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CS131

Mathematics for Computer Scientists II

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January 11, 2022

1 Number System

1.1 Binary

Definition 1.1. Binary number system

The binary number system uses the digits $0, 1$ to express itself. In particular the positive integers are represented as:

$$\sum_{i=0}^{n} a2^i \tag{1}$$

where $a \in \mathbb{B}$ and $\mathbb{B} = \{0, 1\}$. Different number systems are usually expressed with subscripts. E.g. 100101_{two}.

1.2 Converting to base n

We can utilise the division algorithm to achieve this. That is, for some base n to convert from base 10 we divide by n to get remainders.

Example 1.1. Division of binary

$$\begin{aligned} 19 \div 2 &= 9R1 && (2) \\ 9 \div 2 &= 4R1 && (3) \\ 4 \div 2 &= 2R0 && (4) \\ 2 \div 2 &= 1R0 && (5) \\ 1 \div 2 &= 0R1 && (6) \end{aligned}$$

1.3 The division algorithm

Theorem 1.1. *The division algorithm*

Given any integers $a, b \in \mathbb{Z}$ and $b \neq 0$, there are unique integers $q, r \in \mathbb{Z}$ such that $a = qb + r$ and $0 \leq r < |b|$.

1.4 The Euclidean algorithm

The euclidean algorithm utilises the division algorithm to find $gcd(m, n) = b$ where $m, n, b \in \mathbb{Z}$.

Definition 1.2. Greatest Common Divisor

The greatest common divisors of two numbers m, n where $m, n \in \mathbb{Z}$ is the greatest number ζ such that $\zeta \mid m$ and $\zeta \mid n$. It is denoted as $gcd(m, n)$.

Then, through division, observe that $n = mb + r$ In particular, the key observation would be $gcd(r, m) = gcd(n, m) = b$. Repeat this process until one of the numbers reaches 0.