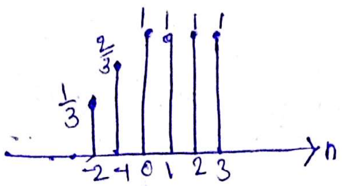
DSP:

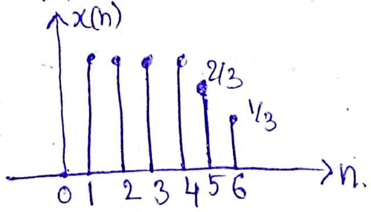
2.1)
$$x(n) = \begin{cases} 1 + \frac{n}{3} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

a)
$$x(n) = \{0, \tfrac{1}{3}, \tfrac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots\}$$

$x(n) =$



b) i) First fold and then delay by 4 samples,

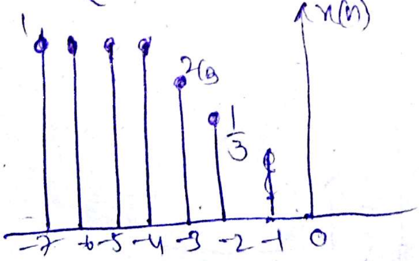


ii) first delay x(n) and then fold., (4 samples delay)

$$x(n-4) = \{\ldots \underset{\uparrow}{0}, 0, \tfrac{1}{3}, \tfrac{2}{3}, 1, 1, 1, 1, 0 \ldots\}$$

folding.

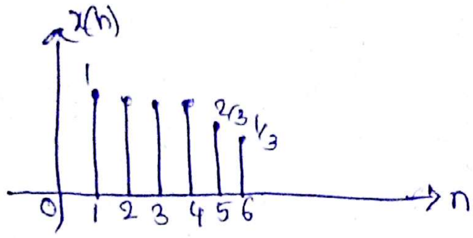
$$x(-n-4) = \{0, 1, 1, 1, 1, \tfrac{2}{3}, \tfrac{1}{3}, 0, \underset{\uparrow}{0} \ldots\}$$



c) $x(-n+4) = ?$

$x(n+4) = \{ \ldots 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, \underset{\uparrow}{0} \ldots \}$

$x(-n+4) = \{ \ldots \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0 \ldots \}$



d) $x(-n+k) \Rightarrow$ we will get this result by first shifting by k units and then folding $x(n+k)$.

e) $x(n)$ in terms of $\delta(n)$ and $u(n)$.

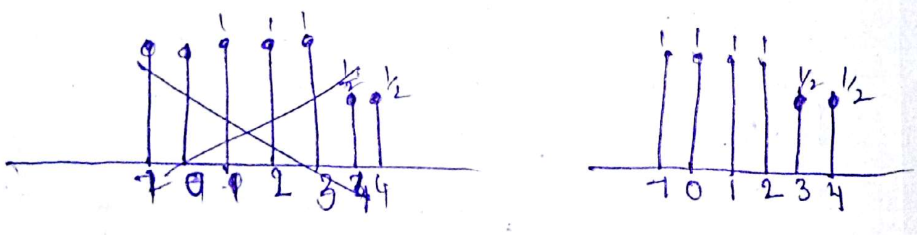
$x(n) = \frac{1}{3}\delta(n+2) + \frac{2}{3}\delta(n+1) + 1\cdot\delta(n) + 1\cdot\delta(n-1) + 1\cdot\delta(n-2) + 1\cdot\delta(n-3)$

(or)

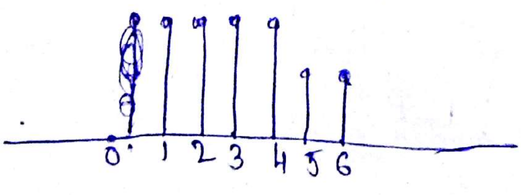
$x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$

2.2)

given $x(n)$.,



a) $x(n-2)$.,

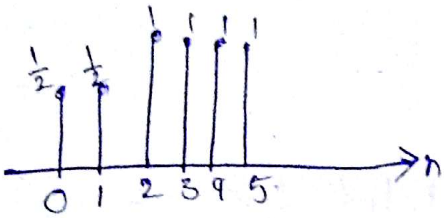


b) $x(4-n) \Rightarrow x(-n+4)$

$x(n) = \{0, 1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}, 0\}$

$x(n+4) = \{0, 1, 1, 1, 1, 1, \frac{1}{2}, \underset{\uparrow}{\frac{1}{2}}, 0\}$

$x(-n+4) = \{0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 1, 0\}$



d) $x(n) \cdot u(2-n)$

$x(n) = \{0, 1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}, 0 \ldots\}$,

$u(2-n) = \{0, \underset{\uparrow}{1}, 1, 1, \ldots\}$

$u(n+2) = \{0, 1, 1, \underset{\uparrow}{1}, 1 \ldots\}$

$u(-n+2) = \{0, 1, \underset{\uparrow}{1}, 1, 1, 0, 0 \ldots\}$

so,

$x(n) \cdot u(n-2) = \{0, 1, \underset{\uparrow}{1}, 1, 1, 0, 0 \ldots\}$

e) $x(n-1) \cdot \delta(n-3)$

$x(n-1) = \{0, \underset{\uparrow}{1}, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0 \ldots\}$

$\delta(n-3) = \{0, \underset{\uparrow}{0}, 0, 0, 1, 0, 0 \ldots\}$

$x(n-1) \cdot \delta(n-3) \Rightarrow x(n-1+3) \cdot \delta(n-3)$

$\Rightarrow x(n+2) \cdot \delta(n-3)$

$\Rightarrow \{\ldots \underset{\uparrow}{0}, 0, 0, 1, 0, \ldots\}$

f) $x(n^2) = \{\ldots 0, x(0), x(1), x(4), x(9), x \ldots\}$

$= \{\ldots 0, 1, \underset{\underset{0}{\uparrow}}{1}, \underset{\underset{1}{\uparrow}}{1}, \underset{\underset{4}{\uparrow}}{1}, \underset{\underset{9}{\uparrow}}{\frac{1}{2}}, \underset{\underset{16}{\uparrow}}{\frac{1}{2}}, 0 \ldots\}$

g) $x_e(n) = ?$

$$\frac{x(n) + x(-n)}{2}$$

$$x(n) \Rightarrow \{ \ldots 0, 1, \underset{\uparrow}{1}, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0 \ldots \}$$

$$x(-n) \Rightarrow \{ 0, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0 \ldots \}$$

$$x(n) + x(-n) \Rightarrow \{ \tfrac{1}{2}, \tfrac{1}{2}, 1, 2, \underset{\uparrow}{2}, 2, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0 \ldots \}$$

$$\frac{x(n) + x(-n)}{2} \Rightarrow \{ \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0 \ldots \}$$

h) $x_o(n) = ?$

$$x(n) - x(-n) = \{ -\tfrac{1}{2}, -\tfrac{1}{2}, -1, 0, \underset{\uparrow}{0}, 0, \tfrac{1}{2}, \tfrac{1}{2}, 0 \ldots \}$$

$$\frac{x(n) - x(-n)}{2} = \{ -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{2}, 0, \underset{\uparrow}{0}, 0, \tfrac{1}{4}, \tfrac{1}{4}, 0 \ldots \}$$

2.3)

a) $u(n) = \{ 0, \underset{\uparrow}{1}, 1, 1, 1, \ldots \}$

$$u(n-1) = \{ 0, \underset{\uparrow}{0}, 1, 1, 1, \ldots \}$$

$$u(n) - u(n-1) = \{ 0, \underset{\uparrow}{1}, 0, 0, \ldots \} \Rightarrow \delta(n)$$

b) $$u(n) = \sum_{k=-\infty}^{\infty} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} \delta(k) = \begin{cases} 0 & , n<0 \\ 1 & , n \geq 0 \end{cases}$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0 ; n<0 \\ 1 , n\geq 0 \end{cases}$$

2.4)

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

Yes decomposition is unique., while,

$x_e(n) \Rightarrow \quad x(-n) = x(n)$

$x_o(n) \Rightarrow \quad x(-n) = -x(n)$

$x_e(n) = ? \quad x_o(e) = ?$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$x(-n) = \{6, 5, \underset{\uparrow}{4}, 3, 2\}$

$x_e(n) = \dfrac{x(n) + x(-n)}{2} = \{4, 4, \underset{\uparrow}{4}, 4, 4\}$

$x_o(n) = \dfrac{x(n) - x(-n)}{2} = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

2.5)

Given, $y(n) = T[x(n)] = x(n^2)$

Time invariant condition $\Rightarrow$ delay input response $=$ delay output

delay input response.,

$x(n-n_0) \Rightarrow y_1(n) = x[(n-n_0)^2]$

delay output for i/p $x(n)$ is.,

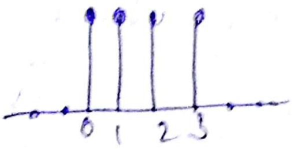
$y(n) \xrightarrow{\text{delaying}} y(n-n_0) = x(n^2-n_0)$

So delay i/p response $\neq$ delayed output so system is time variant.

b) given,

$$x(n) = \begin{cases} 1 & , 0 \le n \le 3 \\ 0 & , \text{else where} \end{cases}$$

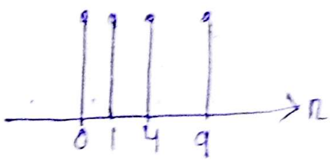
i) $x(n) = \{0, \underset{\uparrow}{1}, 1, 1, 1, 0 \cdots\}$



ii) $y(n) = T(x(n)) = x(n^2)$

$$= \{0, \underset{\uparrow}{1}, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0 \cdots\}$$

so, $y(n) =$



iii) $y_2'(n) = y(n-2)$

$$= \{\underset{\uparrow}{0}, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0 \cdots\}$$

iv) $x_2(n) = x(n-2)$

$$= \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \cdots\}$$

v) $y_2(n) = T[x_2(n)]$

$$= \{0, 0, 1, 1, 0, 0, 0, \cdots\}$$

vi) $\boxed{y_2(n) \neq y_2'(n)}$

So system is time variant.

c) $y(n) = x(n) - x(n-1)$

ii) $y(n) = \{0, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots\} - \{\underset{\uparrow}{0}, 1, 1, 1, 1, 0 \ldots\}$

~~$\{0, -1, \underset{\uparrow}{0}, 0, 1, 0 \ldots\}$~~

$y(n) = \{0, \underset{\uparrow}{1}, 0, 0, 0, -1, 0 \ldots\}$

iii) $y(n-2) = \{\underset{\uparrow}{0}, 0, 1, 0, 0, 0, -1, 0 \ldots\} = y_2'(n)$

iv) $x(n-2) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \ldots\} = x_2(n)$

v) $y_2(n) = T[x_2(n)]$

$y_2(n) = x_2(n) - x_2(n-1)$

$= \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \ldots\} - \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1, 0 \ldots\}$

$= \{\underset{\uparrow}{0}, 0, 1, 0, 0, 0, -1, 0 \ldots\}$

vi) $y_2'(n) = y_2(n) \Rightarrow$ Time invariant

d) $y(n) = n\, x(n)$

i) $x(n) = \{0, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots\}$

ii) $y(n) = n\, x(n) = \{0, \underset{\uparrow}{0}, 1, 2, 3, 0 \ldots\}$

iii) $y(n-2) = y_2'(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 2, 3, 0 \ldots\}$

iv) $x(n-2) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \ldots\} = x_2(n)$

v) $y_2(n) = T[x_2(n)] \Rightarrow n\, x_2(n)$

$\Rightarrow \{0, 0, 2, 3, 4, 5, 0 \ldots\}$

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vi) $y_2(n) \neq y_2'(n) \Rightarrow$ Time invariant

Q.1)

a) $y(n) = \cos[x(n)]$

→ static, non linear, time invariant, causal, stable.

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

→ Static, linear, time invariant, non causal, unstable

c) $y(n) = x(n) \cdot \cos(\omega_0 n)$

→ Static, non linear, time invariant, causal, stable

d) $y(n) = x(-n+2)$

→ dynamic, linear, time invariant, non causal, Stable.

e) $y(n) = \text{Trun}[x(n)]$

→ Static, non linear, time invariant, causal, Stable.

f) $y(n) = \text{Round}[x(n)]$

→ static, non linear, time invariant, Causal, stable

g) $y(n) = |x(n)|$

→ Static, non linear, time invariant, causal, stable

h) $y(n) = x(n) \cdot u(n)$

→ static, linear, time invariant, causal, stable.

i) $y(n) = x(n) + n\,x(n+1)$

→ ~~static~~ dynamic, linear, time invariant, non causal, unstable

j) $y(n) = x(2n)$

→ Dynamic, linear, time variant, noncausal, stable.

k) $y(n) = \begin{cases} x(n) & \text{if } x(n) \geq 0 \\ 0 & \text{if } x(n) < 0 \end{cases}$

→ Static, nonlinear, time invariant, causal, Stable

l) $y(n) = x(-n)$

→ Dynamic, linear, time invariant, non causal, stable

m) $y(n) = \text{sign}(x(n))$

→ Static, non linear, time invariant, causal, stable.

n) $x(n) = x_a(nT)$ → ideal sampling

→ Static, linear, time invariant, causal, stable.

2.9

a) $y(n) = \sum_{k=-\infty}^{n} h(k) \cdot x(n-k)$, $x(n) = 0$, $n < 0$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k) \cdot x(n+N-k) = \sum_{k=-\infty}^{n+N} h(k) \cdot x(n-k)$$

$$= \sum_{k=-\infty}^{n} h(k) \cdot x(n-k) + \sum_{k=n+1}^{n+N} h(k) \cdot x(n-k)$$

$$= y(n) + \sum_{k=n+1}^{n+N} h(k) \cdot x(n-k)$$

For BIBO system $\lim_{n\to\infty} |h(n)| = 0$,

$\therefore \lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k) \cdot x(n-k) = 0$ and

$$\lim_{n\to\infty} y(n+N) = y(n).$$

b)

Let $x(n) = x_0(n) + a\,u(n)$; where a is constant, and $x_0(n)$ is a bounded signal with $\lim_{n\to\infty} x_0(n) = 0$.

Then,
$$y(n) = a\sum_{k=0}^{\infty} h(k) \cdot u(n-k) + \sum_{k=0}^{\infty} h(k) \cdot x_0(n-k)$$

$$= a\sum_{k=0}^{n} h(k) + y_0(n)$$

Clearly, $\sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$ Hence

$$\lim_{n\to\infty} |y_0(n)| = 0$$

c) $y(n) = \sum_k h(k) \cdot x(n-k)$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_k h(k) \cdot x(n-k) \right]^2$$

$$= \sum_k \sum_l h(k) \cdot h(l) \cdot \sum_n x(n-k) \cdot x(n-l)$$

But, $\sum_n x(n-k) \cdot x(n-l) \leq \sum_n x^2(n) = E_x$

$$\therefore \sum_n y^2(n) \leq E_x \sum_k |h(k)| \cdot \sum_l |h(l)|$$

For BIBO system. $\sum_k |h(k)| < M$

Hence, $E_y \leq M^2 E_x$; So that

$E_y < 0$ if $E_x < 0$

2.10)

$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\}$

$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$

For time invariant systems for $y_2(n)$ should have only 3 elements and $y_3(n)$ should have 4 elements

So, it is non-linear.

2.11) $x_1(n) + x_2(n) = \delta(n)$

System is linear, the impulse response of the system

is $y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$.

If the system were time invariant, the response to $x_3(n)$ would be

$\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$ ∴ But this is not a case.

2.12)

a) Any weighted linear combination of the signals $x_i(n)$,

$x_i(n)$. $i = 1, 2 \ldots N$.

b) Any $x_i(n-k)$ where k is any integer and $i = 1, 2, \ldots N$.

2.13

response of LTI system.,

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(k-n)$$

for BIBO stability.,

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)| \, |x(k-n)|$$

↳ max value of signal let M_x

$$\infty \leq \sum_{n=-\infty}^{\infty} M_x \cdot |h(n)|$$

for stable, condition is,

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_b \leq \infty$$

2.14) a)

Causal system $\Leftrightarrow$ o/p becomes non zero after the i/p becomes non zero.

Hence, $x(n) = 0$ for $n < n_0 \Rightarrow y(n) = 0$ for $n < n_0$

b)

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k) \text{ where } x(n) = 0 \text{ for } n<0$$

If $h(k) = 0$ for $k<0$, then

$$y(n) = \sum_{k=0}^{n} h(k) \cdot x(n-k) \text{ and hence } y(n) = 0 \text{ for } n<0.$$

(or)

$$\sum_{-\infty}^{n} h(k) \cdot x(n-k) \Rightarrow h(k) = 0, \; k<0 \text{ when } y(n) = 0$$

2.15)

$$\sum_{n=M}^{N} a^n.$$

Series will be, $a^M, a^{M+1} \ldots a^N$

for $a \neq 1$,

by Sum of N terms in GP $\Rightarrow \dfrac{a^M - a^{N+1}}{1-a}$ $\left(\because \text{first term} = a^M,\; r = \dfrac{a^{M+1}}{a^M} = a\right)$

for $a=1$,

Series will be, $\underbrace{1, 1, \ldots 1}$

$\Rightarrow$ ~~M+N+1~~ $N-M+1$

b) $\displaystyle\sum_{n=0}^{\infty} a^n \Rightarrow$ it is GP and given $|a|<1$,

Series will be, $1, a, a^2 \ldots$

$\uparrow$ first term, $r = a$

And $r<1$ $(\because |a|<1)$ So, $\displaystyle\sum_{n=0}^{\infty} a^n = \frac{a}{1-r} = \frac{1}{1-a}$

2.10 a) Given, $\tilde{y}(n) = x(n) * h(n)$

$$y(n) = \sum_{k} h(k) \cdot x(n-k)$$

$$\sum_{n} y(n) = \sum_{n} \sum_{k} h(k) \cdot x(n-k)$$

$$= \sum_{k} h(k) \cdot \sum_{n=-\infty}^{\infty} x(n-k)$$

$$= \left(\sum_{k} h(k)\right)\left(\sum_{n} x(n)\right)$$

b)

1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

$$\sum y(n) = \sum x(n) * h(n)$$

$$= \sum_{k} h(n) \cdot \sum_{n} x(n)$$

$$= 5 \cdot (1+2+4) \Rightarrow 35$$

2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

$$y(n) = (1+2-1) \cdot (1+2-1) \Rightarrow 4$$

3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

$$\sum y(n) = (0+1-2+3-4) \cdot \left(\frac{1}{2}+\frac{1}{2}+1+\frac{1}{2}\right)$$

$$= (-2)(2.5) \Rightarrow -5$$

4) $x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = (1)$

$$\sum y(n) = (1+2+3+4+5) \cdot (1)$$

$$= 15$$

5) $x(n) = \{1, -2, 3\}$, $h(n) = \{0, 0, 1, 1, 1, 1\}$

$$\sum y(n) = (1 + (-2) + 3) \cdot (0 + 0 + 1 + 1 + 1 + 1)$$

$$= (2) \cdot (4) \Rightarrow 8$$

6) $x(n) = \{0, 0, 1, 1, 1, 1\}$, $h(n) = (1, -2, 3)$

$$\sum y(n) = (1 + 1 + 1 + 1) \cdot (1 - 2 + 3) \Rightarrow 8$$

7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$$\sum y(n) = (0 + 1 + 4 - 3) \cdot (1 + 0 - 1 - 1)$$

$$= (2)(-1) \Rightarrow -2$$

8) $x(n) = (1, 1, 2)$, $h(n) = u(n)$

$$\sum y(n) = (1 + 1 + 2) \cdot \sum_{K=0}^{\infty} (1)$$

$$= 4 \cdot \infty \Rightarrow \infty$$

9) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, -3, \underset{\uparrow}{4}\}$

$$\sum y(n) = [1 + 1 + 0 + 1 + 1] \cdot [1 - 2 - 3 + 4]$$

$$= (4) \cdot 0 \Rightarrow 0$$

10) $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$, $h(n) = x(n)$

$$\sum y(n) = (1 + 2 + \underset{\uparrow}{0} + 2 + 1) \cdot (1 + 2 + 0 + 2 + 1)$$

$$= (6) \cdot (6) \Rightarrow 36$$

11) $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$.

$$\sum x(n) = \sum_{0}^{\infty} \left(\frac{1}{2}\right)^n \cdot 1 \; ; \; \sum h(n) = \sum_{0}^{\infty} \left(\frac{1}{4}\right)^n$$

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So., $\sum x(n) = \frac{1}{1-\frac{1}{2}} \Rightarrow 2$ (where $a=1$, $r=\frac{1}{2}$)

$\sum h(n) = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$ (where $a=1$, $r=\frac{1}{4}$)

$\sum y(n) = \sum x(n) \cdot \sum h(n) = 2 \times \frac{4}{3} \Rightarrow 8/3$

Q.17)

a) $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$

$h(n) = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1\}$

$y(n) = \sum_{k=0}^{n} x(k) \cdot h(n-k)$

$y(0) = x(0) \cdot h(0) \Rightarrow 6$

$y(1) = x(0) \cdot h(1) + x(1) \cdot h(0) \Rightarrow 11$

$y(2) = x(0) \cdot h(2) + x(1) \cdot h(1) + x(2) \cdot h(0) \Rightarrow 15$

$y(3) = x(0) \cdot h(3) + x(1) \cdot h(2) + x(2) \cdot h(1) + x(3) \cdot h(0) \Rightarrow 18$

$y(4) = x(0) \cdot h(4) + x(1) \cdot h(3) + x(2) \cdot h(2) + x(3) \, h(1) + x(4) \cdot h(0)$
$\Rightarrow 14$

$y(5) = x(0) \cdot h(5) + x(1) \cdot h(4) + x(2) \cdot h(3) + x(3) \cdot h(2) + x(4) \cdot h(1)$
$+ x(5) \cdot h(0) \Rightarrow 10$

$y(6) = x(1) \cdot h(5) + x(2) \cdot h(4) + x(3) \cdot h(3) \Rightarrow 1+2+3 \Rightarrow 6$

$y(7) = x(2) \cdot h(5) + x(3) \cdot h(4) \Rightarrow 1+2 \Rightarrow 3$

$y(8) = x(3) \cdot h(5) = 1$

$y(n) = 0 \; ; \; n \geq 9$

So., $y(n) = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$

(b) Similarly as in (a).,

b) $y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

c) $y(n) = \{1, \underset{\uparrow}{2}, 2, 2, 1\}$

d) $y(n) = \{\underset{\uparrow}{1}, 2, 2, 2, 1\}$

2.18)

Given,

$$x(n) = \begin{cases} \frac{1}{3}n & ; \ 0 \le n \le 6 \\ 0 & ; \ \text{elsewhere.} \end{cases}$$

$$h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere.} \end{cases}$$

$y(n) = x(n) * h(n)$

$$x(n) = \left\{ \underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3} \right\}$$

$$h(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$$

$$y(n) = \sum_{k=0}^{n} x(k) \cdot h(n-k)$$

$$= \left\{ 0, \frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2 \right\}$$

$\because$

h(n) \ x(n)	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2

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b) $x(n) = \frac{1}{3}n\{u(n) - u(n-7)\}$

$h(n) = u(n+2) - u(n-3)$

$y(n) = x(n) * h(n)$

$= \frac{1}{3}n\{u(n) - u(n-7)\} * [u(n+2) - u(n-3)]$

$= \frac{1}{3}n[u(n) * u(n+2) - u(n) * u(n-3) - u(n-7) * u(n+2)$
$+ u(n-7) \cdot u(n-3)]$

$= \frac{1}{3}\delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3)$
$+ \frac{20}{3}\delta(n-4) + 6\delta(n-5) + 5\delta(n-6) + 5\delta(n-6)$
$+ \frac{11}{6}\delta(n-7) + \delta(n-8)$

2.20

a) $131 \times 122 \Rightarrow 15982$

b) $\{1, 3, 1\} * \{1, 2, 2\}$

$\Rightarrow \{1, 5, 9, 8, 2\}$

c) $(1 + 3z + z^2) \cdot (1 + 2z + 2z^2) = \{1 + 5z + 9z^2 + 8z^3 + 2z^4\}$

d) $(1.31)(12.2) \Rightarrow 15.982$

e) There are different ways to perform convolution

2.21 a)

$x(n) = a^n u(n)$

$h(n) = b^n u(n)$

$y(n) = x(n) * h(n)$ where $a = b$
$a \neq b$

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case i) $a \neq b$

$$y(n) = \sum_{k=0}^{n} a^k (u(k)) \cdot b^{n-k} \cdot u(n-k)$$

$$= b^n \sum_{k=0}^{n} (a b^{-1})^k$$

$$= b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k$$

$\hookrightarrow$ GP $\Rightarrow 1, \frac{a}{b}, \left(\frac{a}{b}\right)^2, \ldots$

$$= b^n \left[\frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\frac{a}{b}} \right]$$

$$= b^n \left[\frac{\frac{b^{n+1}-a^{n+1}}{b^{n+1}}}{\frac{b-a}{b}} \right]$$

$$= \cancel{b^{n+1}} \left[\frac{b^{n+1}-a^{n+1}}{\cancel{(b^{n+1})}(b-a)} \right]$$

$$\boxed{y(n) = \frac{b^{n+1}-a^{n+1}}{b-a}}$$

case ii) $a = b$,

$$y(n) = b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k$$ for $a = b \Rightarrow \frac{a}{b} = 1$

$$= b^n \sum_{k=0}^{n} 1.$$

$$\boxed{y(n) = b^n (n+1) \; u(n)}$$

b) $x(n) = \{1, 2, 1, 1\}$

$h(n) = \{1, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$

c) $x(n) = \{1, \underset{\uparrow}{1}, 1, 2, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -2, -1\}$

d) $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h'(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$ where

$y'(n) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

2.28)

$y(n) = n\, y(n-1) + x(n)$ for $n \geq 0$

and also given $y(-1) = 0$.

If $y_1(n) = n\, y_1(n-1) + x_1(n)$ &

$y_2(n) = n\, y_2(n-1) + x_2(n)$ then.

$x(n) = a x_1(n) + b x_2(n)$

produces o/p as,

$y(n) = n\,y(n-1) + x(n)$ where $y(n) = a y_1(n) + b y_2(n)$.

Hence System is linear.. If the input is $x(n-1)$ we have,

$y(n-1) = (n-1)\,y(n-2) + x(n-1)$ But

$y(n-1) = n\,y(n-2) + x(n-1)$

Hence system is time variant. If $x(n) = u(n)$; then $|x(n)| \leq 1$. But for this bounded input, the output is

$y(0) = 1, \quad y(1) = 1+1 = 2, \quad y(2) = 2 \times 2 + 1 = 5, \ldots$

which is unbounded; Hence system is unstable.

2.25) With $x(n) = 0$, we have,

$y(n-1) + \frac{4}{3}\,y(n-2) = 0$

$y(-1) = -4/3\; y(-2)$

$y(0) = (-4/3)^2\; y(-2)$

$y(1) = \left(\frac{-4}{3}\right)^3 y(-2)$

$\vdots$

$y(k) = \left(-\frac{4}{3}\right)^{k+2} y(-2)$ ← zero i/p response.

2(b) Given homogeneous eq'n,

$$y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = 0$$

The characteristic eq'n is,

$$\lambda^n - \frac{5}{6} \lambda^{n-1} + \frac{1}{6} \lambda^{n-2} = 0$$

$$\lambda^{n-2} \left[\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} \right] = 0$$

$$\lambda^2 - 5/6 \lambda + \frac{1}{6} = 0$$

roots, $\lambda = \frac{1}{2}, \frac{1}{3}$

Hence, $y_h(n) = C_1 \left(\frac{1}{2}\right)^n + C_2 \left(\frac{1}{3}\right)^n \longrightarrow ①$

particular sol'n to,

$x(n) = 2^n u(n)$ is,

$$y_p(n) = k(2^n) \cdot u(n) \longrightarrow ②$$

Substituting into differential eq'n,

$$k\, 2^n u(n) - 5/6 \cdot k \cdot 2^{n-1} \cdot u(n-1) + k \frac{1}{6} 2^{n-2} u(n-2) = 2^n \cdot u(n)$$

for $n=2$,

$$4k - \frac{5k}{3} + k/6 = 4 \Rightarrow k = 8/5$$

$\therefore$ The total sol'n is,

$y(n) = y_p(n) + y_h(n)$ (from eq① & eq②)

$$= \frac{8}{5} \cdot 2^n u(n) + C_1 \left(\frac{1}{2}\right)^n u(n) + C_2 \left(\frac{1}{3}\right)^n u(n)$$

Now assume $y(-2) = y(-1) = 0$ Then,

$y(0) = 1 \quad \left(\because y(0) = \frac{5}{6}\, y(0-1) - \frac{1}{6}\, y(0-2) + x(0)\right)$

&

$x(0) = 2^{0}\, u(0) \Rightarrow 1)$

&

$y(1) = \frac{5}{6}\, y(0) + 2 = \frac{17}{6}$

Thus.,

$$\frac{8}{5} + C_1 + C_2 = 1 \Rightarrow C_1 + C_2 = -\frac{3}{5}$$

$$\frac{16}{5} + \frac{1}{2}C_1 + \frac{1}{3}C_2 = \frac{17}{6} \Rightarrow 3C_1 + 2C_2 = -\frac{11}{5}$$

and therefore,

$$C_1 = -1, \quad C_2 = \frac{2}{5}$$

The total sol'n is.,

$$y(n) = \left[\left(\frac{8}{5}\right) 2^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right] u(n) \quad \text{Ans.}$$

2.27)

Given eq'n,

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

to the i/p $x(n) = 4^n u(n)$.

Characteristic equation is.,

$$\lambda(n) - 3\lambda(n-1) - 4\lambda(n-2) = 0$$

$$\lambda(n-2)\left[\lambda^2 - 3\lambda - 4\right] = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

4

-4 +1

$$\lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda+1) - 4(\lambda+1) = 0$$

$$\lambda = -1 \text{ (or) } \lambda = 4$$

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so, $y_h(n) = C_2(-1)^n + C_1(4)^n$

Particular sol'n is,

$$y_p(n) = k\, n\, 4^n\, u(n)$$

Then,

$$kn\,4^n u(n) - 3k(n-1)\,4^{n-1} u(n-1) - 4k(n-2)\,4^{n-2} u(n-2)$$

$$= 4^n u(n) + 2(4)^{n-1} u(n-1)$$

for $n=2$,

$$k(32-12) = 4^2+8 \Rightarrow 24 \Rightarrow k = 6/5$$

total sol'n is,

$$y(n) = y_p(n) + y_h(n)$$

$$= \left[\frac{6}{5} n\, 4^n + C_1 4^n + C_2(-1)^n\right] u(n).$$

now to solve C_1 and C_2, assume that $y(-1) = y(-2) = 0$. Then,

$y(0) = 1$ and

$y(1) = 3y(0) + 4 + 2 = 9$

Hence $C_1 + C_2 = 1$ and

$$\frac{24}{5} + 4C_1 - C_2 = 9$$

$$4C_1 - C_2 = 21/5$$

Therefore, $C_1 = 26/25$ and $C_2 = -1/25$

The total sol'n is

$$y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$$

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2.28) From 2.27,

$\lambda = 4, -1$ Hence,

$y_h(n) = C_1 4^n + C_2 (-1)^n$

when $x(n) = \delta(n)$ we find that,

$y(0) = 1$ and

$y(1) - 3y(0) = 2$ or

$y(1) = 5$

Hence $C_1 + C_2 = 1$ and $4C_1 - C_2 = 5$

by solving $C_1 = 6/5$ and $C_2 = -1/5$ Therefore

$$h(n) = \left[6/5\, 4^n - \frac{1}{5}(-1)^n\right] u(n)$$

2.29)

a) $L_1 = N_1 + M_1$ & $L_2 = N_2 + M_2$

b) partial overlap from left:

low $N_1 + M_1$, high $N_1 + M_2 - 1$

Full overlap: low $N_1 + M_2$, high: $N_2 + M_1$

partial overlap from right:-

low $N_2 + M_2 + 1$, high $N_2 + M_2$

c) $x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$

$N_1 = -2, \quad N_2 = 4, \quad M_1 = -1, \quad M_2 = 2$

Partial overlap from left: $n = -3$, $n = -1$, $L_1 = -3$

Full overlap $\Rightarrow$ $n = 0$, $n = 3$

partial overlap from right: $n = 4$, $n = 6$, $L_2 = 6$

(2.30) a) $y(n) - 0.6\, y(n-1) - 0.08\, y(n-2) = x(n)$

The characteristic eqn is,

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$\lambda = 0.2, 0.4$, Hence,

$$y_h(n) = C_1\left(\frac{1}{5}\right)^n + C_2\left(\frac{2}{5}\right)^n$$

* with $x(n) = \delta(n)$, the initial conditions are,

$y(0) = 1 \qquad (\because x(0) = 1)$

$y(1) - 0.6\, y(0) = 0 \Rightarrow y(1) = 0.6$

Hence, $C_1 + C_2 = 1$ (for $y(0)$)

$\frac{1}{5}C_1 + \frac{2}{5} = 0.6$ (for $y(1)$)

$C_1 = -1,\ C_2 = 3$

Therefore $h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$

* Step response, $x(n) = u(n)$ The initial conditions are,

$$s(n) = \sum_{k=0}^{n} h(n-k).\ ,\ n \geq 0$$

$$= \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\frac{1}{5}^{n+1} - 1\right]\right\} u(n)$$

2.31)

Given, $h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$

$x(n) = ?$

$y(n) = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0 ..\}$

$h(n) = \{\underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$

$y(n) = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0\}$

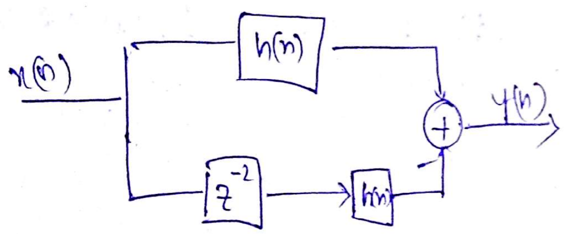
$x(0) \cdot h(0) = y(0) \Rightarrow x(0) = \frac{y(0)}{h(0)} = 1$

$\frac{1}{2} x(0) + x(1) = y(1) \Rightarrow x(1) = 3/2$

By continuing this process, we obtain

$x(n) = \{1, 3/2, 3/2, 7/4, 3/2, \ldots\}$

2.33)



given $x(n) = u(n+5) - u(n-10)$, $h(n) = a^n u(n)$ for $-1<a<$

first we need to determine

$s(n) = x(n) * h(n)$ where $x(n) = u(n)$

$$= \sum_{k=0}^{n} x(k) \cdot h(n-k)$$

$$= \sum_{k=0}^{n} 1 \cdot a^{n-k}$$

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$$= \frac{a^{n+1} - 1}{a - 1} \quad , n \geq 0$$

for $x(n) = u(n+5) - u(n-10)$ we have the response

$$s(n+5) - s(n-10) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$$

from given fig.,

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

Hence
$$y(n) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$$

$$- \frac{a^{n+4} - 1}{a-1} u(n+3) + \frac{a^{n-11} - 1}{a-1} \cdot u(n-12).$$

2.34)

$$h(n) = [u(n) - u(n-M)] / M$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k) \cdot h(n-k)$$

$$= \sum_{k=0}^{\infty} 1 \cdot h(n-k) \Rightarrow \begin{cases} \frac{n+1}{M} & , n < M \\ 1 & , n \geq M \end{cases}$$

2.35)

Given
$$h(n) = \begin{cases} a^n & ; n \geq 0, \quad n \text{ even} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0, \text{even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1 - |a|^2} \quad \text{for stable condition is,}$$

$$|a| < 1$$

//..

$$= \frac{a^{n+1}-1}{a-1}, \quad n \geq 0$$

for $x(n) = u(n+5) - u(n-10)$ we have the response

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

from given fig.,

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

Hence
$$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$- \frac{a^{n+4}-1}{a-1} u(n+3) + \frac{a^{n-11}-1}{a-1} \cdot u(n-12).$$

2.34)

$$h(n) = \{u(n) - u(n-M)\}/M$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k) \cdot h(n-k)$$

$$= \sum_{k=0}^{\infty} 1 \cdot h(n-k) \Rightarrow \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}$$

2.35)

Given
$$h(n) = \begin{cases} a^n & ; n \geq 0, \ n \text{ even} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,\text{even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$

for stable condition is,

$$|a| < 1$$

2.36) we have response $h(n) = a^n u(n)$ then result will be, $(\because x(n) = u(n))$

$$y'(n) = \sum_{k=0}^{n} u(n-k) \cdot h(k)$$

$$= \sum_{k=0}^{n} 1 \cdot a^{k}$$

$$= \frac{1-a^{n+1}}{1-a} u(n).$$

now

for $x(n) = u(n) - u(n-10)$

result obtained $\Rightarrow$

$$y(n) = \frac{1-a^{n+1}}{1-a} u(n) - \frac{1-a^{n-9}}{1-a} u(n-10)$$

2.37)

Given, $h(n) = \left(\frac{1}{2}\right)^n u(n) \Rightarrow a = \frac{1}{2}$

i/p $\Rightarrow$ $x(n) = \begin{cases} 1; & 0 \le n \le 10 \\ 0; & \text{otherwise} \end{cases}$

$x(n) = u(n) - u(n-10)$

so. $$y(n) = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} u(n) - \frac{1-\left(\frac{1}{2}\right)^{n-9}}{1-\frac{1}{2}} u(n-10)$$

$$= 2\left[\left(1-\left(\frac{1}{2}\right)^{n+1}\right)u(n) - \left(1-\left(\frac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

2.38

Given $h(n) = \left(\frac{1}{2}\right)^n u(n)$

i/p's $\Rightarrow$ (a) $x(n) = 2^n u(n)$

(b) $x(n) = u(-n)$

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a) $$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$\cancel{= \sum_{k=0}^{n} 2^{k} \left(\frac{1}{2}\right)^{n-k}}$$

$$= \sum_{k=0}^{n} x(n-k) \cdot h(k)$$

$$= \sum_{k=0}^{n} 2^{n-k} \cdot \left(\frac{1}{2}\right)^{k}$$

$$= 2^{n}\left[\sum_{k=0}^{n}\left(\frac{1}{4}\right)^{k}\right]$$

$$= 2^{n}\left[1-\left(\frac{1}{4}\right)^{n+1}\right]\left(\frac{4}{3}\right)$$

$$= \left[2^{n} - 2^{n} \cdot \frac{1}{4^{n}} \cdot \frac{1}{4}\right]\frac{4}{3}$$

$$= \left(2^{n} - \left(\frac{1}{2}\right)^{n} \cdot \frac{1}{4}\right)\frac{4}{3}$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n).$$

b) $x(n) = u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{k} = 2 \,, \quad n<0 \quad \phi$$

$$y(n) = \sum_{k=n}^{\infty} h(k)$$

$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^{k}$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1}\left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}}\right)$$

$$= 2\cdot\left(\frac{1}{2}\right)^n, \quad n \geq 0.$$

2.39)

Given
$$\left.\begin{aligned} h_1(n) &= \delta(n) - \delta(n-1) \\ h_2(n) &= h(n) \\ h_3(n) &= u(n) \end{aligned}\right\} \Rightarrow \text{cascade}$$

a) $h_c(n) = ?$

$$h_c(n) = h_1(n) * h_2(n) * h_3(n)$$
$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$
$$= [u(n) - u(n-1)] * h(n)$$
$$= \delta(n) * h(n)$$
$$= h(n)$$

b) No.

2.40)

a) $x(n) \cdot \delta(n-n_0) = x(n_0) \Rightarrow x(n)$ value at n_0 is only obtained in the result, rest of part will be zero since $\delta(n-n_0) = 1$ only at n_0.

b) $x(n) * \delta(n-n_0) = x(n-n_0) \Rightarrow$ we obtain shifted version seq of $x(n)$ while $\delta(n-n_0)$ is moving throughout $x(n)$ during convolution

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b) $y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$

$= h(n) * x(n)$

Linearity: $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$

Then $x(n) = \alpha x_1(n) + \beta x_2(n) \rightarrow$

$y(n) = h(n) * x(n)$

$y(n) = h(n) * (\alpha x_1(n) + \beta x_2(n))$

$= \alpha\, h(n) * x_1(n) + \beta\, h(n) * x_2(n)$

Time Invariance:-

$x(n) \rightarrow y(n) \Rightarrow h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$= \sum_k h(k) \cdot x(n-n_0-k)$

$= y(n-n_0)$

c) $y(n) = x(n-n_0)$

$h(n) = \delta(n-n_0)$

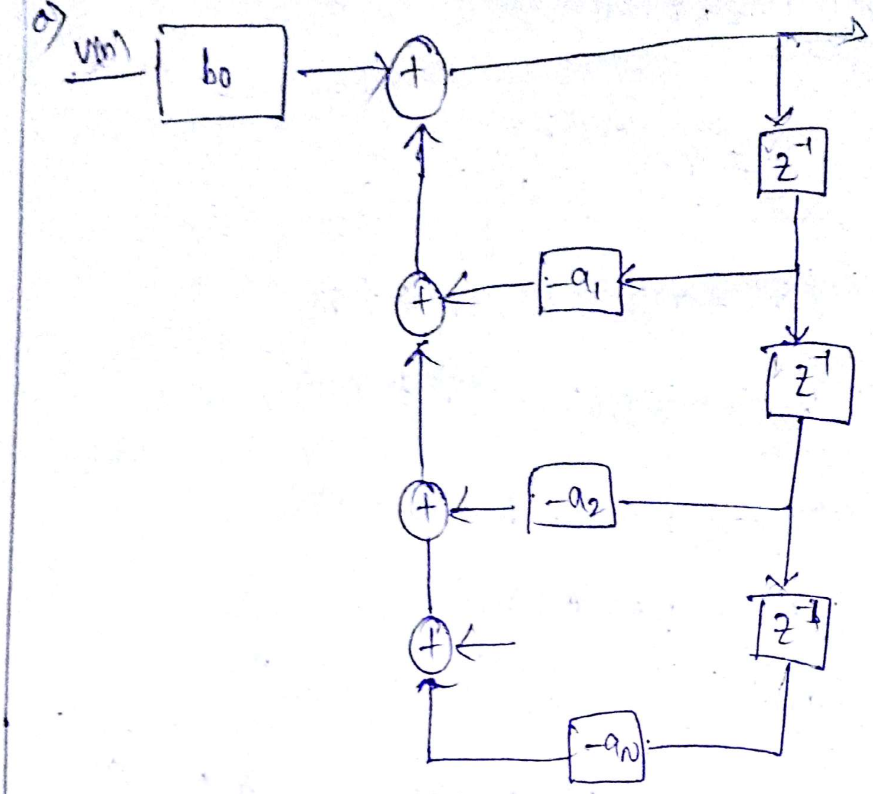
2.(a)

Given functions,

a) $s(n) = -a_1 s(n-1) - a_2 s(n-2) - \cdots - a_N s(n-N) + b_0 v(n)$

b) $s(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \cdots a_N s(n-N)\right]$

a)



2(b)

Zero state response of the System.

$y(n) + \frac{1}{2} y(n-1) = x(n) + 2x(n-2)$; $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$

$y(n) = -\frac{1}{2} y(n-1) + x(n) + 2x(n-2)$

$y(-2) = -\frac{1}{2} y(-3) + x(-2) + 2x(-4) \Rightarrow 1$

$y(-1) = -\frac{1}{2} y(-2) + x(-1) + 2x(-3) \Rightarrow 3/2$ $\boxed{\because -\frac{1}{2}(1)+2+2(0)}$

$y(0) = -\frac{1}{2} y(-1) + x(0) + 2x(-2) \Rightarrow \frac{17}{4}$

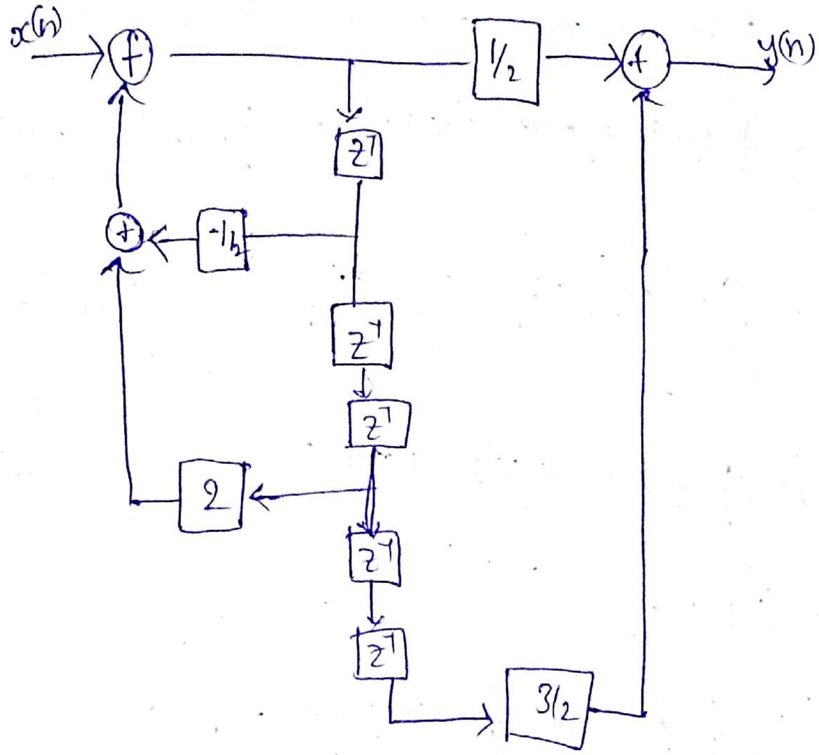
$\boxed{-\frac{1}{2}(3/2) + 3 + 2(1)}$

$y(1) = -\frac{1}{2} y(0) + x(1) + 2x(-1) \Rightarrow \frac{47}{8}$, etc.

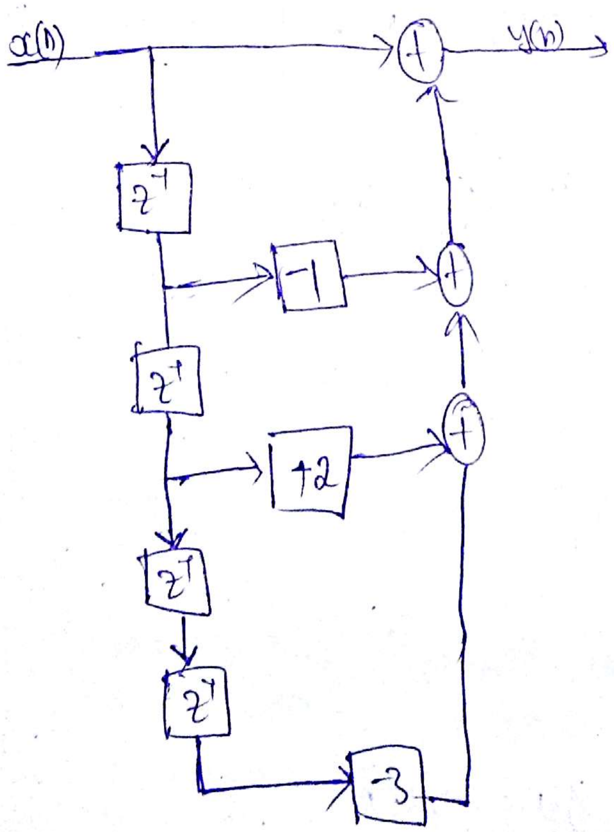
$\boxed{\because y(1) = -\frac{1}{2} \cdot \frac{17}{4} + 4 + 2(2)}$

a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$

$$y(n) = \frac{x(n) + 3x(n-5) + 4y(n-3) - y(n-1)}{2}$$



b) $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$



2.41)

a) $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$x(n) = \{\underset{\uparrow}{1}, 0, 0 \cdots\}$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) \Rightarrow \frac{1}{2} + 0 + 1 \Rightarrow 3/2$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) \Rightarrow \frac{1}{2} \cdot 3/2 + 0 + 0 \Rightarrow 3/4$

silly.,

$y(n) = \{1, 3/2, 3/4, 3/8, 3/16, 3/32, 3/64, \cdots\}$

b) $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

c) $x(n) = \{1, 1, 1 \cdots\}$

$y(0) = \frac{1}{2} y(-1) + x(0) + x(-1) \Rightarrow x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) \Rightarrow \frac{1}{2}(1) + 1 + 1 \Rightarrow 5/2$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) \Rightarrow \frac{1}{2}(5/2) + 1 + 1 \Rightarrow 13/4$

silly..

$y(n) = \{1, 5/2, 13/4, \frac{29}{8}, \frac{61}{16}, \cdots\}$

d) $y(n) = u(n) * h(n)$

$= \sum_{k} u(k) \cdot h(n-k)$

$= \sum_{k=0}^{n} h(n-k)$

$y(0) = h(0) = 1, \quad y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = 13/4$ etc.

e) from a), $h(n) = 0$ for $n < 0$ $\Rightarrow$ System is causal.

$$\sum_{n=0}^{\infty} |h(n)| = 1 + 3/2\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right)$$

$$= 1 + 3/2\left[\frac{1}{1 - 1/2}\right]$$

$$\cancel{= 1 + 3/2\left[\frac{2}{1}\right]} \Rightarrow$$

$$= 1 + 3/2\left[\frac{2}{1}\right] \Rightarrow 4 \Rightarrow \text{system is stable.}$$

2.15)

Given

$$y(n) = a\,y(n-1) + b\,x(n)$$

a) $$\sum_{n=-\infty}^{\infty} h(n) = 1.$$

$$\Rightarrow h(n) = b a^n \cdot u(n)$$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$$

$$\boxed{b = 1-a}$$

b)

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

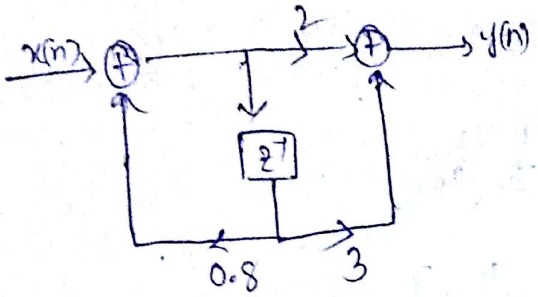
$$= b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$$

$$S(\infty) = \frac{b}{1-a} = 1$$

$$\boxed{b = 1-a}$$

c) $b = 1-a$ in both cases

2. (a)



$$y(n) = 0.8\, y(n-1) + 2x(n) + 3x(n-1)$$

$$y(n) - 0.8\, y(n-1) = 2x(n) + 3x(n-1)$$

Characteristic eqn is.,

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8$$

$$y_h(n) = c(0.8)^n$$

Let us first consider the response of the System.,

$$y(n) - 0.8\, y(n-1) = x(n)$$

to $x(n) = \delta(n)$; since $y(0) = 1$, it follows that $c = 1$

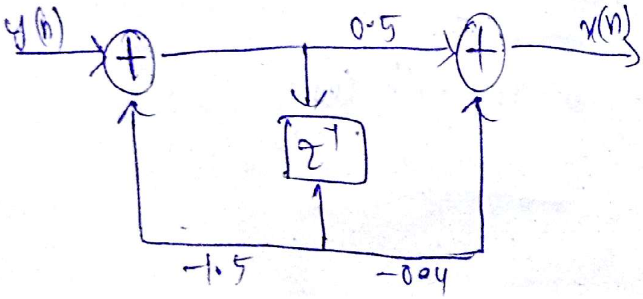
Then impulse response of the Signal System is

$$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$$

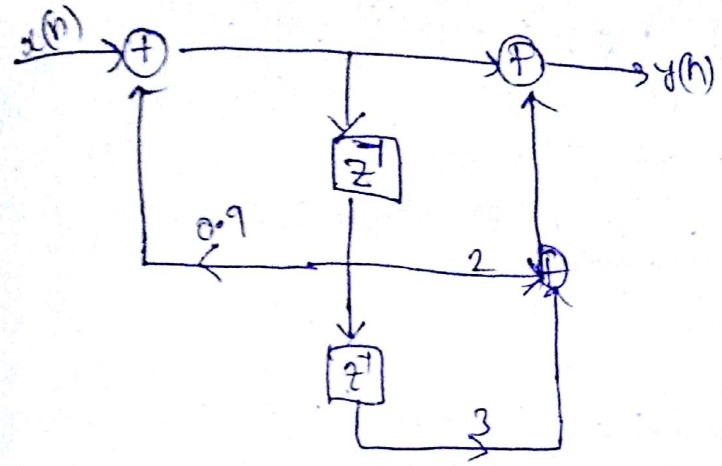
$$= 2\delta(n) + 4.6(0.8)^{n-1} u(n-1).$$

b) The inverse system is characterized by the diff. eqn.

$$x(n) = -1.5x(n-1) + \frac{1}{2} y(n) - 0.4\, y(n-1)$$



2.47)



a) $y(n) = 0.9\,y(n-1) + x(n) + 2\,x(n-1) + 3\,x(n-2)$

For $x(n) = \delta(n)$, we have

$y(0) = 1$

$y(1) = 2.9,$

$y(2) = 5.61$

$y(3) = 5.049$

$y(4) = 4.544$

$y(5) = 4.090 \ldots$

b) Zero state response,

$s(0) = y(0) = 1$

$s(1) = y(0) + y(1) = 3.91$

$s(2) = y(0) + y(1) + y(2) = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2).$

2.49

a) $y(n) = \frac{1}{3}x(n) + \frac{1}{3}x(n-3) + y(n-1)$

for $x(n) = \delta(n)$ we have

$$h(n) = \left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 2/3, 2/3, 2/3, 2/3 \cdots\right\}$$

b) $y(n) = \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + \frac{1}{2}x(n-2)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$ we obtain

$$h(n) = \left\{0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \frac{15}{256}, \cdots\right\}$$

c) $y(n) = 1.4\,y(n-1) - 0.48\,y(n-2) + x(n)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774 \cdots\}$$

d) All three systems are IIR.

e) $y(n) = 1.4\,y(n-1) - 0.48\,y(n-2) + x(n)$

The characteristic eqn is,

$$\lambda^2 - 1.4\lambda + 0.48 = 0$$

$\lambda = 0.8, 0.6$ so,

$y_h(n) = c_1(0.8)^n + c_2(0.6)^n$. for $x(n) = \delta(n)$ we have

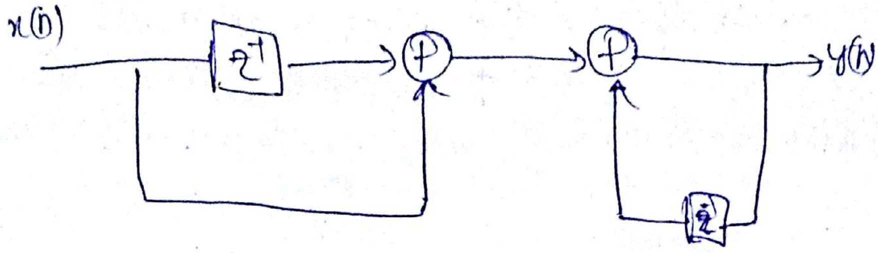
$c_1 + c_2 = 1$ & $0.8c_1 + 0.6c_2 = 1.4$

$c_1 = 4, c_2 = -3$.

$\therefore \quad h(n) = \left[4(0.8)^n - 3(0.6)^n\right]u(n)$ Ans.

2.50)

a)



$$y(n) = \frac{1}{2}\,y(n-1) + x(n) + x(n-1)$$

for $y(n) - \frac{1}{2}\,y(n-1) = \delta(n)$ The sol'n is

$$h(n) = \left(\frac{1}{2}\right)^{n} u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$$

b)

$$h_1(n) * \left[\delta(n) + \delta(n-1)\right] = \left(\frac{1}{2}\right)^{n} u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$$

//..

2.51)

a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$, $h_1(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

convolution ⇒ $\sum_{k=0}^{n} x_1(n) \cdot h(n-k)$

$y_1(n) \Rightarrow \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

correlation ⇒ $r_1(n) = \{1, 3, 7, \underset{\uparrow}{7}, 6, 4\}$

b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$, $h_2(n) = \{1/2, \underset{\uparrow}{1}, 2, 1, \frac{1}{2}\}$

convolution: $y_2(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

correlation $r_2(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

note: convolution = correlation ($\because h_2(-n) = h(n)$)

Convolution $y_3(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

correlation $\gamma_1(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

c)

Convolution $y_4(n) = \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$

correlation $\gamma_4(n) = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

hence $\gamma_3(n) = y_4(n+3)$

and $h_4(-n) = h_3(n+3)$

hence $\gamma_4(n) = y_3(n+3)$

2.52)

$x(n) = \{1, 3, 3, 1\}$.

Zero state response of $x(n)$ is, $y(n) = [1, 4, 6, 4, 1]$

Impulse response = ?

$h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$

$h_0 = 1, \quad h_1 = 1$

2.53)

$y(n) = ?$ for $n \geq 0$.

given 2nd order D.Eqn $\Rightarrow y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-$

I/P $\Rightarrow x(n) = (-1)^n u(n)$

and Initial conditions $y(-1) = y(-2) = 0$.

homogeneous eq'n.,

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda-2) - 2(\lambda-2) = 0$$

$$\lambda = 2, 2$$

$4 \to -2, -2$

$$y_h(n) = C_1 2^n + C_2 n 2^n \quad , \quad y_p(n) = k(-1)^n u(n).$$

~~for $n=0$,~~

$$\cancel{y(0) = x(0) - x(-1)}$$

$$\cancel{= 1 - (-1)}$$

Substituting $y_p(n)$ into eq'n, we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

for $n=2$,

$$k + 4k + 4k = 1 + 1$$

$$k(1+4+4) = 2$$

$k = 2/9$, the total sol'n is,

$$y(n) = \left(C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

from initial conditions, we obtain

$y(0) = 1$, $y(1) = 2$ (by substituting into given D.Eqn)

$$C_1 + 2/9 = 1 \Rightarrow C_1 = 7/9$$

$$2C_1 + 2C_2 - 2/9 = 2 \Rightarrow C_2 = 1/3$$

So, $$y(n) = \left[\frac{7}{9} 2^n + \frac{1}{3} n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

2.55) differential eqn ⇒

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1); \quad x(n) = \text{impulse}$$

from previous problem,

$$h(n) = [C_1 2^n + C_2 n 2^n]\, u(n)$$

with $y(0) = 1$

$y(1) = 3$ we have

$\because$ $n=0$, $y(0) - 4\cancel{y(-1)}^{0} + 4\cancel{y(-2)}^{0} = x(0) - \cancel{x(-1)}^{0}$ (for impulse)

$y(0) = 1 - 0$

$n=1$, $y(1) = 4y(0) - 4\cancel{y(-1)}^{0} + \cancel{x(1)}^{0} - x(0) \Rightarrow 4 - 1 \Rightarrow 3$ //.

$$C_1 = 1$$

$$2C_1 + 2C_2 = 3$$

$$C_2 = 1/2$$

Thus $h(n) = \left(2^n + \frac{1}{2} n 2^n\right) u(n)$ //.

2.56

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * [u(n) - u(n-1)]$$

$$= [x(n) - x(n-1)] * u(n)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k)$$

2.57) Let $h(n)$ be the impulse response of the System,

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = s(k) - s(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k)$$ //.

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$$x(n) = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$

The range of non zero values of $r_{xx}(l)$ is determined by

$$n_0 - N \le n \le n_0 + N$$

$$n_0 - N \le n - l \le n_0 + N$$

Which implies,

$$-2N \le l \le 2N \quad (\because \text{by solving both eqns})$$

for given shift l, the no. of terms in the summation for which both $x(n)$ and $x(n-l)$ are non-zero is $2N+1-|l|$ and the value of each term is 1. Hence,

$$r_{xx}(l) = \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

for $r_{xy}(l)$ we have,

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0|, & n_0 - 2N \le l \le n_0 + 2N \\ 0, & \text{otherwise} \end{cases}$$

2.69)

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$

$$r_{xx}(-3) = x(0) \cdot x(3) = 1$$

$$r_{xx}(-2) = x(0)\, x(2) + x(1)\, x(3) = 3$$

$$r_{xx}(-1) = x(0)\, x(1) + x(1) \cdot x(2) + x(2)\, x(3) = 5$$

$$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$$

and $r_{xx}(l) = r_{xx}(-l)$ so,

$$r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$$

b) $$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n) \cdot y(n-l)$$

we obtain

$$r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$$

we observe that,

$y(n) = x(-n+3)$, which is equivalent reversing the sequence $x(n)$. This has not changed autocorrelation sequence.

2.60)

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$

$$= \begin{cases} 2N+1-|l| & , -2N \le l \le 2N \\ 0 & , \text{otherwise} \end{cases}$$

$$r_{xx}(0) = 2N+1$$

$\therefore$ normalized auto correlation is,

$$\rho_{xx}(l) = \frac{1}{2N+1}\left[2N+1-|l|\right] \quad , -2N \le l \le 2N$$

$$= 0 \quad , \text{otherwise}$$

2.61)

$x(n) = s(n) + \gamma_1 s(n-k_1) + \gamma_2 s(n-k_2)$

k_1, k_2 are delays

γ_1, γ_2 are relelection coefficients.

a) $\gamma_{xx}(l) = ?$

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} \left[s(n) + \gamma_1 s(n-k_1) + \gamma_2 s(n-k_2)\right] \times \left[s(n-l) + \gamma_1 s(n-l-k_1) + \gamma_2 s(n-l-k_2)\right]$$

$$= (1+\gamma_1^2+\gamma_2^2)\,\gamma_{xx}(l) + \gamma_1\left[\gamma_{xx}(l+k_1) + \gamma_{xx}(l-k_1)\right] + \gamma_2\left[\gamma_{xx}(l+k_2) + \gamma_{xx}(l-k_2)\right] + \gamma_1\gamma_2\left[\gamma_{xx}(l+k_1-k_2) + \gamma_{xx}(l+k_2-k_1)\right]$$

b) $\gamma_{xx}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$. Suppose $k_1 < k_2$. Then, we can determine γ_1 and k_1. The problem is to determine γ_2 and k_2 from the other peaks.

c) If $\gamma_2 = 0$, the peaks occur at $l=0$ and $l = \pm k_1$. Then, it is easy to obtain γ_1 and k_1

2.(4)