$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$f(\theta)=e^{-i\theta}(\cos\theta+i\sin\theta)$$
$$f(\theta)=g(\theta)h(\theta)$$
$$g(\theta)=e^{-i\theta}$$
$$h(\theta)=(\cos\theta+i\sin\theta)$$
$$f'(\theta)=g'(\theta)h(\theta)+g(\theta)h'(\theta),\ \text{by product rule}$$

$$g'(\theta)=-ie^{-i\theta}$$
$$h'(\theta)=(-\sin\theta+i\cos\theta)$$
$$f'(\theta)=-ie^{-i\theta}(\cos\theta+i\sin\theta)+e^{-i\theta}(-\sin\theta+i\cos\theta)$$
$$f'(\theta)=e^{-i\theta}(-i\cos\theta-i^2\sin\theta-\sin\theta+i\cos\theta)$$
$$f'(\theta)=e^{-i\theta}(-i\cos\theta+\sin\theta-\sin\theta+i\cos\theta)$$
$$f'(\theta)=e^{-i\theta}(0)=0$$
$$f(\theta)=k,\ \forall\theta$$

$$e^{-i\theta}(\cos\theta+i\sin\theta)=k,\ \forall\theta$$
if $\theta=0$, then
$$e^{-i(0)}(\cos(0)+i\sin(0))=e^0(1+0)=1=k$$
$$e^{-i\theta}(\cos\theta+i\sin\theta)=1$$
$$\cos\theta+i\sin\theta=e^{i\theta}$$