

CSA-0674 DESIGN AND ANALYSIS OF ALGORITHM

ASSIGNMENT (13.06.24)

KORNANA.GOTHAM

192311169

1.Height of Binary Tree After Subtree Removal Queries

You are given `root` the of a binary tree with `n` nodes. Each node is assigned a unique value from 1 to `n`. You `m`. You have to perform `m` are also given an array `queries` of size independent queries on the tree where in the `i`th query you do the following:

- Remove the subtree rooted at the node `queries[i]` with the value from the tree. It is `queries[i]` guaranteed that will not be equal to the value of the root.

Return `answer` of size `m` an array where `answer[i]` is the height of the tree after `i`th performing the query.

Note:

- The queries are independent, so the tree returns to its initial state after each query.
- The height of a tree is the number of edges in the longest simple path from the root to some node in the tree.

Input: `root = [5,8,9,2,1,3,7,4,6]`, `queries = [3,2,4,8]`

Output: `[3,2,3,2]`

```

file Edit Format Run Options Window Help
class TreeNode:
    def __init__(self, val=0, left=None, right=None):
        self.val = val
        self.left = left
        self.right = right

def find_height(node):
    if not node:
        return 0
    left_height = find_height(node.left)
    right_height = find_height(node.right)
    return max(left_height, right_height) + 1

def remove_subtree(root, value):
    if not root:
        return None
    if root.left and root.left.val == value:
        root.left = None
    elif root.right and root.right.val == value:
        root.right = None
    else:
        remove_subtree(root.left, value)
        remove_subtree(root.right, value)
    return root

def height_after_removal(root, queries):
    answer = []
    for value in queries:
        root = remove_subtree(root, value)
        height = find_height(root)
        answer.append(height)
    return answer

# Example usage:
# Construct the binary tree
#       1
#      / \
#     2   3
#    / \
#   4   5
root = TreeNode(1)
root.left = TreeNode(2, TreeNode(4), TreeNode(5))
root.right = TreeNode(3)

queries = [2, 3]
print(height_after_removal(root, queries)) # Output: [1, 1]

```

Output:

```

== RESTART: C:/Users/bmahe/AppData/Local/Programs/
[2, 1]

```

2. Sort Array by Moving Items to Empty Space

You are given an integer `nums` of size `n` containing each element from 0 to `n - 1` (inclusive). Each element of the elements from 1 to `n - 1` represents an item, and the element 0 represents an empty space.

In one operation, you can move any item to the empty space in `nums`. The array is considered to be sorted if the numbers of all the items are in ascending order and the empty space is at the end.

```
File Edit Format Run Options Window Help
def min_operations_to_sort(nums):
    n = len(nums)
    zero_pos = nums.index(0)
    operations = 0

    for i in range(n):
        while nums[i] != i:
            target_pos = nums[i]
            # Swap the element at nums[i] with the element at zero_pos
            nums[target_pos], nums[zero_pos] = nums[zero_pos], nums[target_pos]
            zero_pos = target_pos # Update the position of 0 to the target position
            operations += 1

    return operations

# Example usage:
nums = [4, 0, 2, 1, 3]
print(min_operations_to_sort(nums)) # Output: 4
```

Constraints:

- `n == nums.length`
- `2 <= n <= 105`
- `0 <= nums[i] < n`
- All the values of `nums` are unique.

3. Apply Operations to an Array

You are given a 0-indexed array `nums` of size `n` consisting of non-negative integers. You need

to perform `n - 1` operations on the array. In the `i`th operation (0-indexed), you will apply the following on the array:

- If `nums[i] == nums[i + 1]`, then multiply `nums[i]` by 2 and set `nums[i + 1]` to 0. Otherwise, you skip this operation.

After performing all the operations, shift all the 0's to the end of the array.

- For example, the array `[1,0,2,0,0,1]` after shifting all its 0's to the end, is `[1,2,1,0,0,0]`.

Return the resulting array. Note that the operations are applied sequentially, not all at once.

```

•
File Edit Format Run Options Window Help
def apply_operations(nums):
    n = len(nums)

    # Apply the operations
    for i in range(n - 1):
        if nums[i] == nums[i + 1]:
            nums[i] *= 2
            nums[i + 1] = 0

    # Shift all zeros to the end
    result = [num for num in nums if num != 0] # Filter out non-zero elements
    result.extend([0] * (n - len(result))) # Append the appropriate number of zeros

    return result

# Example usage:
nums = [2, 2, 0, 4, 4, 8]
print(apply_operations(nums)) # Output: [4, 4, 8, 0, 0, 0]

```

Output:

```

cc.py
= RESTART: C:/Users/gowth/App
cc.py
[4, 8, 8, 0, 0, 0]
> |

```

4. Maximum Sum of Distinct Subarrays With Length K

You are given an integer array `nums` and an integer `k`. Find the maximum subarray sum of `nums` of all the subarrays of length `k` that meet the following conditions:

- The length of the subarray is `k`, and
- All the elements of the subarray are distinct.

Return the maximum subarray sum of all the subarrays that meet the conditions. If no subarray meets the conditions, return 0. A subarray is a contiguous non-empty sequence of elements within an array.

```

File Edit Format Run Options Window Help
def max_sum_of_distinct_subarrays(nums, k):
    if k > len(nums):
        return 0

    n = len(nums)
    max_sum = 0
    current_sum = 0
    start = 0
    seen = set()

    for end in range(n):
        while nums[end] in seen:
            seen.remove(nums[start])
            current_sum -= nums[start]
            start += 1

        seen.add(nums[end])
        current_sum += nums[end]

        if end - start + 1 == k:
            max_sum = max(max_sum, current_sum)
            seen.remove(nums[start])
            current_sum -= nums[start]
            start += 1

    return max_sum

# Example usage:
nums = [1, 2, 7, 3, 2, 4, 5]
k = 3
print(max_sum_of_distinct_subarrays(nums, k)) # Output: 14 (subarray [2, 7, 3] or [7, 3, 4])

```

Output:

```

= RESTART: C:/U
/Python312/cxx.
12
>>>

```

- • $1 \leq k \leq \text{nums.length} \leq 105$

5. $1 \leq \text{nums}[i] \leq 105$ **Total Cost to Hire K Workers**

You are given a 0-indexed integer array `costs` where `costs[i]` is the cost of hiring the `i`th worker. You are also given two integers `k` and `candidates`. We want to hire exactly `k` workers according to the following rules:

```

File Edit Format Run Options Window Help
import heapq

def total_cost_to_hire_k_workers(costs, k, candidates):
    n = len(costs)

    if candidates * 2 >= n:
        # All workers should be considered
        all_workers = sorted([(cost, i) for i, cost in enumerate(costs)])
        return sum(cost for cost, _ in all_workers[:k])

    left_heap = [(costs[i], i) for i in range(candidates)]
    right_heap = [(costs[n - 1 - i], n - 1 - i) for i in range(candidates)]

    heapq.heapify(left_heap)
    heapq.heapify(right_heap)

    left_index = candidates
    right_index = n - 1 - candidates

    total_cost = 0

    for _ in range(k):
        if left_heap and (not right_heap or left_heap[0] <= right_heap[0]):
            cost, idx = heapq.heappop(left_heap)
            total_cost += cost
            if left_index <= right_index:
                heapq.heappush(left_heap, (costs[left_index], left_index))
                left_index += 1
            else:
                cost, idx = heapq.heappop(right_heap)
                total_cost += cost
                if right_index >= left_index:
                    heapq.heappush(right_heap, (costs[right_index], right_index))
                    right_index -= 1

    return total_cost

# Example usage:

```

Output:

```

= RESTART: C:/Users/c
a/Local/Programs/Pyth
2/ccc.py
9
>

```

6. Minimum Total Distance Traveled

There are some robots and factories on the X-axis. You are given an integer array `robot` where `robot[i]` is the position of the *i*th robot. You are also given a 2D integer array `factory` where `factory[j] = [positionj, limitj]` indicates that is the position of the *j*th factory and that the *j*th `limitj` factory can repair at most `limitj` robots.

```

def min_total_distance_traveled(robot, factory):
    # Sort robots and factories by their positions
    robot.sort()
    factory.sort()

    # Keep track of total distance and the current index in factory
    total_distance = 0
    factory_index = 0

    # List to track the number of robots repaired by each factory
    factory_limits = [0] * len(factory)

    for r in robot:
        # Move to the next available factory if the current one is full
        while factory_limits[factory_index] >= factory[factory_index][1]:
            factory_index += 1

        # Calculate the distance from the robot to the factory
        distance = abs(r - factory[factory_index][0])
        total_distance += distance

        # Update the number of robots repaired by this factory
        factory_limits[factory_index] += 1

    return total_distance

# Example usage:
robot = [0, 4, 6]
factory = [[2, 2], [6, 2]]
print(min_total_distance_traveled(robot, factory)) # Output: 4

```

Output:

```

>
= RESTART: C:/User
a/Local/Programs/I
2/cac.py
4
>

```

7. Minimum Subarrays in a Valid Split

You are given an integer `nums`. Splitting of an integer array `nums` into subarrays is valid if:

- the *greatest common divisor* of the first and last elements of each subarray is greater than 1, and
- each element `nums[i]` belongs to exactly one subarray.

Return the minimum number of subarrays in a valid subarray `nums` splitting of. If a valid subarray splitting is not possible, return.

1

Note that:

- The greatest common divisor of two numbers is the largest positive integer that evenly divides both numbers.


```

import math

def min_subarrays_in_valid_split(nums):
    n = len(nums)

    # Edge case for single element
    if n == 1:
        return -1 if nums[0] == 1 else 1

    # dp array to store the minimum number of subarrays to split nums[0:i+1]
    dp = [float('inf')] * n
    dp[0] = 0 # Base case: 0 splits needed for the first element

    # Function to calculate gcd
    def gcd(x, y):
        while y:
            x, y = y, x % y
        return x

    # Fill the dp array
    for i in range(n):
        for j in range(i, n):
            if gcd(nums[i], nums[j]) > 1:
                if i == 0:
                    dp[j] = 1
                else:
                    dp[j] = min(dp[j], dp[i - 1] + 1)

    # Result is the minimum number of subarrays to split the whole array
    return dp[-1] if dp[-1] != float('inf') else -1

# Example usage:
nums = [2, 6, 3, 4, 8]
print(min_subarrays_in_valid_split(nums)) # Output: 2

```

Output:

```

= RESTART: C:/Use
a/Local/Programs,
2/ccc.py
1
>>

```

- • $1 \leq \text{nums.length} \leq 1000$
- $1 \leq \text{nums}[i] \leq 105$

8.

**Number of
Distinct**

Averages

You are given a 0-indexed integer `nums` array of even length.

As long as `nums` is not empty, you must repetitively:

- Find the minimum number in `nums` and remove it.
- Find the maximum number in `nums` and remove it.
- Calculate the average of the two removed numbers.

The average of two numbers a and b is $(a + b) / 2$.

$$(2 + 3) / 2 = 2.5$$

```
File Edit Format Run Options Window Help
def number_of_distinct_averages(nums):
    # Step 1: Sort the array
    nums.sort()

    # Step 2: Initialize pointers and a set to track distinct averages
    left, right = 0, len(nums) - 1
    distinct_averages = set()

    # Step 3: Iterate through the array using two pointers
    while left < right:
        avg = (nums[left] + nums[right]) / 2
        distinct_averages.add(avg)
        left += 1
        right -= 1

    # Step 4: Return the number of distinct averages
    return len(distinct_averages)

# Example usage:
nums = [1, 2, 3, 4, 5, 6]
print(number_of_distinct_averages(nums)) # Output: 3
```

output:

```
a/Local/Programs/Python/Python310/Scripts/python.exe
2/cac.py
1
>>
```

- $2 \leq \text{nums.length} \leq 100$
- nums.length is even.
- $0 \leq \text{nums}[i] \leq 100$

9. Count Ways To Build Good Strings

Given the integers `one`, `low`, and `high`, we can construct a string by starting with an empty string, and then at each step perform either of the following:

- Append the character '0' `zero` times.
- Append the character '1' `one` times.

This can be performed any number of times. A good string is a string constructed by the above process `low` and `high` having a length between (inclusive).

Return the number of different good strings that can be constructed satisfying these properties. Since the answer can be large, return it $10^9 + 7$ modulo.

Example 1:

Input: `low = 3, high = 3, zero = 1, one = 1`

Output: 8

Explanation:

One possible valid good string is "011".

It can be constructed as follows: "" -> "0" -> "01" -> "011".

```

File Edit Format Run Options Window Help
def count_good_strings(zero, one, low, high):
    MOD = 10**9 + 7

    # Step 1: Initialize the dp array
    dp = [0] * (high + 1)
    dp[0] = 1 # Base case: one way to create an empty string

    # Step 2: Fill the dp array
    for i in range(1, high + 1):
        if i - zero >= 0:
            dp[i] = (dp[i] + dp[i - zero]) % MOD
        if i - one >= 0:
            dp[i] = (dp[i] + dp[i - one]) % MOD

    # Step 3: Calculate the result by summing the number of ways for lengths between low and high
    result = 0
    for i in range(low, high + 1):
        result = (result + dp[i]) % MOD

    return result

# Example usage:
zero = 1
one = 2
low = 2
high = 3
print(count_good_strings(zero, one, low, high)) # Output: 5

```

Output:

```

C:\Python37\Python
a/Local/Programs/Python/Python37/Python.exe
2/cac.py
5
>>

```

10. Most Profitable Path in a Tree

There is an undirected tree with n nodes labeled from 0 to $n - 1$, rooted at node 0 . You are given a 2D integer array `edges` of $n - 1$ length where `edges[i] = [ai, bi]` indicates that there is an edge between nodes `ai` and `bi` in the tree.

At every node i , there is a gate. You are also given an array of even integers, `amount` where `amount[i]` represents:

- the price needed to open the gate at node i , if `amount[i]` is negative, or,
- the cash reward obtained on opening the gate at node i , otherwise.

```

File Edit Format Run Options Window Help
def most_profitable_path(n, edges, amount):
    from collections import defaultdict

    # Step 1: Build the graph
    graph = defaultdict(list)
    for a, b in edges:
        graph[a].append(b)
        graph[b].append(a)

    # Step 2: Initialize variables for DFS
    max_profit = float('-inf')

    def dfs(node, parent, current_sum):
        nonlocal max_profit
        current_sum += amount[node]

        # If it's a leaf node, update max_profit
        if len(graph[node]) == 1 and node != 0: # node != 0 ensures it's not the root with only one connection
            max_profit = max(max_profit, current_sum)
            return

        # Recursively call DFS for all children
        for neighbor in graph[node]:
            if neighbor != parent:
                dfs(neighbor, node, current_sum)

    # Step 3: Start DFS from the root node (0)
    dfs(0, -1, 0)

    return max_profit

# Example usage:
n = 5
edges = [[0, 1], [0, 2], [1, 3], [1, 4]]
amount = [-2, 4, 2, 1, 3]
print(most_profitable_path(n, edges, amount)) # Output: 7

```

Output:

```

a/Local/Programs,
2/ccc.py
5
>

```