МОСКОВСКИЙ АВИАЦИОННЫЙ ИНСТИТУТ
(НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ)

Институт №8 «Информационные технологии и прикладная математика»
Кафедра 804 «Теория вероятностей и компьютерное моделирование»

**Лабораторная работа №2**
**по курсу «Математическая экономика»**

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Текст программы:

```
library(Matrix)
library(gurobi)
# =========
#   p two
# =========

l <- 8
k <- 21
n <- 10 + k%/%4
#n <- 3
m <- 30 - k%/%4
#m <- 3

cat("l =", l, '\n')
cat("k =", k, '\n')
cat("n =", n, '\n')
cat("m =", m, '\n')

tmp <- 0


#A <- c(-4, 13, 4, 13, -4, 4, 10, 10, 8)
A <- c()
for (i in 1:m)
  for ( j in 1:n){
        tmp = -k + ( 1451*i + 1571*j + 2081*k + 2543*l  ) %% (30 + k%/%5)
        A = append(A, tmp)
  }

A = matrix(A, ncol=n, byrow=T)



# sum (xi) -> min
# x*ai^t >=1
#


solve <- function(AA, n, m){
  cat("A :\n")
  print(AA)
  cat("=======================================================================\n\n")

  b <- matrix(1, 1, m)

  c <- matrix(1, 1, n)
```

```r
max_min = min(AA[1,])
max_i = 1
for (i in 2:m){
        if (min(AA[i,]) > max_min){
        max_min = min(AA[i,])
        max_i = i
        }
}

min_max = max(AA[,1])
min_j = 1
for (j in 2:n){
        if (max(AA[,j]) < min_max){
        min_max = max(AA[,j])
        min_j = j
        }
}


cat("min_max (win for the first) = ", min_max, "\nmax_min (win for the second) = ",
-max_min, '\n\n\n')

cat('garantiruyuchaia dlya pervogo',max_i,'\n')

cat('garantiruyuchaia dlya vtorogo', min_j, '\n\n\n')

cat('solution in clear strategies ')
if (min_max == max_min){
        cat('exists\n\n\n')
}else{
        cat('does not exists\n\n\n')
}

model=list()
model$A = t(AA) - min(AA) + 1
model$obj = b
model$modelsense = 'min'
model$rhs = c
model$sense = matrix('>=', 1, n)
model$vtype = 'C'

params = list()
params$outputflag = 0

result = gurobi(model, params)
print('x=')
print(result$x) # выведем решение задачи
print('y=')
```

```
  print(result$pi) # выведем решение двойственной задачи
  game_price = ((t(result$x)%*%AA)%*%result$pi)
  cat('game pirce is ', game_price, '\n')

  cat("=============================================================\n\n")
}

solve(A, n, m)
solve(-A, n, m)
solve(t(A)%*%A, n, n)



rm(list=ls())
```

Результаты исчислений:

```
schizophrenia@home:~/labs/4kurs/MathEc/2lab$ Rscript main.R
Загрузка требуемого пакета: slam
l = 8
k = 21
n = 15
m = 25
A :
         [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
 [1,]   -2     5   12  -15  -8   -1    6  -21  -14    -7     0     7   -20
 [2,]  -13    -6    1    8  -19  -12   -5    2    9   -18   -11    -4     3
 [3,]   10   -17  -10   -3    4   11  -16   -9   -2     5    12   -15    -8
 [4,]   -1     6  -21  -14   -7    0    7  -20  -13    -6     1     8   -19
 [5,]  -12    -5    2    9  -18  -11   -4    3   10   -17   -10    -3     4
 [6,]   11   -16   -9   -2    5   12  -15   -8   -1     6   -21   -14    -7
 [7,]    0     7  -20  -13   -6    1    8  -19  -12    -5     2     9   -18
 [8,]  -11    -4    3   10  -17  -10   -3    4   11   -16    -9    -2     5
 [9,]   12   -15   -8   -1    6  -21  -14   -7    0     7   -20   -13    -6
[10,]    1     8  -19  -12   -5    2    9  -18  -11    -4     3    10   -17
[11,]  -10    -3    4   11  -16   -9   -2    5   12   -15    -8    -1     6
[12,]  -21   -14   -7    0    7  -20  -13   -6    1     8   -19   -12    -5
[13,]    2     9  -18  -11   -4    3   10  -17  -10   -3     4    11   -16
[14,]   -9    -2    5   12  -15   -8   -1    6  -21   -14    -7     0     7
[15,]  -20   -13   -6    1    8  -19  -12   -5    2     9   -18   -11    -4
[16,]    3    10  -17  -10   -3    4   11  -16   -9   -2     5    12   -15
[17,]   -8    -1    6  -21  -14   -7    0    7  -20   -13    -6     1     8
[18,]  -19   -12   -5    2    9  -18  -11   -4    3    10   -17   -10    -3
[19,]    4    11  -16   -9   -2    5   12  -15   -8   -1     6   -21   -14
[20,]   -7     0     7  -20  -13   -6    1    8  -19   -12    -5     2     9
[21,]  -18   -11   -4    3   10  -17  -10   -3    4    11   -16    -9    -2
[22,]    5    12  -15   -8   -1    6  -21  -14   -7    0     7   -20   -13
```

```
[23,] -6     1      8 -19 -12 -5 2     9 -18 -11   -4    3    10
[24,] -17 -10 -3    4 11 -16 -9 -2   5    12 -15       -8   -1
[25,]  6 -21 -14 -7 0     7 -20 -13 -6 1    8 -19 -12
        [,14] [,15]
 [1,]  -13    -6
 [2,]   10  -17
 [3,]   -1    6
 [4,]  -12    -5
 [5,]   11  -16
 [6,]   0     7
 [7,]  -11    -4
 [8,]   12  -15
 [9,]    1     8
[10,]  -10    -3
[11,]  -21  -14
[12,]   2     9
[13,]   -9    -2
[14,]  -20  -13
[15,]   3    10
[16,]   -8    -1
[17,]  -19  -12
[18,]   4    11
[19,]   -7    0
[20,]  -18  -11
[21,]   5    12
[22,]   -6    1
[23,]  -17  -10
[24,]   6  -21
[25,]   -5    2
```

===========================================================

min_max (win for the first) =  9
max_min (win for the second) =  17


garantiruyuchaia dlya pervogo 3
garantiruyuchaia dlya vtorogo 8


solution in clear strategies does not exists


[1] "x="
 [1] 0.001594896 0.000000000 0.007974482 0.000000000 0.000000000 0.001594896
 [7] 0.000000000 0.007974482 0.001594896 0.000000000 0.001594896 0.000000000
[13] 0.000000000 0.001594896 0.000000000 0.007974482 0.000000000 0.000000000
[19] 0.001594896 0.000000000 0.006379585 0.001594896 0.009569378 0.003189793
[25] 0.000000000

[1] "y="
 [1] 0.006379585 0.007974482 0.007974482 0.004784689 0.009569378 0.001594896
 [7] 0.001594896 0.006379585 0.001594896 0.000000000 0.001594896 0.001594896
[13] 0.000000000 0.001594896 0.001594896
game pirce is  -0.01046476
=========================================================

A :

        [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
 [1,]    2  -5 -12  15 8      1  -6  21  14 7      0     -7    20
 [2,]   13       6  -1  -8 19  12      5  -2  -9   18    11    4      -3
 [3,]  -10  17  10    3  -4 -11  16 9      2    -5  -12 15    8
 [4,]    1  -6  21  14 7      0  -7  20  13 6     -1     -8    19
 [5,]   12       5  -2  -9 18  11      4  -3 -10   17    10    3      -4
 [6,]  -11  16  9      2  -5 -12  15 8      1    -6    21    14    7
 [7,]    0  -7  20  13 6  -1  -8  19  12      5    -2     -9    18
 [8,]   11       4  -3 -10  17  10      3  -4 -11   16     9    2      -5
 [9,]  -12  15  8      1  -6  21  14 7      0    -7    20    13    6
[10,]   -1  -8  19  12 5  -2  -9  18  11      4    -3  -10 17
[11,]   10       3  -4 -11  16 9      2  -5 -12   15     8    1      -6
[12,]   21  14  7      0  -7  20  13 6  -1  -8   19    12    5
[13,]   -2  -9  18  11 4  -3 -10  17  10      3    -4  -11 16
[14,]    9       2  -5 -12  15 8      1  -6  21   14     7    0      -7
[15,]   20  13  6  -1  -8  19  12      5  -2  -9   18    11    4
[16,]   -3 -10  17  10 3  -4 -11  16 9      2    -5  -12 15
[17,]    8       1  -6  21  14 7      0  -7  20   13     6    -1      -8
[18,]   19  12  5  -2  -9  18  11      4  -3 -10   17    10    3
[19,]   -4 -11  16  9      2  -5 -12  15 8      1    -6    21    14
[20,]    7       0  -7  20  13 6  -1  -8  19  12   5    -2     -9
[21,]   18  11  4  -3 -10  17  10      3  -4 -11   16     9    2
[22,]   -5 -12  15  8      1  -6  21  14 7      0    -7    20    13
[23,]    6  -1  -8  19  12      5  -2  -9  18  11   4    -3  -10
[24,]   17  10  3  -4 -11  16 9      2  -5 -12   15     8    1
[25,]   -6  21  14  7      0  -7  20  13 6     -1     -8    19    12
        [,14] [,15]
 [1,]    13       6
 [2,]   -10      17
 [3,]    1      -6
 [4,]    12       5
 [5,]   -11      16
 [6,]    0      -7
 [7,]    11       4
 [8,]   -12      15
 [9,]    -1      -8
[10,]   10       3
[11,]   21      14
[12,]   -2      -9
[13,]    9       2

```
[14,]    20      13
[15,]    -3   -10
[16,]     8       1
[17,]    19      12
[18,]    -4   -11
[19,]     7       0
[20,]    18      11
[21,]    -5   -12
[22,]     6      -1
[23,]    17      10
[24,]    -6      21
[25,]     5      -2
```
==========================================================


min_max (win for the first) =  18
max_min (win for the second) =  8



garantiruyuchaia dlya pervogo 4
garantiruyuchaia dlya vtorogo 10



solution in clear strategies does not exists


[1] "x="
 [1] 0.003189793 0.009569378 0.000000000 0.007974482 0.000000000 0.001594896
 [7] 0.000000000 0.000000000 0.001594896 0.000000000 0.001594896 0.006379585
[13] 0.000000000 0.001594896 0.000000000 0.000000000 0.007974482 0.000000000
[19] 0.001594896 0.000000000 0.000000000 0.001594896 0.000000000 0.001594896
[25] 0.007974482
[1] "y="
 [1] 0.001594896 0.001594896 0.001594896 0.001594896 0.000000000 0.001594896
 [7] 0.001594896 0.000000000 0.001594896 0.006379585 0.007974482 0.007974482
[13] 0.009569378 0.004784689 0.006379585
game pirce is  0.01599984
==========================================================


A :
        [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
 [1,] 3040  874 -442 -500  530 2512  584 -494 -314   410  2018   328  -614
 [2,]  874 2653  -56 -793 -680 1303 2164 -307 -806  -761  1766  1709  -388
 [3,] -442  -56 3220 1090 -428 -314  310 2668  776  -572  -152   710  2524
 [4,] -500 -793 1090 2905  470 -571 -626 1495 2392   365  -608  -425  1390
 [5,]  530 -680 -428  470 2660  226 -746 -256  880  2560   -44  -778  -356
 [6,] 2512 1303 -314 -571  226 3097  970 -409 -428    97  2560   671  -538
 [7,]  584 2164  310 -626 -746  970 2788   16 -682  -836  1390  2290   -74
 [8,] -494 -307 2668 1495 -256 -409   16 3229 1138  -409  -290   373  3076
```

```
 [9,] -314 -806  776 2392  880 -428 -682 1138 2992   766 -508 -524  1024
[10,]  410 -761 -572  365 2560   97 -836 -409  766 2485 -182 -877  -484
[11,] 2018 1766 -152 -608  -44 2560 1390 -290 -508 -182 3136 1048  -428
[12,]  328 1709  710 -425 -778  671 2290  373 -524 -877 1048 2905   274
[13,] -614 -388 2524 1390 -356 -538  -74 3076 1024 -484 -428  274  2948
[14,]  -94 -785  496 1913 1324 -251 -704  815 2470 1201 -374 -589   692
[15,]  868 -502 -614  124 2086  512 -620 -494  482 2002  190 -704  -578
        [,14] [,15]
 [1,]  -94   868
 [2,] -785  -502
 [3,]  496  -614
 [4,] 1913   124
 [5,] 1324  2086
 [6,] -251   512
 [7,] -704  -620
 [8,]  815  -494
 [9,] 2470   482
[10,] 1201  2002
[11,] -374   190
[12,] -589  -704
[13,]  692  -578
[14,] 3061   874
[15,]  874  2632
========================================================

min_max (win for the first) =  2560
max_min (win for the second) =  494


garantiruyuchaia dlya pervogo 8
garantiruyuchaia dlya vtorogo 10


solution in clear strategies does not exists


[1] "x="
 [1] 0.000000e+00 1.053003e-04 0.000000e+00 8.993518e-05 1.712715e-04
 [6] 0.000000e+00 2.156753e-05 1.266564e-04 1.268678e-06 0.000000e+00
[11] 9.959123e-05 6.378632e-05 0.000000e+00 0.000000e+00 5.391882e-05
[1] "y="
 [1] 4.091487e-05 1.560474e-04 0.000000e+00 1.040316e-04 0.000000e+00
 [6] 0.000000e+00 2.156753e-05 0.000000e+00 2.347055e-05 8.817313e-05
[11] 0.000000e+00 7.548635e-05 1.008599e-04 0.000000e+00 1.227446e-04
game pirce is  0.0002611752
========================================================
```

Краткая сводка

| Матрица | A | -A | A^T * A |
|---|---|---|---|
| Выигрыши первого и второго игрока | 9 и 17 | 18 и 8 | 2560 и 494 |
| Их гарантирующие стратегии | 3 и 8 | 4 и 10 | 8 и 10 |
| Решение в чистых стратегиях? | нет. | нет. | нет |
| Смешанная стратегия первого игрока | 0.001594896<br>0.000000000<br>0.007974482<br>0.000000000<br>0.000000000<br>0.001594896<br>0.000000000<br>0.007974482<br>0.001594896<br>0.000000000<br>0.001594896<br>0.000000000<br>0.000000000<br>0.001594896<br>0.000000000<br>0.007974482<br>0.000000000<br>0.000000000<br>0.001594896<br>0.000000000<br>0.006379585<br>0.001594896<br>0.009569378<br>0.003189793<br>0.000000000 | 0.003189793<br>0.009569378<br>0.000000000<br>0.007974482<br>0.000000000<br>0.001594896<br>0.000000000<br>0.000000000<br>0.001594896<br>0.000000000<br>0.001594896<br>0.006379585<br>0.000000000<br>0.001594896<br>0.000000000<br>0.000000000<br>0.007974482<br>0.000000000<br>0.001594896<br>0.000000000<br>0.000000000<br>0.001594896<br>0.000000000<br>0.001594896<br>0.007974482 | 0.000000e+00<br>1.053003e-04<br>0.000000e+00<br>8.993518e-05<br>1.712715e-04<br>0.000000e+00<br>2.156753e-05<br>1.266564e-04<br>1.268678e-06<br>0.000000e+00<br>9.959123e-05<br>6.378632e-05<br>0.000000e+00<br>0.000000e+00<br>5.391882e-05 |
| Смешанная стратегия второго игрока | 0.006379585<br>0.007974482<br>0.007974482 | 0.001594896<br>0.001594896<br>0.001594896 | 4.091487e-05<br>1.560474e-04<br>0.000000e+00 |

| | | | |
|---|---|---|---|
| | 0.004784689 | 0.001594896 | 1.040316e-04 |
| | 0.009569378 | 0.000000000 | 0.000000e+00 |
| | 0.001594896 | 0.001594896 | 0.000000e+00 |
| | 0.001594896 | 0.001594896 | 2.156753e-05 |
| | 0.006379585 | 0.000000000 | 0.000000e+00 |
| | 0.001594896 | 0.001594896 | 2.347055e-05 |
| | 0.000000000 | 0.006379585 | 8.817313e-05 |
| | 0.001594896 | 0.007974482 | 0.000000e+00 |
| | 0.001594896 | 0.007974482 | 7.548635e-05 |
| | 0.000000000 | 0.009569378 | 1.008599e-04 |
| | 0.001594896 | 0.004784689 | 0.000000e+00 |
| | 0.001594896 | 0.006379585 | 1.227446e-04 |
| Цена игры | -0.01046476 | 0.01599984 | 0.0002611752 |