Design and Analysis of Algorithms

L21: Job Scheduling

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Resources

- Text book 2: Sec 4.1, 4.3, 4.4
- Text book 1: Sec 9.1-5.4 - Levitin
- R1: Introduction to Algorithms
  - Cormen et al.
- MIT Open Course Ware
  – https://www.geeksforgeeks.org/job-sequencing-using-disjoint-set-union/

Example Case

- In college fest which starts at 9:00am, there are a number of available events as below to participate, and each event takes `1` unit of time (e.g. 1hr).
  - Each event has different awards values
  - Each event has its own closing timeline.

Event	Closing	Award
`Mimcry`	12:00	200
`Drama`	11:00	100
`Painting`	12:00	90
`Dance`	10:00	50
`JAM`	11:00	125
`Singing`	10:00	60

Deadline
3
2
3
1
2
1

Q: What is the max award you can get?

Greedy Job Scheduling

- A set of n jobs to run on a computer
- Each job i has a deadline $d_i \geq 1$ and profit $p_i \geq 0$
- There is only one computer
- Each job takes one unit of time (simplification)
- Profit is earned when job is completed by deadline
- Find the subset of jobs that maximizes the profit, i.e.

  Maximize $\Sigma_{i \in J} P_i$

Note: It belongs to subset paradigm since we are looking at subset of jobs.

Example: Job Scheduling

Job	Profit	Dead-line
1	100	2
2	10	1
3	15	2
4	27	1

Feasible Solutions	Profit
1	100
2	10
3	15
4	27
1,2	110
1,3	115
1,4	127
2,3	25
3,4	42

Optimal Solution: 1,4

Job Scheduling: Greedy Approach

- What should be the optimization measure to schedule the next job?
- First attempt:
  - Choose $\Sigma_{i \in J} P_i$ as the optimization measure
  - i.e. choose a job that increases this value maximum
    - Subject to constraint of the deadline i.e. J (set of jobs) should be feasible solution.
  - How to choose jobs:
    - Order jobs in decreasing order of profit
    - Choose job one at a time as per this order and add to the solution if solution remains feasible.

Job Scheduling: Greedy Approach

Job	Pro fit	De ad- line
1	100	2
4	27	1
3	15	2
2	10	1

- Application of First Greedy approach
  - Job 1 is added to J. Feasible {1}
  - Next: Job 4 is considered as per order.
    - Is set J={1,4} feasible.
      - Yes if schedule is 4-1, No if 1-4
    - Thus {1,4} is feasible solution.
  - Next: Job 3 is considered,
    {1,4,3} is infeasible, thus J remains {1,4}
  - Next: Job 2 is considered
    {1,4,2} is infeasible thus J remains {1,4}
  - The max profit is 127 for J={1,4}
- Time complexity :
  - Evaluate feasibility for a given set: n!

Job Scheduling: Feasible Solution

- How to determine that a given set of jobs constitute feasible solution.
- Try out all possible permutations in jobs J
  – Check for each permutation if jobs can be scheduled meeting the deadlines.
- Easy to check for a given permutation $\sigma = i_1, i_2, \ldots, i_k$
  - Job i_q must be completed by time q, $1 \le q \le k$
  - If for some job i_q, $q > d_{i_q}$,then job i_q is not completed by d_{i_q}.
- When $|J| = k$, all $k!$ permutations must be checked
- Can we find one permutation that meets the need?
  - Order the jobs in non-decreasing order of deadlines

Proof for Feasible Solution

- Theorem 1:
  - Let J be the set of k jobs and $\sigma = i_1, i_2, \ldots, i_k$ is a permutation of jobs in J such that $d_{i_1} \le d_{i_2} \le \ldots \le d_{i_k}$.

    Then J is a feasible solution if and only if (*iff*) the jobs in J can be processed in the order σ without violating any deadline.
- Theorem 2:
  - The greedy method (order jobs in non-increasing order of profit) always obtains an optimal solution to the job scheduling problem.

Algo High Level

```
Algo GreedyJob(int d[], set J, int n) {
    // J is set of jobs that can be completed in deadlines d[]
    J={1}
    for i=2 to n {
        if all jobs in J U {i} can be completed, then
            // by their deadlines
            J = J U {i}
    }
}
```

Algo 01: Example (Ex 3a, Bk-2)

- n = 7 jobs, J_1, J_2, J_3, J_4, J_5, J_6, J_7
- Profits: $(p_1,p_2,p_3,p_4,p_5,p_6,p_7)$ =3,5,20,18,1,6,30
- deadlines: $(d_1,d_2,d_3,d_4,d_5,d_6,d_7)$ =1,3,4,3,2,1,2
- Deadlines sorted in non-increasing order of profit

i	1	2	3	4	5	6	7
P_i	30	20	18	6	5	3	1
D_i	2	4	3	1	3	1	2
J_i	J_7	J_3	J_4	J_6	J_2 X	J_1 X	J_5 X

0	1	2	3	4	5	6	7
	J_7 J_6	J_3 J_4					
	30	20	18	6			

J_4

Algo-1: Job Scheduling

int `JobSchedule2`(*int* `d[]`, *int* `J[]`, *int* `n`) {

//$n \geq 1$, and deadlines $d[i] \geq 1$, $1 \leq i \leq n$

//Jobs are ordered such that their profits are in non-increasing order i.e. $p[1] \geq p[2] \geq \ldots \geq p[n]$.

//`J[i]` is the i^{th} job in the optimal solution with $k \leq n$ jobs

// At algo termination, $d[J[i]] \leq d[j[i+1]]$, $1 \leq i < k$

// Initialize

`d[0] = 0` // fictitious job with deadline of 0

// allows for job insertion at position `1` later.

`J[0] = 0` // this job is boundary and can't be scheduled

`J[1] = 1` // start with job `1` with highest profit

`k = 1` // job set size is `1` to start with

Algo1: Job Scheduling

```
for(i=2; i≤n; i++) {
  // consider jobs in non-increasing order of p[i]
  // find pos for J[i] and check for feasibility of insertion
  int r = k //job set size
  while ((d[J[r]]>d[i]) &&(d[J[r]!=r))
      r–; //find position where job i can be considered.
  if ((d[J[r]]≤d[i]) &&(d[i]>r)){
      //insert i into J[]
      for (int q=k; q≥(r+1); q--)
          J[q+1]=J[q] // increase deadline of jobs by 1.
      J[r+1]=i
      k++ // since job i is feasible, increase the set size.
  }//end if
}//end for i
return k
}
```

Algo-1: Time Complexity

- For loop run n times.
  - Each job needs to be considered.
- if K is the value of max deadline, then
  - Inside while loop plus for loop (for shifting slots) may run K times.
- Time complexity: $O(nK)$
- Considering K is of order of n (if all jobs can be scheduled)
- Time complexity: $O(n^2)$

Algo 02: Example (Ex 3a, Bk-2)

- n = 7 jobs, J_1, J_2, J_3, J_4, J_5, J_6, J_7
- Profits: $(p_1,p_2,p_3,p_4,p_5,p_6,p_7) = 3, 5, 20, 18, 1, 6, 30$
- deadlines: $(d_1,d_2,d_3,d_4,d_5,d_6,d_7) = 1, 3, 4, 3, 2, 1, 2$
- Deadlines sorted in non-increasing order of profit

i	1	2	3	4	5	6	7
P_i	30	20	18	6	5	3	1
D_i	2	4	3	1	3	1	2
J_i	J_7	J_3	J_4	J_6	J_2 X	J_1 X	J_5 X

0	1	2	3	4	5	6	7
	J_6	J_7	J_4	J_3			
	30	20	18	6			

J_4

Algo 02: Example (Ex 4.6, Bk-2)

- n = 5 **jobs,** J_1, J_2, J_3, J_4, J_5
- **Profits:** $(p_1,p_2,p_3,p_4,p_5) = 20,15,10,5,1$
- **deadlines:** $(d_1,d_2,d_3,d_4,d_5) = 2,2,1,3,2$
- Deadlines sorted in non-increasing order of profit

i	1	2	3	4	5
P_i	20	15	10	5	1
D_i	2	2	1	3	2
J_i	J_1	J_2	J_3 X	J_4	J_5 X

0	0-1	1-2	2-3	3-4	4-5
	J_2	J_1	J_4		
	15	20	5		

Algo-2: Job Scheduling

//Approach: schedule a job in the slot where it meets deadline.
// If no slot is available before deadline, then job is not scheduled.
// jobs are ordered in non-increasing order as per deadlines.

int `JobSchedule-1`(*int* `d[]`, *int* `j[]`, *int* `n`) {

//$n \geq 1$, and deadlines $d[i] \geq 1$, $1 \leq i \leq n$

//Jobs are ordered such that their profits are in non-increasing order i.e. $p[1] \geq p[2] \geq \ldots \geq p[n]$.

//`Job[i]` is i^{th} job in the optimal solution with $k \leq n$ jobs

// At algo termination, $d[Job[i]] \leq d[job[i+1]]$, $1 \leq i < k$

// Initialization

```
k=0;  // size of Job schedule
for i=1 to n
   slot[i]=False // all slots are initialized to false
```

Algo-2: Job Scheduling

```
for (i=1; i≤n; i++) {
  // consider jobs in non-increasing order of p[i]
  //check if any slot available before deadline
  while (j=d[i]; j>0; j—) {
      //find position where job i can be considered.
      if (slot[j] == False{
        //Add jobs to the slot
        slot[j] = True;
        Job[j] = i;
        k++;
        break;  // from while
      }//end if
    }//end while
  } // end for
  return  k
}
```

Algo-2: Time Complexity

- For loop run `n` times.
  – Each job needs to be considered.
- if `K` is the value of max deadline, then
  – while loop may run `K` times.
- Time complexity: $O(nK)$
- Considering `K` is of order of `n` (if all jobs can be scheduled)
- Time complexity: $O(n^2)$

Fast Job Scheduling (Union-Find)

- Let `i` denote the timeslot `i`
  - At the start time, each time slot is its own set
- There are `m` timeslots, where

  $m = \min(n, \max(d_i))$ #i.e. the latest deadline

  example: $n=4$, and $(d_1, d_2, d_3, d_4)=2,5,3,2$

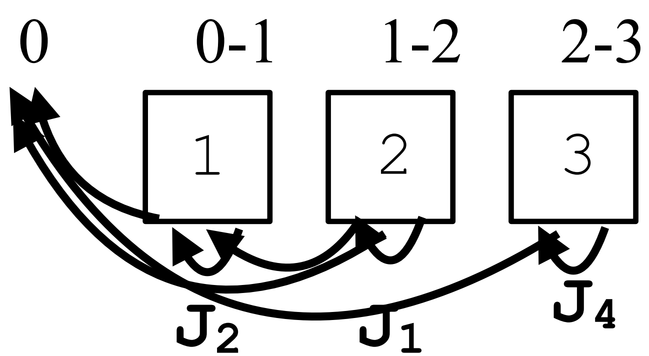
  $m = \min(4,5)=4$

  example: $n=4$, and $(d_1, d_2, d_3, d_4)=2,3,1,2$

  $m = \min(4,3)=3$
- Each set of `k` slots has a value `F(k)` for all slots `i` in set `k`
  - `F(k)` : Stores highest free timeslot before this time
  - `F(k)` : Defined only for root node in set
- Initially all slots are free

Algo 02: Example (Ex 4.6, Bk-2)

- $n = 5$ **jobs**, J_1, J_2, J_3, J_4, J_5
- **Profits:** $(p_1,p_2,p_3,p_4,p_5) = 20, 15, 10, 5, 1$
- **deadlines:** $(d_1,d_2,d_3,d_4,d_5) = 2, 2, 1, 3, 2$
- Deadlines sorted in non-increasing order of profit



Path Compression

Summary

- Job Scheduling
  - Greedy approach: Schedule as per profit and deadline
- Two approaches
  - Schedule the job in earliest slot and then keep shifting right
  - Schedule the job in the deadline slot or look for slots earlier than the deadline