

# PROPORTIONAL CALCULUS

GOSSA AHMED  
gossaahmed@yahoo.fr

November12, 2011

## **preface:**

The main objective of deferential calculus is to calculate the deference of all quantities varies we called mathematical functions is that by creating relationships between the infinitely small  $dy_1$ ,  $dy_2$  ... ... ... ... ...  $dy_n$  variations of its functions with infinitely small  $dx$  independent of the value  $x$  following functions can be seen that in the infinitely small that is to say  $dy$   $dx$  tends to 0 is the only right relationship between the deference  $dy$  and  $dx$  is a relationship of multiplication:

$$dy = y'(x)dx$$

( $y'$ ) called deferential derivative connecting the deference  $dy$  at deference  $dx$  We can calculate the quantity of  $dy$  with a simple algebraic operation if the ( $y'$ ) remains constant with  $dx$ . We can not calculate the quantity of  $dy$  when ( $y'$ ) variable with  $dx$  at simple algebraic operation Leibniz and newton had established a new calculus By which we can calculate the quantity  $dy$  when ( $y'$ ) variable with  $dx$  is called deferential calculus the deferential calculus which has made a revolution in the science of mathematics and its applications in the field of physics and chemistry and other In addition, this knowledge led to the emergence of new concepts such as physical sciences and chemical and other the concept of continuity and the infinitely small and derivatives and integration Based on the philosophical principle that says that each quantity is a collection of Quantities infinitesimals deferential Calculus said that the deference and  $D$  equal the sum of deference infinitesimal  $di$  which can be expressed dy the following mathematical expression: ... . . . ....

$$D = d_1 + d_2 + d_3 + d_n$$

$$d_1 = y'(x_1)dx_1$$

$$d_2 = y'(x_2)dx_2$$

$$dn = y'(x_n)dx_n$$

It is through this calculus we managed to calculate areas and volumes and other quantities which can not be calculated by simple algebraic operations. It is through this calculus we finally managed to establish that equations called differential equations by the solutions of its equations we found the functions of all physical quantities such as heat and pressure and forces ... ... After this brief summary of differential calculus, we have the right to ask

is this—that the differential calculus and the only one of the mathematical calculation Which can be computed with him surfaces and forces ... ... Is that—with this calculation can only be expressed with it the chemical physics equations statistics... ... .... Finally, do this calculation is the only language of physical science and chemical ... .... the calculation is not the only one with which we can express the physical and chemical laws there are several mathematical calculations we can express the physical and chemical laws and other science among these calculations it is the calculation of proportions (fractions) and also the logarithmic calculation but the subject of this document only proportional calculus

. The main purpose of proportional calculus is it to calculate the proportions that we denote by R of all quantity that varied called mathematical functions is that by creating relationship between proportions that we denote by  $ry$   $ry_1, ry_2, \dots, ry_n$  tends to 1 with proportions  $rx_1, rx_2, \dots, rx_n$  that tends to 1 as it notes that the value of  $rx$  independent of the value  $x$  after its functions we see that in the field of proportions tends to the 1 that is to say, tends to 1  $ry$   $rx$  tends to 1 is the only right relationship between these proportions  $ry$  and  $rx$  is an exponential relationship that is to say.

$$ry = (rx)^{y^*(x)}$$

$y^*(x)$  is called the proportional derivative connects the proportions ( $rx$ ) and ( $ry$ ) with the proportion that the derivative called the proportional derivative can be constant or varied. So the proportion (R) equals the multiplication of a ( $ry$ ):

$$R = \prod_{i=1}^{i=\infty} ry_i$$

In what follows we are interested only in the functions of one variable  $x$  as well all numbers are real numbers that is to say all  $x$  in  $R$ . and the applications proportional calculus of the functions of several variables and complex numbers we apply them in another document. Also demonstrations of formulas and calculation steps was the fact when you asked me what I want in documents by not only essential to the calculation with concepts such as recalculation derivative proportional. Theorem of the mean proportional. integral proportional development limited by the series of proportion. which gives the approximation faster than Taylor series differential. Also as in the calculus has created equations called the equations Proportional so we will study it in this text and we will make a few resolutions Examples

### principle of proportional calculus :

The proportional calculus is based on the principle that says that every mathematical proportion is R equals the multiplication of infinite proportions (ri) tend to 1 which is expressed mathematically by ry the following equation:

$$R = r_1 \cdot r_2 \cdot r_3 \cdot \dots \cdot r_n$$

so  $r_n$  tend to 1 and n tend to infinity

### **Proportional derivative :**

let  $f(x)$  interval containing x we say that  $f(x)$  has proportional derivative to x if the quantity

$$\frac{\ln r f(x)}{\ln r x}$$

admits a finite limit called proportional derivative at number x we denote this limit by  $f(x)$  which given following equation:

$$rf(x) = (r(x))^{f^*(x)}$$

we have

$$rf(x) = \frac{f(x)}{f(x_1)}$$

as  $x_1$  tend at x and

$$rx = \frac{x}{x_1}$$

as  $x_1$  tend at x

### **proportional derivative of high order :**

As in the deferential calculus are derivative from higher-order functions

$$f', f'', f''', \dots, f^n$$

in the proportional calculus exist also the derivatives proportional of high order :

$$f^*.f^{**}.f^{***} \dots, f^{*n}$$

### **proportional derivative of some functions:**

$$\begin{aligned} f(x) = e^x &\Rightarrow f^*(x) = x \Rightarrow f^{**}(x) = 1 \Rightarrow f^{***}(x) = 0 \\ f(x) = x^2 &\Rightarrow f^*(x) = 2 \Rightarrow f^{**}(x) = 0 \\ f(x) = x^n &\Rightarrow f^*(x) = n \Rightarrow f^{**}(x) = 0 \end{aligned}$$

### **operation on the proportional derivative :**

Let f and g be two real functions of one variable and let f (x) and g (x) is different:

$$\begin{aligned} \left(\frac{f(x)}{g(x)}\right)^* &= f^*(x) - g^*(x) \\ (f(x).g(x))^* &= f^*(x) + g^*(x) \\ ((f(x))^k)^* &= k.(f(x))^* \end{aligned}$$

example:

$$\begin{aligned} (3x)^* &= (3)^* + (x)^* = 0 + 1 = 1 \\ (x^9)^* &= 9(x)^* = 9(1) = 9 \end{aligned}$$

. fundamental relationship between the derivative and proportional derivative differential changes in the subtraction. Following are true:

After some simple calculation we find the following relation which links the derivative proportional  $f(x)$  and that  $f(x)$  and that derivative differential  $f'(x)$  and  $x$  this independent value

$$f^*(x).f(x) = x.f'(x)$$

relationship is very important Especially in solving differential equations . When we convert the equations proportional And also to calculate the derivatives according to the proportional derivative differential. ... ...

### **Examples of proportional derivatives :**

Calculate the derivatives of the following functions proportional:

$$f(x) = (\ln x).e^x/(x+1)$$

According to the logarithmic property of the derivative was proportionally

$$(\ln(x))^* + (e^x)^* - (x+1)^* = (1/\ln x) + x - (x/(x+1))$$

$$f(x) = (x+1)^k/(\ln x)^c$$

According to the logarithmic property of the derivative was proportional

$$f(x) = ((x+1)^k)^* - ((\ln x)^c)^* = k(x+1)^* c(\ln x)^* = k(x/(x+1)) - c/\ln(x)$$

**Theorem of the proportional mean:**

Let  $f$  be a function defined on a segment  $[a, b] \subset \mathbb{R}$

\*  $f$  is continuous on  $[a, b]$

\*  $f$  is differentiable proportional  $\left] a, b \right[$

then there exists at least one number  $c$  in  $\left] a, b \right[$  [as:

$$\frac{f(b)}{f(a)} = \left(\frac{b}{a}\right)^{f^*(c)}$$

. Example:

$$f(x) = e^x$$

.  $f(x)$  is continuous in the interval  $[10, 8]$  was based on the theorem of the mean proportional :

$$(f(10))/(f(8)) = (10/8)^{f^*(c)}$$

was

$$f^*(x) = (e^x)^* = x$$

$$f^*(c) = \ln(f(10)/f(8))/\ln(10/8) \Rightarrow c = 8,964[10, 8]$$

**Proportional series :**

In differential calculus the Taylor series of a function  $f$  near at point  $(a)$  is an entire series built from  $f$  its successive differential derivatives were:

$$f(x) - f(a) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \dots + \frac{f^n(a)(x-a)^n}{n!}$$

Note that the Taylor series is built by  $f$  deference of:

$$(f(x) - f(a))$$

equals the sum of deference independent value:

$$(x - a)$$

multiplied by its successive deferential derivatives at point  $a$  But in proportional calculus  $(f(x)-f(a))$  is converted to the proportion (fraction) there:

$$\frac{f(x)}{f(a)}$$

and the deference of the value  $(x-a)$  is converted to independent rx proportion that is to say:

$$\frac{x}{a}$$

the derivative successive multiplication of differential derivative to convert to the exponent proportional derivative from successive we find the following relation

$$\frac{f(x)}{f(a)} = \left(\frac{x}{a}\right)^{f^*(a)} \cdot \left(\frac{x}{a}\right)^{\frac{f^*(a) \cdot f^{**}(a) \cdot (\ln(\frac{x}{a}))}{1!}} \cdot \left(\frac{x}{a}\right)^{\frac{f^*(a) \cdot f^{**}(a) \cdot (f^*(a) \cdot f^{**}(a))^* \cdot (\ln(\frac{x}{a}))^2}{2!}} \dots$$

So after simplification we can write it like this:

$$\frac{f(x)}{f(a)} = 1 + \frac{f^*(a) \ln(\frac{x}{a})}{1!} + \frac{f^*(a)((f^*(a)f(a))^*(\ln(\frac{x}{a}))^2)}{2!} + \dots$$

As done in the Maclaurin series of Taylor replaced (a) by 0 in the Proportional series is replaced by 1 include:

$$\frac{f(x)}{f(1)} = 1 + \frac{f(1)^* \ln(x)}{1!} + \frac{f^*(1).((f(1)^* f(1))^*(\ln(x))^2)}{2!} + \dots$$

### **example of Development the function by the proportional series:**

$$f(x) = \sqrt{x}$$

About the calculations are:

$$\frac{\sqrt{x}}{1} = 1 + \frac{\ln x}{2} + \frac{(\ln x)^2}{4.2!} + \frac{(\ln x)^3}{8.3!} + \dots + \frac{(\ln x)^n}{2^n.n!}$$

Witt made a comparison with the Taylor series:

$$\sqrt{2}$$

is calculated by the Taylor series:

$$\sqrt{(x+1)} = 1 + \frac{x}{2} - \frac{1}{8.x^2} + \frac{1}{16.x^3} - \frac{5}{128.x^4}$$

To calculate

$$\sqrt{2}$$

we set  $x = 1$

$$\sqrt{2} = 1 + 0.5 - 0.125 + 0.0625 - 0.0390625 = 1.3984375$$

is calculated by the proportional series:

$$1 + \ln 2 / 2 + (\ln 2)^2 / 8 + (\ln 2)^3 / 48 + ((\ln 2))^4 / 384 = 1 + 0.34657359 + \\ 0,06005662 + 0,00693801 + 6,011 \cdot 10^{-4} = 1.41416$$

We see that

$$\sqrt{2} = 1.41421$$

According to the Taylor series we found

$$\sqrt{2} = 1.3984375$$

According to the series was found proportional

$$\sqrt{2} = 1.41416$$

According to the Taylor series of the error calculation is

$$1.41421 - 1.3984375 = 0.0157725$$

According to the proportional error range

$$1.41421 - 1.41416 = 0.00005$$

Look at the great deference

### **proportional Primitive :**

As in the deferential calculus  $F(x)$  is primitive proportional of  $f(x)$  is that we derive  $F(x)$  in proportion we find  $f(x)$  which is written mathematically by the following equation:

$$(F(x))^* = f(x)$$

Examples:

$$f(x) = x$$

the proportional primitive are:

$$F(x) = k \cdot e^x$$

with  $k$  any real

$$f(x) = 1 / \ln x$$

the proportional primitive are:

$$F(x) = k \cdot \ln x$$

with k any real

$$f(x) = 2$$

- proportional primitive are:

$$F(x) = k \cdot x^2$$

with k any real

$$f(x) = x/(x + 1)$$

the proportional primitive are:

$$F(x) = k(x + 1)$$

with k any real

$$f(x) = x/\operatorname{tg} x$$

proportional primitive are:

$$F(x) = k \cdot \sin x$$

with k any real

### Proportional integral :

To better understand the full proportional integral we will compare it with the full differential integral which we understand well. In the deferential calculus to calculate the integral of f (x) from c to d we calculate the differential primitive:

$$F(x)$$

then find the following deference:

$$(F(d) - F(c))$$

that it should be noted by the following mathematical relationship :

$$F(d) - F(c) = \int_c^d f(x) dx$$

is an infinite sum of deference that the worm tends (0) In the proportional calculus to calculate the proportional integral f (x) from c to d we calculate the primitive proportional F(x) then made calculate the following division:

$$F(d)/F(c)$$

it should be noted that the mathematical relationship follows:

$$\frac{F(d)}{F(c)} = \{_c^d(rx)^{f(x)}}$$

designated the multiplication of infinite proportions which tends to 1. We see that the deference:

$$F(d) - F(c)$$

changes in proportion

$$F(d)/F(c)$$

We see that the deference:

$$dx$$

is transformed proportion:

$$rx$$

We change the sign of the sum:

$$\int$$

by the new multiplication sign:

$$\{$$

has to distinguish the differential calculus

### **example of proportional integral:**

Calculates the proportional integral at the function:

$$f(x) = x$$

2 to 4 we note that mathematically by the following equation:

$$\{_2^4(rx)^x$$

This integral equals proportionate:

$$F(4)/F(2)$$

or  $F(x)$  is proportional primitive of  $f(x)$  We calculated the primitive proportional was equal:

$$\begin{aligned} F(x) &= k \cdot e^x \\ e^4/e^2 &= e^{(4-2)} = e^2 \end{aligned}$$

So the integral proportional at 2 to 4 equal:  $e^2$

**proportional integral laws:**

$$\begin{aligned}\{^a_a(rx)\}^{f(x)} &= 1 \\ (\forall f(x), a, \in \mathbb{R}) \\ \{^b_a(rx)\}^{f(x)} \{^a_b(rx)\}^{f(x)} &= 1 \\ \forall f(x), a, b, \in \mathbb{R} \end{aligned}$$

**Proportional equations:**

equations are proportional to the relationships that bind a function  $f(x)$  with proportional derivatives:

$$F(f \cdot f^* \cdot f^{**} \cdots f^{*n})$$

by mathematical equations:.....

$$F(f \cdot f^* \cdot f^{**} \cdot f^{***} \cdots f^{*n}) = R(x)$$

. Examples of proportional equations:

$$y^*x + y = 0$$

y are unknown

$$y^{**} + y^*x + y = tg(x)$$

y are unknown

**Examples of convert the differential equations to proportional equation:**

Transforms the differential equation in the following equation proportional:

$$y' + yx = 0$$

The fundamental relationship this equation becomes:

$$y^* \cdot y = x \cdot y'$$

to solve it out common factor multiplication:

$$y \cdot \left( \frac{y'x}{y} + x^2 \right) = 0$$

was

$$y^* = y'x/y \Rightarrow yy^* + yx^2 = 0 \Rightarrow y(y^* + x^2) = 0 \Rightarrow y = 0$$

and

$$(y^* + x^2) = 0$$

which implies :

$$y^* = -x^2$$

So the solutions are:

$$y_1 = 0$$

and

$$y_2 = ke^{(-\frac{x^2}{2})}$$