Fast Analysis of Nonideal Switched Capacitor Circuits Using Convolution

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A new fast analysis method of nonideal switched capacitor circuits based on convolution integral is introduced. Convolution integral is used to create an equivalent circuit which is solved by a general-purpose circuit simulator program. The method solves as well time domain presentation in steady state as frequency response for nonideal circuits. The circuit may be composed of all kinds of linear components. All switches have to have same period, but they can be opened and closed at any time. Example is given to demonstrate that the proposed method is efficient enough to be used in practical circuit design. The simulation results show good agreement with those obtained by conventional slow transient analysis. Simulations are made using a general-purpose circuit simulator **APLAC** [1] (orginally Analysis Program for Linear Active Circuits).

1. DESCRIPTION OF THE METHOD

The circuit can be divided into time-dependent and frequency-dependent parts, Fig. 1 a). Switches and sources are included in the time-dependent part while other components remain in the frequency-dependent part.

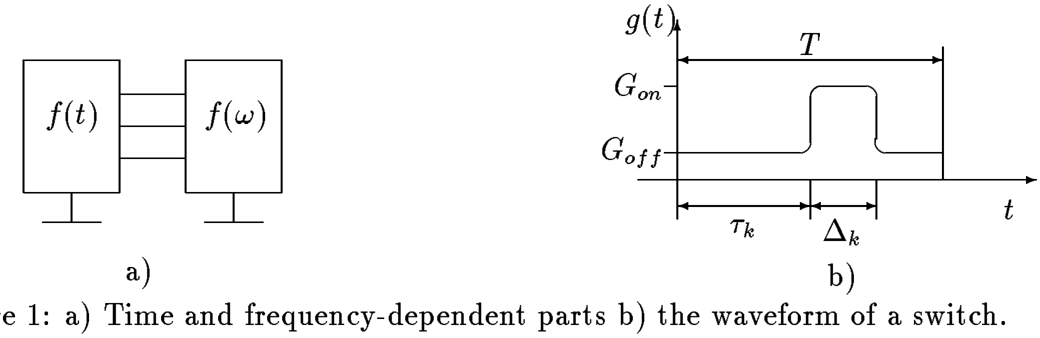

Figure 1: a) Time and frequency-dependent parts b) the waveform of a switch.

Usually the frequency response is under interest. The frequency response of the frequency-dependent part can be calculated in a straightforward manner. The frequency response of the time-dependent part is computed by creating a frequency-domain equivalent circuit with the aid of the convolution.

1.1. Fourier series

All signals are described using Fourier series. For simplicity and without restricting the generality, the formulation is derived for circuits which have only one excitation at frequency $f_s = \frac{\omega_s}{2\pi}$. The source can thus be expressed by one sine- or cosine-term (or a linear combination):

$$h(t) = \begin{cases} \sin(\omega_s t) \\ \cos(\omega_s t), \end{cases} \tag{1}$$

where $h(t)$ refers to voltage or current. If the circuit has sources at different frequencies, the superposition method can be used.

The basic element of the switched capacitor circuits is a switch. The switch, having on- and off-resistancies R_{on} and R_{off}, is described by a time-dependent conductance using Fourier series. Conductance is used instead of resistance in order to have more accurate representation of R_{on}, this being more critical than R_{off}. Transition between R_{on} and R_{off} is described by tanh-function, as shown in Fig. 1 b). The conductance of the switch is given by

$$g(t) = \begin{cases} G_{off} + \frac{1}{2}\{G_{on} - G_{off}\}\{1 + \tanh[k(t - \tau_k)]\}, & t < \tau_k + \frac{\Delta_k}{2} \\ G_{off} + \frac{1}{2}\{G_{on} - G_{off}\}\{1 - \tanh[k(t - (\tau_k + \Delta_k))]\}, & t \geq \tau_k + \frac{\Delta_k}{2}, \end{cases} \tag{2}$$

where k determinates the slope of $g(t)$ at point τ_k, $G_{on} = \frac{1}{R_{on}}$, and $G_{off} = \frac{1}{R_{off}}$. Fourier series of the conductance is

$$g(t) \approx \tilde{g}(t) = a_0 + \sum_{n=1}^{2N} [a_n \cos(n\omega_c t) + b_n \sin(n\omega_c t)], \tag{3}$$

where $\omega_c = 2\pi f_c$, f_c is the switching frequency, $T = \frac{1}{f_c}$ is the switching period, $2N$ is number of Fourier series components, and coefficients a_0, a_n, and b_n can be calculated numerically.

1.2 Fourier transformation

Fourier transformation of source (1) is given by

$$H(\omega) = \begin{cases} \int_{-\infty}^{\infty} \sin(\omega_s t) e^{-j\omega t} dt = \frac{1}{2j}[\delta(\omega - \omega_s) - \delta(\omega + \omega_s)] \\ \int_{-\infty}^{\infty} \cos(\omega_s t) e^{-j\omega t} dt = \frac{1}{2}[\delta(\omega - \omega_s) + \delta(\omega + \omega_s)]. \end{cases} \tag{4}$$

and that one of switch (3) by

$$\begin{aligned} G(\omega) &= \int_{-\infty}^{\infty} \left\{ a_0 + \sum_{n=1}^{2N} [a_n \cos(n\omega_c t) + b_n \sin(n\omega_c t)] \right\} e^{-j\omega t} dt \\ &= a_0 \delta(\omega) + \frac{1}{2} \sum_{n=1}^{2N} [(a_n - jb_n)\delta(\omega - n\omega_c) + (a_n + jb_n)\delta(\omega + n\omega_c)], \end{aligned} \tag{5}$$

In short

$$G(\omega) = \sum_{n=-2N}^{2N} G_n \delta(\omega - n\omega_c), \text{ where } G_{-n} = G_n^*. \tag{6}$$

1.3. Convolution

Let's denote the voltage over the switch by $u(t)$, the current through the switch by $i(t)$ and the conductance by $g(t)$. In the time domain

$$i(t) = g(t)u(t), \tag{7}$$

which in the frequency domain yields

$$I(\omega) = G(\omega) * U(\omega) = \int_{-\infty}^{\infty} G(\xi)U(\omega - \xi)d\xi. \tag{8}$$

The main point in this paper is to represent (8) as an equivalent circuit. In the frequency domain the voltages and currents have components only at discrete frequencies. The equivalent circuit may thus be found by copying original circuit once for each frequency (see Fig. 2). If two sine- and cosine-functions at frequencies ω_c and ω_s are multiplied the result will appear at frequencies $\omega_c \pm \omega_s$. For example, $\sin(\omega_c t)\cos(\omega_s t) = \frac{1}{2}[\sin(\omega_c + \omega_s) + \sin(\omega_c - \omega_s)]$ Thus the voltage and current components are given by

$$\begin{aligned} U(\omega) &= \sum_{k=-N}^{N} U_k \delta(\omega - k\omega_c - \omega_s), && (9)\\ I(\omega) &= \sum_{l=-N}^{N} I_l \delta(\omega - l\omega_c - \omega_s). && (10) \end{aligned}$$

Substitutig (9), (10), and (6) into (8) gives

$$\begin{aligned} \sum_{l=-\infty}^{\infty} I_l \delta(\omega - l\omega_c - \omega_s) &= \int_{-\infty}^{\infty} \sum_{n=-2N}^{2N} G_n \delta(\xi - n\omega_c) \sum_{k=-N}^{N} U_k \delta(\omega - \xi - k\omega_c - \omega_s) d\xi \\ &= \sum_{n=-2N}^{2N} \sum_{k=-N}^{N} G_n U_k \delta(\omega - n\omega_c - k\omega_c - \omega_s). && (11) \end{aligned}$$

Equation (11) cannot be solved explicitly and it is true only when the impulse on the right hand side is at the same frequency as the impulse on the left hand side, i.e.

$$n\omega_c + k\omega_c + \omega_s = l\omega_c + \omega_s \Leftrightarrow n + k = l. \tag{12}$$

Frequencies which satisfy equation (12) transforms (11) as follows

$$I_l = G_n U_k. \tag{13}$$

Subscripts l, n, k of the current, conductance, and voltage denote the respective frequency components. Thus G_n describes the (trans)conductance at frequency $n\omega_c$, I_l the current at $l\omega_c + \omega_s$, and U_k the voltage at $k\omega_c + \omega_s$. Equation (13) gives transformation from one frequency to another. If the current and voltage appear at the same frequency, i.e. $k = l$, and thus at the same copied circuit, then equation (13) describes a conductance, while for $l \neq k$, (13) describes a voltage-controlled current source, i.e. a coupling between those copied circuits representing the respective frequencies. The complete frequency-domain equivalent circuit of (11) is shown in Fig. 2. The circuit is copied $S = 2N + 1$ times.

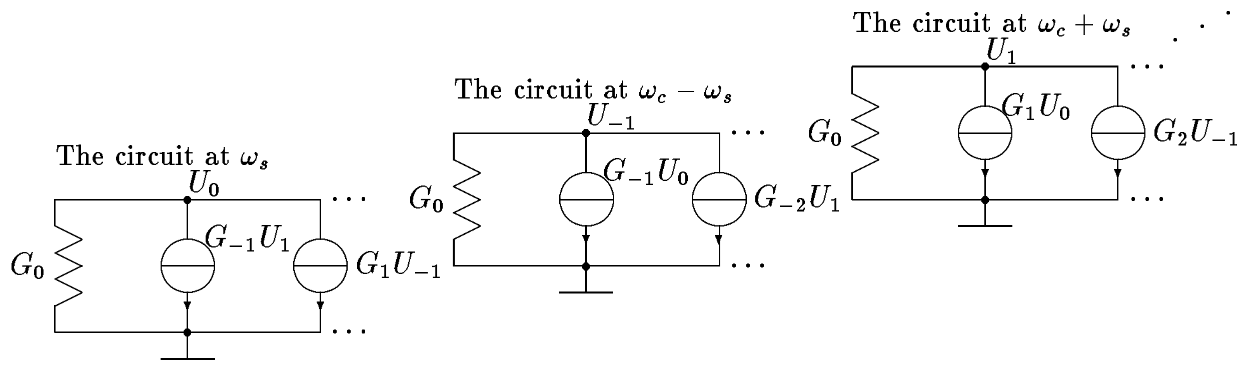

Figure 2: Creation of the equivalent circuit.

In the original circuit there is only one excitation at frequency $f_s = \frac{\omega_s}{2\pi}$. The equivalent circuit is thus excited with a source between those nodes which describe signal frequency ω_s. In other multiplied nodes a current source is replaced by an open-circuit whereas a voltage source must be described by a short-circuit in order to have zero excitation at corresponding frequencies.

Any frequency-dependent (or time-invariant) components connected to the switch nodes have to be multiplied as well. For example, a capacitor at frequency $k\omega_c + \omega_s$ is described by impedance $Z_c = \frac{1}{j(k\omega_c+\omega_s)C}$. When the circuit is excited only using one frequency ω_s, the impedance of the components are determined using this frequency. In order to have correct impedance, the capacitor values in the multipled circuits have to be modified as follows

$$Z_C = \frac{1}{j(k\omega_c + \omega_s)C} = \frac{1}{j\omega_s C_k} \Leftrightarrow C_k = \frac{k\omega_c + \omega_s}{\omega_s}C. \tag{14}$$

1.4. Frequency response and waveform

Coefficients U_k of the impulses are obtained from the equivalent circuit, and they are

$$U(\omega) = \sum_{k=-N}^{N} [U_k\delta(\omega - k\omega_c - \omega_s) + U_k^*\delta(\omega + k\omega_c + \omega_s)]. \tag{15}$$

Once the equivalent circuit has been created the frequency response is obtained by repeating conventional AC analyses at desired signal frequencies.

The steady state time-domain waveform of any voltage is obtained directly from the inverse Fourier transformation of equation (15)

$$u(t) = \sum_{k=0}^{N}\{2\Im\{U_k\}\sin[(-k\omega_c - \omega_s)t] + 2\Re\{U_k\}\cos[(k\omega_c + \omega_s)t]\}. \tag{16}$$

2. EXAMPLES

In this section it is shown how biquad filter can be simulated using the convolution method. The circuit is shown in Fig. 3. The sampling frequency is $f_c = 1$ MHz (i.e. $T = 1$ μs), $R_{on} = 100$ Ω, and $R_{off} = 100$ kΩ. The component values are $C_A = 0.51$ nF, $C_B = 3.68$ nF, $C_C = 0.22$ nF, $C_D = 1.61$ nF, $C_E = 0.1$ nF, and $C_J = 0.1$ nF.

The frequency response is shown in Fig. 4. The analysis with twenty harmonics took 160 CPU-seconds on an HP-9000/720.

In Fig. 4 the results are also compared with those obtained using conventional transient analysis. The transient analysis is carried out at every desired frequency. This is extremely tedious since one has to wait until the steady state has been reached.

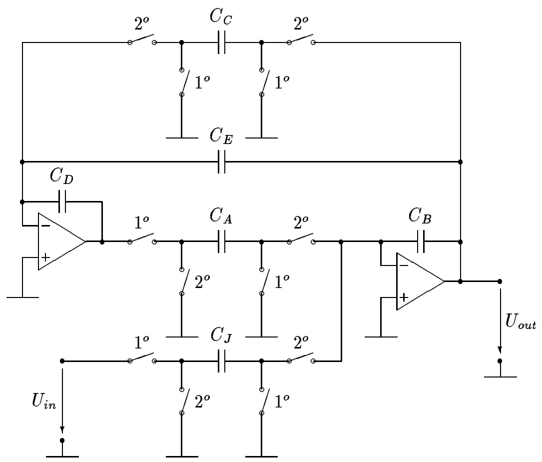

Figure 3: Biquad filter.

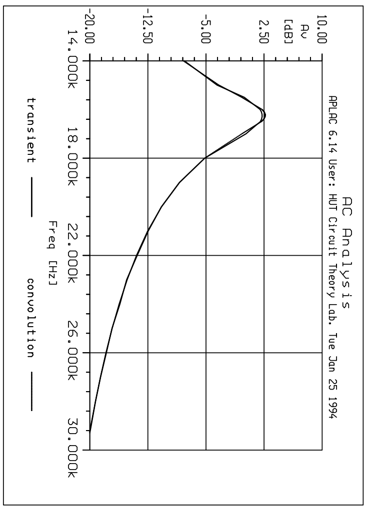

Figure 4: The frequency response before optimization.

Fig. 4 shows that peak 2.6 dB is at frequency $f_0 = 16.3$ kHz. The circuit was designed to have peak 10 dB at $f_0 = 22$ kHz. The reason for the deviation is the fact that the circuit was designed for ideal switches. However, the deteriorating effect of the realistic switches can be corrected by optimizing the capacitor values. This was done using **MinMax** optimization of **APLAC** [2]. The circuit was optimized for twenty harmonics. The new component values are $C_A = 800$ pF, $C_B = 3.81$ nF, $C_C = 340$ pF, $C_D = 2.28$ nF, $C_E = 114$ pF, and $C_J = 179$ pF. The corrected frequency response is shown in Fig. 5. This example shows that the convolution method is fast enough for the optimization of circuits. Even though the example circuit is small much larger practical circuits have been analyzed using the proposed convolution method.

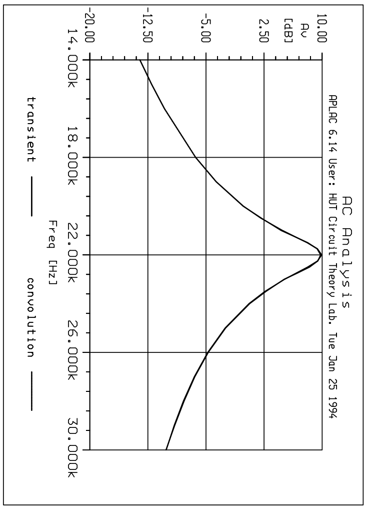

Figure 5: The frequency response after optimization.

The steady state response of the corrected circuit at frequency $f_0 = 22$ kHz is shown in Fig. 6. Also the transient response is shown in Fig. 6. It can be seen that convolution analysis using twenty harmonics ($N = 20$) gives the same results as the tedious transient response.

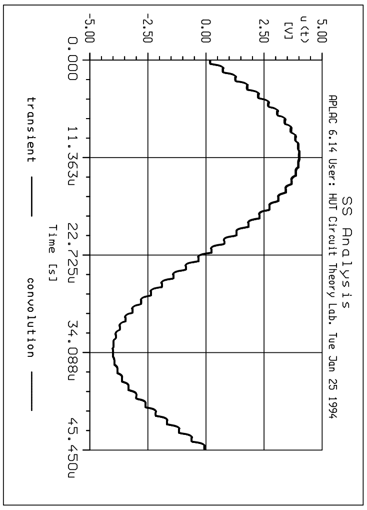

Figure 6: The waveform of the steady state.

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[2] R. Niutanen, M. Valtonen, and K. Mannersalo, "Optimization Methods in APLAC" Helsinki University of Technology, Circuit Theory Laboratory, Report CT-15, 1992, 49 p.