Design and Analysis of Algorithms

L02: Algorithm Specifications and Analysis

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Resources

- Python turtle graphics
  - https://docs.python.org/3/library/turtle.html
- Textbook :
  - T1 (Levitin)
  - T2 (Horowitz, Sahani)

Observation about Programs

- Consider following programming problems
  - Count number of digits
  - Reverse the digits
  - Binary representation of a number N
  - Prime factors
  - Generate haromonic series
  - Integer display in binary
  - Fibonacci series generation
  - Ramaujan number computation

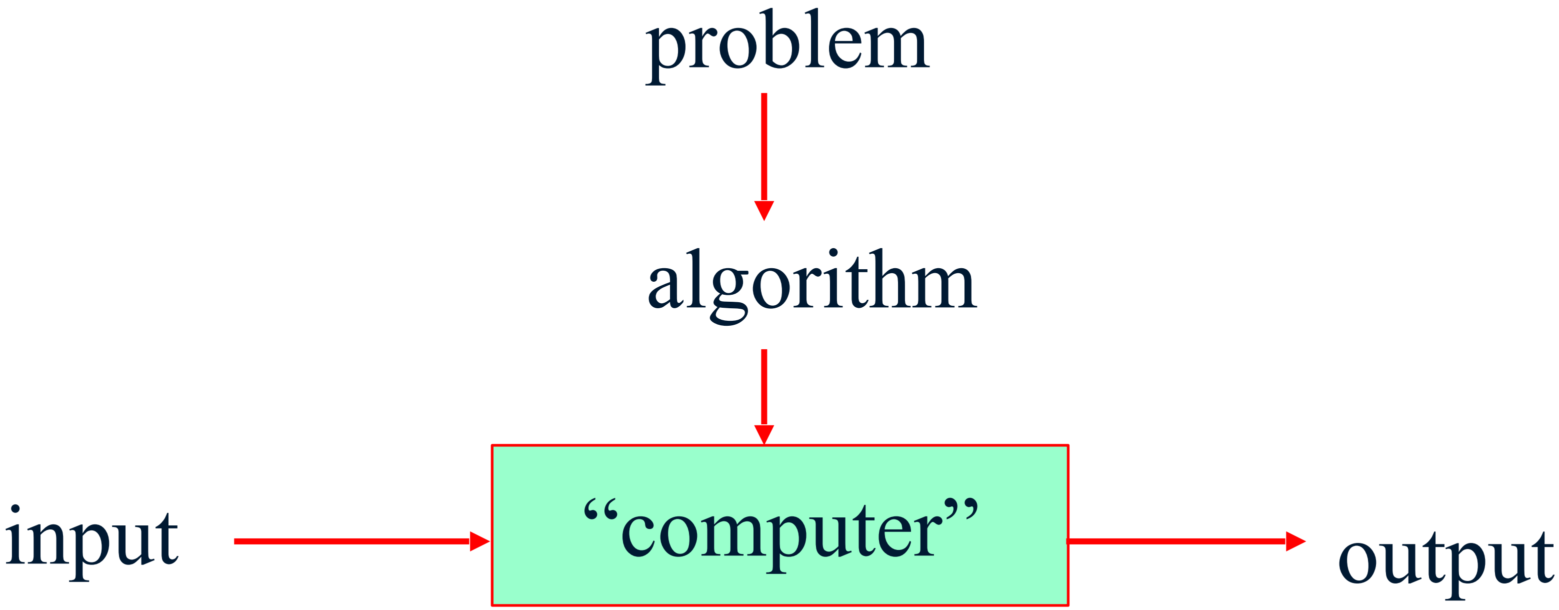
Observation about Programs

- All programs require a computer to run
  - Can we run these with pen and paper
- Input : hard coded or specified as argument
- Each program produces some output
- Program <u>terminates</u> after some time.
  - Does not run for ever.
    - Can we write a program to print all integers?
- Each instruction is <u>unambiguous</u>.
  - There is no confusion on how it should be executed
- Each step is executable. It is <u>feasible</u>.
- These are **attributes** of algorithm

Algorithm

- Defn: A finite set of instructions that when executed (followed) accomplishes a particular task
  - *An algorithm is a sequence of unambiguous instructions for solving a problem, i.e. for obtaining a required output for any legitimate input in a finite amount of time.*
- Algorithm must satisfy following criteria
  - **Input** : zero or more input
  - **Outout**: At least one output is produced
  - **Definite** : unambiguity
  - **Finite**: termination
  - **Effectiveness**: Feasible to execute

Algorithm



Algorithm Criteria: Examples

- Example for illustrating criteria importance
  - Ambiguity:
    - Go to college (KSIT) or movie theatre
    - Withdraw some money from ATM/Bank
  - Infeasible
    - Divide by zero
    - Represent 1/3 in decimals without loss of precision
  - Infinite:
    - Print all integers
    - What about "Count number of hairs on your head"
- Computational procedures: Algorithms that are definite and feasible.
- Program: expression of an algorithm in a prog. lang.

Differences with Data Structures

- All program work with (manipulate) data and requires some form of representation of data
- Data structure is concerned with following for data
  - Representation
  - Organization
  - Manipulation
- Data manipulation requires an algorithm
- Study of `Data Structures` and `Algorithm` is fundamental to Computer Science

Algorithms: Areas of Study

- How to **design** algorithms
  - The key focus of this course
  - What approaches to take: e.g. divide and conquer
- Algorithm validation
  - Program validation
  - How to ensure it outputs correct value
- **Performance analysis**:
  - How much time it takes, how effective it is.
    - Best case, worst case, average case
    - Example: finding a seat for new student who enters the classroom
- How to test the programs
- Optimality: Is the solution optimal?

Design Strategies

- Brute force
- Divide and conquer
- Decrease and conquer
- Greedy approach
- Dynamic programming
- Backtracking
- Branch and bound
- Space and time trade offs

Algorithm Specification

- How should an algorithm be specified?
- Can it be described in plain english or any other natural language?
  - Will it be unambiguous?
  - Does it offer critieria of finiteness, feasibility etc.
- Should it be defined via a flow chart?
  - Does graphical representation covers all criteria?
  - Works well when it is short and simple/.
- Should we use a programming language? C/C++/Java?
  - What would be difference between program and algorithm?
- A mix and match of all of above?

Example: Algorithm Specification

- Selection sort:
  - Description in english:
    - To sort collection of n unsorted elements, find the shortest element and place it ahead of sorted list.
  - Issues with this description
    - How are the elements initially stored in a collection?
      - In an array, linked list or something else?
    - How is sorted list maintained?
      - Meaning of ahead of sorted list?
    - How the result is placed?

Selection Sort: Specification

- Assumption:
  - Input: All unsorted elements are stored in an array
    - i^{th} element is stored in i^{th} position i.e. a[i]
  - Output: Sorted elements will be in the same array
- Specification

  for `i=1` *to* `n` *do*

  examine `a[i]` *to* `a[n],` *and*

  find the smallest element `a[j]`

  Interchange `a[i]` *and* `a[j]`
- Does it meet all criteria?
  - Input, output, unambiguity, feasibility, finiteness,

Selection Sort: Java Code

- // I/p: array `a[1:n]` of n elements contains n integers.

```
for (int i=1; i<=n; i++) {
  int j=i;
  for (int k=i+1; k<10; k++) {
    if (a[k] < a[j])
      j = k;
  }
  int x = a[i];
  a[i] = a[j];
  a[j] = x;
}
```

- Does it work correctly?

Selection Sort: Example

- python turtle graphics
  - Module: sorting_animate.py
  - Modified to define number of items to sort
  - Invocation:
    - python selectionsort.py <n>

Recursive Algorithms Specification

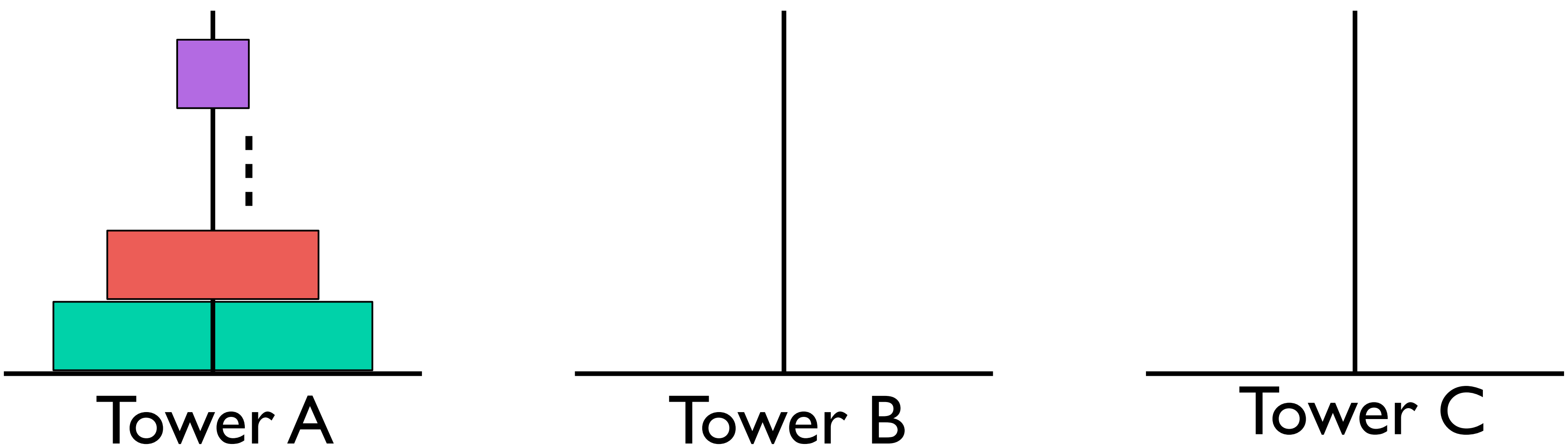
- Recursion:
  - Efficient mechanism to specify a good number of problems
    - Factorial: `n! = n * (n-1)!`
    - Binomial coefficient:
      - write ${}^{n}C_{k}$ recursively (Use pascal triangle)

$${}^{n}C_{k} = n!/(k!(n-k)!) = {}^{n-1}C_{k} + {}^{n-1}C_{k-1}$$

- Recursion invocation
  - Direct recursion: by a function itself e.g. factorial
  - Indirect recursion: a function calls another function which in turn calls the calling function.
- Works well where problem can be described using recursion

Hanoi Towers

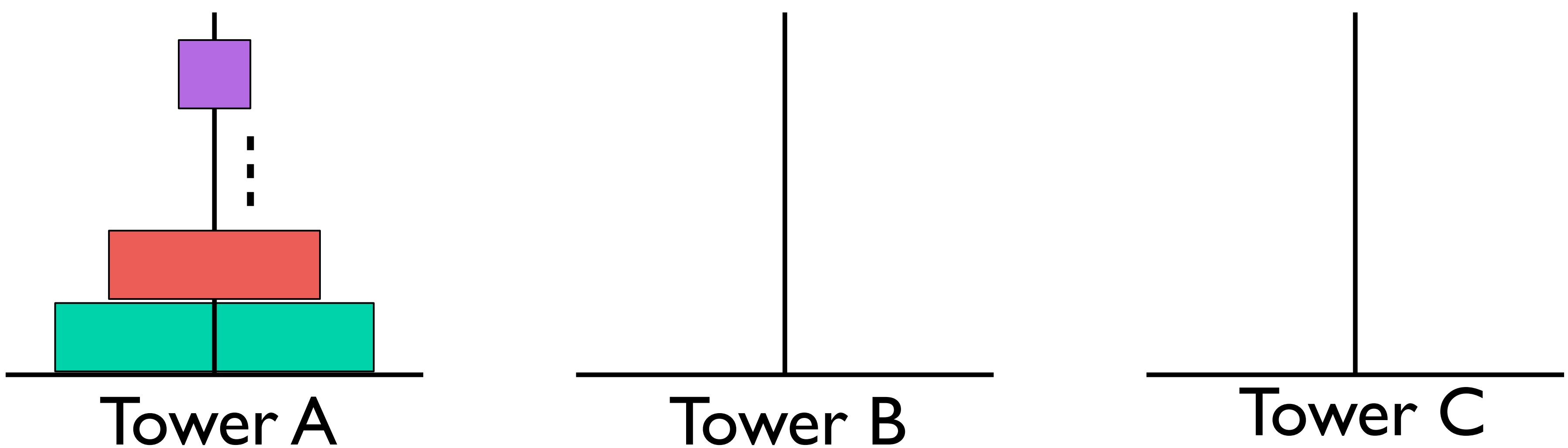
- python turtle graphics
  - Module: minimal_hanoi.py
  - Modified to define number of disks to sort as command line argument
  - Invocation:
    - python hanoi.py <n>



Tower of Hanoi

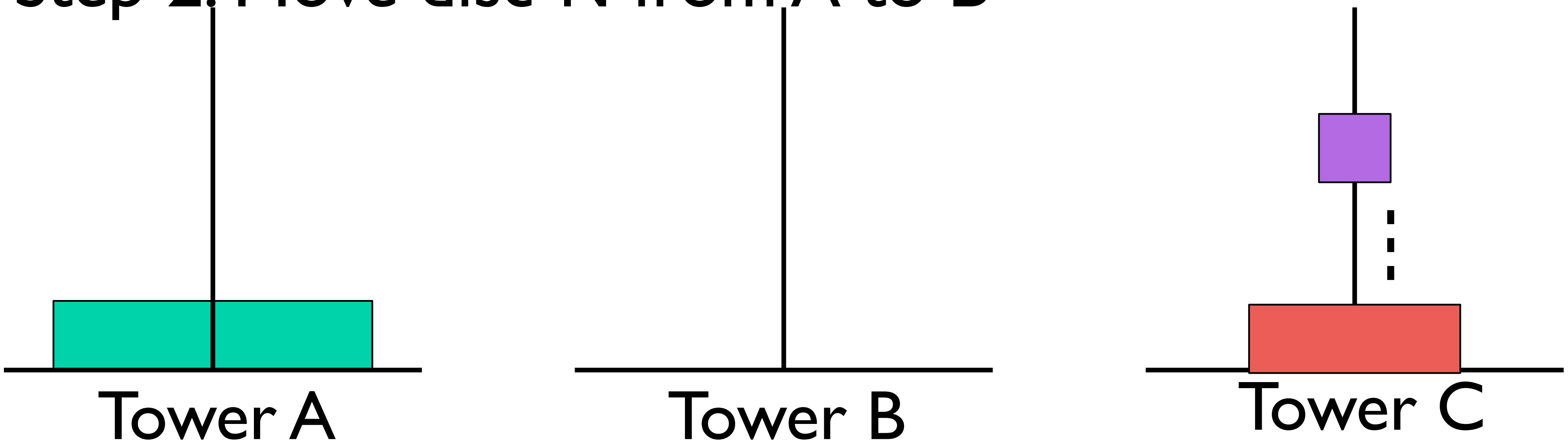
- Task: move N discs from A to B using C
- Input: N disks on Tower A, o/p:: N discs on Tower B
- Algo specification
  - `S1`: Move top `N-1` discs from A to C using B
  - `S2`: Move largest disc `N` to tower B
  - `S3`: Move top `N-1` discs from C to B using A

Step 1: Move N-1 discs from A to C using B

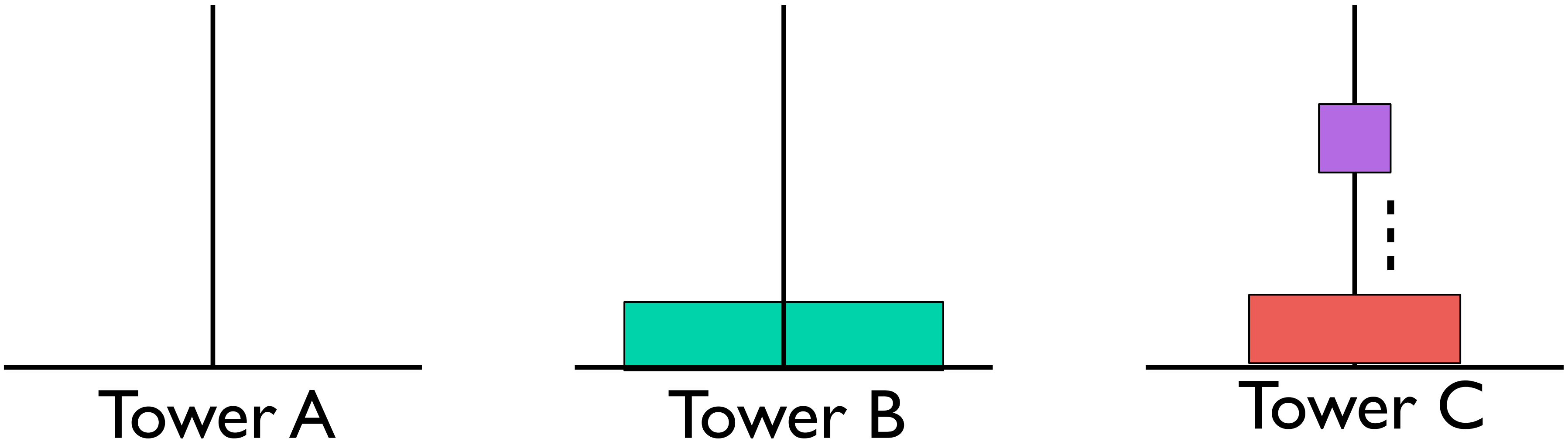


Tower of Hanoi: Algo Specification

- Step 2: Move disc-N from A to B



- Step 3: N-1 discs from C to B using A



Permutation of N number

- Tasks: Given N items, print all of its permutations
- Example: given S = {a,b,c}
- Output: {a,b,c}, {a,c,b},{b,a,c},{b,c,a},{c,a,b},{c,b,a}
- Algo specification for n items in an array `a[1]`...`a[n]`

```
for i = 1 to n
```

`print a[i]`

Let `b[1..n-1] = a[1..i-1] + a[i+1..n]` i.e. excluding `a[i]`

print all permulations of `b[1..n-1]` # recursion part

Exercises Using Recursion

- Generate n sequences for Fibonacci series, given

  $a_1=1,\ a_2=1$

  $F(n) = F(n-1) + F(n-2)$

- Specify Horner's rule in recursive way
  - $A(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$
- Given N boolean variables, print all possible combinations of possible truth values
  - e.g. for 3 boolean variables, the possible values are

    TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF

- Generate Power set of S
  - e.g. if S ={a,b,c}, then the power set is

    {},{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}

Summary

- Algorithm critiera
- Design strategies
- Algorithm specification
- Sample algorithms (with recursion)
  – Hanoi's tower
  – Permuation of N numbers
  – Fibonacci series
  – Horner's rule
  – All possible truth values for N variabes
  – Power set of S.