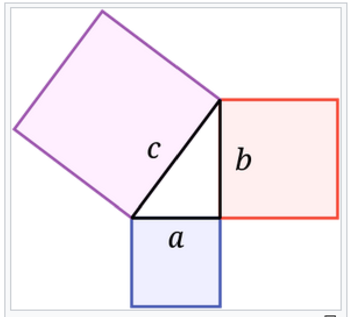
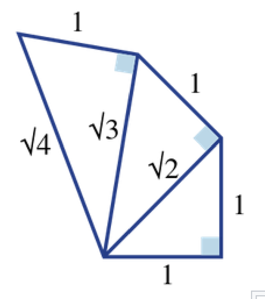
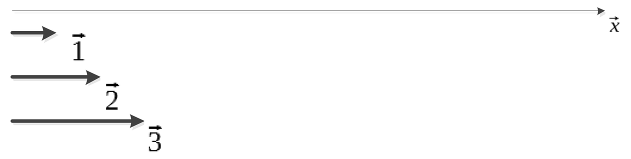
Pythagorean theorem



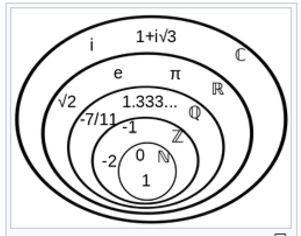
Pythagorean theorem
The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c).

$a^2 + b^2 = c^2,$





$\vec{1}+\vec{2}=\vec{3}$



Subsets of the complex numbers

Imaginary number

From Wikipedia, the free encyclopedia

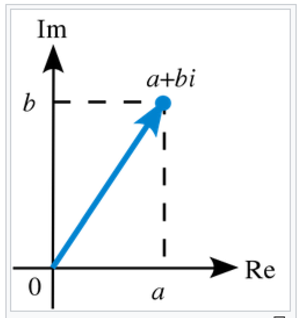
"Imaginary Numbers" redirects here. For the 2013 EP by The Maine, see Imaginary Numbers (EP).

An **imaginary number** is a complex number that can be written as a real number multiplied by the imaginary unit i,[note 1] which is defined by its property $i^2 = -1$.[1][2] The square of an imaginary number bi is $-b^2$. For example, $5i$ is an imaginary number, and its square is -25. By definition, zero is considered to be both real and imaginary.[3] The set of imaginary numbers is sometimes denoted using the blackboard bold letter $\mathbb{I}$.[4]

Complex number

From Wikipedia, the free encyclopedia

A **complex number** is a number that can be expressed in the form $a + bi$, where a and b are real numbers, and i represents the imaginary unit, satisfying the equation $i^2 = -1$. Because no real number satisfies this equation, i is called an imaginary number. For the complex number $a + bi$, a is called the **real part**, and b is called the **imaginary part**. The set of complex numbers is denoted using the symbol $\mathbb{C}$. Despite the historical nomenclature "imaginary", complex numbers are regarded in the mathematical sciences as just as "real" as the real numbers, and are fundamental in many aspects of the scientific description of the natural world.[note 1][1][2][3][4]



A complex number can be visually represented as a pair of numbers (a, b) forming a vector on a diagram called an Argand diagram, representing the complex plane. "Re" is the real axis, "Im" is the imaginary axis, and i satisfies $i^2 = -1$.

Complex numbers allow solutions to certain equations that have no solutions in real numbers. For example, the equation

$$(x+1)^2 = -9$$

has no real solution, since the square of a real number cannot be negative. Complex numbers, however, provide a solution to this problem. The idea is to extend the real numbers with an indeterminate i (sometimes called the imaginary unit) taken to satisfy the relation $i^2 = -1$, so that solutions to equations like the preceding one can be found. In this case, the solutions are $-1 + 3i$ and $-1 - 3i$, as can be verified using the fact that $i^2 = -1$:

$$((-1+3i)+1)^2 = (3i)^2 = (3^2)(i^2) = 9(-1) = -9,$$
$$((-1-3i)+1)^2 = (-3i)^2 = (-3)^2(i^2) = 9(-1) = -9.$$

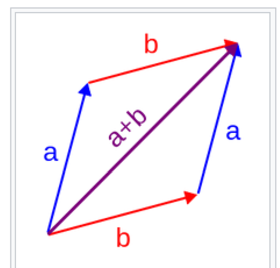
Addition and subtraction [edit]

Two complex numbers a and b are most easily added by separately adding their real and imaginary parts of the summands. That is to say:

$$a + b = (x + yi) + (u + vi) = (x + u) + (y + v)i.$$

Similarly, subtraction can be performed as

$$a - b = (x + yi) - (u + vi) = (x - u) + (y - v)i.$$



Addition of two complex numbers can be done geometrically by constructing a parallelogram.

Using the visualization of complex numbers in the complex plane, the addition has the following geometric interpretation: the sum of two complex numbers a and b, interpreted as points in the complex plane, is the point obtained by building a parallelogram from the three vertices O, and the points of the arrows labeled a and b (provided that they are not on a line). Equivalently, calling these points A, B, respectively and the fourth point of the parallelogram X, the triangles OAB and XBA are congruent. A visualization of the subtraction can be achieved by considering addition of the negative subtrahend.

Multiplication [edit]

Since the real part, the imaginary part, and the indeterminate i in a complex number are all considered as numbers in themselves, two complex numbers, given as $z = x + yi$ and $w = u + vi$ are multiplied under the rules of the distributive property, the commutative properties and the defining property $i^2 = -1$ in the following way

$$\begin{aligned}
z \cdot w &= (x + yi) \cdot (u + vi) \\
&= x(u + vi) + yi(u + vi) && \text{by the (right) distributive law} \\
&= xu + xvi + yiu + yivi && \text{by the (left) distributive law} \\
&= xu + yivi + xvi + yiu && \text{by the commutativity of addition} \\
&= xu + yvi^2 + xvi + yui && \text{by the commutativity of multiplication} \\
&= (xu + yvi^2) + (xvi + yui) && \text{by the associativity of addition} \\
&= (xu - yv) + (xvi + yui) && \text{by the defining property of } i \\
&= (xu - yv) + (xv + yu)i && \text{by the distributive law.}
\end{aligned}$$

Euler's formula [edit]

Euler's formula states that, for any real number x,

$$e^{ix} = \cos x + i \sin x.$$

The functional equation implies thus that, if x and y are real, one has

$$e^{x+iy} = e^x \cos y + ie^x \sin y,$$

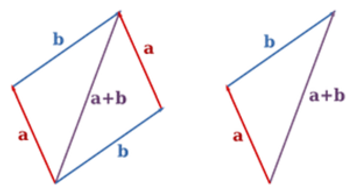
which is the decomposition of the exponential function into its real and imaginary parts.

* Elementary Number
  $0, 1, i, e, \pi$
* Elementary Function

Function Type	Differentiation	Inverse
Power: x^2, x^n	$2x^1, nx^{n-1}$	Root: $\sqrt{x}$

Implementation: Vector2

```
#pragma once
#include <math.h>

class KVector2
{
public:
    static KVector2 zero;
    static KVector2 one;
    static KVector2 right;
    static KVector2 up;

    static KVector2 Lerp(const KVector2& begin, const KVector2& end, double
ratio);

public:
    double  x;
    double  y;

public:
    KVector2(double tx = 0.0, double ty = 0.0) { x = tx; y = ty; }
    KVector2(int tx, int ty) { x = (double)tx; y = (double)ty; }
    double Length() const
    {
        return sqrt(x*x + y*y);
    }
    void Normalize()
    {
        const double length = Length();
        x = x / length;
        y = y / length;
    }
};

inline KVector2 operator+( const KVector2& lhs, const KVector2& rhs )
{
    KVector2 temp( lhs.x + rhs.x, lhs.y + rhs.y );
    return temp;
}

inline KVector2 operator-( const KVector2& lhs, const KVector2& rhs )
{
    KVector2 temp( lhs.x - rhs.x, lhs.y - rhs.y );
    return temp;
}

inline KVector2 operator*(double scalar, const KVector2& rhs)
{
    KVector2 temp(scalar*rhs.x, scalar*rhs.y);
    return temp;
}

inline KVector2 operator*(const KVector2& lhs, double scalar)
{
    KVector2 temp(scalar*lhs.x, scalar*lhs.y);
    return temp;
}
```



$$\mathbf{a} + \mathbf{b} = (a_1 + b_1)\mathbf{e}_1 + (a_2 + b_2)\mathbf{e}_2 + (a_3 + b_3)\mathbf{e}_3$$

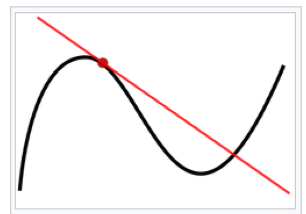
$$\mathbf{a} - \mathbf{b} = (a_1 - b_1)\mathbf{e}_1 + (a_2 - b_2)\mathbf{e}_2 + (a_3 - b_3)\mathbf{e}_3$$

Derivative

From Wikipedia, the free encyclopedia
(Redirected from Differentiation (mathematics))

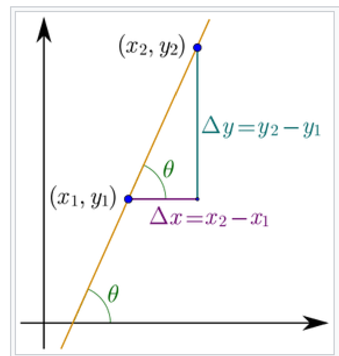
This article is about the term as used in calculus. For a less technical overview of the subject, see differential calculus. For other uses, see Derivative (disambiguation).

The **derivative** of a function of a real variable measures the sensitivity to change of the function value (output value) with respect to a change in its argument (input value). Derivatives are a fundamental tool of calculus. For example, the derivative of the position of a moving object with respect to time is the object's velocity: this measures how quickly the position of the object changes when time advances.



The graph of a function, drawn in black, and a tangent line to that function, drawn in red. The slope of the tangent line is equal to the derivative of the function at the marked point.

The derivative of a function of a single variable at a chosen input value, when it exists, is the slope of the tangent line to the graph of the function at that point. The tangent line is the best linear approximation of the function near that input value. For this reason, the derivative is often described as the "instantaneous rate of



Slope of a linear function: $m = \dfrac{\Delta y}{\Delta x}$

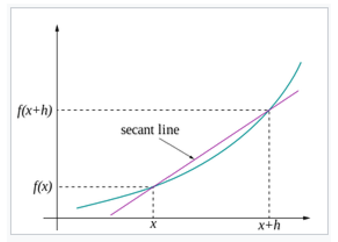


Figure 2. The secant to curve $y = f(x)$ determined by points $(x, f(x))$ and $(x + h, f(x + h))$

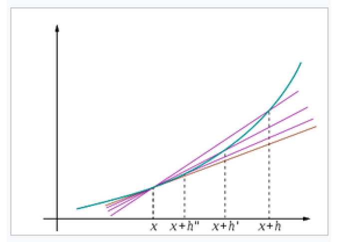


Figure 3. The tangent line as limit of secants

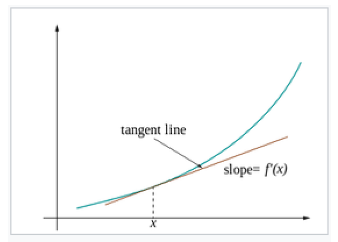
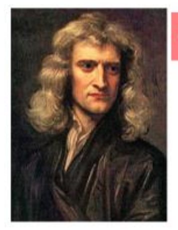


Figure 1. The tangent line at $(x, f(x))$

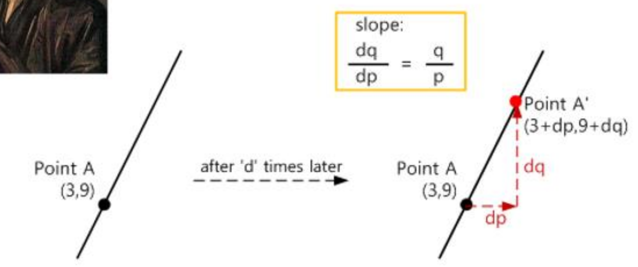
$$m = \frac{\Delta f(a)}{\Delta a} = \frac{f(a+h) - f(a)}{(a+h) - (a)} = \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

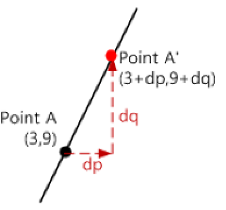
```
double NewtonsDifference(FUNCTION f, double a, double h = 0.0001)
{
   const double y0 = f(a);
   const double y1 = f(a + h);
   return (y1 - y0) / h;
}
```



Get the slope of (3,9) of y=x²



$$y = x^2$$
$$(9 + dq) = (3 + dp)^2$$
$$\cancel{9} + dq = \cancel{9} + 6dp + d^2p^2$$
$$\cancel{d}q = 6\cancel{d}p + \cancel{d}dp^2$$
$$q = 6p + dp^2$$

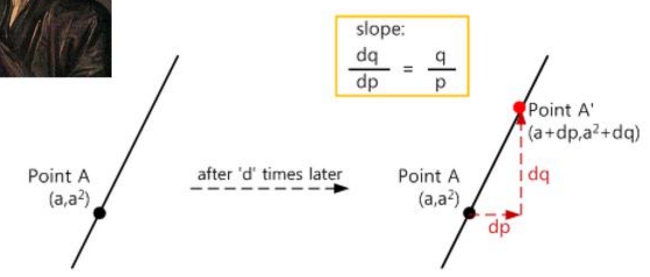


$$q/p = 6 + dp$$

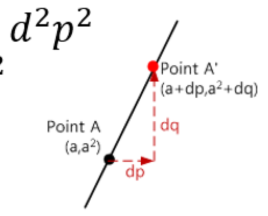
(dp is infinitely small, so)

$$q/p = 6$$

Get the slope of (a,a²) of y=x²



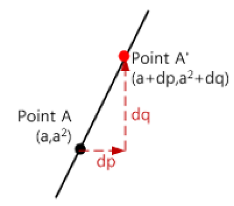
$$y = x^2$$
$$(a^2 + dq) = (a + dp)^2$$
$$\cancel{a^2} + dq = \cancel{a^2} + 2adp + d^2p^2$$
$$\cancel{d}q = 2adp + \cancel{d}dp^2$$
$$q = 2ap + dp^2$$



$$q/p = 2a + dp$$

(dp is infinitely small, so)

$$q/p = 2a$$



$$dy/dx = F'(x^2) = 2x$$

Rules for basic functions

Derivatives of powers:

if

$$f(x) = x^r,$$

where r is any real number, then

$$f'(x) = rx^{r-1},$$

wherever this function is defined. For example, if

$$f(x) = x^{1/4}$$

then

$$f'(x) = (1/4)x^{-3/4},$$

$$f'(x^n) = nx^{n-1}$$

* Elementary Number
  $0, 1, i, e, \pi$
* Elementary Function

Function Type	Differentiation	Inverse
Power: x^2, x^n	$2x^1, nx^{n-1}$	Root: $\sqrt{x}$

Implementation: Numerical Differentiation

```
double NewtonsDifference(FUNCTION f, double a, double h = 0.0001)
{
    const double y0 = f(a);
    const double y1 = f(a + h);
    return (y1 - y0) / h;
}
```

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Implementation: Draw Tangent Line