Quiz 1

Q1 Induction - Base Case (1 point)

Version 1

Suppose we are tyring to prove $\Pi_{i=2}^{n}(1-\frac{1}{i^2})=\frac{n+1}{2n}$. The smallest base case is when $n=$

Answer: 2

Version 2

Supose we are trying to prove $\Sigma_{i=1}^{n}(2i-1)=n^2$. The smallest base case is when $n=$

Answer: 1

Q2 Weak Induction (4 points)

Version 1

Suppose we are tyring to prove $\Pi_{i=2}^{n}(1-\frac{1}{i^2})=\frac{n+1}{2n}$ using weak induction.

1. (2 points) What is the inductive hypothesis?

   Answer: Let $k\geq 2$ be an arbitrary integer. Assume $\Pi_{i=2}^{k}(1-\frac{1}{i^2})=\frac{k+1}{2k}$.

2. (2 points) Given the hypothesis, what statement do we need to prove? (Only write the statement. No need to write the entire proof.)

   Answer: To prove that $\Pi_{i=2}^{k+1}(1-\frac{1}{i^2})=\frac{k+2}{2(k+1)}$

Version 2

Supose we are trying to prove $\Sigma_{i=1}^{n}(2i-1)=n^2$ using weak induction.

1. (2 points) What is the inductive hypothesis?

   Answer: Let $k\geq 2$ be an arbitrary integer. Assume $\Sigma_{i=1}^{k}(2i-1)=k^2$

2. (2 points) Given the hypothesis, what statement do we need to prove? (Only write the statement. No need to write the entire proof.)

   Answer: To prove that $\Sigma_{i=1}^{k+1}(2i-1)=(k+1)^2$

Q3 Strong Induction (5 points)

Version 1

Let the "Tribonacci sequence" be defined by $T_1 = T_2 = T_3 = 1$ and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$. Suppose we are trying to use strong induction to prove that $T_n < 2^n$ for all positive integers . n.

1. (3 points) What is the inductive hypothesis?

   Answer: Let $k \geq 4$ be an arbitrary integer. Assume $T_k = T_{k-1} + T_{k-2} + T_{k-3}$.

2. (2 points) Given the hypothesis, what statement do we need to prove? (Only write the statement. No need to write the entire proof.)

   Answer: To prove that $T_{k+1} = T_k + T_{k-1} + T_{k-2}$

Version 2

Let a_n be the sequence defined by $a_1 = 1$, $a_2 = 8$, $a_n = a_{n-1} + 2a_{n-2} (n \geq 3)$. Suppose we are trying to use strong induction to prove that $a_n = 3 \times 2^{n-1} + 2(-1)^n$ for all positive integers . n.

1. (3 points) What is the inductive hypothesis?

   Answer: Let $k \geq 3$ be an arbitrary integer. Assume $a_k = 3 \times 2^{k-1} + 2(-1)^k$.

2. (2 points) Given the hypothesis, what statement do we need to prove? (Only write the statement. No need to write the entire proof.)

   Answer: To prove that $a_{k+1} = 3 \times 2^k + 2(-1)^{k+1}$.