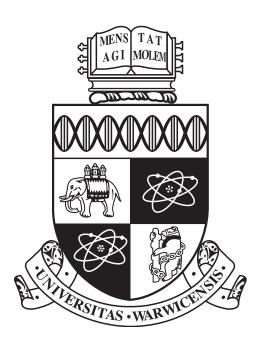
University of Warwick Department of Computer Science

CS131

Mathematics for Computer Scientists II



Cem Yilmaz January 11, 2022

1 Number System

1.1 Binary

Definition 1.1. Binary number system

The binary number system uses the digits 0,1 to express itself. In particular the positive integers are represented as:

$$\sum_{i=0}^{n} a2^{i} \tag{1}$$

where $a \in \mathbb{B}$ and $\mathbb{B} = \{0, 1\}$. Different number systems are usually expressed with subscripts. E.g. 100101_{two} .

1.2 Converting to base n

We can utilise the division algorithm to achieve this. That is, for some base n to convert from base 10 we divide by n to get remainders.

Example 1.1. Division of binary

$$19 \div 2 = 9R1 \tag{2}$$

$$9 \div 2 = 4R1\tag{3}$$

$$4 \div 2 = 2R0 \tag{4}$$

$$2 \div 2 = 1R0 \tag{5}$$

$$1 \div 2 = 0R1 \tag{6}$$

1.3 The division algorithm

Theorem 1.1. The division algorithm

Given any integers $a,b\in\mathbb{Z}$ and $b\neq 0$, there are unique integers $q,r\in\mathbb{Z}$ such that a=qb+r and $0\leq r<|b|$.

1.4 The Euclidean algorithm

The euclidean algorithm utilises the division algorithm to find gcd(m,n) = b where $m,n,b \in \mathbb{Z}$.

Definition 1.2. Greatest Common Divisor

The greatest common divisors of two numbers m, n where $m, n \in \mathbb{Z}$ is the greatest number ζ such that $\zeta \mid m$ and $\zeta \mid n$. It is denoted as gcd(m, n).

Then, through division, observe that n=mb+r In particular, the key observation would be $\gcd(r,m)=\gcd(n,m)=b$. Repeat this process until one of the numbers reaches 0.