SPATIO-TEMPORAL MODELING OF PRECIPITATION IN WEST AFRICA: ANALYSIS OF CLIMATE RISKS

Kossivi GNOZIGUE, Mamadou Lamine DIOP, El Hadji DEME & Aliou DIOP

Gaston Berger University of Saint-Louis

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Introduction

Precipitation is vital for West Africa, but its spatio-temporal variability poses a major challenge, exacerbated by climate change. Understanding and forecasting these precipitations is crucial for managing water resources and mitigating flood risks. Our objectives are to:

- Analyze flood risks associated with extreme precipitation in West Africa.
- Develop a predictive precipitation model integrating geographical data.

Study Area Presentation

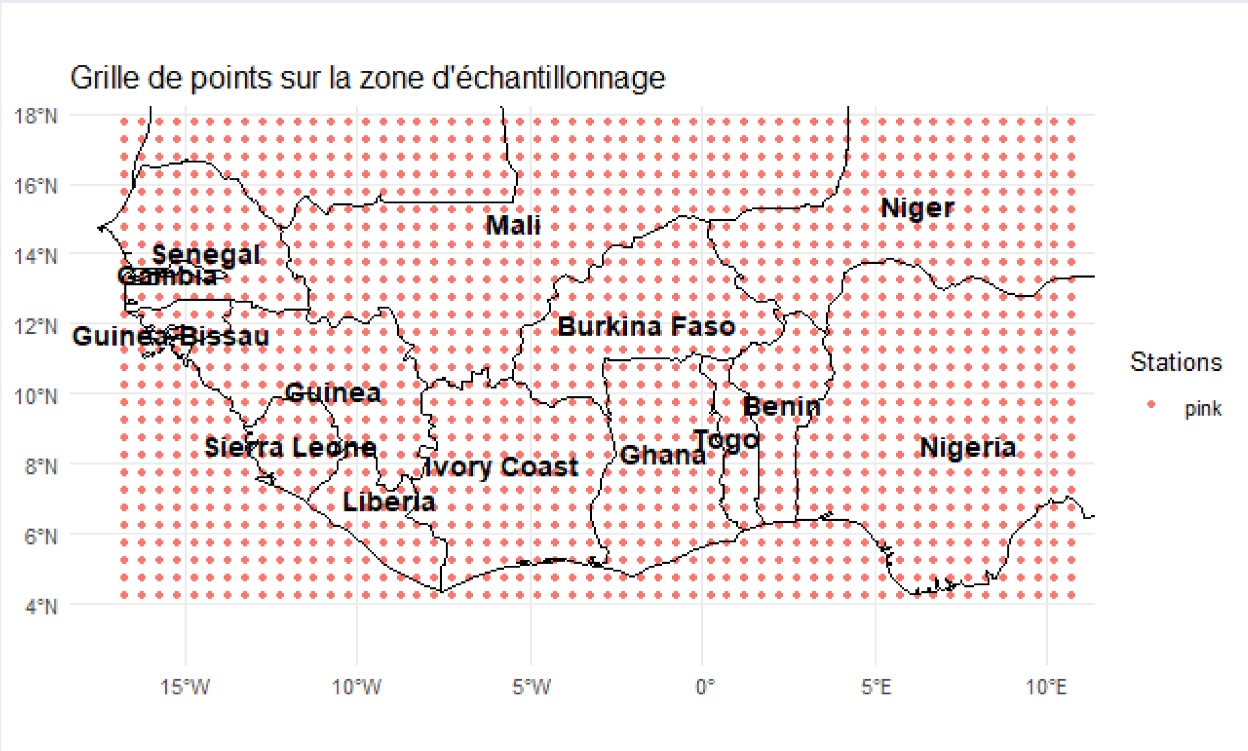

Figure: Illustration of the method.

Data Presentation

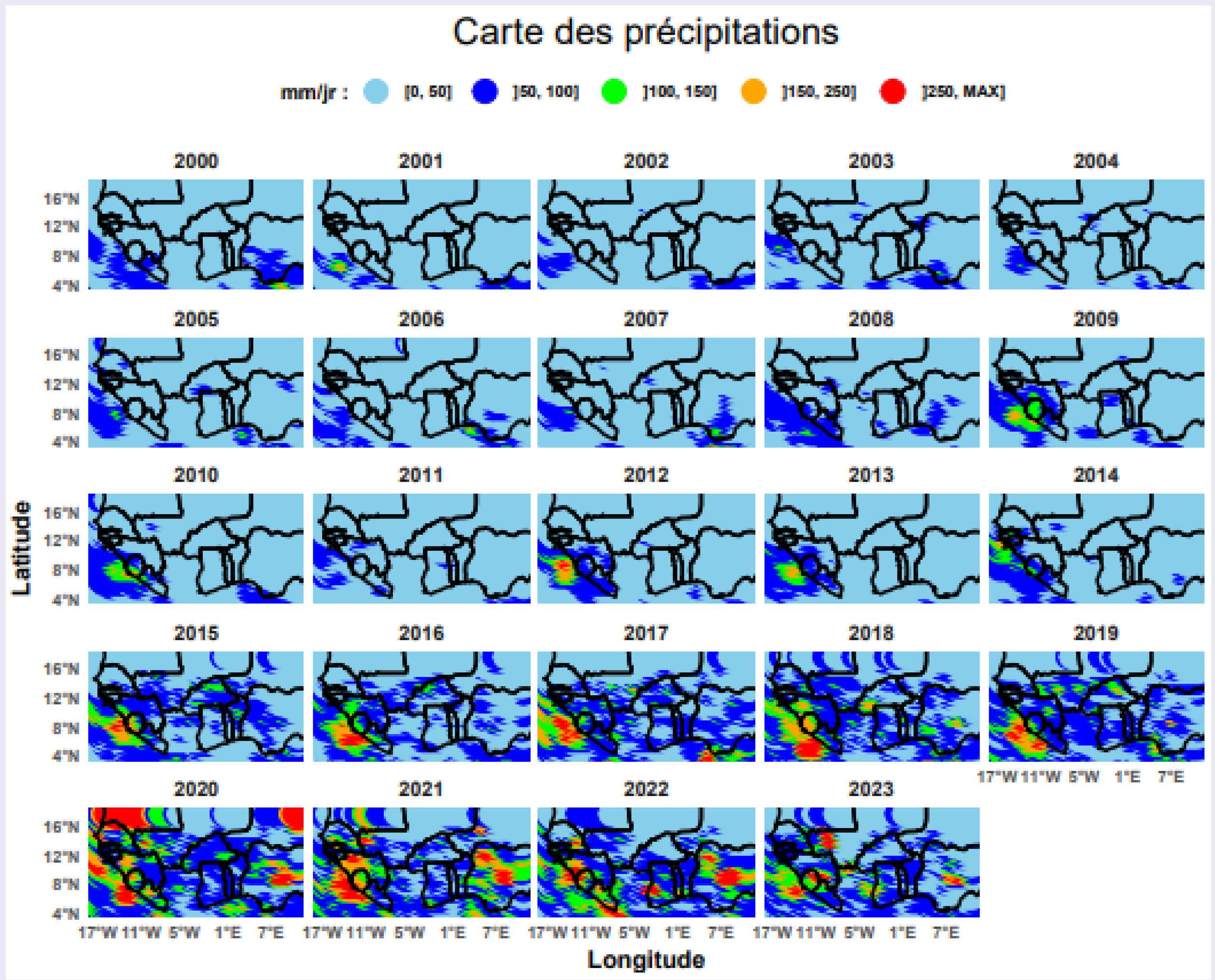

Figure: Illustration of the method.

Methodology

This research employs two extreme value modeling methods:

- Classical GEV: Using the maximum likelihood estimator on the GEV distribution.
- Hierarchical Bayesian Modeling: Aiming to capture uncertainty and spatial dependence using the Hamiltonian Monte Carlo algorithm implemented in R via Stan.

Classical GEV Models

Table: Classical GEV Models.

Models	Location	Scale	Shape
Model 1	$\mu_0 + \mu_1 lat + \mu_2 lon$	$\sigma_0 + \sigma_1 lat + \sigma_2 lon$	ξ_0
Model 2	$\mu_0 + \mu_1 lat + \mu_2 lon$	$\sigma_0 + \sigma_1 lat$	ξ_0
Model 3	$\mu_0 + \mu_1 lat + \mu_2 lon + \mu_3 t$	$\sigma_0 + \sigma_1 lat + \sigma_2 lon + \sigma_3 t$	ξ_0
Model 4	$\mu_0 + \mu_1 lat + \mu_2 lon + \mu_3 t$	$\sigma_0 + \sigma_1 lat + \sigma_2 t$	ξ_0
Model 5	$\mu_0 + \mu_1 lat + \mu_2 lon + \mu_3 t$	$\sigma_0 + \sigma_1 lat + \sigma_2 lon + \sigma_3 t$	$\xi_0 + \xi_1 t$
Model 6	$\mu_0 + \mu_1 lat + \mu_2 lon + \mu_3 t$	$\sigma_0 + \sigma_1 lat + \sigma_2 t$	$\xi_0 + \xi_1 t$

Classical GEV Results: Model 3 Selected

Table: Criterion Values for Each Model

Models	Deviance	AIC	BIC
Model 1	339324.5	339338.5	339398.2
Model 2	340677.0	340689.0	340740.2
Model 3	339256.2	339274.2	339351
Model 4	340676.9	340692.9	340761.2
Model 5	339277.4	339297.4	339382.7
Model 6	340676.9	340694.9	340771.7

Classical GEV Results: Return Level Map

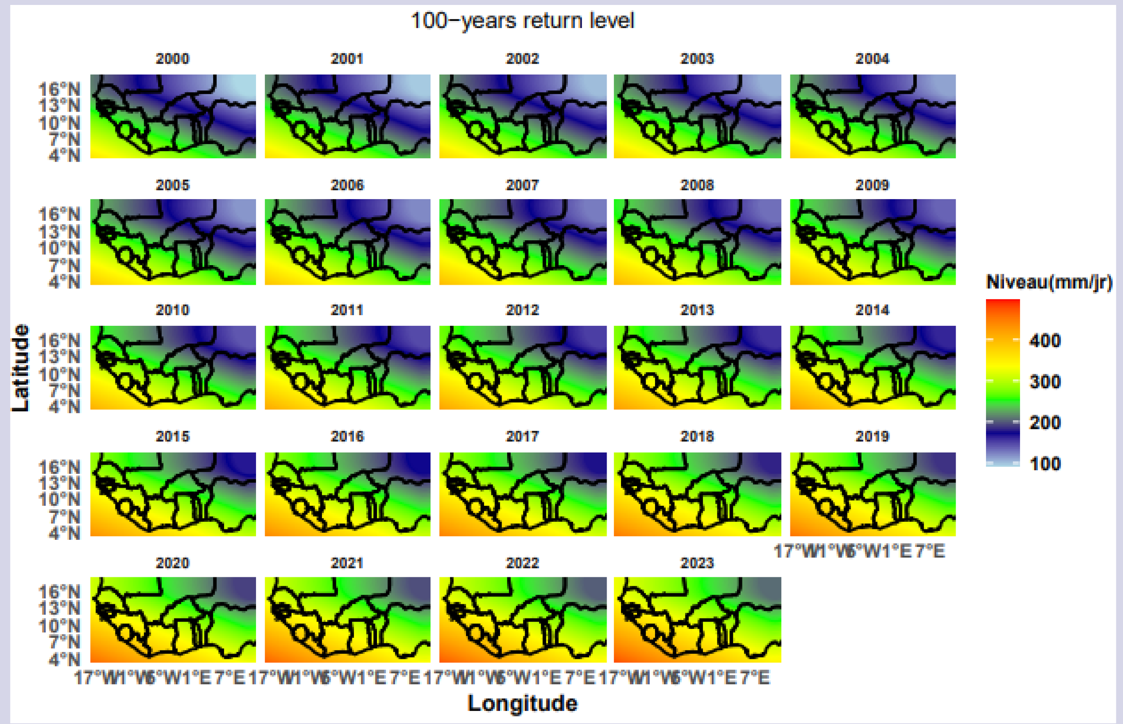

Figure: 100-year return levels from the classical GEV model.

Bayesian Models

- **Simple GEV Model: Capturing the overall trend.** For each observation y_i:

$$y_i \sim \text{GEV}(\mu, \sigma, \xi) \text{ where } \mu = \beta_0^\mu, \sigma = \beta_0^\sigma \text{ and } \xi = \beta_0^\xi \quad (1)$$

Priors: $\beta_0^\mu \sim \mathcal{N}(0, 10)$, $\beta_0^\sigma \sim \mathcal{LN}(0, 5)$ and $\beta_0^\xi \sim \mathcal{N}(0, 0.5)$.

- **GEV Model with Linear Spatial Effects: Capturing linear spatial gradients.**

$$y_i \sim \text{GEV}(\mu_i, \sigma_i, \xi) \text{ where } \mu_i = \beta_0^\mu + \beta_1^\mu \text{lat}_i + \beta_2^\mu \text{lon}_i, \sigma_i = \beta_0^\sigma + \beta_1^\sigma \text{lon}_i \text{ and } \xi = \beta_0^\xi \quad (2)$$

Priors: $\beta_0^\mu, \beta_1^\mu, \beta_2^\mu, \sim \mathcal{N}(0, 5)$, $\beta_0^\sigma, \beta_1^\sigma \sim \mathcal{LN}(0, 1)$ and $\beta_0^\xi \sim \mathcal{N}(0, 0.5)$

-**Multilevel GEV Model with Station-Specific Random Effects: Capturing inter-station variation.** For station j and observation i:

$$y_{ij} \sim \text{GEV}(\mu_{ij}, \sigma_{ij}, \xi) \quad (3)$$

Where $\mu_{ij} = \beta_0^\mu + u_j^\mu, u_j^\mu \sim \mathcal{N}(0, \tau_\mu)$, $\sigma_{ij} = \beta_0^\sigma + u_j^\sigma, u_j^\sigma \sim \mathcal{N}(0, \tau_\sigma)$ and $\xi = \beta_0^\xi$

Priors: $\beta_0^\mu \sim \mathcal{N}(0, 10)$, $\beta_0^\sigma \sim \mathcal{LN}(0, 1)$, $\beta_0^\xi \sim \mathcal{N}(0, 0.5)$, $\tau_\mu, \tau_\sigma \sim \mathcal{LN}(0, 1)$

Bayesian Model Results: Multilevel GEV Model with Station-Specific Random Effects

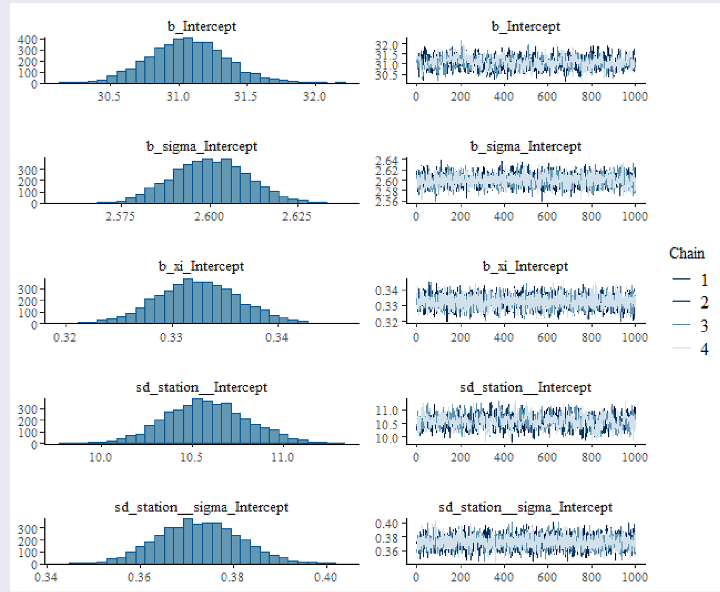

Figure: Chain Convergence

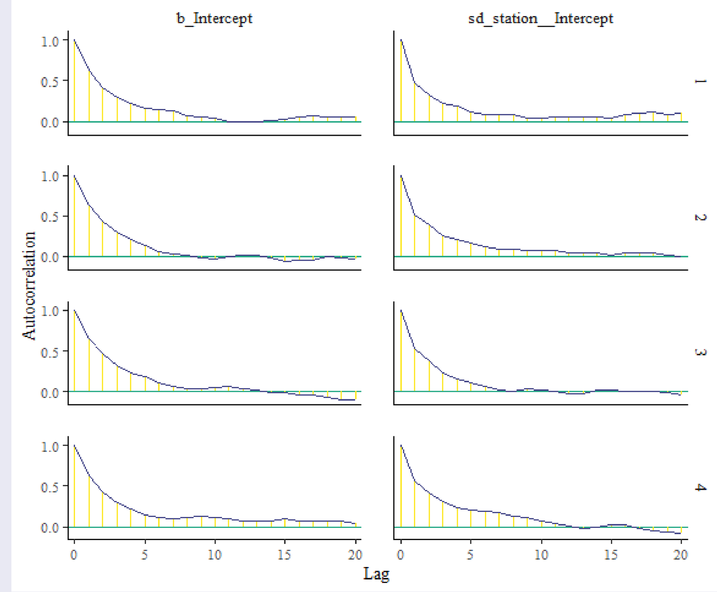

Figure: Autocorrelation

Bayesian Model Results: Return Level Map

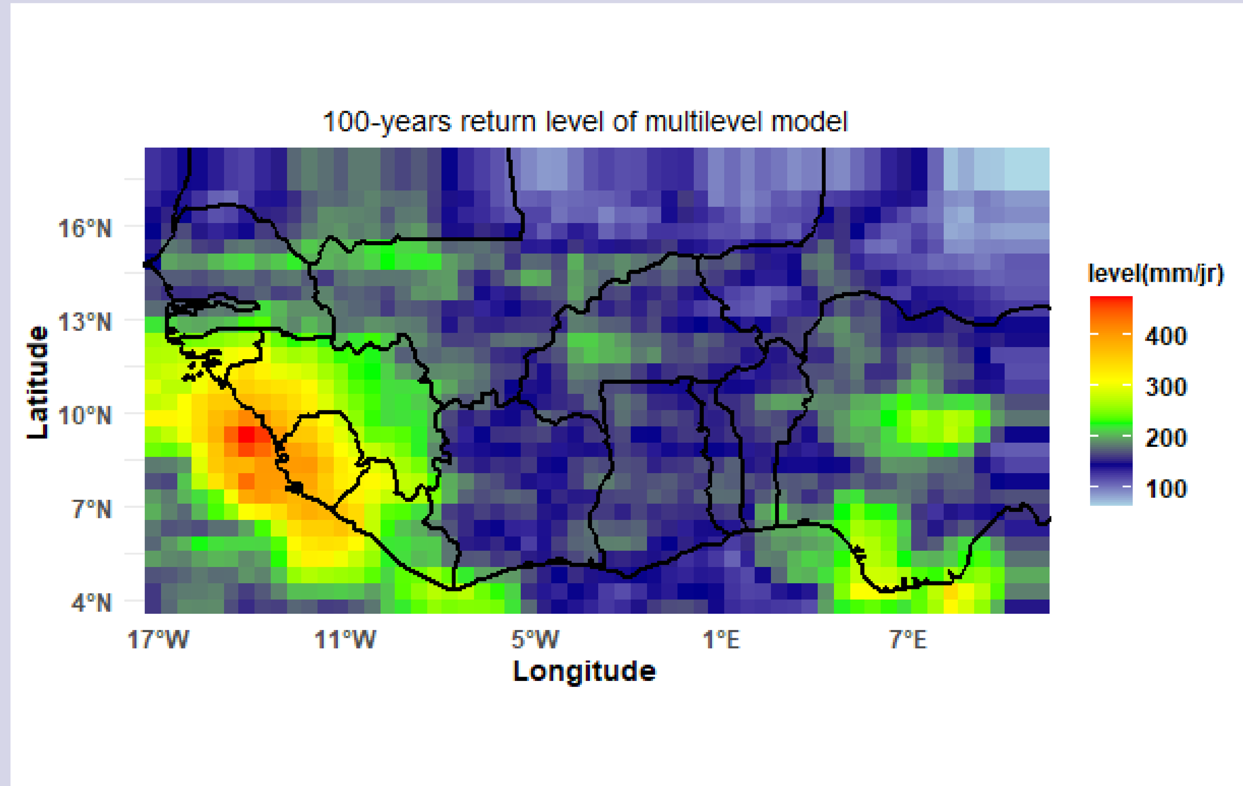

Figure: 100-year return levels from model with Station-Specific Random Effects.

Conclusion

- This work demonstrates the superiority of hierarchical Bayesian modeling over classical GEV methods through the analysis of extreme precipitation in West Africa.
- 100-year return level maps are an essential tool, offering a clear visualization of risks. This information is crucial for water resource planning, agriculture, and climate adaptation strategies in the region.
- This model paves the way for future research to refine the forecasting of extreme events by integrating new predictors and more complex dependence structures (e.g., models with Gaussian processes, machine learning tools).

References

- Coles, S. (2001). *An Introduction to Statistical Modeling of Extreme Values. Springer Series in Statistics.* London: Springer-Verlag
- Cooley, D., Nychka, D., et Naveau, P. (2007). *Bayesian Spatial Modeling of Extreme Precipitation Return Levels.* Journal of the American Statistical Association
- Neal, R. M. (2011). *MCMC using Hamiltonian dynamics*, In S. Brooks, A. Gelman, G. Jones, and X. L. Meng (Eds.), Handbook of Markov Chain Monte Carlo (pp. 113-162). Chapman and Hall/CRC.