Vector Basics

The fundamental root of it all building block for linear algebra is the **vector**, so it is worth making sure that were all on the same page about what exactly a vector is. You see broadly speaking. There are three distinct but related ideas about **vectors** which i will call the physic student prospective, the computer science student perspective and the mathematician prospective

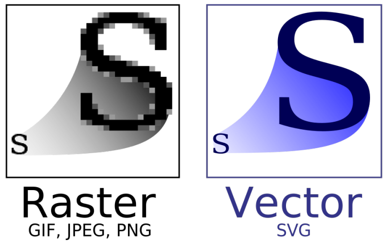

figure: Vector graphics

The physic student perspective is that **vectors** are arrows, pointing and space. What defines a given **vector** is its length and the direction its pointing. But as long, as those two facts are the same: you can move it all around and it's still the same vector

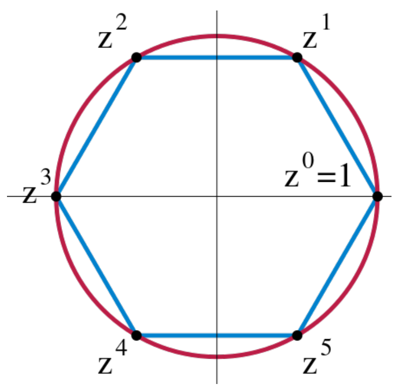

figure: Linear algebraic group

Vectors that live in the flat plane are two dimensional and those sitting in broader space. That you and i live in are three dimensional

The computer science perspective is that **vectors** are ordered lists of numbers

For example, let us say you are doing some analytics about house prices and the only features you carred about were square footage and price. You might model each house with a pair of numbers, the first indicating square footage and the second indicating price

Notice the order matters here

In the lingo you be modelling houses as two dimensional **vectors**. Where, in this context, **vector** is pretty much, just a fancy word for list. And what makes it two dimensional? is the fact that the length of that list is two.

The mathematician on the other hand seeks to generalize both these views, basically saying that a **vector** can be anything where there is a sensible notion of adding two **vectors** and multiplying a vector by a number operations that will talk about later on in this video

The details of this view are rather abstract. I actually think it is healthy to ignore it until the last vadeo of the series favoring a more concrete setting in the interim. But the reason i bring it up here is that it hints at the fact that the ideas of **vector** addition and multiplication by numbers will play an important role throughout linear algebra

But before i talk about these operations, let us just settle in on a specific thought to have in mind. When i say the word **vector**

Given the geometric focus that i am shooting for here. Whenever, i introduce a new topic involving **vectors** i want you to first think about an arrow and specifically think about that arrow inside a coordinate system like bx by plane with its tail sitting at the origin

This is a little bit different from the physic student perspective where **vectors** can freely sit anywhere they want in space in linear algebra. It is almost always the case that your **vector** will be rooted at the origin

Then, once you understand a new concept in the context of arrows in space will translate it over to the list of numbers point of view which we can do by considering the coordinante of the **vector**

Now, while i am sure that many of you are already familiar with this coordinate system, it is worth walking through explicitly. Since this is where all of the important back and forth happens between the two perspectives of linear algebra

Focusing your attention on two dimensions for the moment. You have a horizontal line called the x axis and a vertical line called the y axis

The place where the intersect is called. The origin which you should think of as the center of space and the root of all **vectors**

After choosing an arbitrary length to represent one you make tick marks on each axis to represent this distance

When i want to convey the idea of two d spaces whole which will s comes up a lot in these videos, i will extend these take marks to make grid lines, but right now, they will actually get a little bit in the way

The coordinance of a **vector** is a pair of numbers that basically gives instructions for how to get from the tail of that vector at the origin to its tip. The first number tells you how far to walk along the x axis positive numbers indicating rightward motion, negative numbers indicating leftward motion and the second number tells you how far to walk parallel though the y axis. After that positive numbers indicating upward motion and negative numbers indicating downward motion

To distinguish **vectors** from points. The convention is to write. This pair of numbers vertically with square brackets around them

Every pair of numbers gives you one and only one **vector**. And every vector is associated with one and only one pair of numbers

What about in three dimensions? well, you add a third axis called the z axis, which is perpendicular to both the x and y axis. And in this case, each **vector** is associated with an ordered triplet of numbers: the first tells you how far to move along the x axis, the second tells you how far to move parallel to the y axis, and the third one tells you how far to then move parallel to this new z axis

Every triplet of numbers gives you one unique **vector** in space and every vector in space gives you exactly one triplet of numbers

Right so back to **vector** addition and multiplication by numbers after all, every topic. And linear algebra is given a center around these two operations

Luckily each one is pretty straight forward to define

Let us say we have two **vectors**, one pointing up and a little to the right, and the other one pointing right and down a bit to add these two vectors move the second one, so that its tail sits. At the tip of the first one

Then, if you draw a new **vector** from the tail of the first one to where the tip of the second one now sits that new vector is there, sum

This definition of addition by the way is pretty much the only time in linear algebra where we let **vectors** stray away from the origin

Now, why is this a reasonable thing to do? why this definition of edition and not some other? one

So the way i like to think about it is that each **vector** represents a certain movement, a step with a certain distance and direction in space

If you take a step along the first factor, then take a step in the direction and distance described by the second **vector**. The overall effect is just the same as if you moved along the sum of those two **vectors** to start with

You can think about this as an extension of how we think about adding numbers on a number line

One way that we teach kids to think about this say with two plus five is to think of moving two steps to the right followed by another five steps to the right. The overall effect is the same, as if you just took seven steps to the right

In fact, let us see how **vector** edition looks. Numerically, the first vector here has coordinates one two

And the second one has coordinates three negative, one, When you take the **vector** sum, using this tip to tail method, you can think of a four step path from the origin to the tip of the second vector walk. One to the right then two up then three to the right then one

Reorganizing these steps so that you first do all of the right word motion. Then do all the vertical motion you can read it as saying first move one plus three to the right then move two minus one

So the new **vector** has coordinates one plus three and two plus negative

In general **vector** edition. In this list of numbers conception looks like matching up their terms and adding each one together

The other fundamental **vector** operation is multiplication by a number now this is best understood, just by looking at a few examples. If you take the number two and multiplied by a given vector, it means you stretch out that vector so that it is two times as long as when you started

If you multiply that **vector** by say one, third, it means you squish it down, so that it is onethird. The original length

When you multiply by a negative number like negative one, then the **vector** first gits flipped around then stretched out by that factor of one.

This process of stretching, or squitshing, or sometimes reversing the direction of a **vector** is called. Scaling

And whenever you catch a number like two or one third or negative, one acting like this. Scaling some **vector** you call it a scaler

In fact throughout linear algebra, one of the main things that numbers do is scale **vectors**. So it is common to use the word scaler pretty much interchangeably with the word number

Numerically stretching out of **vector** by a factor of say. Two corresponds with multiplying each of its components. By that factor two

So in the conception of **vectors** as lists of numbers. Multiplying a given **vector** by a scaler means multiplying each one of those components by that scaler

We will see in the following videos what i mean when i say that linear algebra topics tend to revolve around these two fundamental operations, **vector** edition and scalar multiplication. And i will talk more in the last video about how and why the mathematician thinks only about these operations independent and abstracted away from, however, you choose to represent **vectors**

In truth. It does not matter whether you think about **vectors** as fundamentally being arrows in space. Like i am suggesting you do that happen to have a nice numerical representation or fundamentally as lists of numbers that happen to have a nice geometric interpretation. The usefulness of linear algebra has less to do with either one of these views than it does with the ability to translate back and forth between them

It gives the data analyst a nice way to conceptualize many lists of numbers in a visual way, which can seriously clarify patterns in data and give a global view of what certain operations do

And on the flip side. It gives people like physicists and computer graphics programmers a language to describe space and the manipulation of space using numbers that can be crunched and run through a computer

When i do mathe animations. For example, i start by thinking about what is actually going on in space, and then get the computer to represent things numerically thereby figuring out where to place the pixels on the screen and doing that usually realize on a lot of **linear algebra understanding**. So there is your **vector** basics, and then the next video will start getting into some pretty neat concepts, surrounding **vectors** like span basis and linear dependence

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https://en.wikipedia.org/wiki/Linear_algebraic_group

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