Review Session: A Money in Utility Model

1 The MiU Model

In this exercise you will solve the dynamic stochastic general equilibrium model with "money in the utility function" assumption (MiU model). This model is based on the idea that preferences are non-separable between real balances and consumption. Therefore, it differs from a RBC model because it features monetary rigidities. For the sake of simplification, we assume ex-ante that labor, capital and assets markets are clear such that supply = demand.

1.1 The Model

1.1.1 Household's preferences

We consider a discrete-time economy populated by large number of identical households distributed on the unit interval. The representative household selects a sequence of consumptions (c_t), worked hours (h_t), real balances (m_t) and real bond holdings (b_t) such that, for all t,

$$\max_{c_t,h_t,k_{t+1},m_t,b_t} \quad \mathcal{W}_t = \mathrm{E}_0 \sum_{j=0}^{\infty} \beta^j \left[\left(c_{t+j}^{1-\gamma} + m_{t+j}^{1-\gamma} \right)^{\frac{1}{1-\gamma}} - \frac{h_{t+j}^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, with $\gamma > 0$ and $\gamma \neq 1$ is the inverse of the elasticity of money demand with respect to the nominal interest rate, and φ is the Frish parameter. $\mathrm{E}_t\{\cdot\}$ is the expectation operator conditional on the information set available as of time t. Notice that holding money provides utility to the representative household. The representative household maximizes (1) subject to the budget constraint

$$P_t[c_t + i_t] + M_t + B_t \leq P_t w_t h_t + P_t r_{k,t} k_t + R_{B,t-1} B_{t-1} + M_{t-1} + P_t \vartheta_t + P_t T_t, \tag{2}$$

where i_t is real investment; k_t is the stock of capital hold in t; P_t is the price index; $r_{k,t} \equiv R_{k,t}/P_t$ is the real return of capital; $w_t \equiv W_t/P_t$ is the real wage rate; $b_t \equiv B_t/P_t$, where B_t denotes the nominal bonds acquired in period t and maturing in period $t+1$; $R_{B,t-1}$ denotes the *gross* nominal interest rate of the economy (i.e $R_{B,t-1}$ equals the interest rate plus one), M_t is nominal value of cash; ϑ_t denotes profits in real terms redistributed by the firm and T_t denotes lump-sum transfers (taxes if negative). The asset market opens first, implying that the households enter in each period with bonds B_{t-1} and money M_{t-1}.
The law of motion of capital, denoted by k_t, is given by

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{3}$$

where δ is the depreciation rate of capital.

1.1.2 Production sector

A representative firm produces an output good, y_t, by using the labor, h_t, and capital, k_t, inputs. We assume it faces the following Cobb-Douglas production function

$$y_t = k_t^{\alpha} h_t^{1-\alpha}, \tag{4}$$

where $0 < \alpha < 1$ is the capital share in output.

1.1.3 Money growth rule

Let denote $u_t \equiv M_t/M_{t-1}$. We assume that money growth is determined by the process

$$\hat{u}_t = \rho_u \hat{u}_{t-1} + \varepsilon_{u,t},$$

where $\varepsilon_{u,t}$ is a monetary innovation, such that $\varepsilon_{u,t} \sim \text{iid}\,(0, \sigma_u)$.

1.2 Household's Optimization Program

1. Let π_t denote the *gross* inflation rate (i.e one plus the inflation rate). Re-write the budget constraint (2) in real terms.

2. Write the Lagrangian associated to the representative household's program. Let Λ_t denotes the Lagrangian multiplier associated to the budget constraint.

3. Compute the first order conditions (FOCs, hereafter) with respect to household's decisions variables.

4. Let denote $\bar{x}$ as the steady state level of x_t, $\forall\ x_t$, a variable of the model. Compute the model's steady-state based on each FOC. Notice that we assume a zero steady-state inflation rate implying that $\bar{\pi} = 1$ (since π_t is the gross inflation rate). Find the expression of $\bar{c}/\bar{m}$ as a function of β and γ.

5. Let denote $\hat{x}_t \equiv \log(\frac{x_t}{\bar{x}})$, the log-deviation of x_t from it steady state level, $\forall\ x_t$, a variable of the model.

   It can be shown that the Euler equation on consumption can be written as

$$\gamma\left(\hat{m}_t - \hat{c}_t\right) = \left[1 + (1-\beta)^{\frac{1-\gamma}{\gamma}}\right]\hat{\Lambda}_t. \tag{5}$$

   and the money demand equation

$$\hat{m}_t = \hat{c}_t - \frac{1}{\gamma}\left[\frac{1 + (1-\beta)^{\frac{1-\gamma}{\gamma}}}{(1-\beta)^{\frac{1}{\gamma}}}\right]\hat{R}_{B,t}. \tag{6}$$

   Interpret these two equations.

6. Log-linearize the following three equations

$$\Lambda_t = \beta \mathrm{E}_t\left\{\frac{\Lambda_{t+1} R_{B,t}}{\pi_{t+1}}\right\}, \tag{7}$$

$$\Lambda_t = \beta \mathrm{E}_t\left\{\Lambda_{t+1}\left[1 - \delta + r_{k,t+1}\right]\right\}, \tag{8}$$

$$h_t^{\varphi} = \Lambda_t w_t. \tag{9}$$

1.3 Rest of the Model

In this sub-section, we describe the remaining equations of the model.

1.3.1 Firm optimization program

The firms chooses the quantity of capital and labor needed to produce output such that

$$\max_{k_t,h_t} \quad \vartheta_t = k_t^{\alpha} h_t^{1-\alpha} - w_t h_t - r_{k,t} k_t.$$

The FOC reads

$$\frac{\partial \vartheta_t}{\partial k_t} = 0 \Leftrightarrow \alpha \frac{y_t}{k_t} = r_{k,t}, \tag{10}$$

$$\frac{\partial \vartheta_t}{\partial h_t} = 0 \Leftrightarrow (1-\alpha) \frac{y_t}{h_t} = w_t. \tag{11}$$

1. How do you interpret these equations?

The log-linear version of these equations and the production function reads

$$\hat{y}_t - \hat{k}_t = \hat{r}_t^k,$$
$$\hat{y}_t - \hat{h}_t = \hat{w}_t,$$
$$\hat{y}_t = \alpha \hat{k}_t + (1-\alpha)\hat{h}_t.$$

1.3.2 Resource constraint

We abstract from any fiscal policy, i.e. the share of public spending on GDP is zero. The resource constraint of the economy is given by (in log-linear version)

$$\hat{y}_t = \gamma_c \hat{c}_t + \gamma_i \hat{\imath}_t, \tag{12}$$

where γ_c and γ_i are functions of parameters.

1.4 Model's summary

1. The log-linearized MiU model can be summarized as:

$$\gamma \left(\hat{m}_t - \hat{c}_t\right) = \left[1 + (1-\beta)^{\frac{1-\gamma}{\gamma}}\right] \hat{\Lambda}_t. \tag{13}$$

$$\hat{m}_t = \hat{c}_t - \frac{1}{\gamma} \left[\frac{1 + (1-\beta)^{\frac{1-\gamma}{\gamma}}}{(1-\beta)^{\frac{1}{\gamma}}}\right] \hat{R}_{B,t}. \tag{14}$$

$$\hat{\Lambda}_t - \hat{R}_{B,t} = \hat{\Lambda}_{t+1} - \hat{\pi}_{t+1}.$$

$$\left[1 - \beta\left(1-\delta\right)\right] \hat{r}_{k,t+1} = \left(\hat{R}_{B,t} - \hat{\pi}_{t+1}\right).$$

$$\varphi \hat{h}_t = \hat{\Lambda}_t + \hat{w}_t.$$

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta \hat{\imath}_t, \tag{15}$$

$$\hat{y}_t - \hat{k}_t = \hat{r}_{k,t}, \tag{16}$$

$$\hat{y}_t - \hat{h}_t = \hat{w}_t, \tag{17}$$

$$\hat{y}_t = \alpha \hat{k}_t + (1-\alpha)\hat{h}_t, \tag{18}$$

$$\hat{y}_t = \gamma_c \hat{c}_t + \gamma_i \hat{\imath}_t, \tag{19}$$

$$\hat{u}_t = \hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t, \tag{20}$$

$$\hat{u}_t = \rho_u \hat{u}_{t-1} + \varepsilon_{u,t}, \tag{21}$$

Check that the log-linear system contains as many equations as variables.

1.5 Model's calibration

The model's parameters are calibrated as follows:

Parameter		Value
β	Discount factor	0.99
φ	Frisch elasticity	1.00
γ	Risk aversion	2.00
α	Capital share on output	0.33
δ	Depreciation rate of capital	0.025
ρ_u	Smoothing parameter in money rule	0.80
σ_u	s.e. of monetary shock	1.00

2 IRFs to a money growth shock

1. Open the .mod file MiUmodel.mod and write the model into Dynare using the list of variables and the system equations defined into Section 1.4 as well as the calibration table.

2. Plot the IRFs of output, consumption, the nominal interest rate and real balances to a money growth rate shock.

3. Assume that the utility function (1) is replaced

$$\max_{c_t,h_t,k_{t+1},m_t,b_t} \quad \mathcal{W}_t = \mathrm{E}_0 \sum_{j=0}^{\infty} \beta^j \left[\log\left(c_{t+j}\right) + (1-\gamma)\, m_{t+j}^{\frac{1}{1-\gamma}} - \frac{h_{t+j}^{1+\varphi}}{1+\varphi} \right], \tag{22}$$

   Do we have money neutrality in this model? Rewrite the .mod file with the new FOCs with respect to consumption and real balance and compare the IRFs. What do you conclude