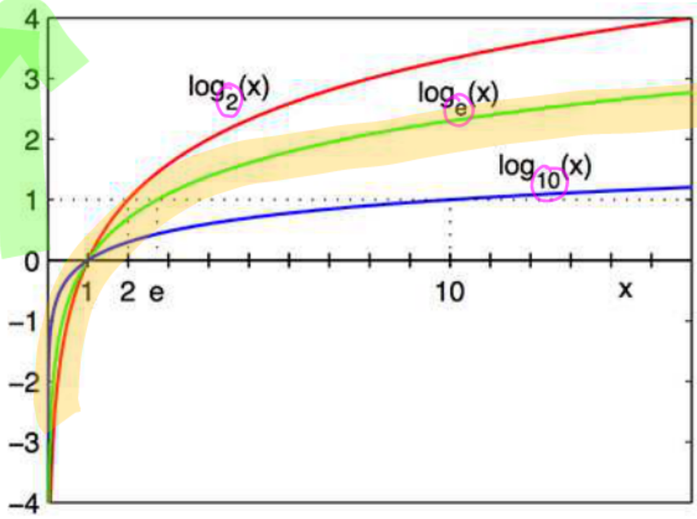
Euler Constant

Logarithm

$\log_b(x) = y$, $b^y = x$

base



Inverse function

$f(x) = x+1$ ----①

$y = x+1$

$x = y-1$

$f(x) = x-1$ ----②

$y = 2^x$ exponential function.

$2^3 = 2 \cdot 2 \cdot 2 = 8$

$\log_2 \boxed{8} = ?\ 3$ $\quad 2^8,\ {}_2 8 = ?$

$\log_2 \boxed{8} = 3$

*Binary Search

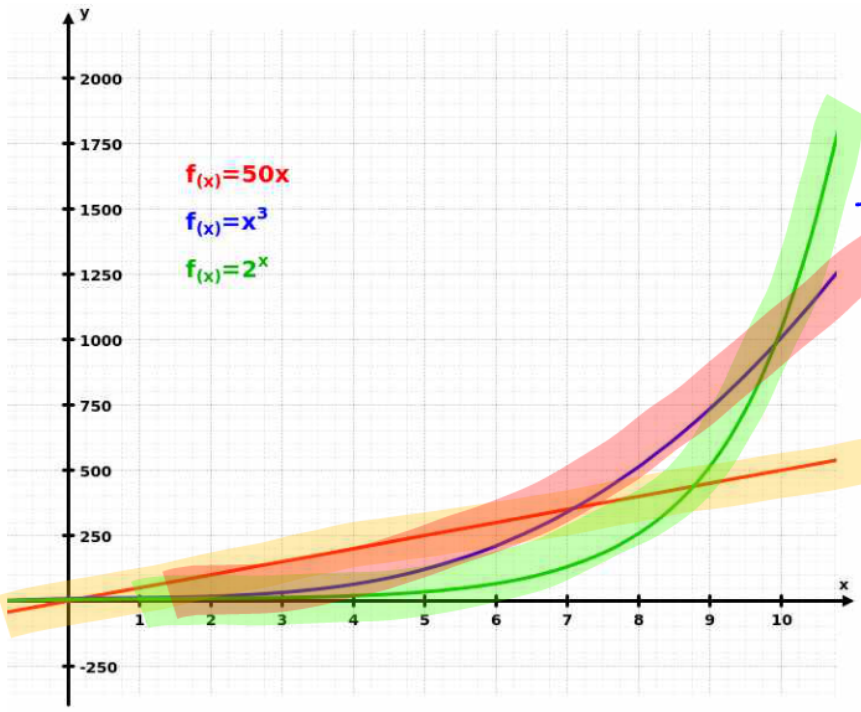
sort (asc)

3번 수행.

$\log_2 8 = 3$

$O(\log_2 n)$

Why Exponential



Exponential growth (green) describes many physical phenomena.

Differentiation of exponential function

$$\frac{d}{dx}a^x = \lim_{h\to 0}\frac{a^{x+h}-a^x}{h}$$

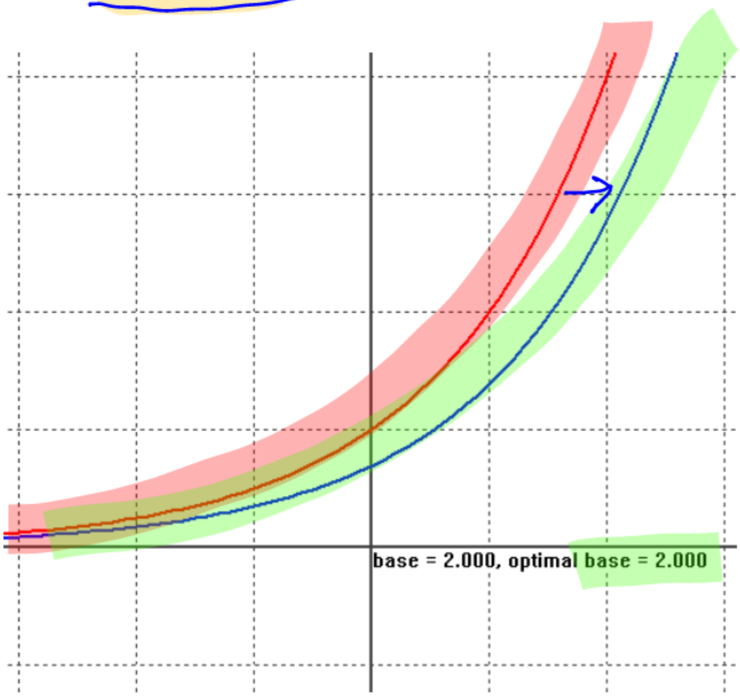
$$= \lim_{h\to 0}\frac{a^x a^h - a^x}{h}$$

$$= a^x\left(\boxed{\lim_{h\to 0}\frac{a^h-1}{h}}\right)$$

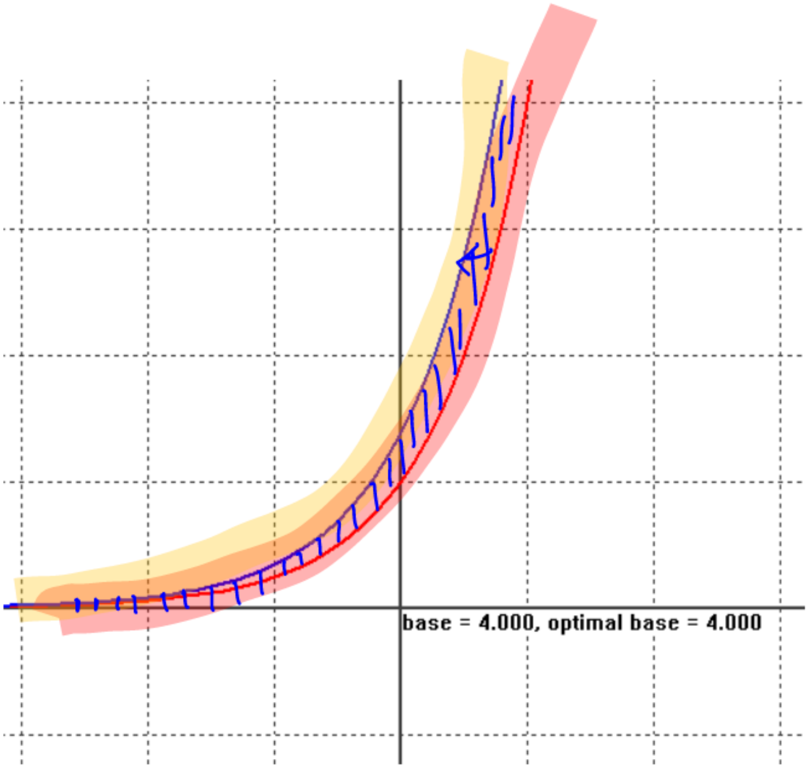
$a^{x+h} = a^x \cdot a^h$

a^h

$\frac{1-1}{0} = \frac{0}{0}$



[Fig] 2^x, $\frac{d}{dx}2^x$



$2 < ? < 4$

$$\frac{d\,e^x}{dx} = e^x.$$

[Fig] $4^x, \dfrac{d}{dx}4^x$ e

$2 < MysteriousIrrationalConstant\ \ e < 4$

Numerical solution

Implementation

standard deviation.

$$s = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N-1}}$$

```
double GetStdDeviation( double base_, double beginX, double endX, double xstep
)
{
    std::vector<double>        vecDiff;

    double x = beginX;
    double ydiff;
```

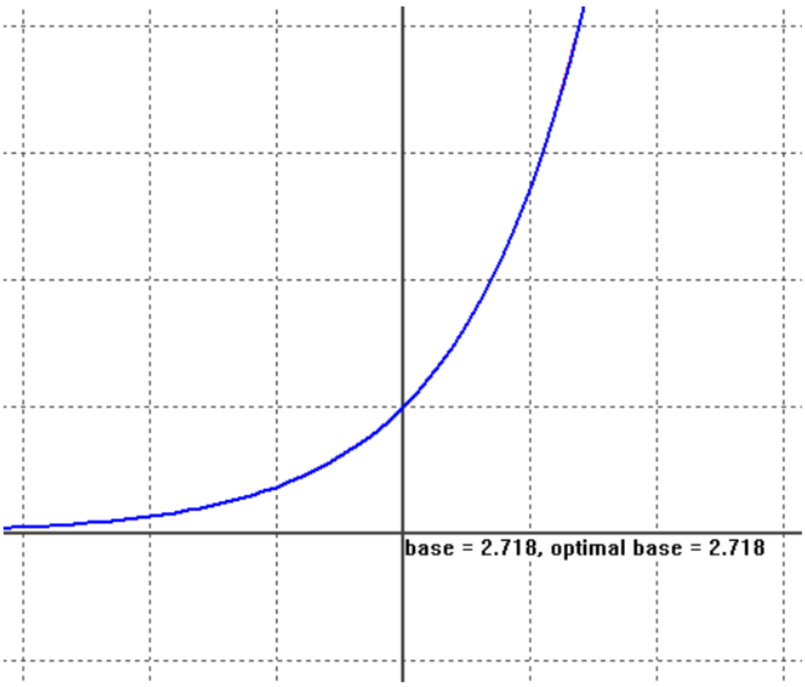
```
    double N = 0;
    while(x < endX)
    {
        ydiff = ExpFunction( base_, x ) - SymmetricDifference( &ExpFunction,
base_, x );
        x += xstep;
        N += 1;
        vecDiff.push_back( ydiff );
    }//while

    double sum = 0;
    for(const double diff : vecDiff)
    {
        sum += ( diff * diff );
    }

    return sqrt( sum / ( N - 1 ) );
}
```

$e \approx 2.718\ldots$

Euler Constant.



$e \approx 2.718$

Why e

$$\frac{d}{dx}e^x = e^x$$

$\frac{d}{dx} e^x = e^x$

std::exp(x)

$$a^x = e^{cx}$$
$$2^x = e^{x\ln(2)}$$
$$3^x = e^{x\ln(3)}$$
$$\dots$$

$\ln 2 = \log_e 2$

$$\frac{d}{dx}e^{cx} = ce^{cx}$$

chain rule

$c \cdot e^{cx}$

$$\frac{d}{dx}a^x = a^x \ln a$$

Complex Number

Quaternion

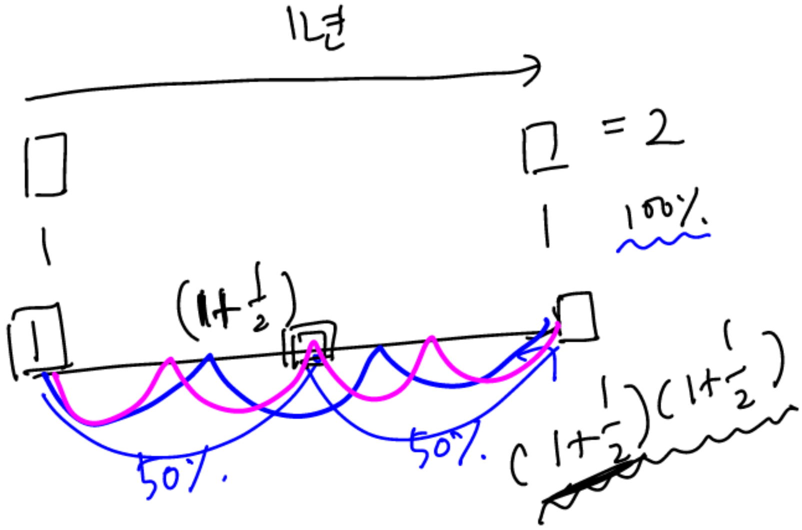
Fourier Transform

$$\frac{d}{dx}log_e x = \frac{1}{x}$$

Meaning of e

Example

$$(1+1) = 2$$
$$(1+\frac{1}{2})(1+\frac{1}{2}) = 2.25$$



$$(1+\frac{1}{3})(1+\frac{1}{3})(1+\frac{1}{3}) = 2.370$$

$$(1+\frac{1}{4})(1+\frac{1}{4})(1+\frac{1}{4})(1+\frac{1}{4}) = 2.441$$

...

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

≈ 2.718

Definition

$$e = \lim_{n\to\infty}(1+\frac{1}{n})^n$$

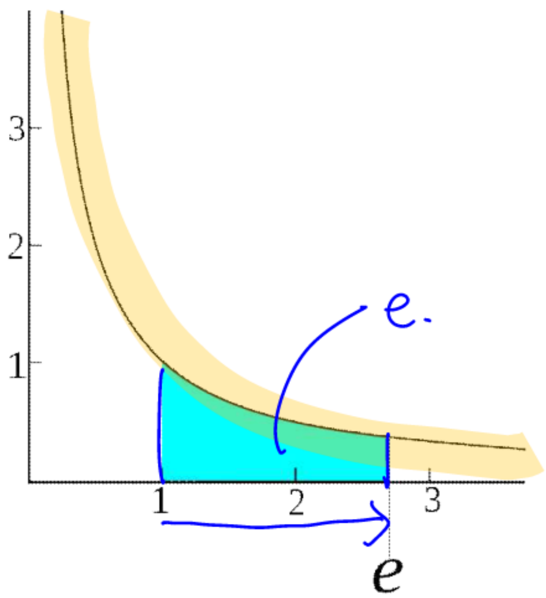
$$e = \lim_{t\to\infty}(1+t)^{\frac{1}{t}}$$

$$e = \sum_{n=0}^{\infty}\frac{1}{n!} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \dots$$

$$\int_1^e \frac{1}{t}\,dt = 1$$

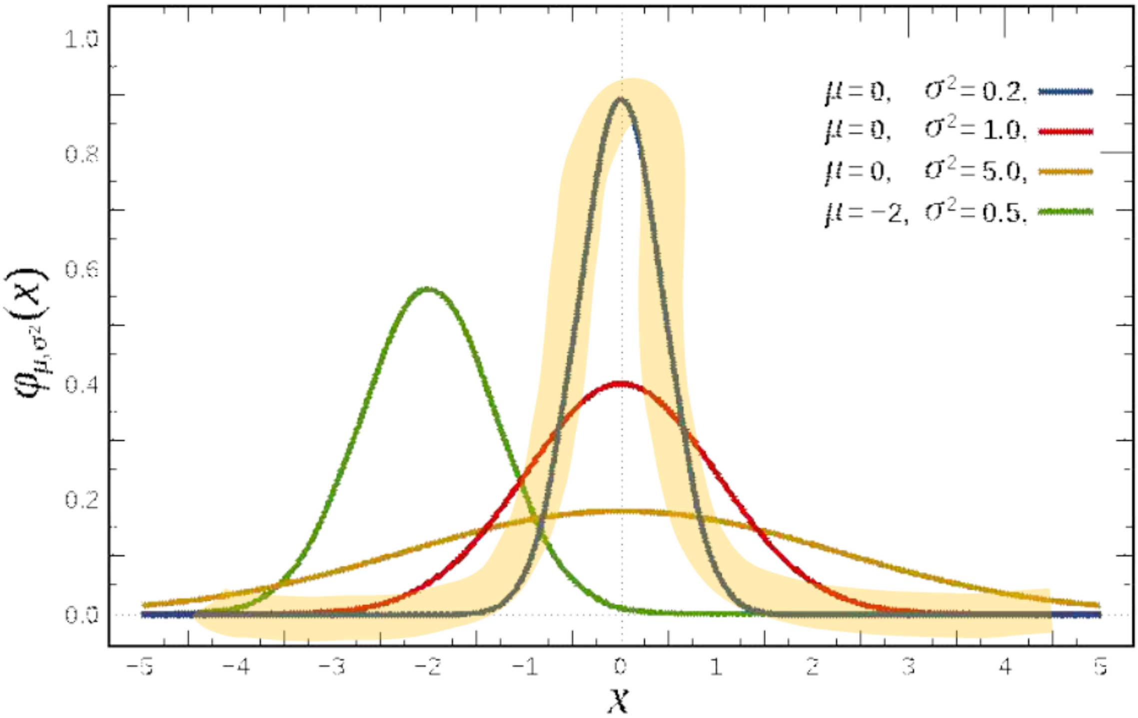
$\left[\log_e t\right]_1^e = \log_e e - \log_e 1$ $\quad e^0$

$= 1 - 0 = 1$



Gaussian function

$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}}$$



Standard normal distribution

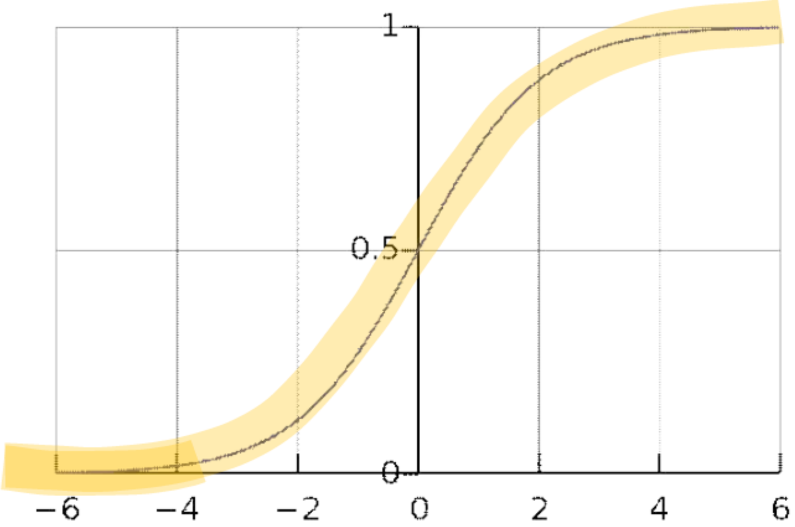
$$S(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$$

Logistic Function

Sigmoid curve

$$f(x) = \frac{L}{1 + e^{-k(x - x_0)}}$$

L=1

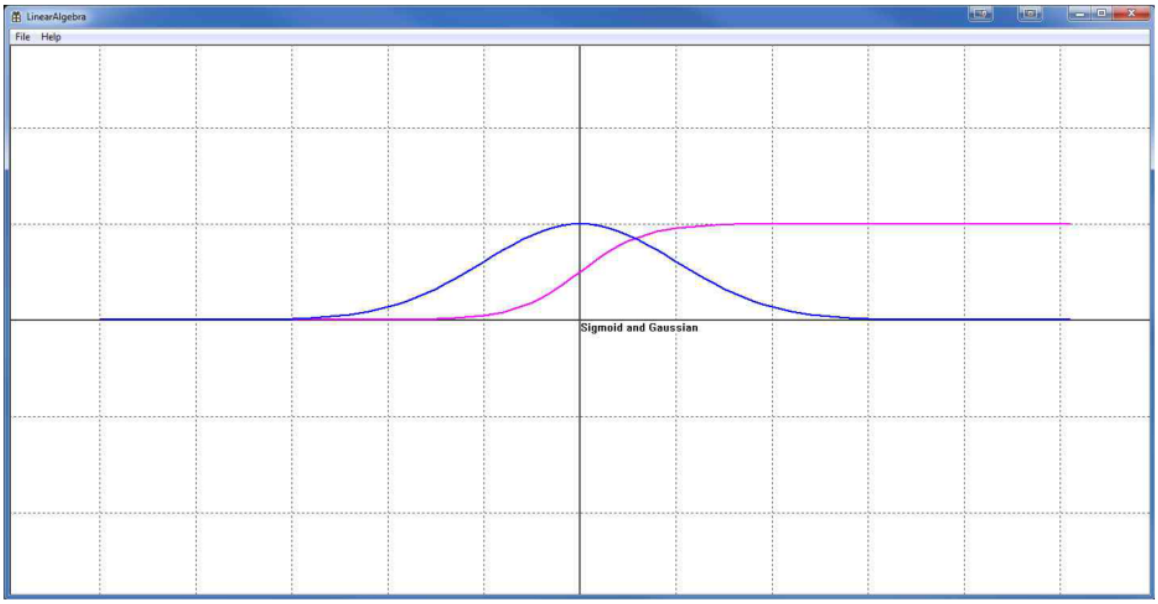


Implementation

```
double Logistic( double x )
{
    // Logistic Function
    const double L = 1.0;
    const double k = 3.0;
    const double x0 = 0.0;
    return L / (1 + std::exp(-k*(x - x0)));
}

double Gaussian( double x )
{
    // Gaussian Function
    const double a = 1.0;
    const double b = 0.0;
    const double c = 1.0;
    return a*std::exp(-((x-b)*(x-b))/(2*c*c));
```

}


LinearAlgebra
File Help
Sigmoid and Gaussian

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