Design and Analysis of Algorithms

L23: Huffman Codes
Optimal Tree Subproblem

Dr. Ram P Rustagi
Sem IV (2020-Even)
Dept of CSE, KSIT
rprustagi@ksit.edu.in

Resources

- Text book 1: Sec 9.1-5.4 - Levitin
- R1: Introduction to Algorithms
    - Cormen et al.

Data Compression

- Consider saving a text file consisting of alphabets
  - It uses ASCII encoding,
  - each character uses 7 bits.
  - Thus, if file has `1000` characters,
    - file size is `7000` bits
- We know in english text certain characters appear more often than others e.g. `a,e,i,t,h,s` etc.
  - Other characters appear less often e.g. `z,x,q` etc.
- Can we use a different representation than ASCII
  - Assign shorter codes to chars occurring frequently
  - Assign longer codes to chars occurring less times.
  - Will we save disk space?

Data Compression: Communication

- Consider choosing electives with percentage of students
  - `17CS561`: Java Programming `(25%)`
  - `17CS562`: Artificial Intelligence `(12.5%)`
  - `17CS563`: Embedded Systems `(6.25%)`
  - `17CS564`: Dot Net framework `(6.25%)`
  - `17CS565`: Cloud computing `(50%)`
- A general coding would require 3 bits, e.g.

  `000` - Java Programming

  `001` - AI

  `010` - Embedded Systems

  `011` - DotNet framework

  `100` - Cloud Computing
- Can we employ better encoding so that average bits becomes less than 3.

Data Compression: Communication

- Consider choosing electives with percentage of students
  - `17CS561`: Java Programming `(25%)`
  - `17CS562`: Artificial Intelligence `(12.5%)`
  - `17CS563`: Embedded Systems `(6.25%)`
  - `17CS564`: Dot Net framework `(6.25%)`
  - `17CS565`: Cloud computing `(50%)`
- Consider following encoding (prefix-free)

  `0` - Cloud Computing

  `10` - Java Programming

  `110` - AI

  `1110` - Embedded Systems

  `1111` - DotNet framework
- What is the average number of bits for encoding these?

`1*1/2+2*1/4+3*1/8+4*1/16+4*1/16` `=15/8=1.875`

What is a Coding Problem…

- Coding: assignment of bit strings to alphabets
- Codewords: bit strings assigned for characters of alphabet. Example:
  - if `P(a)=0.4,P(b)=0.3,P(c)=0.2,P(d)=0.1`
  - `ASCII` codes can be assigned as
    `a:00, b:01, c:10, d:11`
    Number of bits for each code is 2 (avg is 2 bits too)
  - Codes can be assigned as
    `a:0, b:10, c:110, d:111`
  - then the average length of this coding scheme is
    = `1*0.4 + 2*0.3 + 3*0.2 + 3*0.1 = 1.9` bits

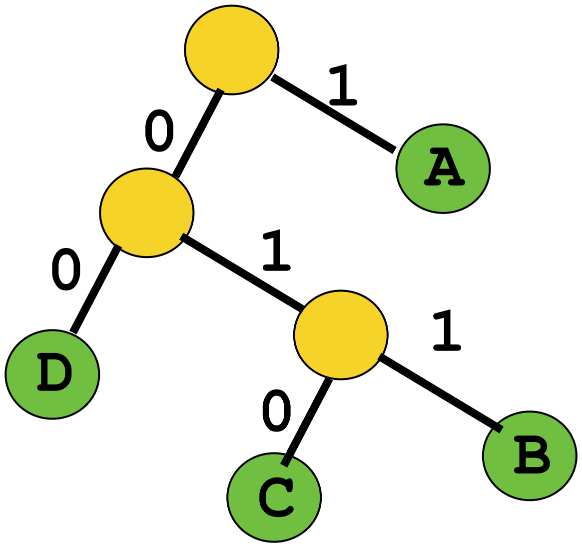
What is a Coding Problem…

- Two kind of encodings:
  - Fixed encoding e.g. `ASCII`
  - Variable length encoding: Morse encoding (dots, dashes)
- Prefix free codes
  - No codeword is prefix of another code
  - Allows for efficient decoding.
- Problem: If the frequency of character occurrences are known, what is the best binary prefix code?
  - Best: Shortest average code length
  - Average code lengths represets expected number of bits required to transmit/store a character.

Huffman Codes

- Any binary tree with edges labeled as `0, 1`
  - Provides a prefix code for characters assigned to leaves
  - Just concatenate the label of edges on the path from root to a vertex
  - Example:

```
A: 1
B: 011
C: 010
D: 00
```



- Optimal binary tree can be constructed using Huffman's algorithm

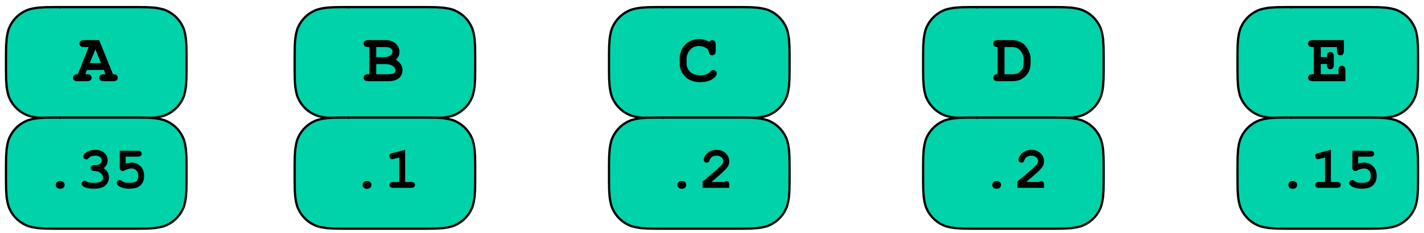
Huffman's Algorithm

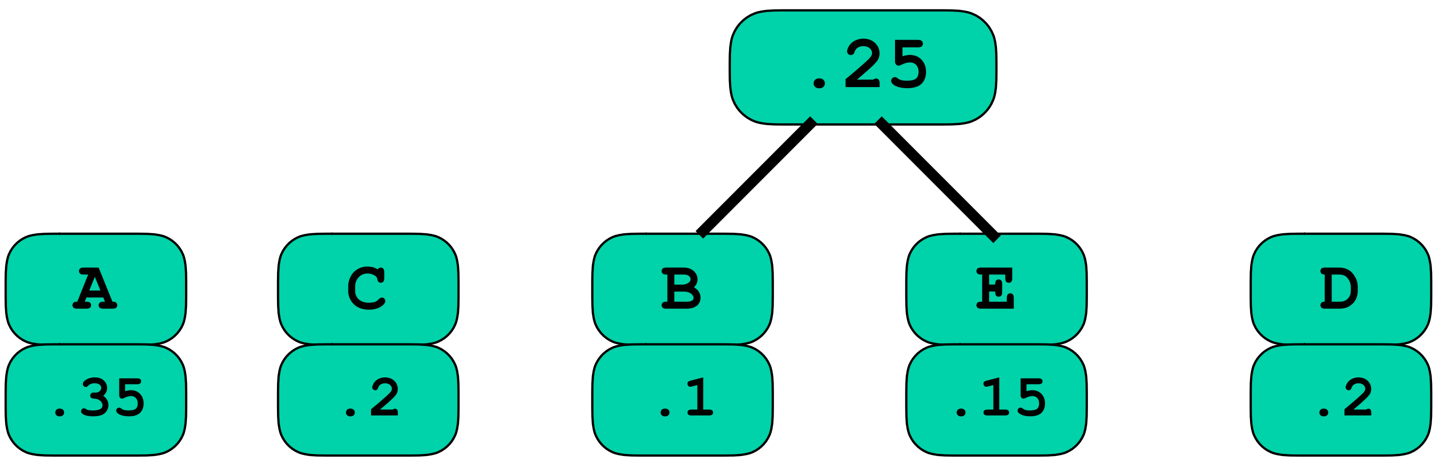
- Initialize `n` one node trees with alphabet characters
  - Assign tree weights as character frequencies
- Repeat the following steps `n-1` times
  - Join two binary trees with smallest weights into one binary tree
    - one tree would become left subtree
    - other tree would become right sub-tree
  - Make the weight of new binary tree (after joining) as equal to sum of weights of its sub trees.
    - Mark the edge joining left subtree with label `0`
    - Mark the edge joining right subtree with label `1`

Example: Huffman Tree

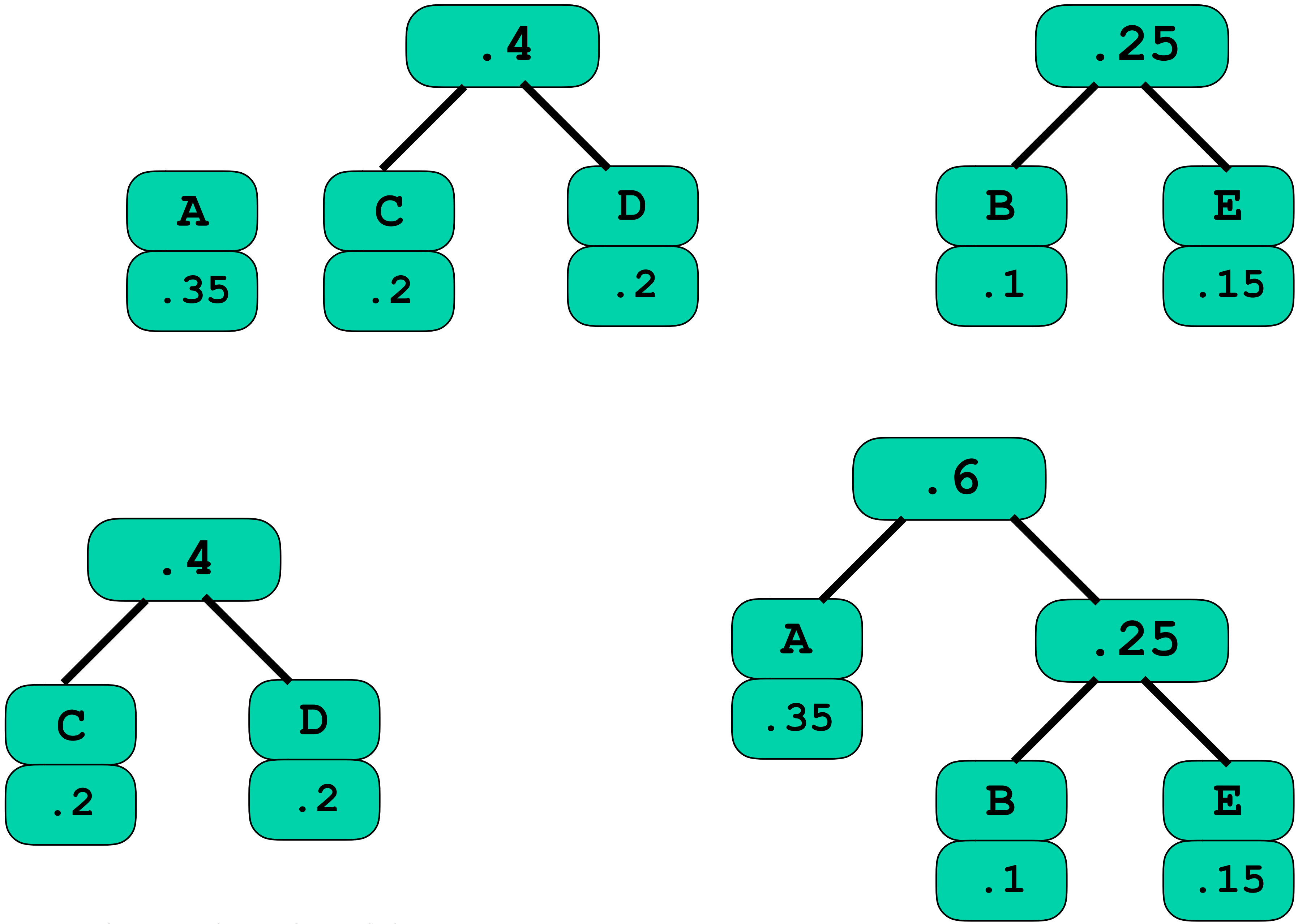
- Character frequences (probabilities)

  A:0.35, B:0.1, C:0.2, D:0.2, E:0.15

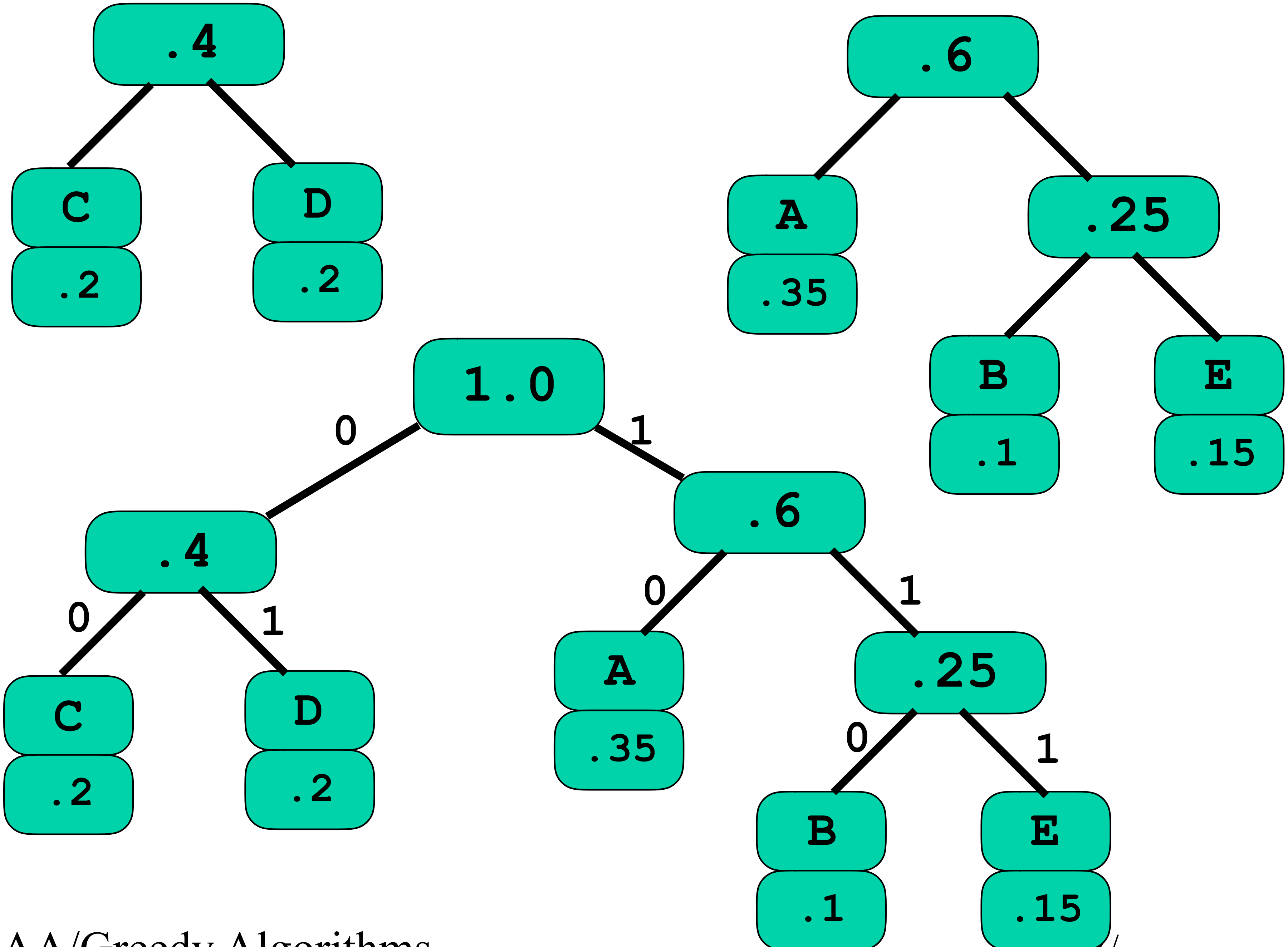




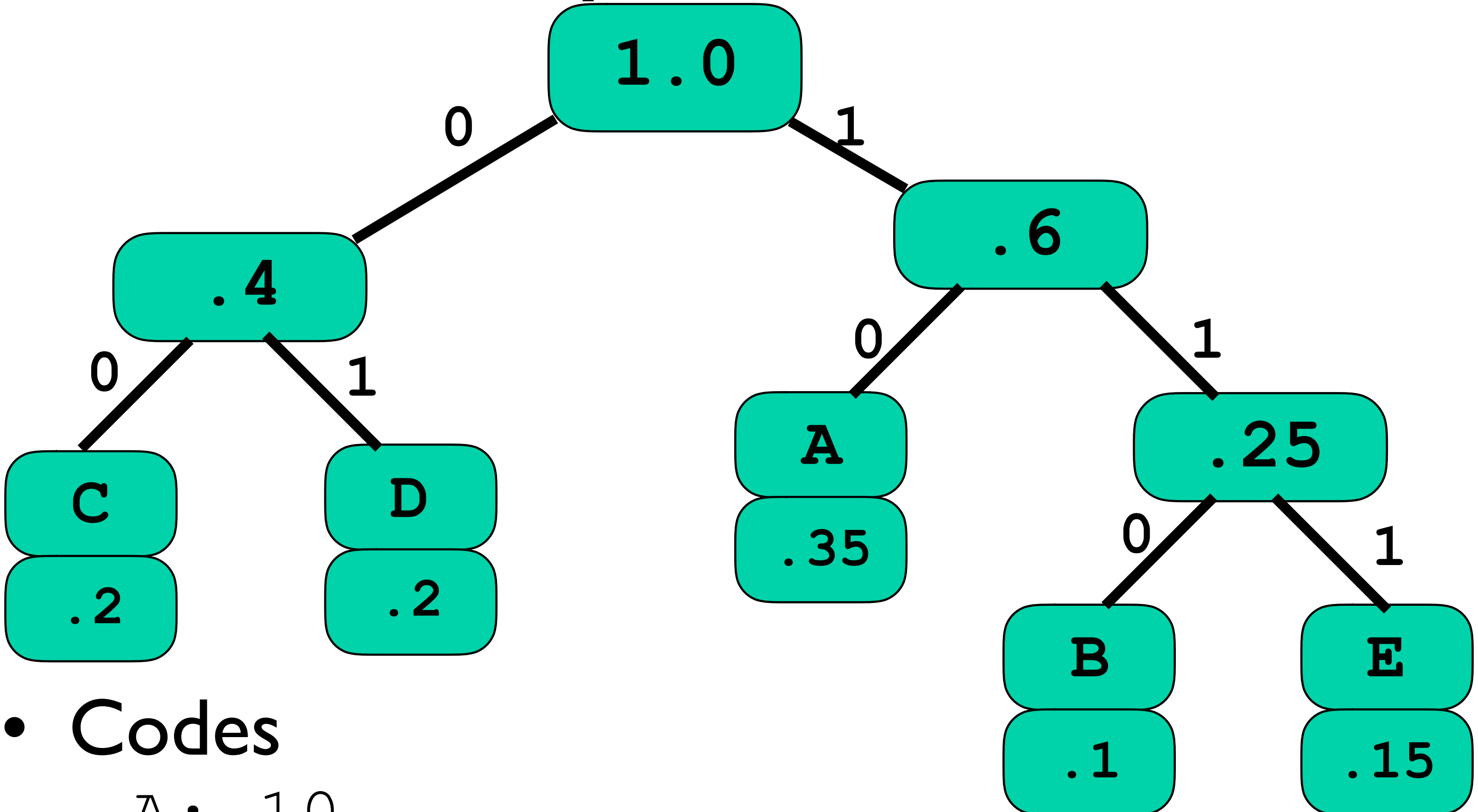
Example: Huffman Tree



Example: Huffman Tree



Example: Huffman Tree



- Codes

```
A:  10
B:  110
C:  00
D:  01
E:  111
```

Note: Characters with

Higher freq. (prob.) have shorter codes

Shorter freq.(prob.) have longer codes.

Example: Huffman Tree

- Character frequences (probabilities)

  `A:0.35, B:0.1, C:0.2, D:0.2, E:0.15`
- Codes

  `A: 10, B: 110, C: 00, D: 01, E: 111`
- Average code length

  `2*.35 + 3*.1 + 2*.2 + 2*.2 + 3*.15`

  `= 0.70 + 0.3 + 0.4 + 0.4 + 0.45 = 2.25`
- Code length for fixed length coding: `3`
  - `5` characters, require 3 bits
- Compression ratio:
  - `(3.0 - 2.25)/3.0*100% = 25%`
- Q: Represent character squence ACDBA

Huffman Tree/Codes

- Some characteristics
  - Codewords of two least frequent characters is same
    - They are at same level
    - What happens if $\exists$ more than 2 least frequent chars?
  - Codeword length of a more frequent character is always smaller than codeword length of less frequent characters. Proof by contradiction.
  - If alphabet's frequency is sorted,
    - Huffman tree can be constructed in Linear time
  - The max length of a codeword in huffman encoding of n characters can be `n-1`. Consider when each frequency is different.

Algorithm: Huffman Code

Algo `Huffman(W[0:n-1])`

// i/p: an array `W[0:n-1]` of weights

// o/p: A Huffman tree with leaves having weights of W

__*Initialize*__ Priority Queue `Q` of size `n` with `1`–node trees and weights equal to elements of `W[0:n-1]`

__*while*__ `Q` has more than `1` element __*do*__

  $T_1 \leftarrow$ minimum weight tree in `Q`

  __*delete*__ T_1 from `Q`

  $T_2 \leftarrow$ minimum weight tree in `Q`

  __*delete*__ T_2 from `Q`

  __*create*__ a new tree `T` with T_1/T_2 as left/right subtree of `T`
    with weight(T)=weight(T_1)+weight(T_2)

  Insert `T` into `Q`

Complexity Analysis (General)

- In each iteration
  - Removing two tree
  - Adding one tree
  - Effectively reducing number of trees by `1`.
  - Thus, total iteration = `n-1`
  - Removing min weight node takes `lg n`
  - Thus, time complexity `O(nlg n)`

Summary

- Huffman codes
- Huffman tree
- Algo
- Complexity analysis