football league analysis

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1 Football team clustering based on league play-graph

This notebook runs spectral clustering on football league play-graph. Clustering results with different k are saved in '/results/football_team_clustering/' folder

```
[1]: import os
     os.chdir(os.getcwd() + '/../../')
[2]: import pandas as pd
     import numpy as np
     from scipy import sparse
     import matplotlib.pyplot as plt
     from scripts.utils import *
     from scripts.SpectralClustering import *
[3]: a = import_graph('data/football/play_graph.txt')
     a = a-1
     a[:5]
[3]: array([[0, 1],
            [2, 3],
            [4, 5],
            [6, 7],
            [8, 9]])
```

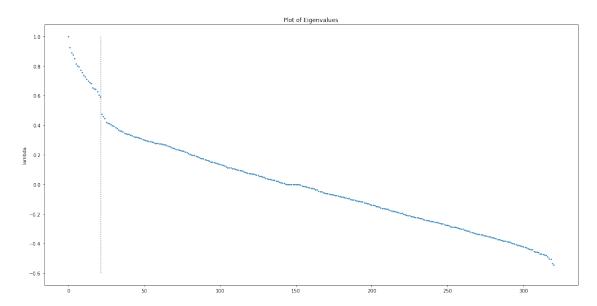
play-graph a is a 2D array where each row indicates team_id in column 1 played team_id in column 2.

```
[4]: # initialize SpectralClust class
spectral = SpectralClust(a)
```

1.1 1. Eigenvalues plot

```
[5]: spectral.construct_similarity_matrix(plot_eigenvalues=True, figsize=(20, 10)) plt.vlines(21.5, ymin=-0.6, ymax=1, colors='black', linestyles='dotted', ⊔ →alpha=0.5)
```

[5]: <matplotlib.collections.LineCollection at 0x18dcbe61070>



Eigenvalues plot shows sharp cutoff at 22 values, a strong evidence of the presence of 22 clusters in the dataset. The plot visualizes connectivity between clusters and in general first k eigenvalues are selected that maximizes the eigengap. The intuition is, well-defined eigengap corresponds to well-defined clusters (with stark difference between intercluster connectivity and intracluster connectivity). Although some clustering problems will have poorly-defined similarity within clusters which will result in relatively smooth eigenvalues plot and the need to use more sophisticated method, in this football team clustering case, number of cluster can be derived using eigengap method.

1.2 2. Different values of k

1.2.1 2.1. Asigned clusters for each node

Assigned clusters for different value of k can be found in '/results/football_team_clustering/' folder with their corresponding filenames.

```
[6]: K = [5, 7, 10, 22]
n_repeats = 10
rng = np.random.default_rng(0)
best_results_list = []
for k in K:
    best_results_list.append(spectral.run_clustering(k, n_repeats, rng=rng,u)
    verbose=False)['best_result'])

df_best_results = pd.DataFrame(best_results_list)
df_best_results['k'] = K
```

```
[6]:
                                                  clusters \
     0 [2, 2, 2, 4, 4, 4, 3, 4, 4, 4, 2, 2, 0, 0, 4, ...
     1 [1, 5, 1, 6, 6, 6, 4, 6, 4, 6, 4, 1, 3, 3, 6, ...
     2 [0, 7, 0, 0, 0, 0, 9, 4, 5, 4, 9, 0, 1, 6, 4, ...
     3 [11, 3, 11, 2, 12, 7, 4, 16, 5, 6, 18, 11, 17,...
                                                 centroids \
    0 [[-0.06046351655345556, 0.11815336708265124, 0...
     1 [[-0.0390576621436381, 0.06495519614449481, -0...
    2 [[-0.05527812331812012, -0.03693667270532918, ...
     3 [[-0.06264290399188291, -0.037091650204747424,...
                                         closest centroids
    0 [[-0.05670445499375304, -0.0518598687808655, 0... 1.240332
     1 [[-0.05909347279734633, -0.05544581126899997, ... 2.055875
     2 [[-0.05527812331812012, -0.03693667270532918, ... 2.724288
     3 [[-0.0619155704642075, -0.057839666132939026, ... 3.889151
        initialization_time optimization_time iterations
                                                              k
     0
                   0.015625
                                       0.000000
                                                          6
                                                              5
     1
                   0.015625
                                       0.000000
                                                         16
                                                              7
     2
                   0.031250
                                       0.015625
                                                         16 10
     3
                   0.171875
                                       0.000000
                                                          8
                                                             22
[7]: # output files
     idx2name = read_team_name('data/football/inverse_teams.txt')
     dirpath = 'results/football_team_clustering/'
     for k in K:
         output_file(a,
                     idx2name,
                     df best results.loc[df best results['k'] == k, 'clusters'].
      \rightarrowiloc[0],
                     dirpath+'nodes_k'+str(k)+'.csv',
                     dirpath+'edges_k'+str(k)+'.csv')
    1.2.2 2.2. Smallest and largest clusters
```

df_best_results.head()

```
[8]: def highest_count(arr):
         count = np.bincount(arr)
         return np.max(count)
     def lowest_count(arr):
         count = np.bincount(arr)
```

```
[8]:
            largest_cluster_size
                                     smallest_cluster_size
     0
         5
                               168
         7
     1
                               110
                                                          10
     2
                                 77
                                                          10
       10
     3
                                 51
                                                           7
        22
```

1.3 3. Consistency between repeats

```
[9]: df_k10 = pd.DataFrame({'label': idx2name})
for i in range(n_repeats):
    results = spectral.run_clustering_once(10, rng=np.random.default_rng(i))
    df_k10[f"iteration_{i+1}"] = results['clusters']
```

Georgia Tech, Georgia State and Georgia teams are used to evaluate consistency across different iterations:

```
[10]: df_k10[df_k10['label'].isin(['Georgia Tech', 'Georgia State', 'Georgia'])]
                           iteration_1
[10]:
                   label
                                        iteration_2 iteration_3
                                                                   iteration_4 \
      3
           Georgia State
                                     5
                                                   2
                                                                 8
                                                                              2
      95
            Georgia Tech
                                     5
                                                   2
                                                                 8
                                                                               2
                                     5
                                                   2
                                                                 8
                                                                               2
      143
                 Georgia
           iteration_5
                        iteration_6 iteration_7 iteration_8 iteration_9
      3
                      3
                                   3
                                                               7
                                                               7
      95
                      3
                                   3
                                                 4
                                                                            9
      143
                      3
                                   3
                                                                            9
           iteration 10
      3
                       0
      95
                       0
      143
```

The three teams are clustered in the same team 10 out of 10 iterations. Different cluster assignments can happen when the algorithm is initialized differenty (randomly).

1.4 Additional remarks

In this analysis, kmeans++ is used as a default method of initialization of centroids proposed in 2007 by David Arthur and Sergei Vassilvitskii (https://en.wikipedia.org/wiki/K-means%2B%2B).

Random_partition method of initialization (where each datapoints are randomly assigned one of the k clusters whose centroids act as the initial centroids) is considered a poor initialization method because most of the time, initial centroids are located in the interior of dataspace. From the 10 iterations spectral clustering with the same seeds, two put Georgia Tech in different cluster from Georgia State and Georgia. This can of course be due to randomness.

```
[11]: df k10 = pd.DataFrame({'label': idx2name})
      for i in range(n_repeats):
          results = spectral.run_clustering_once(10, init_method='random_partition',__
       →rng=np.random.default_rng(i))
          df k10[f"iteration {i+1}"] = results['clusters']
      df_k10[df_k10['label'].isin(['Georgia Tech', 'Georgia State', 'Georgia'])]
Γ11]:
                   label
                           iteration 1
                                        iteration_2 iteration_3
                                                                   iteration 4
      3
           Georgia State
      95
            Georgia Tech
                                     5
                                                   0
                                                                3
                                                                              8
      143
                 Georgia
                                     7
                                                   0
                                                                3
                                                                              8
           iteration_5
                        iteration_6 iteration_7
                                                    iteration_8 iteration_9
      3
                                   7
                                                 3
                                                              2
                                                                            5
      95
                      9
                                   2
                                                 3
                                                              2
                                                                            5
                                   7
                                                 3
                                                              2
      143
                      9
                                                                            5
           iteration 10
      3
                      1
      95
                      1
      143
                       1
     and just for completeness, the other two initialization methods provided in the script are explored
[12]: df k10 = pd.DataFrame({'label': idx2name})
      for i in range(n repeats):
          results = spectral.run_clustering_once(10, init_method='random_data',_
       →rng=np.random.default_rng(i))
          df_k10[f"iteration_{i+1}"] = results['clusters']
      df_k10[df_k10['label'].isin(['Georgia Tech', 'Georgia State', 'Georgia'])]
[12]:
                   label
                           iteration_1
                                        iteration_2
                                                      iteration_3
                                                                   iteration_4 \
```

```
Georgia Tech
                                7
                                              0
                                                             0
                                                                           2
95
                                7
                                                             0
                                                                           2
143
            Georgia
     iteration_5
                   iteration_6 iteration_7 iteration_8 iteration_9
3
                8
                              9
                                            6
                                                          9
                                                                         8
95
                8
                              9
                                            3
                                                          9
                                                                         8
143
                8
                              9
                                            6
                                                          9
                                                                         8
     iteration 10
```

Georgia State

```
3
                      6
      95
                      6
      143
                      6
[13]: df_k10 = pd.DataFrame({'label': idx2name})
      for i in range(n_repeats):
          results = spectral.run_clustering_once(10, init_method='random_uniform', __
       →rng=np.random.default_rng(i))
          df_k10[f"iteration_{i+1}"] = results['clusters']
      df_k10[df_k10['label'].isin(['Georgia Tech', 'Georgia State', 'Georgia'])]
[13]:
                   label
                          iteration_1 iteration_2 iteration_3
                                                                  iteration_4 \
      3
           Georgia State
      95
            Georgia Tech
                                    6
                                                  3
                                                               2
                                                                            8
      143
                 Georgia
                                    6
                                                  3
                                                               2
                                                                            8
           iteration_5 iteration_6 iteration_7 iteration_8 iteration_9 \
      3
                                  1
                                                7
                                                                          5
                                                7
                     3
                                                                          5
      95
                                  1
                                                             9
      143
                     3
                                  1
                                                7
                                                             9
                                                                          5
           iteration_10
      3
                      9
                      9
      95
      143
                      9
```

In general, the three initialization methods with k=10 yield expected results that the three mentioned teams are in the same cluster.

[]: