$\sim$ Assignment 4 Computational Models Spring 2022 $\sim$

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Exercise 1

(a)

Definition I. for every $x \in \Sigma^*$

- If $x \in \mathcal{L}$, then there exists $c \in \Sigma^*$ such that V accept (x, c)
- If $x \notin \mathcal{L}$, then V rejects (x, c) for every $c \in \Sigma^*$

Definition II. for every $x \in \Sigma^*$

- If $x \in \mathcal{L}$, then there exists $c \in \Sigma^*$ such that V accept (x, c)
- If $x \notin \mathcal{L}$, then V *does not accept* (x, c) for every $c \in \Sigma^*$

I $\Rightarrow$ **II** first property hold since its identical. and the second since when V reject $(x, c) \Rightarrow$ V *does not accept* (x, c).
II $\Rightarrow$ **I** Let $V_{\mathbf{II}}$ define a verifier. to prove that for some $\mathcal{L}$ exists a verifier by **I**, its sufficient[1] to prove that $\mathcal{L} \in \mathcal{RE}$. now lets define TM that run as follow.

Algorithm 1 $\mathcal{M}_{\mathcal{L}}$ on input x

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Let c_1, c_2 ... denote lexicographic order for ∀c ∈ Σ*
i ← 1
while i < ∞ do
    for j ← 1 to i do
        simulate V_II(x, c_j) for i steps
        Accept if V_II(x, c_j)
    end for
end while
```

Let $x \in \mathcal{L}$ exists V s.t V accept $(x, c) \Rightarrow \mathcal{M}_{\mathcal{L}}$ halt and accept at some point. and If $x \notin \mathcal{L}$ then $\mathcal{M}_{\mathcal{L}}$ *does not accept* matter how many steps we run. Hence $L(\mathcal{M}_{\mathcal{L}}) = \mathcal{L}$

(b) Definition III

- If $x \in \mathcal{L}$, then there exists $c \in \Sigma^*$ such that V accept (x, c)
- If $x \notin \mathcal{L}$, then V rejects (x, c) for every $c \in \Sigma^*$
- V *always halt*

III $\Rightarrow$ **I** is trivial, since its weaker condition.

[1] Recitation 8 ex 3

I $\Rightarrow$ **III** Let V define a verifier for some $\mathcal{L}$,same as above by **Def 1** its following that $\mathcal{L} \in \mathcal{RE}$, and let $M_{\mathcal{L}}$ define some TM that accept it. we can write the following:

$$\mathcal{L} = \{x : \exists c \text{ such that } (x, c) \in L(V)\}$$

where V has the property that given input $(x, c), V$ reject any input if $x \notin \mathcal{L}$.

Algorithm 2 V_{III} on input (x, c)

If (x,c) is invalid input then **Reject**
Emulate in parallel $M_{\mathcal{L}}(x)$ and $V(x, c)$
Accept if $M_{\mathcal{L}}$ accept
Reject if V Reject

By the we define the verifier if $x \in \mathcal{L}$ $M_{\mathcal{L}}$ halt and accept. and if $x \notin \mathcal{L}$ then V reject any input i.e halt and reject . Hence V *always halt*

Exercise 2

Let $\mathcal{C} \subset \mathcal{RE}$ be such that $\emptyset \in C$. then

$$\mathcal{L}_{\mathcal{C}} = \{\langle M \rangle : M \text{ is a TM and } \mathcal{L}(M) \in \mathcal{C}\} \notin \mathcal{RE}$$

Since $\mathcal{C} \subset \mathcal{RE}$ then exist some $\mathcal{L}' \in \mathcal{RE}$ such that $\mathcal{L}' \notin \mathcal{C}$. Let $M_{\mathcal{L}'}$ denote TM that accept $\mathcal{L}'$