Design and Analysis of Algorithms

L12: Quicksort

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Resources

- Text book 1: Levitin (QuickSort)
- NPTel: DAA by Prof Madhavan Mukund
  - https://onlinecourses.nptel.ac.in/noc20_cs27/unit?unit=12&lesson=18
  - https://onlinecourses.nptel.ac.in/noc20_cs27/unit?unit=12&lesson=19

Sort Algorithms

- Bubble sort
- Selection sort
- Insertion sort
- Mergesort
- **Quicksort**
- Shell sort
- Heap sort
- Radix sort

Quick Sort

- Introduced by Hoare in 1960
- MergetSort shortcomings
  - No inplace sort
  - Extra space could be costly
  - Inherently recursive
    - recursive calls and returns are expensive
- Purpose of quicksort
  - Overcome shortcomings of mergesort
  - Extra space is caused by Merge operation
    - Can we avoid merge operation
    - Merging happens because elements in left half move to right half and vice versa
  - Can we divide such elements in left half are always less than elements in right half?
    - No need to merge (no extra space required)

Quick Sort

- What results in merging?
  - Some elements in right side need to move to left side
    - These are smaller than some elements in left side
- Objective
  - Divide in such a way that elements in left are always smaller than elements in right
  - Can we use divide and conquer?
  - Can we find middle value (median) and put in center?
- Method
  - Assume we have median `m` and placed in the middle
  - Move everything less than it to the left
  - Move everything greater than it to the right
- Claim: we can move everything in linear time.
  - Use the process recursively (divide and conquer)

Quick Sort

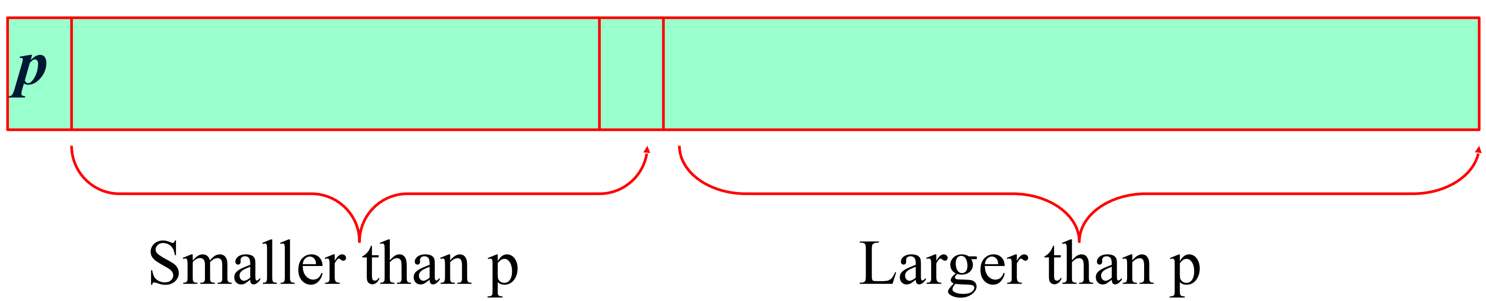
- Division into parts,
  - Each part of size n/2,
  - n operations (moving to left and right of median)
  - Time complexity same as that of mergesort, i.e.
  - $T(n) = 2T(n/2) + \Theta(n)$

    $= \ldots$

    $= \Theta(n\log_2 n)$
- Challenge: how to find the median?
  - Finding median requires sorting of elements
    - Which is what we want to achieve
    - A classic chicken and egg problem?
- How to approach?
  - Pick some value (not necessarily the median)
  - Follow the steps as if it is median (pivot)

QuickSort

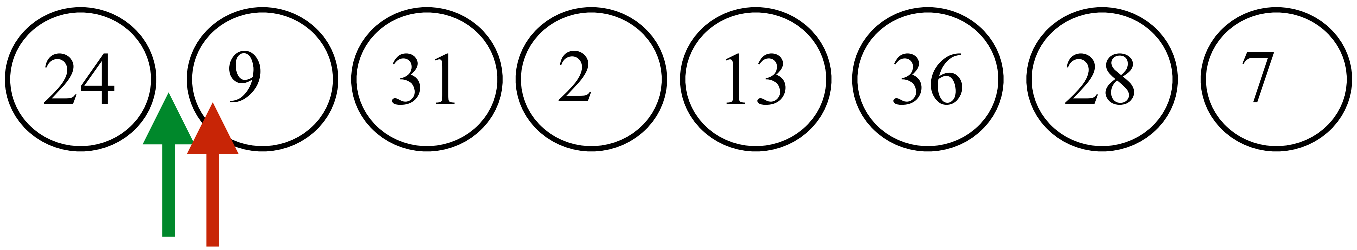
- A highly efficient algorithm
- Pick a pivot, divide input array in smaller arrays using the pivot (a specified value)
  – One array contains smaller values
  – Other array contains larger values
- Exchange the pivot with last element in first array
  – pivot is in its final position
- Sort the sub arrays recursively



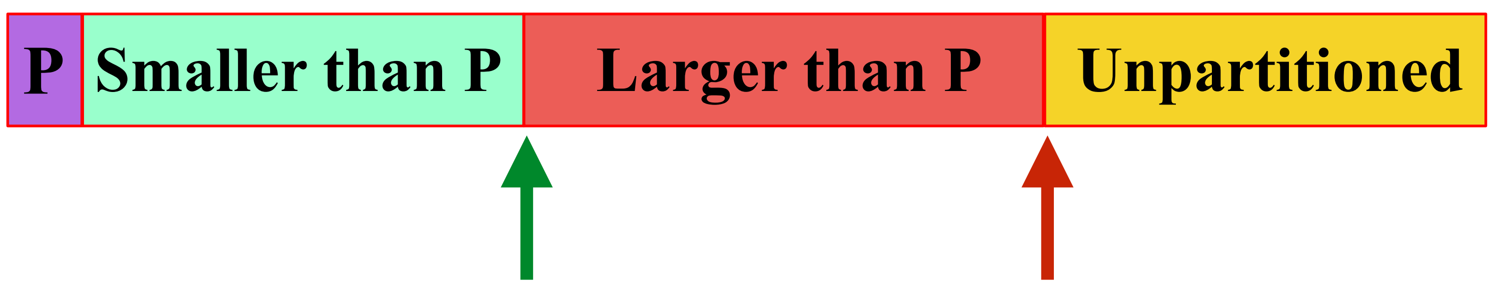
QuickSort Algorithm: Steps

- Select a *pivot* (partitioning element)
  - e.g. `1st` element or last element.
  - You can choose any element and swap it with last element
- Rearrange the array as follows i.e move pivot between lower and upper partition
  - All elements in first `s-1` positions are ≤ pivot
  - All elements in remaining `n-s` positions ≥ pivot
- Repeat the process

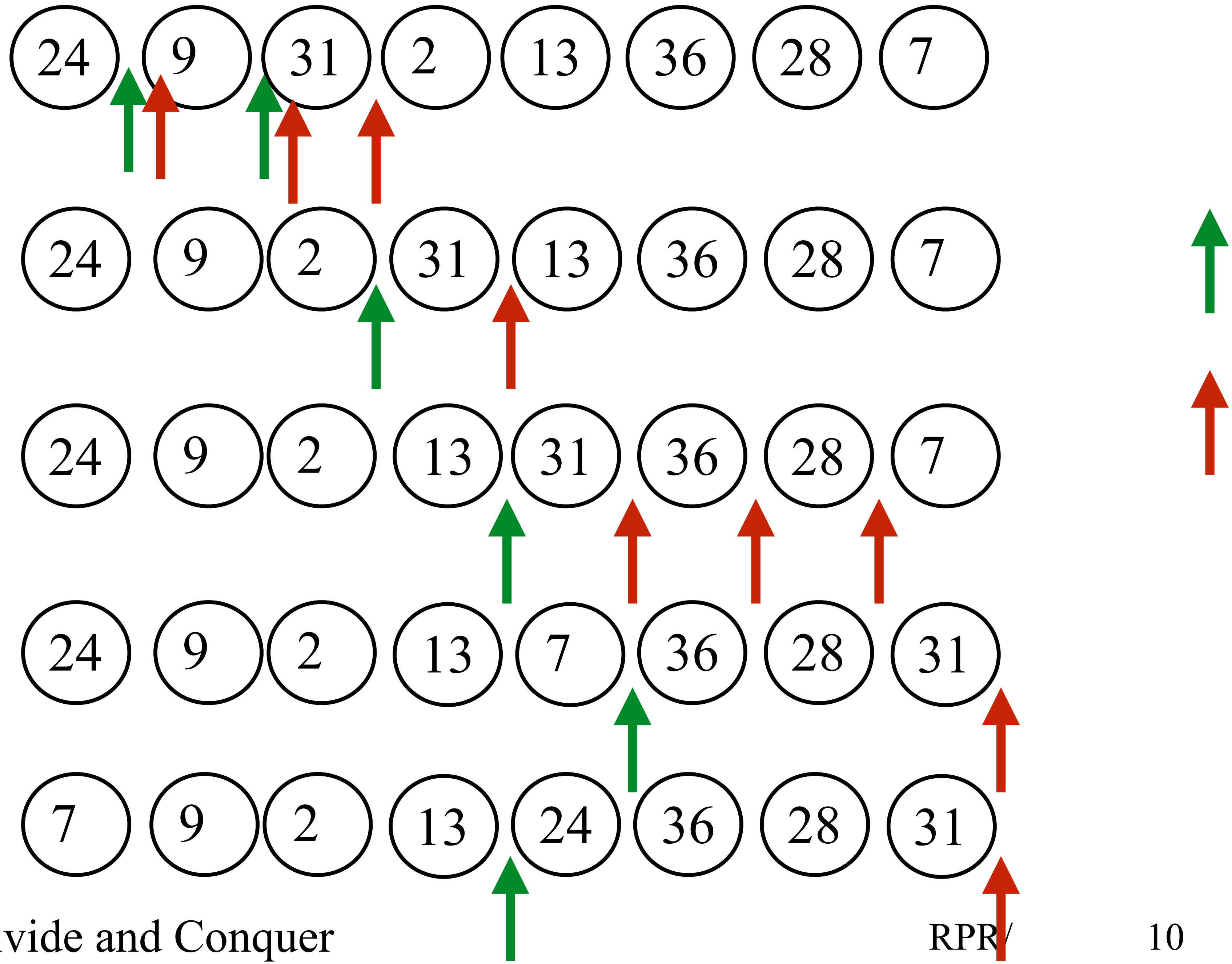
Quicksort



- Define two pointers
  - Green: indicates end of lower partition
  - Red: end of current partitioning i.e. elements to the right are yet to be partitioned



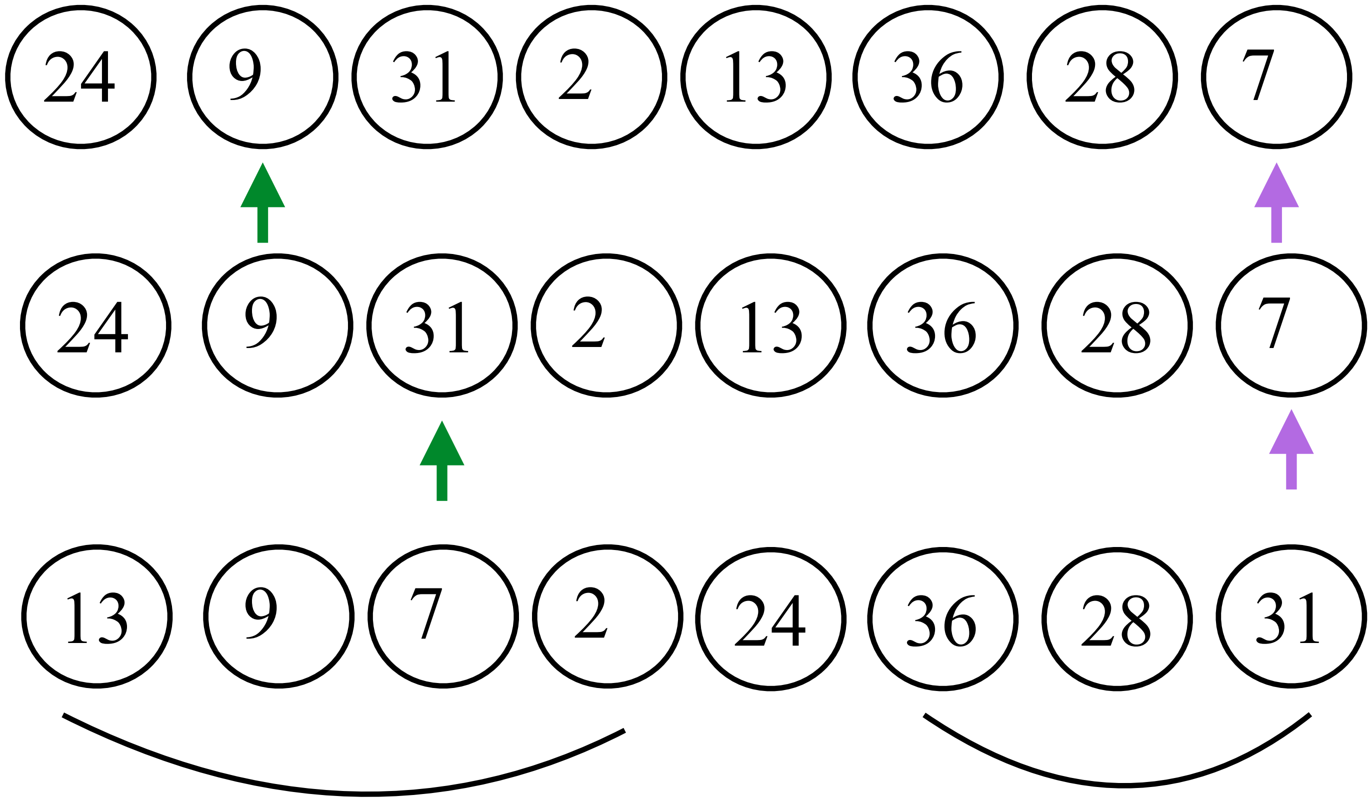
Quicksort


24 9 31 2 13 36 28 7
24 9 2 31 13 36 28 7
24 9 2 13 31 36 28 7
24 9 2 13 7 36 28 31
7 9 2 13 24 36 28 31

Quicksort algo

```
QSort(A,L,R) # sort A[L..R-1]
if R-L<=1 #base case
   return
#partition w.r.t. pivot
green=L+1
for red=L+1, red<R, red++)
   if A[red]<=A[L] #pivot
      swap(A[green], A[red])
      green++
swap(A[L], A[green-1]) #move pivot into place
QSort(A, L, green)
QSort(A, green+1, R)
```

Quicksort (Another Partitioning)



Quicksort (Book)

- Algo `quicksort(left, right,A[])`

```
# array index starts from 0 to n-1
#i/p: left - array index to start from
     right - array index up to which to consider
     array[] defined by left and right indices
#o/p: array[]] sorted in ascending order

if left < right
  s ← partition(left, right)
  quicksort(left, s-1)
  quicksort(s+1, right)
return
```

Quicksort

- Algo `partition(L,R,A[])`

```
pivot←A[L]; i←L; j←R+1
repeat
   repeat
      i←i+1
   until A[i] >= pivot
   repeat
      j←j-1
   until A[j] <= pivot
   swap(A[i], A[j])
until i>=j
swap(A[i], A[j])  #undo last swap when i>=j
swap(A[L], A[j])  #put pivot in its place
return j
```

Analysis: QuickSort

- Best case: split is approximately in the middle
  $T(n) = 2T(n/2) + \Theta(n)$
  $= \Theta(n \log_2 n)$
- Worst case: split is at the end (or beginning)
  - e.g. sorted array
  $T(n) = T(n-1) + \Theta(n)$
  $= \Theta(n^2)$
- Average case:
  $T(n) = \Theta(n \log_2 n)$
- Improvements (20-25%)
  – Better pivot selection : take median
  – Use insertion sort on smallar array size
  – Eliminate recursion and use iteration

Analysis: QuickSort

- Recursive calls works on two segments of the array and elements of one segment are not exchanged with elements of other segments.
- Essentially, no combination of results are required
- In practice quicksort is very fast
  - Typically, the default algorithm for in-built sort functions
    - e.g. spreadsheets
  - Programming languages use this sort for built-in sort

Summary

- Mergesort
  - Not in place sort
- Quicksort
  - In place sort
  - Practically used on large data

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