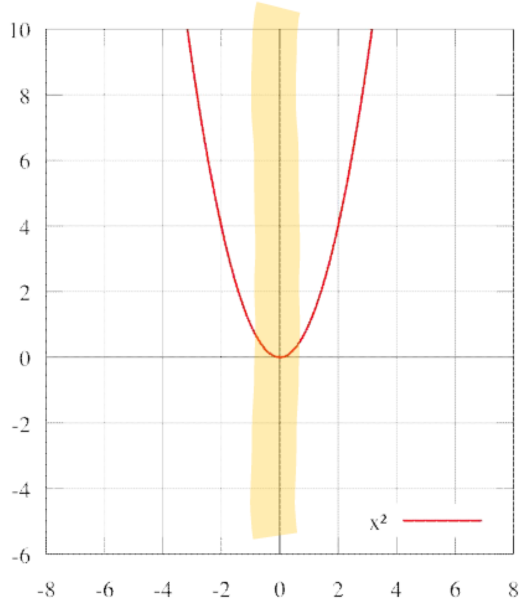
Euler's Formula

Even function

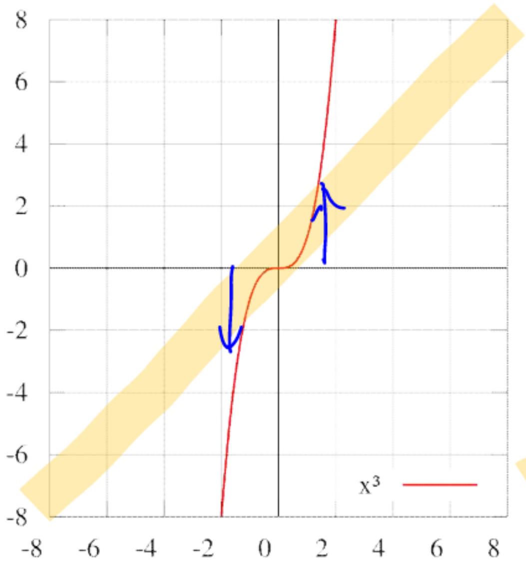
$f(x) = f(-x)$



$f(x) = x^2$

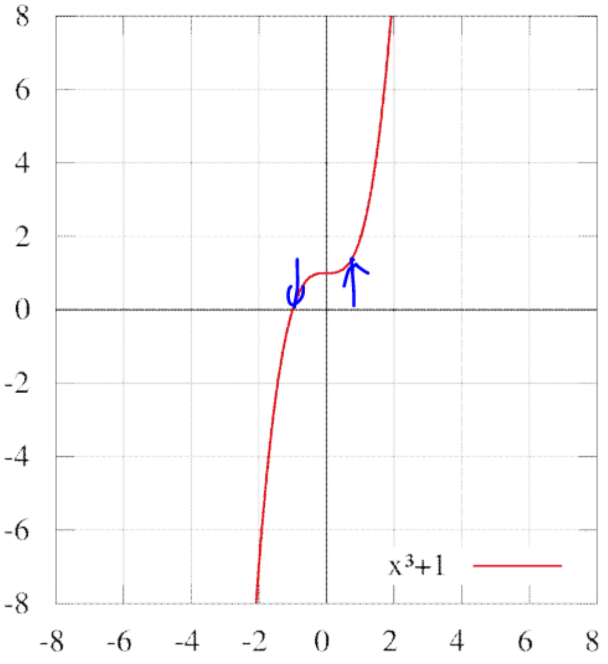
Odd function

$-f(x) = f(-x)$



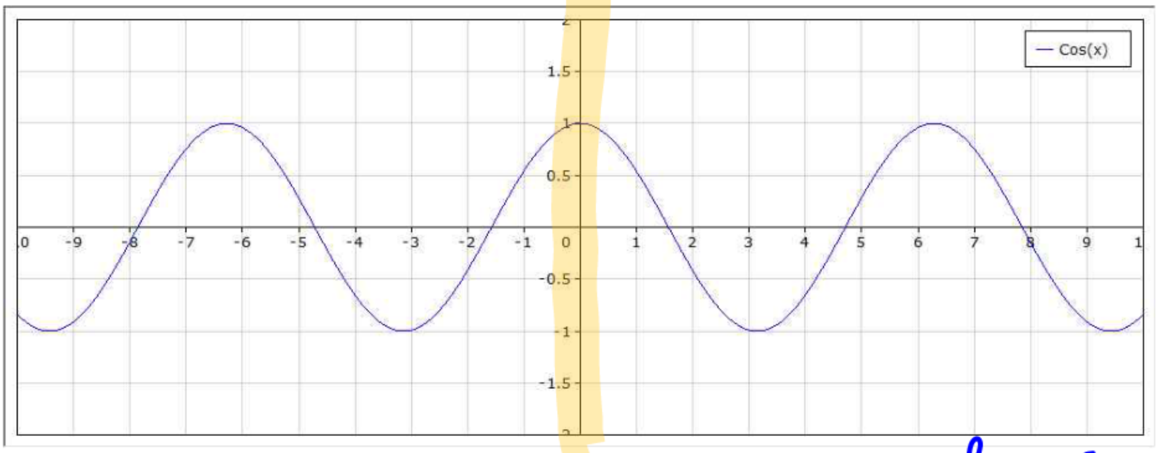
$f(x) = x^3$

not even or odd



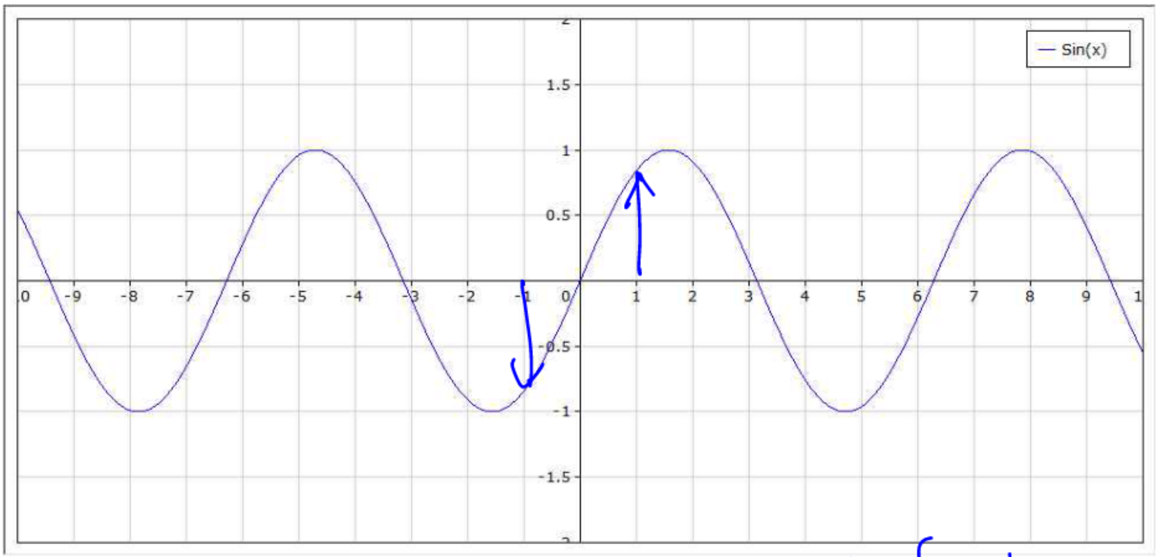
$$f(x) = x^3 + 1$$

Cosine and Sine



$\cos(x)$

even function



$\sin(x)$

odd function.

Complex number

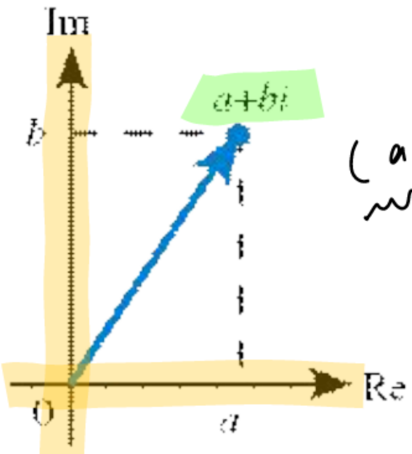
A complex number is a number that can be expressed in the form $a + bi$, where a and b are real numbers, and i is a solution of the equation $x^2 = -1$.

Imaginary number

Im

i

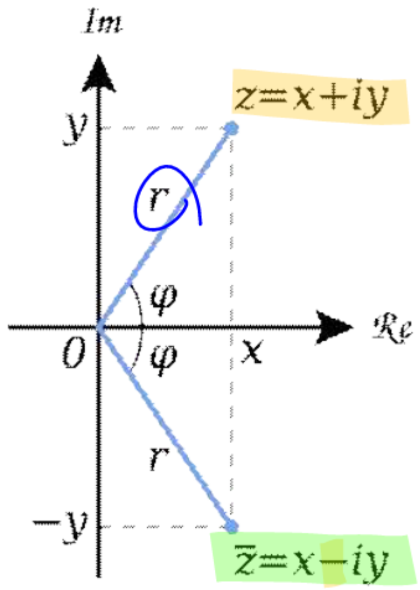
$i \cdot i = i^2 = -1$



(a, b) complex number
= number

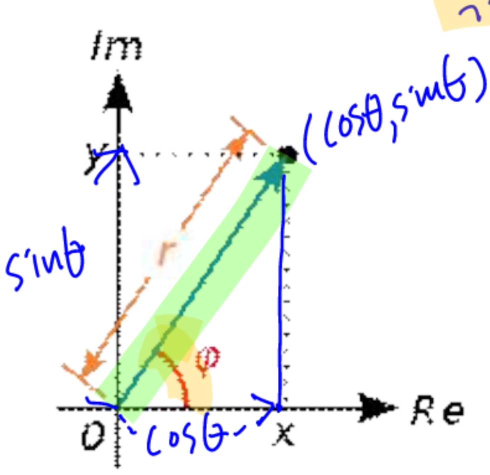
Re

Conjugate 켤레



r^2

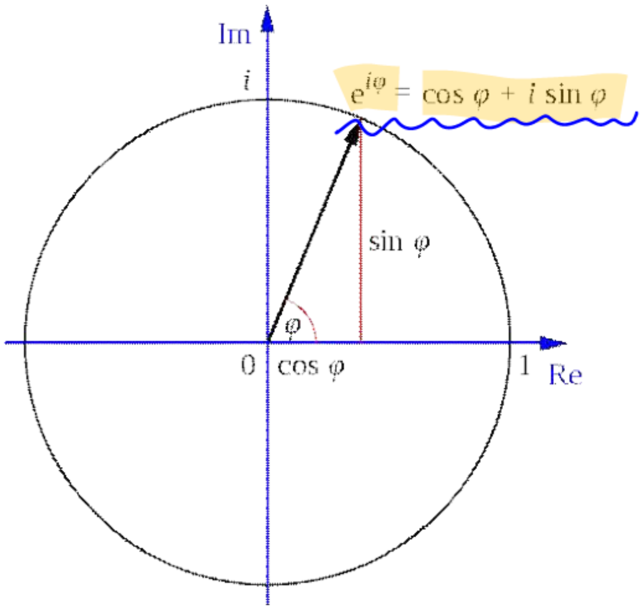
Polar Coordinate 극좌표



$(\phi,\ r)$

$r(\cos(\theta) + i\sin(\theta))$

Euler's formula



$e^2 = e \cdot e \qquad e^i = ?$

$$e^{ix} = \cos(x) + i\sin(x)$$

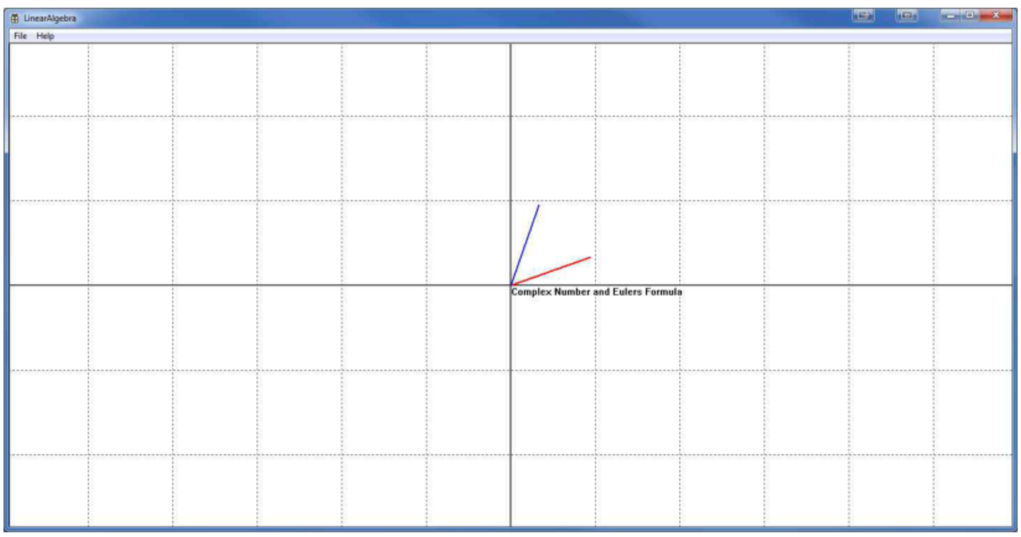
$e^{\pi i} = \cos\pi + i\sin\pi$
$= -1 + 0$

$e^{\pi i} = -1$

$$(\cos(x) + i\sin(x))(\cos(y) + i\sin(y)) = ?$$

$$e^{ix}e^{iy} = e^{i(x+y)} = \cos(x+y) + i\sin(x+y)$$

Implementation



```
    static double timer = 0;
    timer += (double)fElapsedTime_;
    const std::complex<double> i( 0, 1 );
    std::complex<double>    c0;
    c0 = std::polar<double>( 1.0, M_PI / 4.0 );

    {
        std::complex<double>    c1;
        c1 = std::polar<double>( 1.0, timer );
        double theta = std::arg( c0 * c1 );

        KMatrix2 m;
        m.SetRotation( theta );
        KVector2 v( 1, 0 );
        v = m * v;
        KVectorUtil::DrawLine(  hdc,  KVector2::zero,  v,  2,  PS_SOLID,
RGB( 255, 0, 0 ) );
    }

    {
        std::complex<double>    c2;
        c2 = std::exp( i * -timer );
        double theta = std::arg( c0 * c2 );

        KMatrix2 m;
        m.SetRotation( theta );
        KVector2 v( 1, 0 );
        v = m * v;
        KVectorUtil::DrawLine(  hdc,  KVector2::zero,  v,  2,  PS_SOLID,
RGB( 0, 0, 255 ) );
    }
```

Conjugate

$e^{-ix} = \cos(-x) + i\sin(-x) = \cos(x) - isin(x)$

$e^{ix}e^{-ix} = e^{i(x-x)} = e^{0i} = e^{0} = 1$

Pythagorean theorem

$$e^{ix}e^{-ix} = (\cos(x) + isin(x))(\cos(x) - isin(x))$$
$$= \cos^2(x) - \cancel{\cos(x)i\sin(x)} + \cancel{isin(x)\cos(x)} - i^2\sin^2(x)$$
$$= \cos^2(x) + \sin^2(x) = 1$$

+1

Proof

Taylor Series.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$$

$$e^{ix} = \sum_{n=0}^{\infty} \frac{(ix)^n}{n!} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \ldots$$

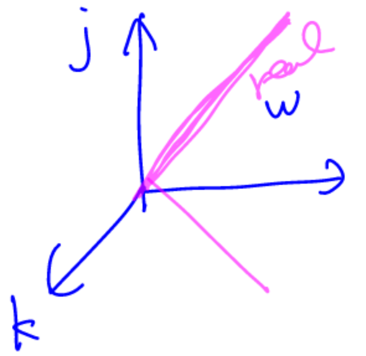
$$e^{ix} = \cos(x) + i\sin(x)$$

Quaternion

$$e^{ix} = \cos(x) + i\sin(x)$$
$$= \cos(x) + \frac{\sin(x)}{x}(ix)$$

imaginary number

$i^2=-1$



$i^2=-1$

$j^2=-1$

$k^2=-1$

Rotating Axis

$\vec{v} = (x,y,z)$

$(w,\vec{v}) = (w,x,y,z)$

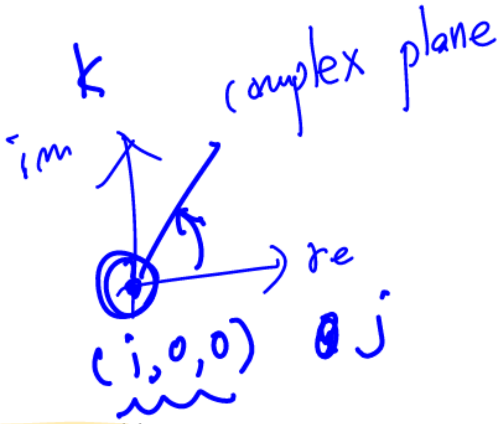
$\vec{v} = xi+yj+zk = (x,y,z)$

$ijk=-1$

$ij=k, ji=-k$

$jk=i, kj=-i$

$ki=j, ik=-j$



Quaternion

$$e^{w+xi+yj+zk} = e^w\left(\cos(|v|) + \frac{\sin(|v|)}{|v|}(xi+yj+zk)\right)$$

$|v| = \sqrt{x^2+y^2+z^2}$ $=1$

$$e^{w+xi+0j+0k} = e^w(\cos(x)+\sin(x)(i))$$

Rotating Idea

(0,a,b,c)

$$e^{(w+xi+yj+zk)}e^{(ai+bj+ck)} = e^w e^{(x+a)i+(y+b)j+(z+c)k}$$

$$e^0 e^{(x+a)i+(y+b)j+(z+c)k} = e^{(x+a)i+(y+b)j+(z+c)k}$$

① $\cos\theta + i\sin\theta$

↓

$\cos\delta + i\sin\delta$

② $\cos\theta + i\sin\theta$

↓

$\cos\delta + \cancel{\theta i}$

③ $\cos\theta + i\sin\theta$

↓

$\cancel{\cos\delta} + i\sin\delta$

Conjugate of Complex Number

$c = a + bi$

$|c| = \sqrt{a^2 + b^2}$

$c^* = a - bi$

$c \bullet c^* = a^2 + b^2 = |c|^2$

Conjugate of Quaternion

$q = (w, x, y, z)$

$q^* = (w, -x, -y, -z)$

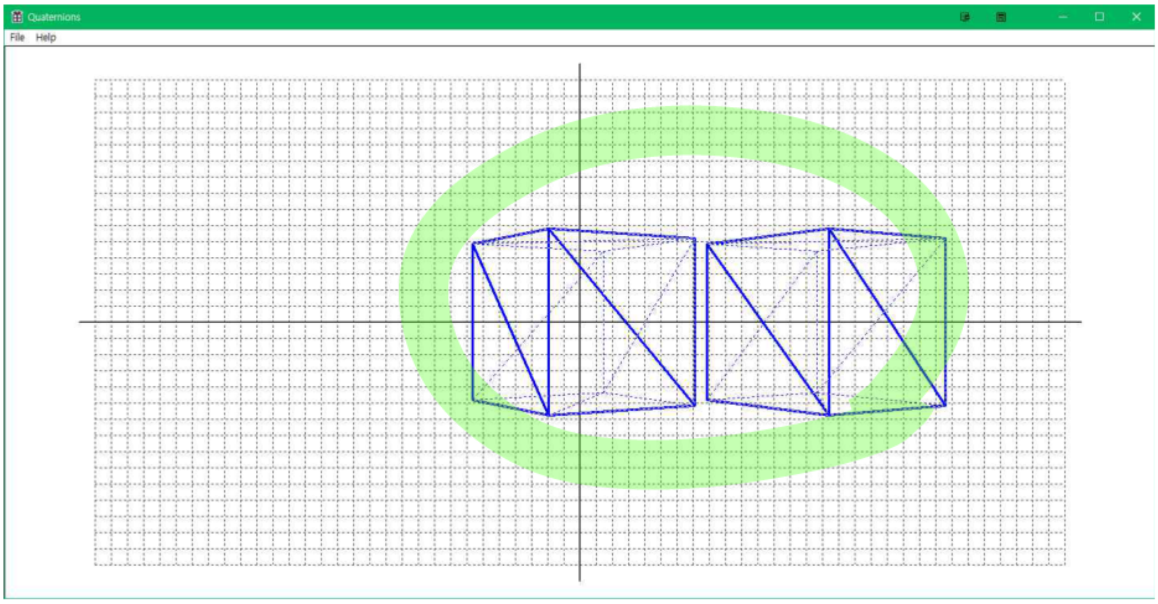
$|q| = \sqrt{w^2 + x^2 + y^2 + z^2}$

$q \bullet q^* = |q|^2$

$v' = qvq^*$ $\quad v' = q \cdot v_0 \cdot q^*$

$v = (a, b, c)$

$\hookrightarrow v_0 = (\emptyset, a, b, c)$



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