 1$

$a=2 \quad b=4 \quad f(n)=1$

$n^{\log_4 2} = \sqrt{n}$

$f(n)=1$

Case 1 $f(n)$ is smaller

Then

$f(n) = O(n^{.5-\varepsilon})$

$T(n) = \Theta(n^{\log_4 2})$

2. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

$a=2 \quad b=4$

$n^{\log_4 2} = \sqrt{n}$

$f(n) = \sqrt{n}$

Case 2 $T(n) = \Theta(\sqrt{n} \log n)$

3. $T(n) = 2T(\frac{n}{4}) + n$

$n^{\log_4 2} = \sqrt{n}$

$f(n) = n$

$f(n) > \sqrt{n}$

Case 3 must ~~satisfy~~ regularity condition

$af(\frac{n}{b}) \leq cf(n)$ where $c > 1$

$2(\frac{n}{4}) \leq c(n)$

$\frac{n}{2} \leq c(n) \rightarrow T(n) = \Theta(f(n)) = n$

4. $T(n) = 2T(\frac{n}{4}) + n^2$

$f(n) = n^2 \quad a = 2 \quad b = 4$

$f(n) > \sqrt{n}$

Case 3 must satisfy regularity condition

$$2\left(\frac{n^2}{4}\right)$$

$$\frac{n^2}{2} \leq c\,n^2 \quad c > 1 \text{ then}$$

$$T(n) = \Theta(n^2)$$

2 Q2 **50 / 50**

✓ **- 0 pts** *Correct*

- 50 pts Missing

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- 7 pts Q3 incorrect

- 3 pts Did not choose epsilon