

# Design and Analysis of Algorithms

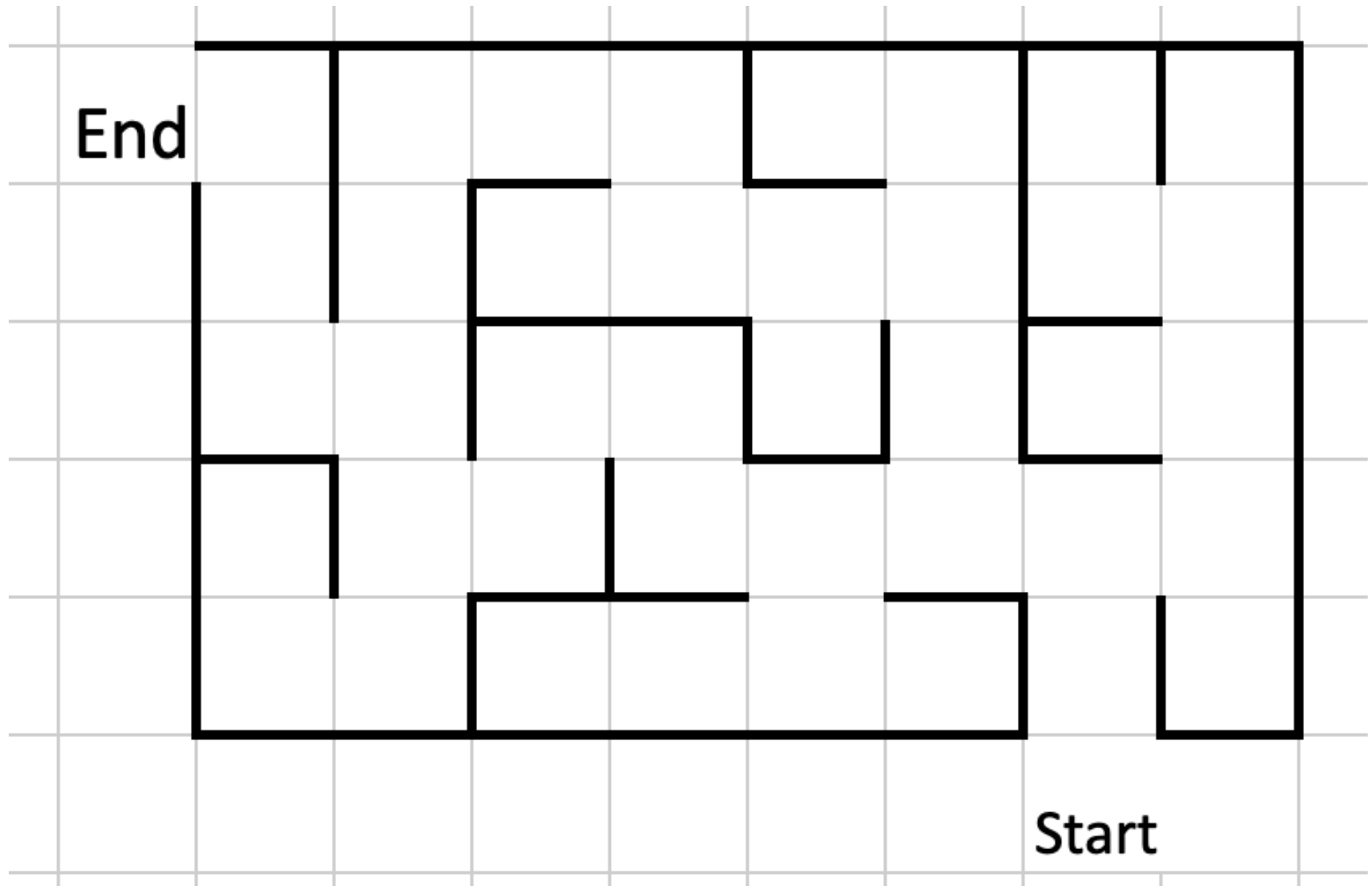
## L33: Intro to Backtracking

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# Resources

- Text book 2: Horowitz
  - Sec 7.1, 7.2, 7.3, 7.4, 7.5, 8.2, 11.1
- Text book 1: Levitin
  - Sec 12.1, 12.2
- R1: Introduction to Algorithms
  - Cormen et al.
- Youtube link of video lecture
  - [https://www.youtube.com/watch?v=Jcnk\\_hwS08A](https://www.youtube.com/watch?v=Jcnk_hwS08A)

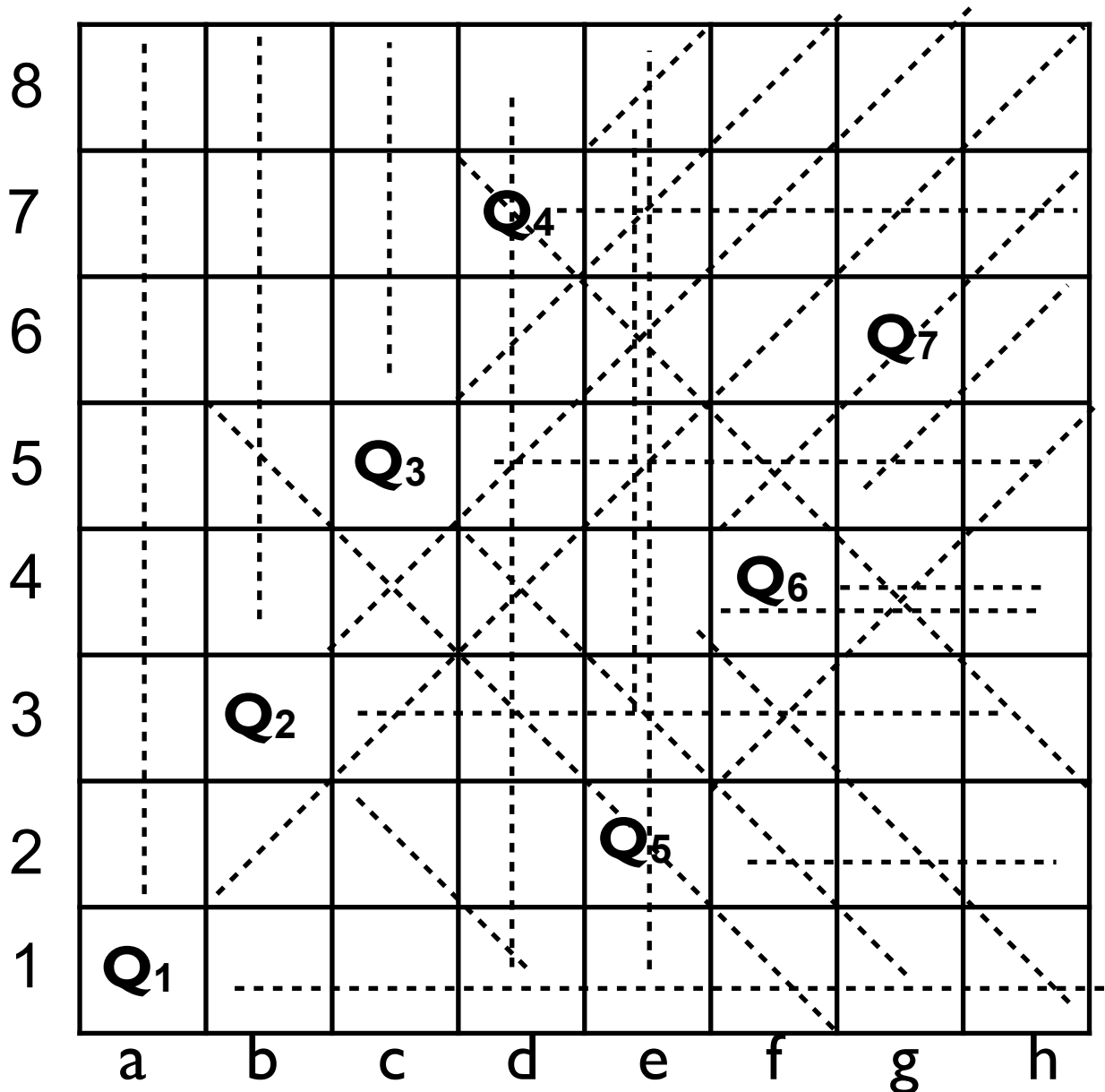
# Finding a way in Maze



# Overview of Backtracking

- Backtracking
  - Start from some solution.
  - Keep exploring for next part of solution
  - When exploration of solution stops (not possible to proceed further)
    - Resume back from the last point where decision was made to explore the current path.
    - Explore with the next path.

# 8-Queens Problem



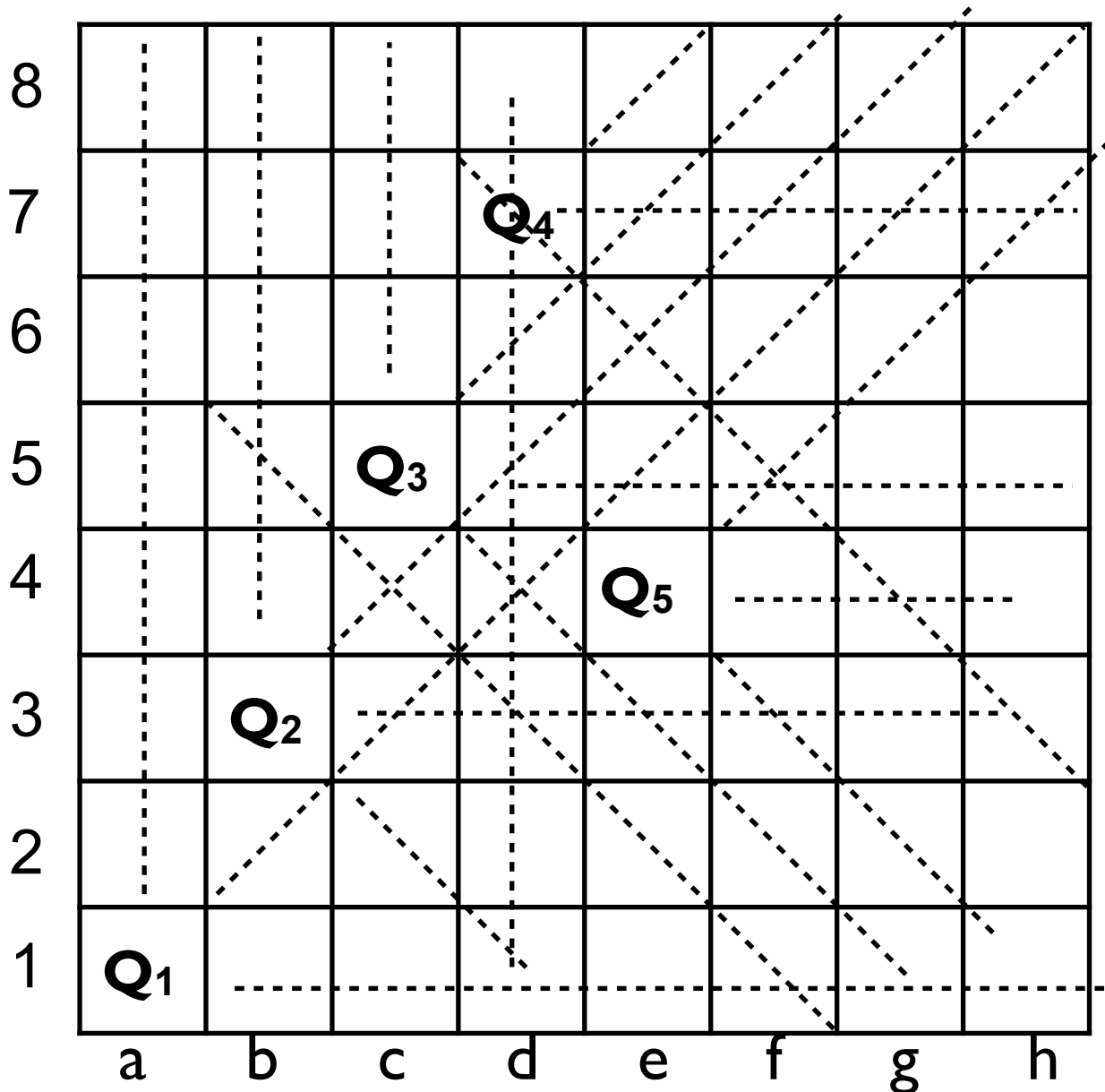
Q<sub>8</sub> can't be placed.

Backtrack to Q<sub>7</sub> which  
can't be placed too

Backtrack to Q<sub>6</sub> which  
can't be placed too

Backtrack to Q<sub>5</sub> which  
can be placed at e4

# 8-Queens Problem



Q<sub>5</sub> is placed at e4

Q<sub>6</sub> can't be placed

Backtrack to Q<sub>5</sub>

Q<sub>5</sub> can't be placed

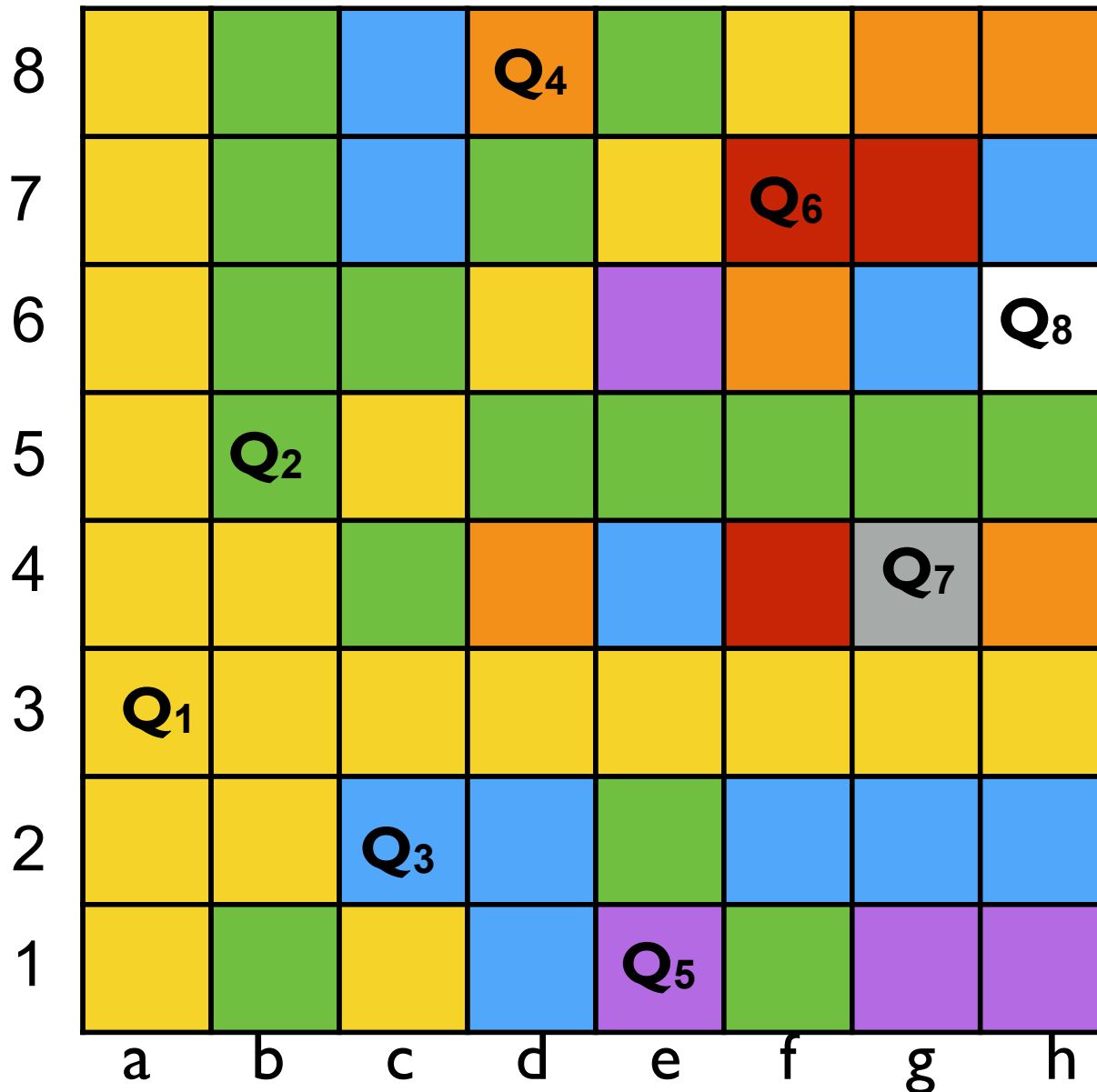
Backtrack to Q<sub>4</sub>

Q<sub>4</sub> is moved to d8

Continue in this way.

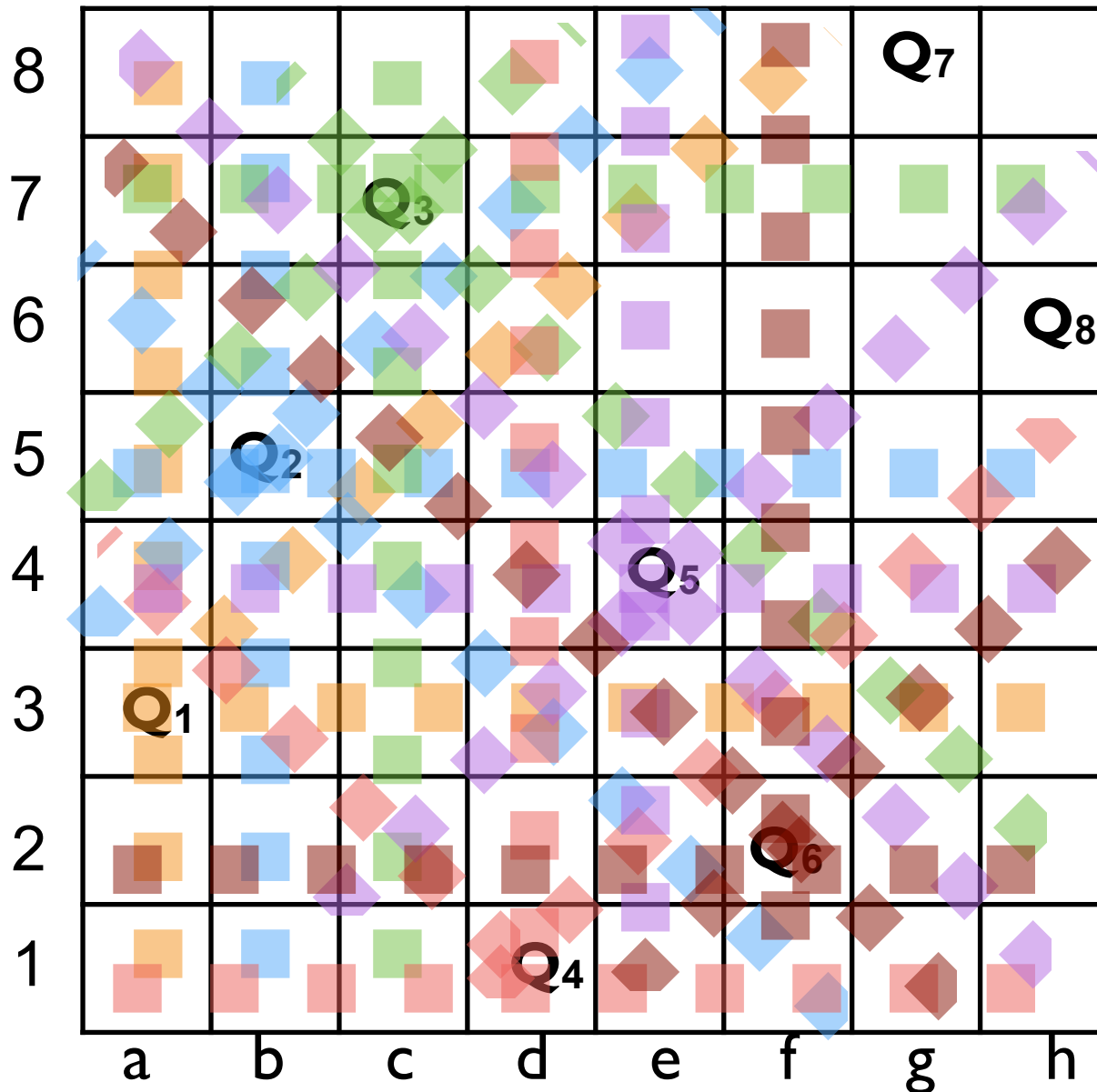
Find the positions  
where 8 queens can  
be placed.

# 8-Queens Problem



Continuing in this way.  
positions of 8 queens.

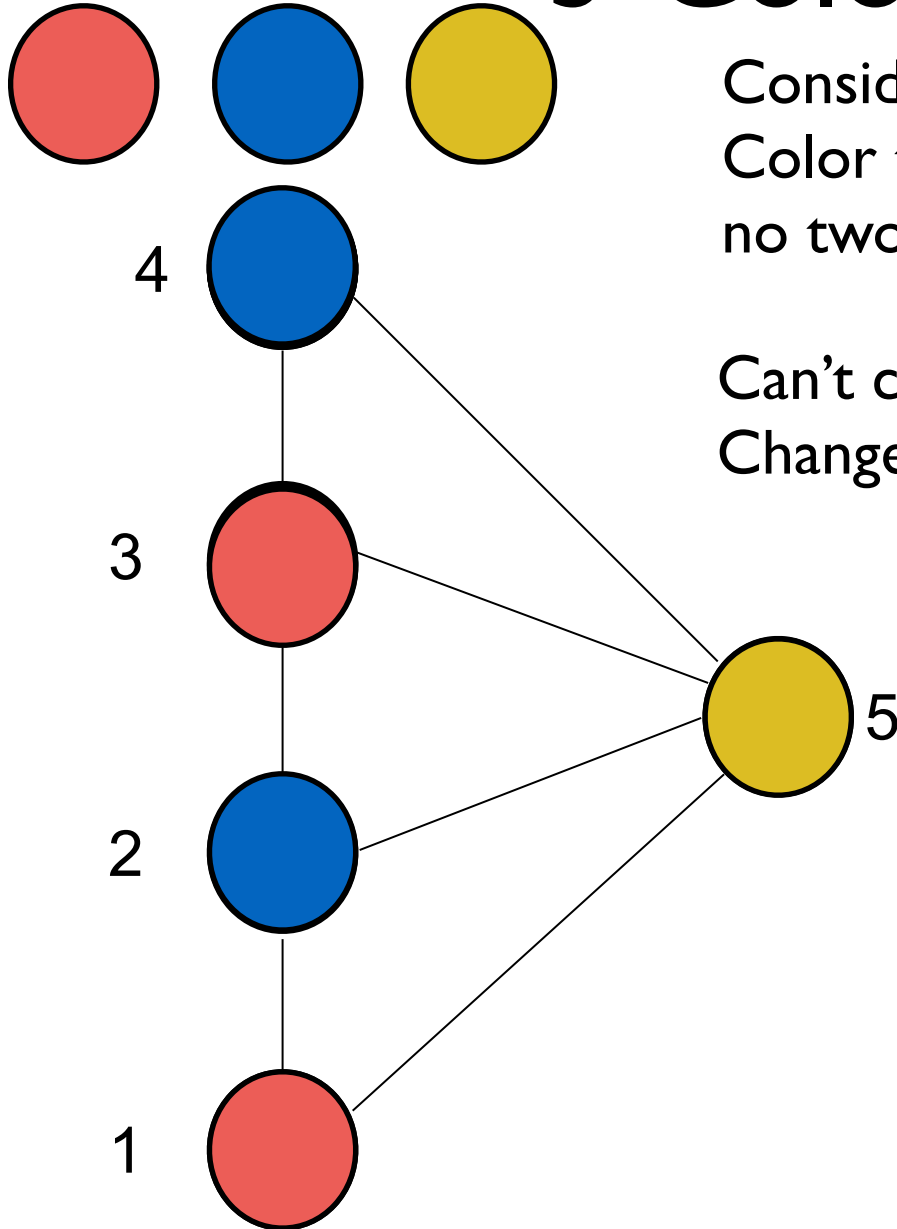
# 8-Queens Problem: Soln 2



Continuing further  
another solution for  
8 queens problem



# 3-Color Problem



Consider three colors are : Red, Blue, Yellow  
Color the nodes with these 3 colors such that  
no two adjacent nodes have same color.

Can't color node 5, so backtrack  
Change 4 to Blue

Can't color node 5, so backtrack  
Can't change 4 to Yellow  
Backtrack to 3

Solution: 1-R, 2-B, 3-R, 4-B, 5-Y

Are there other solutions?

# Sum of Subset Problem

- Given a set  $S$  of numbers and a value  $m$ ,
  - Find all subsets  $S_i \subseteq S$  so that their sum of elements in  $S_i$  equals  $m$ .
  - An element in a subset is to be considered only once.
- Example
  - $S = \{11, 13, 24, 7\}$ , and  $m = 31$
  - Possible subsets are
    - $S_1 = \{11, 13, 7\}$
    - $S_2 = \{24, 7\}$

# Backtracking: General Method

- General solution is an  $n$ -tuple  $(x_1, \dots, x_n)$ , where
  - $x_i$  is chosen from some finite set  $S_i$ .
  - While choosing  $x_i$ , it has to follow some constraints
    - or meet a criterion function  $P(x_1, \dots, x_n)$
- Suppose, the size of each set  $S_i$  is  $m_i$
- Then, total number of possible tuples are
$$M = m_1 * m_2 * \dots * m_n$$
- Identify those tuples that satisfies the constraints i.e. Criterion function.
- Backtracking approach provides the answer in far fewer trials than  $M$ .

# Backtracking: 8-Queens Method

- Let queens are numbered 1 thru 8, i.e.  $Q_1, \dots, Q_8$
- Each queen must be on a separate column (and row)
  - For simplicity, let's say  $Q_i$  is placed on  $i^{\text{th}}$  column.
- Thus, solution can be represented by an 8-tuple
$$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$$
  - where  $x_1 \approx a, x_2 \approx b, x_3 \approx c, x_8 \approx h$
- Each queen must be on a separate row.
- Thus, each  $x_i$  can have a value from 1 to 8.
  - Thus, constraint is  $x_i \in S_i = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- Solution space size before and after the constraint
  - before:  $8^8$ , after:  $8!$
- Representation for solution-1
  - $\{3, 5, 2, 8, 1, 7, 4, 6\}$

# 8-Queens Problem

$\{3, 5, 2, 8, 1, 7, 4, 6\}$

8			<b>Q<sub>4</sub></b>					
7					<b>Q<sub>6</sub></b>			
6							<b>Q<sub>8</sub></b>	
5		<b>Q<sub>2</sub></b>						
4						<b>Q<sub>7</sub></b>		
3	<b>Q<sub>1</sub></b>							
2			<b>Q<sub>3</sub></b>					
1				<b>Q<sub>5</sub></b>				
	a	b	c	d	e	f	g	h

# Backtracking: Sum of Subsets

- Problem:  $S = \{11, 13, 24, 7\}$ , and  $m=31$
- Solution approach 1 :
  - Consider 4-tuple  $\{x_1, x_2, x_3, x_4\}$ 
    - where,  $x_i \in S_i = \{0, 1\}$
    - Size of solution space:  $2^n$
  - possible solutions
    - $\{1, 1, 0, 1\}$
    - $\{0, 0, 1, 1\}$
- Solution approach 2
  - Solution contains the index values of elements.
  - Solution is a vector of varying dimensions
  - Possible solutions
    - $(1, 2, 4)$
    - $(3, 4)$

# Backtracking: 3-Color problem

- Problem:  $G = \{V, E\}$ , and 3 colors to color the graph
- Solution vector:  $n$ -tuple  $(x_1, \dots, x_n)$ 
  - where  $x_i \in S_i = \{R, B, Y\}$
- Size of total solution space:  $3^n$ 
  - An edge reduces solution space from  $3^2$  to  $3 * 2 = 6$
  - Any path of length  $k$  reduces the solution space from  $3^{k+1}$  to  $3 * 2^k$

# Summary

- Overview of backtracking
- Problem examples for backtracking
  - 8-queens problems
  - Sum of subsets
  - 3-color problem
- Solution space
- Possible solution space.