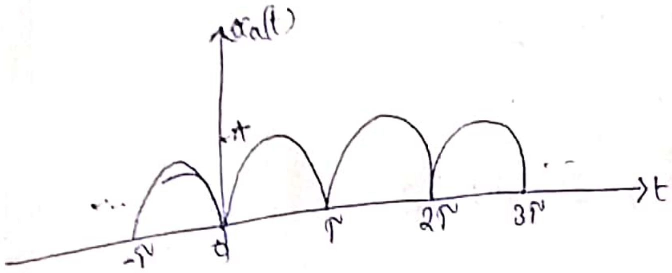
R161385

Assignment-2

1) Sol.



(a) $X_a(F) = ?$

$x_a(t)$ is periodic so using Fourier series.

$$x_a(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi kt/T}$$

where $$C_k = \frac{1}{T} \int_0^T A \sin(\pi t/T)\, e^{j2\pi kt/T}\, dt$$

$$= \frac{A}{j2T} \int_0^T \left(e^{j\pi t/T} - e^{-j\pi t/T}\right) e^{-j2\pi kt/T}\, dt$$

$$= \frac{A}{j2T} \left[\frac{e^{j\pi(1-2k)t/T}}{j\pi/_2 (1-2k)} - \frac{e^{-j\pi(1+2k)t/T}}{-j\pi/_2 (1+2k)} \right]_0^T$$

$$= \frac{A}{\pi} \left[\frac{1}{1-2k} + \frac{1}{1+2k} \right]$$

$$\boxed{C_k = \frac{2A}{\pi(1-4k^2)}}$$

Then $$X_a(F) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi(F-k/T)t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} C_k \int_{-\infty}^{\infty} e^{-j2\pi(F-k/T)t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} C_k\, \delta(F - k/T)$$

Hence, the Spectrum of $x_a(t)$ consists of Spectral lines of frequencies k/T ; $k = 0, \pm 1, \pm 2 \cdots$ with amplitude $|C_k|$ and phases $\angle C_k$.

(b) $$P_x = \frac{1}{T} \int_0^T x_a^2(t)\, dt$$

$$= \frac{1}{T} \int_0^T A^2 \sin^2(\pi t/T)\, dt$$

$$= \frac{1}{T} \int_0^T A^2 \left(\frac{1 - \cos\left(\frac{2\pi t}{T}\right)}{2}\right) dt$$

$$= \frac{A^2}{2T} \left[t - \frac{\sin\left(\frac{2\pi t}{T}\right)}{\left(\frac{2\pi}{T}\right)}\right]_0^T$$

$$= \frac{A^2}{2T} \left[T - \frac{\sin\left(\frac{2\pi T}{T}\right)}{\left(\frac{2\pi}{T}\right)} - 0 - \sin(0)\right]$$

$$= \frac{A^2}{2T} [T]$$

$$\boxed{P_x = \frac{A^2}{2}} \Rightarrow \text{power}$$

(c) The p. spectral density spectrum is $|C_k|^2, k = 0, \pm 1, \pm 2 \cdots$

$$C_k = \frac{2A}{\pi[1-4k]}$$

$$|C_k|^2 = \frac{4A^2}{\pi^2} \cdot \left|\frac{1}{1-4k}\right| \Rightarrow \frac{4A^2}{\pi^2}\left[\frac{1}{(4k^2-1)^2}\right]$$

$$\Rightarrow \frac{4A^2}{\pi^2}\left[\frac{1}{1} + \frac{2}{3^2} + \frac{2}{15^2} + \cdots\right]$$

now., $1 + \frac{2}{3^2} + \frac{2}{15^2} + \cdots = 1.2337$ (inf series sum to $\pi^2/8$)

Hence $\sum_{k=-\infty}^{\infty} |c_k|^2 = \frac{4A^2}{\pi^2}(1.2337)$

$= \frac{A^2}{2}$.

(d) parsevals relation.,

$$P_x = \frac{1}{T_p} \int_0^{T_p} x_a^2(t)\, dt$$

$$= |c_k|^2.$$

$$\sum_{k=-\infty}^{\infty} |c_k|^2 = \frac{4A^2}{\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{(4k^2-1)^2}$$

$$= \frac{4A^2}{\pi^2}\left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \cdots\right]$$

$$= \frac{4A^2}{\pi^2}(1.2337)$$

$$= A^2/2.$$

2)

(a) $x_a(t) = \begin{cases} A e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$X_a(F) = \int_0^{\infty} A e^{-at} e^{-j2\pi F t}\, dt$$

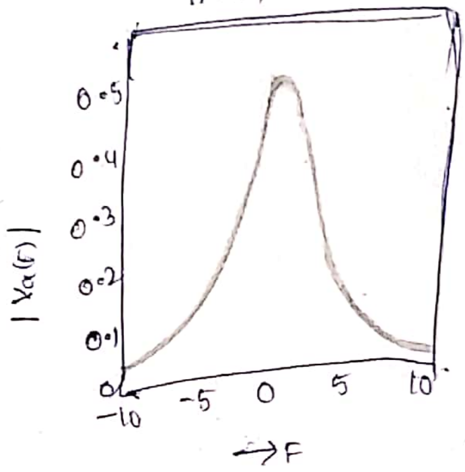
$$= \left[\frac{A}{-a - j2\pi F} e^{-(a+j2\pi F)t}\right]_0^{\infty}$$

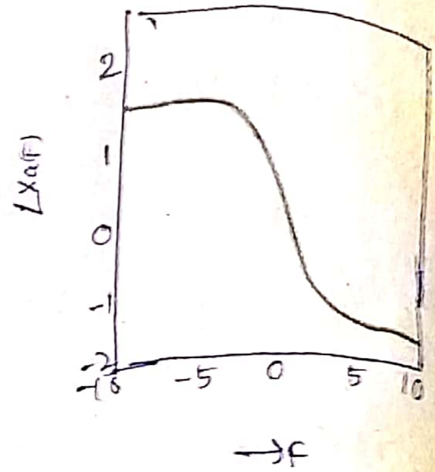
$$= \frac{A}{a + j2\pi F}$$

$$|X_a(F)| = \frac{A}{\sqrt{a^2 + (2\pi F)^2}}$$

$$\angle X_a(F) = -\tan^{-1}\left(\frac{2\pi F}{a}\right)$$

$A = 2,\ a = 4$





(b) $x_a(t) = A e^{-a|t|}$

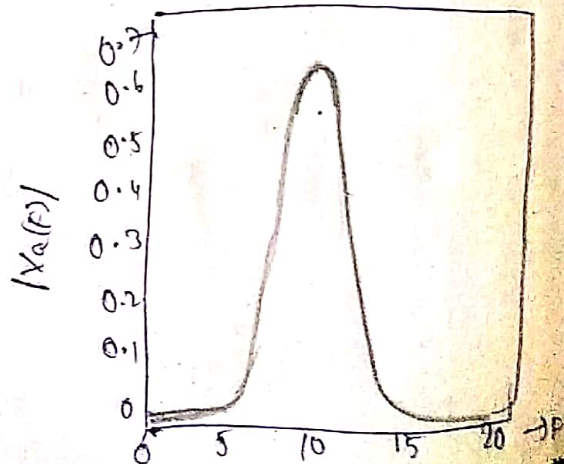
$$X_a(F) = \int_0^{\infty} A e^{-at} e^{-j2\pi F t}\, dt + \int_0^{\infty} A e^{-at} e^{-j2\pi F t}\, dt$$

$$= \frac{A}{a - j2\pi F} + \frac{A}{a + j2\pi F}$$

$$= \frac{2aA}{a^2 + (2\pi F)^2}$$

$$|X_a(F)| = \frac{2aA}{a^2 + (2\pi F)^2}$$

$$\angle X_a(F) = 0.$$



1.14

$x(n) = \{\ldots 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1 \ldots\}$

ⓐ $N = 6$

$$C_K = \frac{1}{6} \sum_{n=0}^{5} x(n) e^{-j2\pi kn/6}$$

$$= \left[3 + 2e^{-j2\pi k/6} + e^{-j2\pi k/3} + e^{-j4\pi k/3} + 2e^{-j10\pi k/6}\right]$$

$$C_k = \frac{1}{6}\left[3 + 4\cos(\pi k/3) + 2\cos(2\pi k/3)\right]$$

Hence $C_0 = 9/6$, $C_1 = 4/6$, $C_2 = 0$, $C_3 = \frac{1}{6}$, $C_4 = 0$, $C_5 = 4/6$

ⓑ
$$P_t = \frac{1}{6} \sum_{n=0}^{5} |x(n)|^2$$

$$= \frac{1}{6}\left(3^2 + 2^2 + 1^2 + 0^2 + 1^2 + 2^2\right) = 19/6$$

$$P_f = \sum_{n=0}^{5} |c(n)|^2$$

$$= \left[\left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + 0^2 + \left(\frac{4}{6}\right)^2\right]$$

$$= 19/6$$

Thus $\boxed{P_t = P_f = \frac{19}{16}}$

1.15

$$x(n) = 2 + 2\cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$$

ⓐ power spectrum density spectrum = ?

(a) $c_k = \frac{1}{8}\sum_{n=0}^{7} x(n)\cdot e^{-j\pi kn/4}$

$x(n) = \{ 1/2,\ 2+\frac{3}{4}\sqrt{2},\ 1,\ 2-\frac{3}{4}\sqrt{2},\ \frac{1}{2},\ 2-\frac{3}{4}\sqrt{2},\ 1, 2+\frac{3}{4}\sqrt{2} \}$

Hence,

$c_0 = 2,\ c_1 = 1,\ c_2 = \frac{1}{2},\ c_3 = \frac{1}{4},\ c_4 = 0,\ c_5 = \frac{1}{4},\ c_6 = \frac{1}{2}$

(b) $P = \sum_{q=0}^{7} |c(q)|^2$

$= 4+1+1+\frac{1}{4}+\frac{1}{4}+\frac{1}{16}+\frac{1}{16}$

$= 53/8$

4.6)

(a) $x(n) = 4\sin\frac{\pi(n-2)}{3}$

$= 4\sin\frac{2\pi(n-2)}{6}$

$c_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\cdot e^{-j2\pi kn/6}$

$= \frac{4}{6}\sum_{n=0}^{5}\sin\left(\frac{2\pi(n-2)}{6}\right)\cdot e^{-j2\pi kn/6}$

$= \frac{1}{\sqrt{3}}\left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{-j\pi k/3} + e^{-j2\pi k/3}\right]$

$= \frac{1}{\sqrt{3}}(-j2)\left[\sin\left(\frac{2\pi k}{6}\right) + \sin\left(\frac{\pi k}{3}\right)\right] e^{-j2\pi k/3}$

Hence,

$C_0 = 0$

$C_1 = -j2e^{-j2\pi/3}$

$C_2 = C_3 = C_4 = 0$, $C_5 = C_1^* = j2e^{+j2\pi/3}$

So., $|C_1| = |C_5|$ & $|C_0| = |C_2| = |C_3| = |C_4| = 0$

$\angle C_1 = \pi + \pi/2 - 2\pi/3 = 5\pi/6$

$\angle C_5 = -5\pi/6$

$\angle C_0 = \angle C_2 = \angle C_3 = \angle C_4 = 0$

(b.) $x(n) = \cos\left(\frac{2\pi n}{3}\right) + \sin\left(\frac{2\pi n}{5}\right) \Rightarrow N = 15$

$C_k = C_{1k} + C_{2k}$

C_{1k}: DTFS Co-efficients of $\cos\left(\frac{2\pi n}{3}\right)$

C_{2k}: DTFS coefficients of $\sin\left(\frac{2\pi n}{5}\right)$

but $\cos\left(\frac{2\pi n}{3}\right) = \frac{1}{2}\left(e^{j2\pi n/3} + e^{-j2\pi n/3}\right)$

hence ;, $C_{1k} = \begin{cases} \frac{1}{2} ; & k = 5, 10 \\ 0 ; & \text{otherwise} \end{cases}$

Silly.,
$\sin\left(\frac{2\pi n}{5}\right) = \frac{1}{2j}\left(e^{\frac{j2\pi n}{5}} - e^{-j2\pi n/5}\right)$

hence $C_{2k} = \begin{cases} \frac{1}{2j} & K = 3 \\ \frac{-1}{2j} & K = 12 \\ 0 & \text{Otherwise.} \end{cases}$

$\therefore$ $C_k = C_{1k} + C_{2k} = \begin{cases} 1/2j & k = 3 \\ 1/2 & k = 5, 10 \\ -1/2j & K = 12 \\ 0 & \text{otherwise} \end{cases}$

(c) $x(n) = \cos\left(\frac{2\pi n}{3}\right) \cdot \sin\left(\frac{2\pi n}{5}\right)$

$= \frac{1}{2}\sin\left(\frac{16\pi n}{5}\right) - \frac{1}{2}\cdot\sin\left(\frac{4\pi n}{5}\right)$

as same as procedure in b we get

$$C_k = \begin{cases} -1/4j, & k = 2,7 \\ 1/4j, & k = 8,13 \\ 0, & \text{otherwise} \end{cases}$$

(d) $x(n) = \{\ldots, -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2, \ldots\}$

$N = 5$.

$C_k = \frac{1}{5}\sum_{n=0}^{4} x(n) \cdot e^{-j2\pi nk/5}$

$= \frac{1}{5}\left[e^{-j2\pi k/5} + 2e^{-j4\pi k/5} - 2e^{-j6\pi k/5} - e^{-j8\pi k/5}\right]$

$= 2j/5\left[-\sin\left(2\pi k/5\right) - 2\sin\left(4\pi k/5\right)\right]$

$\therefore$ $C_0 = 0$

$C_1 = 2j/5\left[-\sin(2\pi/5) + 2\sin(4\pi/5)\right]$

$C_2 = 2j/5\left(\sin(4\pi/5) - 2\sin(8\pi/5)\right)$

$C_3 = -C_2$

$C_4 = -C_1$

(e) $x(n) = \{\ldots, -1, 2, \underset{\uparrow}{1}, 2, -1, 0, -1, 2, 1, 2 \ldots$

$N = 6$

$$C_k = \frac{1}{6} \sum_{n=0}^{5} x(n) \cdot e^{-j2\pi nk/6}$$

$$= \frac{1}{6}\left[1 + 2e^{-j\pi k/3} - e^{-j2\pi k/3} - e^{-j4\pi k/3} + 2e^{-j5\pi k/3}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos(\pi k/3) - 2\cos(2\pi k/3)\right]$$

Therefore $C_0 = 1/2$

$C_1 = 2/3$, $C_2 = 0$, $C_3 = -5/6$, $C_4 = 0$, $C_5 = 2/3$

(g) $x(n) = 1 \quad -\infty < n < \infty$

$N = 1$, $C_k = x(0) = 1$ (or) $C_0 = 1$

(h) $x(n) = (-1)^n$; $-\infty < n < \infty$

$N = 2$. $x(n) = \{\ldots -1, +1, -1, +1, \ldots\}$

$$C_k = \frac{1}{2} \sum_{n=0}^{1} x(n) \cdot e^{-j\pi nk}$$

$$= \frac{1}{2}\left(1 - e^{-j\pi k}\right)$$

$$\boxed{C_0 = 0,\ C_1 = 1}$$

1.7)

Sol: Finding periodic signals, given.

(a) $C_k = \cos k\pi/4 + \sin 3k\pi/4$

$$x(n) = \sum_{k=0}^{7} C_k\, e^{j2\pi nk/8}$$

given, $c_k = e^{j2\pi pk/8}$ then,

$$\sum_{k=0}^{7} e^{j2\pi pk/8} \cdot e^{j2\pi nk/8} = \sum_{n=0}^{7} e^{\frac{j2\pi(p+n)k}{8}}$$

$$= \begin{cases} 8 & p = -n \\ 0 & p \neq -n \end{cases}$$

Since $c_k = \frac{1}{2}\left[e^{j2\pi k/8} + e^{-j2\pi k/8}\right] + \frac{1}{2j}\left[e^{j6\pi k/8} - e^{-j6\pi k/8}\right]$

we have $x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\,\delta(n+3) +$

$4j\,\delta(n-3)$; $-3 \leq n \leq 5$

(b)

$$c_k = \begin{cases} \sin(k\pi/3) & ; 0 \leq k \leq 6 \\ 0 & ; k = 7 \end{cases}$$

$c_0 = 0,\ c_1 = \sqrt{3}/2,\ c_2 = \sqrt{3}/2,\ c_3 = 0,\ c_4 = -\sqrt{3}/2,\ c_5 = -\sqrt{3}/2,$

$c_6 = c_7 = 0$

$$x(n) = \sum_{k=0}^{7} c_k\, e^{j2\pi nk/8}$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j2\pi n}{4}} - e^{\frac{j\pi 4n}{4}} - e^{\frac{j5\pi n}{4}}\right]$$

$$x(n) = \sqrt{3}\left[\sin\left(\frac{\pi n}{2}\right) + \sin\left(\frac{\pi n}{4}\right)\right] e^{\frac{j\pi(3n-2)}{4}}$$

(c) $\{c_k\} = \{\ldots 0, \frac{1}{4}, \frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}, \frac{1}{4}, 0 \ldots\}$

$$x(n) = \sum_{k=-3}^{4} c_k\, e^{j2\pi nk/8}$$

$$= 2 + e^{j\pi n/4} + e^{-j\pi n/4} + \frac{1}{2} e^{\frac{j\pi n}{2}} + \frac{1}{2} e^{-\frac{j\pi n}{2}}$$

$$+ \frac{1}{4} e^{j3\pi n/4} + \frac{1}{4} e^{\frac{-j3\pi n}{4}}$$

$= 2 + 2\cos\left(\frac{7\pi n}{4}\right) + \cos\left(\frac{\pi n}{5}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$

Q.9) Fourier transform for the signals..

(a) $x(n) = u(n) - u(n-6)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{5} e^{-j\omega n}$$

$$\boxed{X(\omega) = \frac{1 - e^{-j6\omega}}{1 - e^{-j\omega}}}$$

(b) $x(n) = 2^n u(-n)$

$$X(\omega) = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n}$$

$$= \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^m$$

$$\boxed{X(\omega) = \frac{2}{2 - e^{j\omega}}}$$

(c) $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$

$$X(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m e^{-j\omega m} \quad 4^4 \cdot e^{j4\omega}$$

$$\boxed{X(\omega) = \frac{4^4 e^{j4\omega}}{1 - \frac{1}{4} e^{-j\omega}}}$$

(d) $x(n) = a^n \sin\omega_0 n \, u(n)$, $|a|<1$

$$X(\omega) = \sum_{n=0}^{\infty} a^n \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} \right] e^{-j\omega n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} [a e^{-j(\omega-\omega_0)}]^n - \frac{1}{2j} \sum_{n=0}^{\infty} [a e^{-j(\omega+\omega_0)}]^n$$

$$= \frac{1}{2j} \left[\frac{1}{1-a e^{-j(\omega-\omega_0)}} - \frac{1}{1-a e^{-j(\omega+\omega_0)}} \right]$$

$$= \cancel{\frac{1}{2j}} \cdot \frac{a \sin\omega_0 e^{-j\omega}}{1-2a\cos\omega_0 e^{-j\omega} + a^2 e^{-j2\omega}}$$

A.

(e)

$x(n) = |a|^n \sin\omega_0 n$, $|a|<1$

note that $\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |a|^n |\sin\omega_0 n|$

Suppose $\omega_0 = \pi/2$, then $|\sin\omega_0 n| = 1$.

$$\sum_{n=-\infty}^{\infty} |a|^n = \sum_{n=-\infty}^{\infty} |x(n)| \longrightarrow \infty \Rightarrow \boxed{\text{diverge}}$$

So. Fourier transform not exist.

(f)

$$x(n) = \begin{cases} 2 - \left(\frac{1}{2}\right)^n ; & |n| \le 4 \\ 0 ; & \text{otherwise} \end{cases}$$

$$X(\omega) = \sum_{n=-4}^{4} x(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} \left(2 - \left(\frac{1}{2}\right)^n\right) e^{-j\omega n}$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} - \frac{1}{2}\Big[-4e^{j4\omega} + 4e^{j4\omega} - 3e^{-j\omega 3} + e^{-j3\omega} + 2e^{-j2\omega} - e^{-j\omega} + e^{-j\omega}\Big]$$

g) $x(n) = \{-2, -1, 0, 1, 2\}$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}$$

$$= -2e^{j2\omega} - e^{j\omega} + e^{j\omega} + 2e^{-j2\omega}$$

$$= -2j\{2\sin 2\omega + \sin\omega\}$$

h)

$$x(n) = \begin{cases} A(2M+1-|n|; & |n| \le M \\ 0 & ; \quad |n| > M \end{cases}$$

$$X(\omega) = \sum_{n=-M}^{M} (2M+1-|n|)e^{-j\omega n}$$

$$= (2M+1)A + A\sum_{k=1}^{M}(2M+1-k)(e^{-j\omega k} + e^{j\omega k})$$

$$= (2M+1)A + 2A\sum_{k=1}^{M}(2M+1-k)\cos\omega k.$$

4.10)

(a) $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) \cdot e^{j\omega n}\, d\omega.$

given $X(\omega) = \begin{cases} 0 & ; \quad 0 \le |\omega| \le \omega_0 \\ 1 & \quad \omega_0 < |\omega| \le \pi \end{cases}$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_0} 1 \cdot e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_0}^{\pi} e^{j\omega n}\, d\omega$$

$$x(0) = \frac{1}{2\pi}(\pi - \omega_0) + \frac{1}{2\pi}(\pi - \omega_0)$$

$$= \frac{\pi - \omega_0}{\pi}$$

for $n \neq 0$, $\int_{-\pi}^{-\omega_0} e^{j\omega n} d\omega = \frac{1}{jn} e^{j\omega n} \Big|_{-\pi}^{-\omega_0}$

$$= \frac{1}{jn}\left(e^{-j\omega_0 n} - e^{-j\pi n}\right)$$

$$\int_{\omega_0}^{\pi} e^{j\omega n} d\omega = \frac{1}{jn} e^{j\omega n} \Big|_{\omega_0}^{\pi}$$

$$= \frac{1}{jn}\left(e^{j\pi n} - e^{j\omega_0 n}\right)$$

Hence, $x(n) = -\frac{\sin n\omega_0}{n\pi}$; $n \neq 0$

(b)

Given $X(\omega) = \cos^2(\omega)$

$$= \left(\frac{1}{2} e^{j\omega} + \frac{1}{2} e^{-j\omega}\right)^2$$

$$= \frac{1}{4}\left(e^{j2\omega} + 2 + e^{-j2\omega}\right)$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) \cdot e^{j\omega n} d\omega$$

$$= \frac{1}{8\pi}\left[2\pi\, \delta(n+2) + 4\pi\, \delta(n) + 2\pi\, \delta(n-2)\right]$$

$$= \frac{1}{4}\left[\delta(n+2) + 2\,\delta(n) + \delta(n-2)\right]$$

//.

(c) $X(\omega) = \begin{cases} 1 & ; \omega_0 - \delta\omega/2 \le |\omega| \le \omega_0 + \delta\omega/2 \\ 0 & ; \text{elsewhere} \end{cases}$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(n) \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{\omega_0 - \delta\omega/2}^{\omega_0 + \delta\omega/2} e^{j\omega n}\, d\omega$$

$$= \frac{2}{\pi}\,\delta\omega\left(\frac{\sin(n\,\delta\omega/2)}{n\,\delta\omega/2}\right) e^{jn\omega_0}$$

d) $$\frac{1}{2\pi} Re\left\{ \int_0^{\pi/8} 2\, e^{j\omega n} d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n} d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n} d\omega + \int_{7\pi/8}^{\pi} e^{j\omega n} d\omega \right\}$$

$$\Rightarrow \frac{1}{\pi}\int_0^{\pi/8} 2\cos\omega n\, d\omega + \int_{\pi/8}^{3\pi/8} \cos\omega n\, d\omega + \int_{6\pi/8}^{7\pi/8} \cos\omega n\, d\omega + \int_{7\pi/8}^{\pi} 2\cos\omega n\, d\omega$$

$$\Rightarrow \frac{1}{n\pi}\left[\sin\left(\frac{7\pi n}{8}\right) + \sin\left(\frac{6\pi n}{8}\right) - \sin\left(\frac{3\pi n}{8}\right) - \sin\left(\frac{\pi n}{8}\right)\right]$$

4.

given $x(n) = \{1, 0, -1, \underset{\uparrow}{2}, 3\}$

$$X(\omega) = X_R(\omega) + j\, X_I(\omega)$$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \left\{\frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \frac{1}{2}\right\}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$= \left\{\frac{1}{2}, 0, -2, \underset{\uparrow}{0}, 2, 0, \frac{1}{2}\right\}$$

Then $X_e(\omega) = \sum_{n=-3}^{3} x_e(n) \cdot e^{-j\omega n}$

$j X_1(\omega) = \sum_{n=-3}^{3} x_o(n) \cdot e^{-j\omega n}$

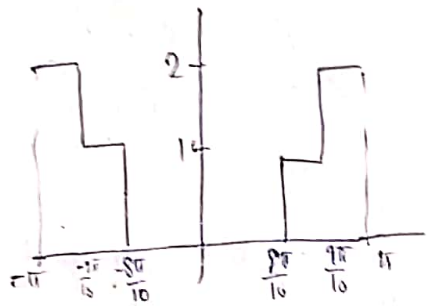
Now, $Y(\omega) = X_1(\omega) + X_e(\omega) \cdot e^{j2\omega}$

$\therefore \quad y(n) = F^{-1}[X_1(\omega)] + F^{-1}\{X_e(\omega)\, e^{j2\omega}\}$

$= -j\, x_o(n) + x_e(n+2)$ Ans.

4.12) Given fourier transform

a)



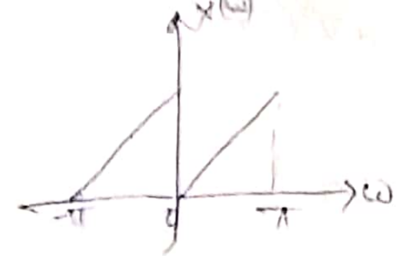
$$x(n) = \frac{1}{2\pi}\left[\int_{8\pi/10}^{9\pi/10} e^{j\omega n}\, d\omega + \int_{-9\pi/10}^{-8\pi/10} e^{j\omega n}\, d\omega + 2\int_{9\pi/10}^{\pi} e^{j\omega n}\, d\omega + 2\int_{-\pi}^{-9\pi/10} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{j9\pi n/10} - e^{-j9\pi n/10} - e^{j8\pi n/10} + e^{-j8\pi n/10}\right) + \frac{2}{jn}\left(-e^{j9\pi n/10} + e^{-j9\pi n/10} + e^{j\pi n} - e^{-j\pi n}\right)\right]$$

$$= \frac{1}{\pi n}\left[\sin \pi n - \sin 8\pi n/10 - \sin 9\pi n/10\right]$$

$$= -\frac{1}{n\pi}\left[\sin 4\pi n/5 + \sin 9\pi n/10\right].$$ Ans.

b) $X(\omega) \Rightarrow$



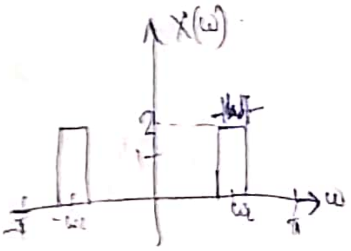
$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} X(\omega)\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi} X(\omega)\cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi}\frac{\omega}{\pi}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{\omega}{jn\pi}\, e^{j\omega n}\Big|_{-\pi}^{\pi} + \frac{e^{j\omega n}}{jn}\Big|_{-\pi}^{0}\right]$$

$$= \frac{1}{\pi n}\sin\left(\frac{\pi n}{2}\right)\cdot e^{-jn\pi/2}.$$

c)



$$x(n) = \frac{1}{2\pi}\int_{\omega_c-\omega/2}^{\omega_c+\omega/2} 2\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{-\omega_c-\omega/2}^{-\omega_c+\omega/2} 2\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{\pi}\left[\frac{1}{jn}\, e^{j\omega n}\Big|_{\omega_c-\omega/2}^{\omega_c+\omega/2} + \frac{e^{j\omega n}}{jn}\Big|_{\omega_c-\omega/2}^{-\omega_c+\omega/2}\right]$$

$$= \frac{2}{\pi n}\left[\frac{e^{j(\omega_c+\omega/2)n} - e^{j(\omega_c-\omega/2)n} + e^{-j(\omega_c-\omega/2)n} - e^{-j(\omega_c}}{2j}\right]$$

$$= \frac{2}{\pi n}\left[\sin\left(\omega_c+\omega/2\right)n - \sin\left(\omega_c-\omega/2\right)n\right]$$

4.13)

$$x_1(n) = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

$$X_1(\omega) = \sum_{n=0}^{M} e^{-j\omega n}$$

$$= \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$$

$$x_2(n) = \begin{cases} 1; & -M \le n \le -1 \\ 0; & \text{otherwise} \end{cases}$$

$$X_2(\omega) = \sum_{n=-M}^{-1} e^{-j\omega n}$$

$$= \sum_{n=1}^{M} e^{j\omega n}$$

$$= \left[\frac{1 - e^{j\omega M}}{1 - e^{j\omega}}\right] \cdot e^{j\omega}$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$= \frac{1 + e^{j\omega} - e^{j\omega} - 1 - e^{-j\omega(M+1)} - e^{j\omega(M+1)} + e^{j\omega M} + e^{-j\omega M}}{2 - e^{-j\omega} - e^{j\omega}}$$

$$= \frac{2\cos\omega M - 2\cos\omega(M+1)}{2 - 2\cos\omega}$$

$$= \frac{2\sin\left(\omega M + \frac{\omega}{2}\right)\cos\omega/2}{2\sin^2(\omega/2)}$$

$$= \frac{\sin\left(M + \frac{1}{2}\right)\omega}{\sin(\omega/2)}$$

1.14) $x(n) = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$

a) $X(0) = \sum_n x(n) = -1$

b) $\angle X(\omega) = \pi$ for all ω

c) $x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, d\omega$

hence, $\int_{-\pi}^{\pi} X(\omega) d\omega = 2\pi \cdot x(0)$

$= 2\pi(-3) \Rightarrow -6\pi$

d) $X(\pi) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-jn\pi} = \sum_n (-1)^n x(n)$

$= -3 - 4 - 2 = -9$

e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = 2\pi \sum_n |x(n)|^2 = 2\pi(19) \Rightarrow 38\pi$

1.15)

Given, $c = \dfrac{\sum_{n=-\infty}^{\infty} n\, x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$

$X(\omega) = \sum_n x(n) \cdot e^{-j\omega n}$

$X(0) = \sum_n x(n)$

$\left.\frac{d X(\omega)}{d\omega}\right|_{\omega=0} = -j \sum n\, x(n) \cdot e^{-j\omega n} \Big|_{\omega=0}$

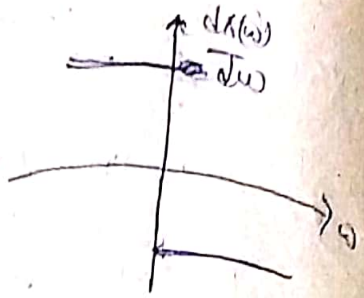
$= -j \sum_n n\, x(n)$

4.4

Therefore, $C = \dfrac{j\frac{d X(\omega)}{d\omega}\Big|_{\omega=0}}{X(0)}$

(5)

$X(0) = 1$, $\frac{dX(\omega)}{d\omega} \to$



So., $C = \frac{0}{1} \Rightarrow 0$

(6)

given., $a^n u(n) \xleftrightarrow{F} \dfrac{1}{1-ae^{-j\omega}}$; $|a|<1$

need to show.,

$x(n) = \dfrac{(n+l-1)!}{n!(l-1)!} a^n u(n) \xleftrightarrow{F} X(\omega) = \dfrac{1}{(1-ae^{-j\omega})^l}$

$x_1(n) = a^n u(n)$

Suppose that,

$x_k(n) = \dfrac{(n+k-1)!}{n!(k-1)!} a^n u(n) \xleftrightarrow{F} \dfrac{1}{(1-ae^{-j\omega})^k}$

holds then,

$x_{k+1}(n) = \dfrac{(n+k)!}{n!\,k!} a^n u(n)$

$= \dfrac{n+k}{k} x_k(n)$

$X_{k+1}(\omega) = \dfrac{1}{k} \sum_n n x_k(n) \cdot e^{-j\omega n} + \sum_n x_k(n) \cdot e^{-j\omega n}$

$= \dfrac{1}{k} j \dfrac{dX_k(\omega)}{d\omega} + X_k(\omega)$

$= \dfrac{ae^{-j\omega}}{(1-ae^{-j\omega})^{k+1}} + \dfrac{1}{(1-ae^{-j\omega})^k}$

4.1.5

a) $x^*(n)$

$$\sum_n x^*(n) \cdot e^{-j\omega n} = \left(\sum_n x(n) \cdot e^{-j(-\omega) \cdot n}\right)^* = X^*(-\omega)$$

b)

$$\sum_n x^*(-n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x^*(n) \cdot e^{j\omega n} = X^*(\omega)$$

c)

$$\sum_n y(n) e^{-j\omega n} = \sum_n x(n) e^{-j\omega n} - \sum_n x(n-1) e^{-j\omega n}$$

$$Y(\omega) = X(\omega) + X(\omega) e^{-j\omega}$$

$$= \left(1 - e^{-j\omega}\right) X(\omega).$$

d)

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)$$

$$= y(n) - y(n-1)$$

hence $X(\omega) = \left(1 - e^{-j\omega}\right) Y(\omega)$

$$Y(\omega) = \frac{X(\omega)}{1 - e^{-j\omega}}$$

e) $Y(\omega) = \sum_n x(n) \cdot e^{-j\omega n}$

$$= \sum_n x(n) \cdot e^{-j\omega/2\, n}$$

$$= X(\omega/2)$$

f) $Y(\omega) = \sum_n x(n/2) e^{-j\omega n}$

$$= \sum_n x(n) \cdot e^{-j2\omega n} \Rightarrow X(2\omega)$$

4.18) Fourier transforms.,

a) $x_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$$X_1(\omega) = \sum_n x(n) \cdot e^{-j\omega n}$$

$$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$$

$$= 1 + 2\cos\omega + 2\cos 2\omega$$

b) $x_2(n) = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

$$X_2(\omega) = \sum_n x_2(n) \cdot e^{-j\omega n}$$

$$= e^{j4\omega} + e^{j2\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$$

$$= 1 + 2\cos 2\omega + 2\cos 4\omega$$

c) $x_3(n) = \{1, 0, 0, 1, 0, 0, \underset{\uparrow}{1}, 0, 0, 1, 0, 0, 1\}$

$$X_3(\omega) = \sum x_3(n) e^{-j\omega n}$$

$$= e^{j6\omega} + e^{j3\omega} + 1 + e^{-j3\omega} + e^{-j6\omega}$$

d) Yes.,

relation, $X_2(\omega) = X_1(2\omega)$ and $X_3(\omega) = X_1(3\omega)$.

e) $x_k(n) = \begin{cases} x\left(\frac{n}{k}\right) & \text{if } n/k \text{ integer} \\ 0 & \text{otherwise,} \end{cases}$ then show that,

$$X_k(\omega) = X(k\omega)$$

Proof.

$$X_k(\omega) = \sum_{n,\ \frac{n}{k}\ \text{an integer}} x_k(n) \cdot e^{-j\omega n}$$

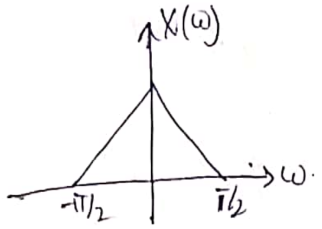
$$= \sum_n x(n)\, e^{-jk\omega n}$$

$$= X(k\omega)$$

4.19)

Sol.

Given $X(\omega) =$



a) $x_1(n) = x(n) \cdot \cos(\pi n/4)$

$$x_1(n) = \frac{1}{2}\left(e^{j\pi n/4} + e^{-j\pi n/4}\right) x(n)$$

$$X_1(\omega) = \frac{1}{2}\left[X\left(\omega - \pi/4\right) + X\left(\omega + \pi/4\right)\right]$$

b) $x_2(n) = \frac{1}{2j}\left(e^{j\pi n/2} + e^{-j\pi n/2}\right) x(n)$

$$X_2(\omega) = \frac{1}{2j}\left(X\left(\omega - \pi/2\right) + X\left(\omega + \pi/2\right)\right)$$

c) $x_3(n) = \frac{1}{2}\left(e^{j\pi n/2} + e^{-j\pi n/2}\right) x(n)$

$$X_3(\omega) = \frac{1}{2}\left[X\left(\omega - \pi/2\right) + X\left(\omega + \pi/2\right)\right]$$

d) $x_4(n) = \frac{1}{2}\left(e^{j\pi n} + e^{-j\pi n}\right) x(n)$

$$X_4(\omega) = \frac{1}{2}\left[X(\omega - \pi) + X(\omega + \pi)\right]$$

$$= X(\omega - \pi)$$

4.21) Let $x_N(n) = \frac{\sin \omega_c n}{\pi n}$; $-N \le n \le N$

$= x(n) \cdot w(n)$.

where $x(n) = \frac{\sin \omega_c n}{\pi n}$, $-\infty \le n \le \infty$

$w(n) = 1$, $-N \le n \le N$

$= 0$, otherwise

Then $\frac{\sin \omega_c n}{\pi n} \xrightarrow{F} X(\omega)$

$= 1$; $|\omega| \le \omega_c$
0 ; otherwise.

$$X_N(\omega) = X(\omega) * W(\omega)$$

$$= \int_{-\pi}^{\pi} X(\theta) \cdot W(\omega - \theta) \, d\theta$$

$$= \int_{-\omega_c}^{\omega_c} \frac{\sin (2N+1) \cdot (\omega-\theta)/2}{\sin(\omega-\theta)/2} \, d\theta$$

4.22)

Given, $X(\omega) = \frac{1}{1 - a e^{-j\omega}}$ then Fourier transforms of,

a) $x(2n+1) = ?$

$$X_1(\omega) = \sum_n x(2n+1) e^{-j\omega n}$$

$$= \sum_k x(k) \cdot e^{-j\omega k/2} e^{j\omega/2}$$

$$= X(\omega/2) \cdot e^{j\omega/2} \Rightarrow \frac{e^{j\omega/2}}{1 - a e^{j\omega/2}}$$

b) $x_2(n) = e^{j\pi n/2}\, x(n+2)$

$$X_2(\omega) = \sum_n e^{j\pi n/2} x(n+2) \cdot e^{-j\omega n}$$

$$= \sum_K x(k) \cdot e^{-jk(\omega + j\pi/2)} e^{j2\omega}$$

$$= -X\left(\omega + \frac{j\pi}{2}\right) e^{j2\omega}$$

c) $X_3(\omega) = ?$

$x_3(n) = x(-2n)$.

$$X_3(\omega) = \sum_n x(-2n)\, e^{-j\omega n}$$

$$= -\sum_K x(k) \cdot e^{-jk\omega/2}$$

$$= X(-\omega/2)$$

d) $x_4(n) = x(n) \cdot \cos(0.3\pi n)$

$$X_4(\omega) = \sum_n \frac{1}{2}\left(e^{j0.3\pi n} + e^{-j0.3\pi n}\right) x(n) \cdot e^{-j\omega n}$$

$$= \frac{1}{2} \sum x(n) \cdot \left[e^{-j(\omega - 0.3\pi)n} + e^{-j(\omega + 0.3\pi)n}\right]$$

$$= \frac{1}{2}\left\{X(\omega - 0.3\pi) + X(\omega + 0.3\pi)\right\}$$

e) $X_5(\omega) = X(\omega) \cdot \left[X(\omega) \cdot e^{-j\omega}\right] = X^2(\omega) \cdot e^{-j\omega}$

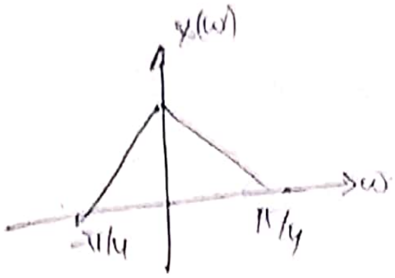
f) $x(n) * x(-n) \Rightarrow X(\omega) \cdot X(-\omega)$

$$\Rightarrow \frac{1}{(1 - ae^{-j\omega})(1 - ae^{j\omega})}$$

$$= \frac{1}{1 - 2a\cos\omega + a^2}$$

Given,

$X(\omega) =$



Fourier transforms of,

a) $y_1(n) = \begin{cases} x(n) & , n \text{ even} \\ 0 & , n \text{ odd} \end{cases}$

and $y_1(n) = x(n) \cdot s(n)$, where $s(n) = \{\ldots 0, 1, 0, 1, 0, 1, \ldots\}$

$$Y_1(\omega) = \sum_n y_1(n) \cdot e^{-j\omega n}$$

$$= \sum_{n, \text{even}} \text{for } x(n) \cdot e^{-j\omega n}$$ obtained by combining res of b and c

b) $y_2(n) = x(2n)$

$$Y_2(\omega) = \sum_n y_2(n) \cdot e^{-j\omega n}$$

$$= \sum_n x(2n) \cdot e^{-j\omega n}$$

$$= \sum_m x(m) \cdot e^{-j\omega m/2}$$

$$= X(\omega/2)$$

c) $y_3(n) = \begin{cases} x(n/2) & ; n \text{ (even)} \\ 0 & ; \text{otherwise} \end{cases}$

$$Y_3(\omega) = \sum_n y_3(n) \, e^{-j\omega n}$$

$$= \sum_{n \text{ even}} x(n/2) \cdot e^{-j\omega n}$$

$$= \sum_m x(m)\, e^{-j2\omega m}$$

$$= X(2\omega).$$

So, now $y_1(n)$ may be expressed as,

$$y_1(n) = \begin{cases} y_2(n/2), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

hence $Y_1(\omega) = Y_2(2\omega)$