ECS 122A – Algorithm & Analysis
Homework 01

Due: October 03, 2021, 11:59pm PT

Note:

- Please identify the corresponding pages for each question according to the outline on Gradescope.
- If you handwrite your solutions, make sure they are clear and readable.

Question 1: Inductive Proof (20 points)

1. (5 points) Find the closed form of $\Sigma_{i=1}^{n} 2^i$.
2. (15 points) Prove your closed-form formula using induction. (Show your work.)

Question 2: Basic Code Analysis (15 points)

What is the asymptotic upper bound (tightest Big-O) of the following algorithm, assume n is the input and is a positive number? (Briefly explain your solution.)

```
i = n
while (i > 1) {
    j = i
    while (j < n) {
        k = 1
        while (k < n) {
            k = k * 2
        }
        j = j + 1
    }
    i = i / 2
}
```

Question 3: Proving Big-O (15 points)

Prove that $T(n) = 2n^4 + 5n^3 + 3n^3 logn + 2n + 5$ is $O(n^4)$ without using the Limit Lemma Theorem. (Show your work.)

Question 4: Limit Lemma Theorem (10 points)

Prove that $T(n) = 5n^6 + n^2 + 3$ is $O(logn + n^6 + n)$ using the Limit Lemma Theorem. (Show your work.)

Question 5: MinHeap Review (40 points)

1. (15 points) Build a (binary) min heap by pushing the following numbers one by one in order:

$$31, 14, 26, 27, 33, 19, 20.$$

   Draw the min heap (as a binary tree) after pushing each number (no need to draw the intermediate steps of swapping).

2. (15 points) Perform the `pop` (or sometimes called `extractMin`) operation on the final resulting min heap in the above. (Show your work, including the intermediate steps of swapping.)

3. (10 points) Given an array `A` of numbers. Let `f(A)` be a function that pushes each element of `A` onto a min heap h, i.e., `f(A)` executes the following statements sequentially:

```
push(A[1])
push(A[2])
...
push(A[n])
```

What is the asymptotic upper bound of `f(A)`? (Show your work.)