ECS 122A B01-B03 FQ 2021 Homework 01

Geoffrey Mohn

TOTAL POINTS

100 / 100

QUESTION 1

1 Inductive Proof **20 / 20**

✓ **+ 20 pts** *All Correct*

+ 0 pts Wrong/No Submission

QUESTION 2

2 Basic Code Analysis **15 / 15**

✓ **+ 15 pts** *Correct*

+ 0 pts Invalid/No Submission

QUESTION 3

3 Proving Big-O **15 / 15**

✓ **+ 15 pts** *Correct*

+ 0 pts No/Invalid Submission

QUESTION 4

4 Limit Lemma Theorem **10 / 10**

✓ **+ 10 pts** *Correct*

+ 0 pts No/Invalid Submission

QUESTION 5

5 MinHeap **40 / 40**

✓ **+ 40 pts** *All Correct*

+ 0 pts No/Invalid Submission

Search to find

Q1 $\sum_{i=1}^{n} 2^i = ?$ let $n=1$ $\sum_{i=1}^{1} 2^i = 2$ $i=3$ $2^3=8$

$a_n = a \cdot r^n$ $2 \cdot 2 \cdot 2$

$\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$ $a_0 = a$ $2(4)$ $n=2 \rightarrow \Sigma=6$

$ar=2$ $a_0=2$ $2i^2$

$a_1 = a_0 r$ if $n=3$ if $n=4$

2) to prove $\sum_{i=1}^{3} 2^i = 14$ $\Sigma=30$

$\sum_{i=1}^{n} 2^i = (2^{n+1}-2)$ $\forall n \geq 1$ by induction $2+4+8+16$

2^4+2^3

Base case let $n=1$ $2^5-2=30$

$\sum_{i=1}^{1} 2^i = 2^2-2=2$ P(C) is true

LHS=2 RHS=2

let $k \geq 1$ be an arbitrary int to prove P(K) assume P(K) is true to prove P(K+1)

P(K) $\sum_{i=1}^{k} 2^i = 2^{k+1}-2$

P(K+1) $\sum_{i=1}^{k+1} 2^i = 2^{k+2}-2$

by inductive Hypothesis

$2^{k+1}-2+k+1 = 2^{k+1}-1+k$

1 Inductive Proof 20 / 20

✓ **+ 20 pts** *All Correct*

+ 0 pts Wrong/No Submission

Q2 $i=n$

loop $i>1$ $\quad\quad\quad\quad\quad$ $i/_2$ $\;$ j++ $\;$ $K \times 2$

i *= $1/2$

Loop2 $j < n$

$j = i$

j++

Loop3 $k < n$

k *= 2

$O(n)$ because $i=n$ & $j=i$ then $j=n$ thus never satisfying the conditional statement $j<n$ $\therefore$ the 2^{nd} & 3^{rd} nested loop is not stepped in.

There exist some constant c for $O(n)$

Q3 $T(n) = 2n^4 + 5n^3 + 3n^3/\log n + 2n + 5$ is $O(n^4)$

for all $n \geq k$ $\exists$ a constant c

Such that $c \cdot n^4 \geq 2n^4 + 5n^3 + 3n^3/\log n + 2n + 5$

Ex let $n = 1$

$$c \cdot 1^4 \geq 2 + 5 + 0 + 2 + 5$$
$$12$$

let $c \geq 12$

then $T(n)$ is asymptotically bounded by $O(n^4)$

2 Basic Code Analysis **15 / 15**

✓ **+ 15 pts** *Correct*

+ 0 pts Invalid/No Submission

Q2 $i = n$

Loop $i > 1$ $\quad i/2 \quad j{+}{+} \quad k \times 2$

$i *= 1/2$

Loop2 $j < n$

$j = i$

$j{+}{+}$

Loop3 $k < n$

$k *= 2$

$O(n)$ because $i = n$ & $j = i$ then $j = n$ thus never satisfying the conditional statement $j < n$ ∴ the 2nd & 3rd nested loop is not stepped in.

There exist some constant C for $O(n)$

Q3 $T(n) = 2n^4 + 5n^3 + 3n^3/\log n + 2n + 5$ is $O(n^4)$

for all $n \geq k$ $\exists$ a constant C

such that $C \cdot n^4 \geq 2n^4 + 5n^3 + 3n^3/\log n + 2n + 5$

Ex let $n = 1$

$$C \cdot 1^4 \geq 2 + 5 + 0 + 2 + 5$$

$$12$$

let $C \geq 12$

then $T(n)$ is asymptotically bounded by $O(n^4)$

3 Proving Big-O 15 / 15

✓ **+ 15 pts** *Correct*

+ 0 pts No/Invalid Submission

Q4 $T(n) = 5n^6 + n^2 + 3$ is $O(\log n + n^6 + n)$
let $g(n) = \log n + n^6 + n$

by def of $O(g(n))$ there exists positive constants c & K such that $0 \leq T(n) \leq c\,g(n)$ for all $n \geq K$

Let $L = \lim_{n \to \infty} \frac{T(n)}{g(n)} \quad \frac{5n^6 + \dots}{n^6 + \dots} = 5$

$L = 5$ is a positive constant then $T(n)$ is $\Theta(g(n))$ $\therefore$

$T(n)$ is $O(\log n + n^6 + n)$

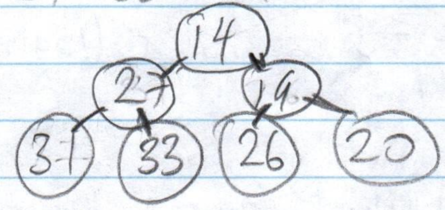
4 Limit Lemma Theorem 10 / 10

✓ **+ 10 pts** *Correct*

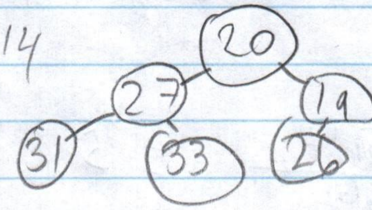
+ 0 pts No/Invalid Submission

31 14 26 27 33 19 20

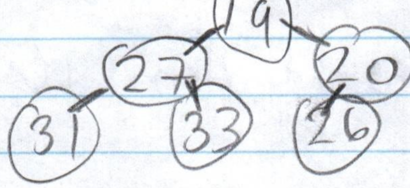
Q5



2)
remove 14



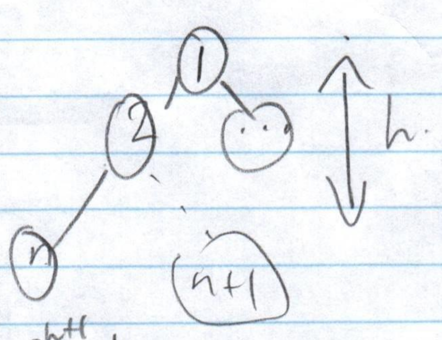
Reheap



3) insert new item in next position at bottom of heap $O(1)$

Reheap up/down $O(h)$ h = Tree height

Minheap (A)



$2^h \le n \le 2^{h+1}-1$

$O(\log n)$

5 MinHeap 40 / 40

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