Elementary function

From Wikipedia, the free encyclopedia

For the complexity class, see ELEMENTARY. For the logical system, see Elementary function arithmetic.

In mathematics, an **elementary function** is a function of a single variable composed of particular simple functions.

Elementary functions are typically defined as a sum, product, and/or composition of finitely many polynomials, rational functions, trigonometric and exponential functions, and their inverse functions (including arcsin, log, $x^{1/n}$).[1]

Elementary functions were introduced by Joseph Liouville in a series of papers from 1833 to 1841.[2][3][4] An algebraic treatment of elementary functions was started by Joseph Fels Ritt in the 1930s.[5]

Basic examples [edit]

The elementary functions (of x) include:

- Constant functions: $2,\ \pi,\ e$, etc.
- Powers of x: $x,\ x^2,\ x^3$, etc.
- Roots of x: $\sqrt{x},\ \sqrt[3]{x}$, etc.
- Exponential functions: e^x
- Logarithms: $\log x$
- Trigonometric functions: $\sin x,\ \ \cos x,\ \ \tan x$, etc.
- Inverse trigonometric functions: $\arcsin x,\ \ \arccos x$, etc.
- Hyperbolic functions: $\sinh x,\ \ \cosh x$, etc.
- Inverse hyperbolic functions: $\operatorname{arsinh} x,\ \ \operatorname{arcosh} x$, etc.
- All functions obtained by adding, subtracting, multiplying or dividing any of the previous functions[6]
- All functions obtained by composing previously listed functions

Some elementary functions, such as roots, logarithms, or inverse trigonometric functions, are not entire functions and may be multivalued.

Composite examples [edit]

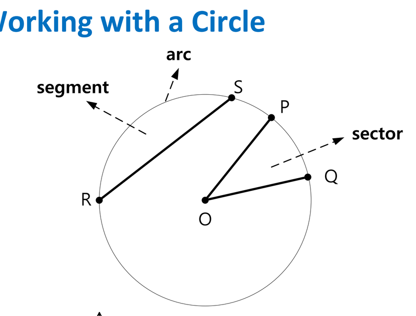
Examples of elementary functions include:

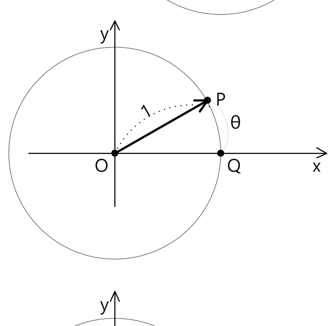
- Addition, e.g. (x+1)
- Multiplication, e.g. (2x)
- Polynomial functions
- $\dfrac{e^{\tan x}}{1+x^2}\sin\left(\sqrt{1+(\ln x)^2}\right)$
- $-i\ln(x+i\sqrt{1-x^2})$

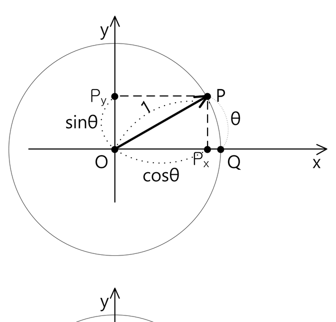
The last function is equal to $\arccos x$, the inverse cosine, in the entire complex plane.

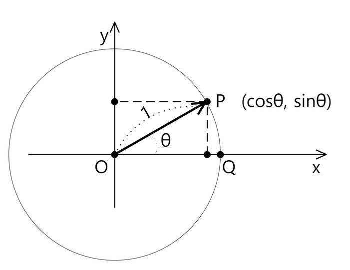
All monomials, polynomials and rational functions are elementary. Also, the absolute value function, for real x, is also elementary as it can be expressed as the composition of a power and root of x: $|x|=\sqrt{x^2}$.

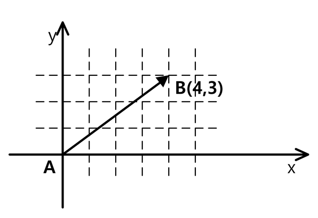
Working with a Circle









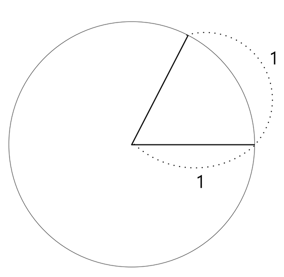


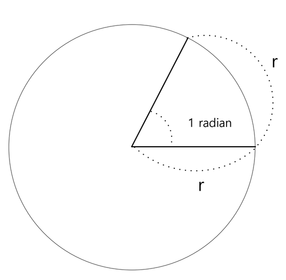
Radian

From Wikipedia, the free encyclopedia

Definition

Radian describes the plane angle subtended by a circular arc, as the length of the arc divided by the radius of the arc. One radian is the angle subtended at





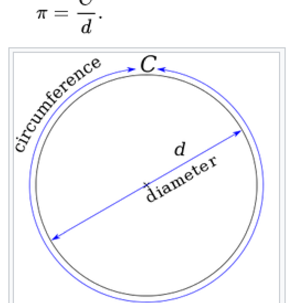
Pi

From Wikipedia, the free encyclopedia

Definition

π is commonly defined as the ratio of a circle's circumference C to its diameter d:[12][2]

$$\pi=\frac{C}{d}.$$



The circumference of a circle is slightly more than three times as long as its diameter. The exact ratio is called π.

$\pi=3.1415\ldots$
$2\pi\ radian=360\ degree$

Trigonometric functions

From Wikipedia, the free encyclopedia
(Redirected from Trigonometric function)

In mathematics, the **trigonometric functions** (also called **circular functions**, **angle functions** or **goniometric functions**[1][2]) are real functions which relate an angle of a right-angled triangle to ratios of two side lengths. They are widely used in all sciences that are related to geometry, such as navigation, solid mechanics, celestial mechanics, geodesy, and many others. They are among the simplest periodic functions, and as such are also widely used for studying periodic phenomena, through Fourier analysis.

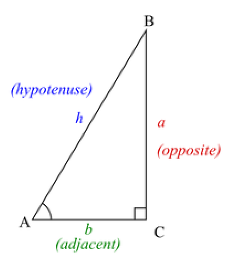
The most widely used trigonometric functions are the sine, the **cosine**, and the **tangent**. Their reciprocals are respectively the **cosecant**, the **secant**, and the **cotangent**, which are less used in modern mathematics. Each of these six trigonometric functions has a corresponding inverse function (called inverse trigonometric function), and an equivalent in the hyperbolic functions as well.[3]

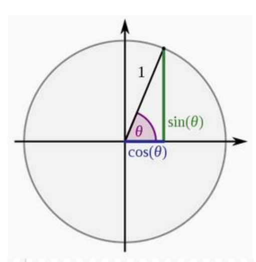
Right-angled triangle definitions [edit]

In this section, the same upper-case letter denotes a vertex of a triangle and the measure of the corresponding angle; the same lower case letter denotes an edge of the triangle and its length.

Given an acute angle $A=\theta$ of a right-angled triangle, the hypotenuse h is the side that connects the two acute angles. The side b *adjacent* to θ is the side of the triangle that connects θ to the right angle. The third side a is said to be *opposite* to θ.

If the angle θ is given, then all sides of the right-angled triangle are well-defined up to a scaling factor. This means that the ratio of any two side lengths depends only on θ. Thus these six ratios define six functions of θ, which are the trigonometric functions. More precisely, the six trigonometric functions are:[4][5]





sine

$$\sin\theta=\frac{a}{h}=\frac{\text{opposite}}{\text{hypotenuse}}$$

cosine

$$\cos\theta=\frac{b}{h}=\frac{\text{adjacent}}{\text{hypotenuse}}$$

tangent

$$\tan\theta=\frac{a}{b}=\frac{\text{opposite}}{\text{adjacent}}$$

cosecant

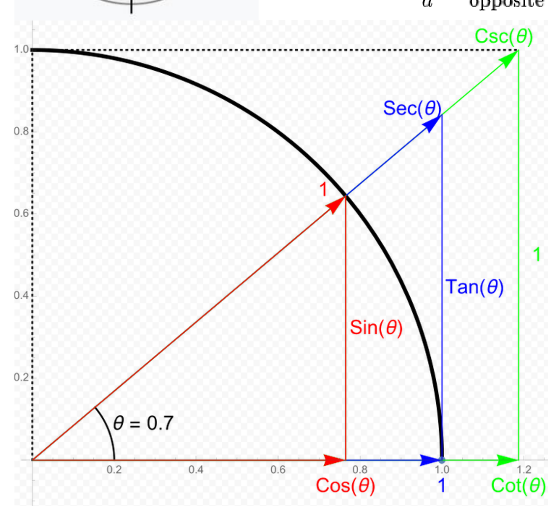
$$\csc\theta=\frac{h}{a}=\frac{\text{hypotenuse}}{\text{opposite}}$$

secant

$$\sec\theta=\frac{h}{b}=\frac{\text{hypotenuse}}{\text{adjacent}}$$

cotangent

$$\cot\theta=\frac{b}{a}=\frac{\text{adjacent}}{\text{opposite}}$$



Sum and difference formulas

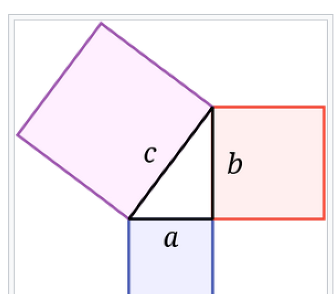
Sum

$$\sin(x+y)=\sin x\cos y+\cos x\sin y,$$
$$\cos(x+y)=\cos x\cos y-\sin x\sin y,$$
$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}.$$

Difference

$$\sin(x-y)=\sin x\cos y-\cos x\sin y,$$
$$\cos(x-y)=\cos x\cos y+\sin x\sin y,$$
$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}.$$

Pythagorean theorem



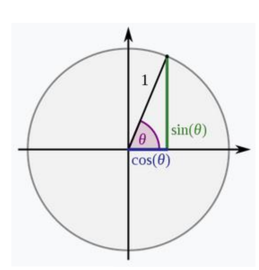
Pythagorean theorem
The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c).

$$a^2+b^2=c^2$$

Pythagorean identity [edit]

The Pythagorean identity, is the expression of the Pythagorean theorem in terms of trigonometric functions. It is

$$\sin^2 x+\cos^2 x=1.$$



Law of cosines

From Wikipedia, the free encyclopedia

This article is about the law of cosines in Euclidean geometry. Fc Lambert's cosine law.

In trigonometry, the **law of cosines** (also known as the **cosine formula**, **cosine rule**, or **al-Kashi's theorem**[1]) relates the lengths of the sides of a triangle to the cosine of one of its angles. Using notation as in Fig. 1, the law of cosines states

$$c^2=a^2+b^2-2ab\cos\gamma,$$

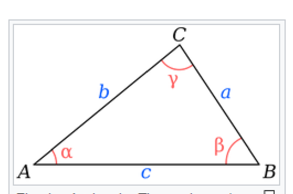
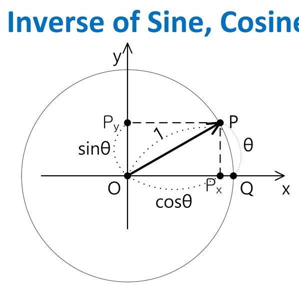


Fig. 1 – A triangle. The angles α (or A), β (or B), and γ (or C) are respectively opposite the sides a, b, and c.

Inverse of Sine, Cosine



$$\cos^{-1}(\overline{OP_x})=\theta$$
$$\sin^{-1}(\overline{OP_y})=\theta$$

$$acos(\overline{OP_x})=\theta$$
$$asin(\overline{OP_y})=\theta$$

Differentiation

Definition by differential equations [edit]

Sine and cosine are the unique differentiable functions such that

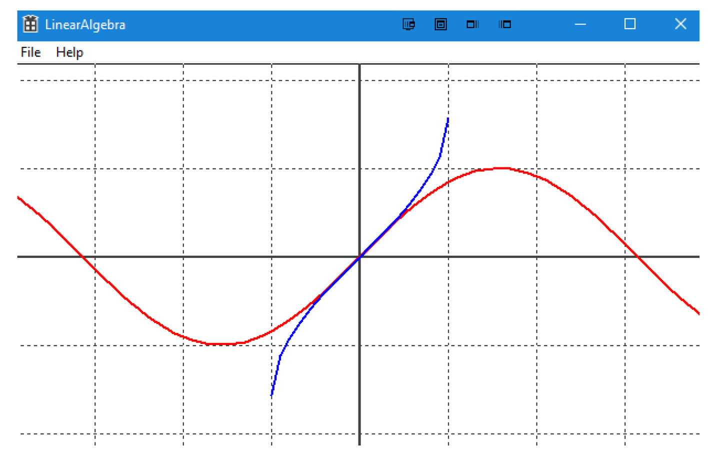
$$\frac{d}{dx}\sin x=\cos x,$$
$$\frac{d}{dx}\cos x=-\sin x,$$

Implementation: Draw Sine and Cosine

```
double SineFunction(double x)
{
    return sin(x);
}

double AsineFunction(double x)
{
    return asin(x);
}


    DrawFunction(hdc, &SineFunction, -20.0, +20.0, 0.1, RGB(255, 0, 0));
    DrawFunction(hdc, &AsineFunction, -1.0, +1.0 - 0.0001, 0.1, RGB(0, 0, 255));
```



- Elementary Number
  $0,1,i,e,\pi$
- Elementary Function

Function Type	Differentiation	Inverse	Differentiation
Power: $x^2,\ x^n$	$2x^1, nx^{n-1}$	Root: $\sqrt{x}$	$\dfrac{1}{2\sqrt{x}}$
Exponential: e^x	e^x	Log: $\ln x$	$\dfrac{1}{x}$
Trigonometric: $\cos(x), \sin(x)$	$-\sin(x), \cos(x)$	Inverse Trigonometric: $acos(x), asin(x)$	...

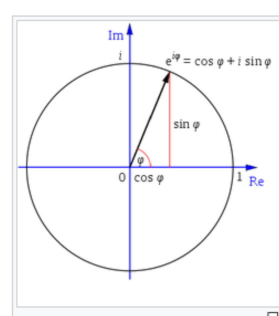
Euler's formula

From Wikipedia, the free encyclopedia

Euler's formula, named after Leonhard Euler, is a mathematical formula in complex analysis that establishes the fundamental relationship between the trigonometric functions and the complex exponential function. Euler's formula states that for any real number x:

$$e^{ix}=\cos x+i\sin x,$$

When $x=\pi$, Euler's formula evaluates to $e^{i\pi}+1=0$, which is known as Euler's identity.



Complex number for rotation

$$e^{i\theta}e^{i\gamma}=e^{i(\theta+\gamma)}$$
$$\cos(\theta+\gamma)+i\sin(\theta+\gamma)$$

https://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/1.-differentiation/part-a-definition-and-basic-rules/session-7-derivatives-of-sine-and-cosine/MIT18_01SCF10_Ses7a.pdf

Derivative of $\sin x$, Algebraic Proof

A specific derivative formula tells us how to take the derivative of a specific function: if $f(x)=x^n$ then $f'(x)=nx^{n-1}$. We'll now compute a specific formula for the derivative of the function $\sin x$.

As before, we begin with the definition of the derivative:

$$\frac{d}{dx}\sin x=\lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin(x)}{\Delta x}$$

You may remember the following angle sum formula from high school:

$$\sin(a+b)=\sin(a)\cos(b)+\sin(b)\cos(a)$$

This lets us untangle the x from the Δx as follows:

$$\frac{d}{dx}\sin x=\lim_{\Delta x\to 0}\frac{\sin x\cos\Delta x+\cos x\sin\Delta x-\sin(x)}{\Delta x}.$$

We can simplify this expression using some basic algebraic facts:

$$\begin{aligned}
\frac{d}{dx}\sin x &= \lim_{\Delta x\to 0}\left[\frac{\sin x\cos\Delta x-\sin x}{\Delta x}+\frac{\cos x\sin\Delta x}{\Delta x}\right]\\
&= \lim_{\Delta x\to 0}\left[\frac{\sin x(\cos\Delta x-1)}{\Delta x}+\frac{\cos x\sin\Delta x}{\Delta x}\right]\\
&= \lim_{\Delta x\to 0}\left[\sin x\left(\frac{\cos\Delta x-1}{\Delta x}\right)+\cos x\left(\frac{\sin\Delta x}{\Delta x}\right)\right]\\
\frac{d}{dx}\sin x &= \lim_{\Delta x\to 0}\sin x\left(\frac{\cos\Delta x-1}{\Delta x}\right)+\lim_{\Delta x\to 0}\cos x\left(\frac{\sin\Delta x}{\Delta x}\right)
\end{aligned}$$

We now have two familiar functions – $\sin x$ and $\cos x$ – and two ugly looking fractions to deal with. The fractions may be familiar from our discussion of removable discontinuities.

$$\begin{aligned}
\lim_{\Delta x\to 0}\frac{\cos\Delta x-1}{\Delta x} &= 0\\
\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x} &= 1.
\end{aligned}$$

Proofs

List of Maclaurin series of some common functions [edit]

See also: List of mathematical series

Several important Maclaurin series expansions follow.[12] All these expansions are valid for complex arguments x.

Exponential function [edit]

The exponential function e^x (with base e) has Maclaurin series

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots.$$

It converges for all x.

Trigonometric functions [edit]

The usual trigonometric functions and their inverses have the following Maclaurin series:

$$\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}\qquad =x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\qquad \text{for all } x$$

$$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}\qquad =1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots\qquad \text{for all } x$$

Proofs [edit]

Various proofs of the formula are possible.

Using power series [edit]

Here is a proof of Euler's formula using power-series expansions, as well as basic facts about the powers of i:[9]

$$\begin{aligned}
i^0&=1, & i^1&=i, & i^2&=-1, & i^3&=-i,\\
i^4&=1, & i^5&=i, & i^6&=-1, & i^7&=-i\\
&\vdots & &\vdots & &\vdots & &\vdots
\end{aligned}$$

Using now the power-series definition from above, we see that for real values of x

$$\begin{aligned}
e^{ix}&=1+ix+\frac{(ix)^2}{2!}+\frac{(ix)^3}{3!}+\frac{(ix)^4}{4!}+\frac{(ix)^5}{5!}+\frac{(ix)^6}{6!}+\frac{(ix)^7}{7!}+\frac{(ix)^8}{8!}+\cdots\\
&=1+ix-\frac{x^2}{2!}-\frac{ix^3}{3!}+\frac{x^4}{4!}+\frac{ix^5}{5!}-\frac{x^6}{6!}-\frac{ix^7}{7!}+\frac{x^8}{8!}+\cdots\\
&=\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\cdots\right)+i\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\right)\\
&=\cos x+i\sin x,
\end{aligned}$$