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y_0 allows us to compute recursively the remaining entries that are now functions of λ. It is easy to see that eigenvalues λ_r are nothing other than the zeros of the last condition $y_m(\lambda) + hy_{m-1}(\lambda) = 0$. For a given eigenvector, define its squared norm by $\alpha_r = \sum_{k=0}^{m-1} a_k |y_k(\lambda_r)|^2$. The following theorem is found in Atkinson (1964) or in Teschl (2000).

Theorem 5. *Assume that we are given* $h \in \mathbb{R}$, $\{a_k > 0\}$, *eigenvalues* $\{\lambda_r\}_{0 \le r \le m-1}$, *norming constants* $\{\rho_r\}_{0 \le r \le m-1}$, *then there exists* $\{c_k\}_{-1 \le k \le m-1}$ *which are positive and constants* $\{b_k\}_{0 \le k \le m-1}$ *such that the boundary value problem has the set* $\{\lambda_r\}_{0 \le r \le m-1}$ *as its eigenvalues.*

AMIN BOUMENIR

See also **Inverse problems; Inverse scattering method or transform; Quantum inverse scattering method**

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GENERAL CIRCULATION MODELS OF THE ATMOSPHERE

Atmospheric general circulation models (AGCMs) simulate the dynamical, physical, and chemical processes of planetary atmospheres. For the Earth's atmosphere (*See* **Atmospheric and ocean sciences**), they are based on the thermo-hydrodynamic equations, which consist of the conservation of momentum, mass, and energy with the ideal gas law in coordinates suitable for the rotating planet.

In the presently used form, they were first derived by Vilhelm Bjerknes in 1904; subsequently, Lewis Fry Richardson (1922) proposed numerical weather prediction (NWP) as a practical application which, in 1950, was successfully performed by Jule Charney, R. Fjørtoft, and John von Neumann on an electronic computer based on a simplified set of these equations. While numerical weather prediction models utilize the atmospheric short-term memory for forecasting, AGCMs developed since the 1960s extend applications to longer time scales simulating seasonal and climate variability (for a personal recollection, see Smagorinsky, 1983).

Since then, numerical weather prediction and atmospheric general circulation modeling have enjoyed continuous advances, which are attributed to the following gains and improvements: (1) observational data accuracy, analysis, and assimilation; (2) insight into dynamical and physical processes, numerical algorithms, and computer power; and (3) the use of model hierarchies to study individual atmospheric phenomena. With simulations of the Earth system and that of other planets envisaged, a broad field of science has been established, which is of vital importance socio-economically, agriculturally, politically, and strategically.

Observations

Since the foundation of the World Meteorological Organization (WMO) and international treaties to monitor and record meteorological and oceanographical data, the collection of global data through local weather and oceanic stations has been systematically organized and has become a truly globalized system through the deployment of satellites and introduction of remote sensing facilities. The availability of extensive global data has enabled the extension of NWP models to complex global GCMs, thus facilitating simulations on larger time scales (months and years instead of hours and days) and validation of NWPs in return. Furthermore, it has become possible to study climate history and make estimates of future climates, which is particularly important due to likely anthropogenic impacts.

General Circulation Models

Atmospheric general circulation models have two basic components. First, the dynamical core consists of the primitive equations (the conservation equations with vertical momentum equation approximated by hydrostatic equilibrium, that is, balancing the vertical pressure gradient and the apparent gravitational forces) under adiabatic conditions. Second, physical processes contribute to the diabatic sources and sinks interacting with the dynamical core. They are incorporated as parameterizations, mostly in a modular format: solar and terrestrial radiation, the hydrological cycle (with phase transitions manifested in evaporation and transpiration, cloud, and precipitation processes), the planetary boundary layer communicating between the free atmosphere and the ground (soil with vegetation, snow and ice cover; ocean with sea ice), and atmospheric chemistry. Most of these parameterizations enter the thermodynamic energy equation as heat sources or sinks.

Dynamical core

State-of-the-art atmospheric general circulation models commonly utilize the so-called primitive equation approximation of the Navier–Stokes equations. In addition to the dry dynamics, equations describing the transport of other constituents such as water vapor, cloud liquid water and ice, trace gases, and particles (aerosols) can be an integral part of the dynamical core. To integrate the equations, they are discretized in space and time where finite differences and spectral methods are the most dominant. For more details on the governing equations; *See* the entries for **Atmospheric and ocean sciences; Fluid dynamics; Navier–Stokes equation**.

(i) Horizontal discretization: In the horizontal, grid point, or spectral representations of the dependent model variables are used. Different grid structures and finite difference schemes have been designed to reduce the error (Messinger & Arakawa, 1976). An alternative approach is the spectral method. The dependent variables are represented in terms of orthogonal functions where appropriate basis functions are the spherical harmonics. The maximum wave number of the expansion defines the resolution of the model. Since the computation of products is expensive, only linear terms are evaluated in the spectral domain. To compute the nonlinear contributions, the variables are transformed into grid point space every time step, where the respective products are computed and transformed back to spectral space. Necessary derivatives are computed during the transformation. This spectral-transform procedure (Eliassen et al.,1970; Orszag, 1970) makes the spectral approach computationally competitive with finite difference schemes. For low resolutions, the spectral method is, in general, more accurate than the grid point method. However, spectral methods are less suitable for the treatment of scalar fields, which exhibit sharp gradients and, for physical reasons, must maintain a positive-definite value (e.g., water vapor, cloud water, chemical tracers). Therefore, selected fields are often treated separately in the grid point domain using, for example, semi-Lagrangian techniques. Recently, with increasing model resolutions and the need for transporting more species (e.g., for chemical submodels), grid point models are attracting more attention again, while novel grid structures are introduced, for instance, the spherical icosahedral grid of the German Weather Service model GME (Majewski et al., 2000).

(ii) Vertical discretization: In general, finite differences and numerical integration techniques are used for the derivatives and integrals in the vertical. The vertical coordinate can be defined in different ways. The isobaric coordinate eliminates the density from the equations and simplifies the continuity equation compared with a z-coordinate system. However, the intersection of low-level pressure surfaces with the orography enforces time-dependent lower boundary conditions which are difficult to treat numerically. This problem can be avoided if terrain following sigma (σ) coordinates are used, where sigma is defined by the pressure divided by the surface pressure $\sigma = p/p_0$. Unfortunately, the sigma coordinate leads to a formulation of the pressure gradient force which, in the presence of steep orography, is difficult to treat. The advantages of both sigma and pressure coordinates are combined by introducing a hybrid coordinate system with a smoothed transition from σ to p with height.

Physical Processes and Parameterizations

Many processes that are important for large-scale atmospheric flow cannot be explicitly resolved by the model due to its given spatial and temporal resolution. These processes need to be parameterized; that is, their effect on the large-scale circulation needs to be formulated in terms of the resolved grid-scale variables. The most prominent processes in building a parameterization package of an atmospheric general circulation model are long- and short-wave radiation, cumulus convection, large-scale condensation, cloud formation and the vertical transport due to turbulent fluxes in the planetary boundary layer, and the effect of different surface characteristics such as vegetation on the surface fluxes. Additional processes such as the excitation of gravity waves and their impact on the atmospheric momentum budget or the effect of vertical eddy fluxes above the boundary layer are often considered. Because the land surface provides a time-dependent boundary condition that acts on time scales comparable to the atmosphere, great effort has been made to include land surface and soil processes in the atmospheric parameterization package. More recently, the effect of the interaction of various chemical species and their reactions with atmospheric circulation are also being considered. In addition to the direct relation between the resolved atmospheric flow and the effect of the parameterized processes, there are various other interactions among the individual processes that have to be taken into account. For typical comprehensive atmospheric general circulation models, Figure 1 displays the interrelations between the adiabatic dynamics, providing the spatial and temporal distribution of the dependent model variables and the various processes being parameterized.

Model Hierarchy

General circulation models of reduced complexity are continuously developed to supplement comprehensive

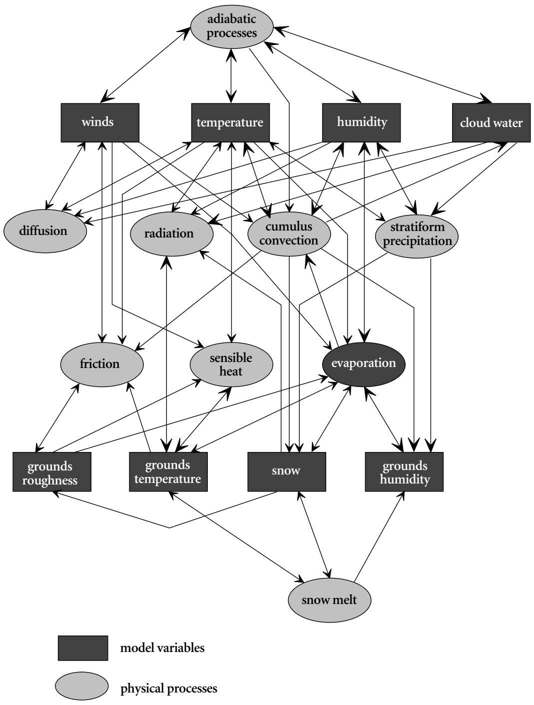

Figure 1. Interactions in comprehensive GCMs (schematic).

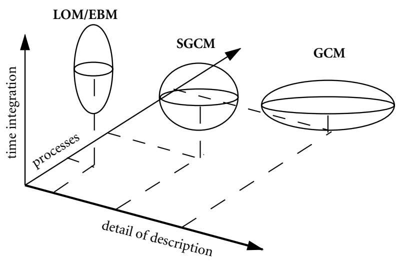

Figure 2. A model hierarchy of general circulation models.

GCMs, to gain insight into atmospheric phenomena (see Figure 2) and for educational purposes: when utilizing the full set of equations, the model spectrum ranges from simple GCMs (SGCMs) with analytic forms of heating and friction to low-order models (LOMs). A prominent LOM is the Lorenz model (*See* **Lorenz equations**), which approximately describes the nonlinear convection dynamics in the vicinity of a critical point for the stream function and temperature in a set of ordinary differential equations. It can be regarded as including first-order nonlinear effects (temperature advection) to a linear model, which leads to chaotic behavior. The Lorenz model is used to study predictability and serves as a paradigm for phase-space behavior of atmospheric GCMs. Utilizing thermal energy conservation only, another spectrum of models (energy balance models, or EBMs) is obtained by averaging in certain spatial directions. This leads to the horizontally averaged one-dimensional radiative-convective models; the one-dimensional energy balance model when averaged vertically and longitudinally for studying climate feedback and stability; and two-dimensional statistical–dynamical models when averaged longitudinally where dynamical processes are being parameterized. A prominent EBM example is the globally averaged or zero-dimensional energy balance model. With ice-albedo and water vapor-emissivity feedbacks included, climate catastrophes leading to a snowball earth and runaway greenhouse can be demonstrated. With random forcing and periodic solar radiation input (e.g., Milankovich cycles), stochastic resonance emerges.

KLAUS FRAEDRICH, ANDREAS A AIGNER, EDILBERT KIRK, AND FRANK LUNKEIT

See also **Atmospheric and ocean sciences; Fluid dynamics; Forecasting; Lorenz equations; Navier–Stokes equation**

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GENERAL RELATIVITY

Called *general relativity*, Albert Einstein's theory of gravitation was created as a generalization of his special relativity theory. As special relativity is a theory of physical space-time (neglecting gravitational effects), general relativity is a theory of physical space-time in the presence of gravitation.

While Maxwell's theory of electromagnetism is a relativistic theory that is covariant with respect to Lorentz transformations, Newton's theory of gravitation is incompatible with special relativity. In 1907,

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