

Inverse Fourier Transform

$$f(t) = \sum_{v=-\infty}^{\infty} F(v) e^{2\pi i v t}$$

$$f(t) = \int_{v=-\infty}^{\infty} F(v) e^{2\pi i v t} dv$$

Forward Fourier Transform

$$F(v) = \sum_{t=-\infty}^{\infty} f(t) e^{-2\pi i v t}$$

$$F(v) = \int_{t=-\infty}^{\infty} f(t) e^{-2\pi i v t} dt$$

Hint

$$\frac{f(t)}{e^{2\pi i v t}}$$

Discrete Fourier Transform

$$F(k) = \sum_{n=0}^{N-1} f(n) e^{-2\pi i k \frac{n}{N}}$$

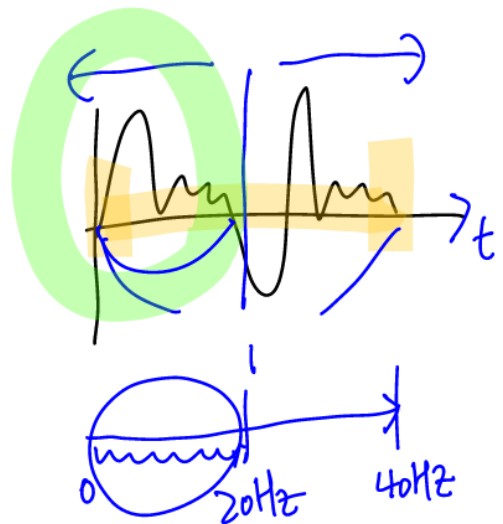
Amplitude

$$\frac{|F(k)|}{N} = \frac{\sqrt{\text{Re}(F(k))^2 + \text{Im}(F(k))^2}}{N}$$

Inverse Transform

$$f(n) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{2\pi i k \frac{n}{N}}$$

conjugate



Implementation

```
using ComplexArray = std::valarray<std::complex<double> >;
```

Preparing Signal

```
std::complex<double> test[BIN_SIZE];  
// fill test signal data  
double x = 0;  
double y;  
for(int i = 0; i < BIN_SIZE; ++i)  
{  
    test[i] = std::complex<double>( SignalFunction( x )  
);  
    x += SAMPLING_STEP;  
}
```

Forward Fourier Transform

```
void dft(ComplexArray& x)  
{  
    const size_t N = x.size();  
  
    ComplexArray xresult(N);  
  
    for (size_t k = 0; k < N; ++k)  
    {  
        xresult[k] = 0;  
        for (size_t n = 0; n < N; ++n)  
        {
```

```

        std::complex<double> e = std::polar(1.0, -2 * M_PI
* k * (double)n / (double)N);
        xresult[k] += x[n] * e;
    }
}
x = xresult;
}

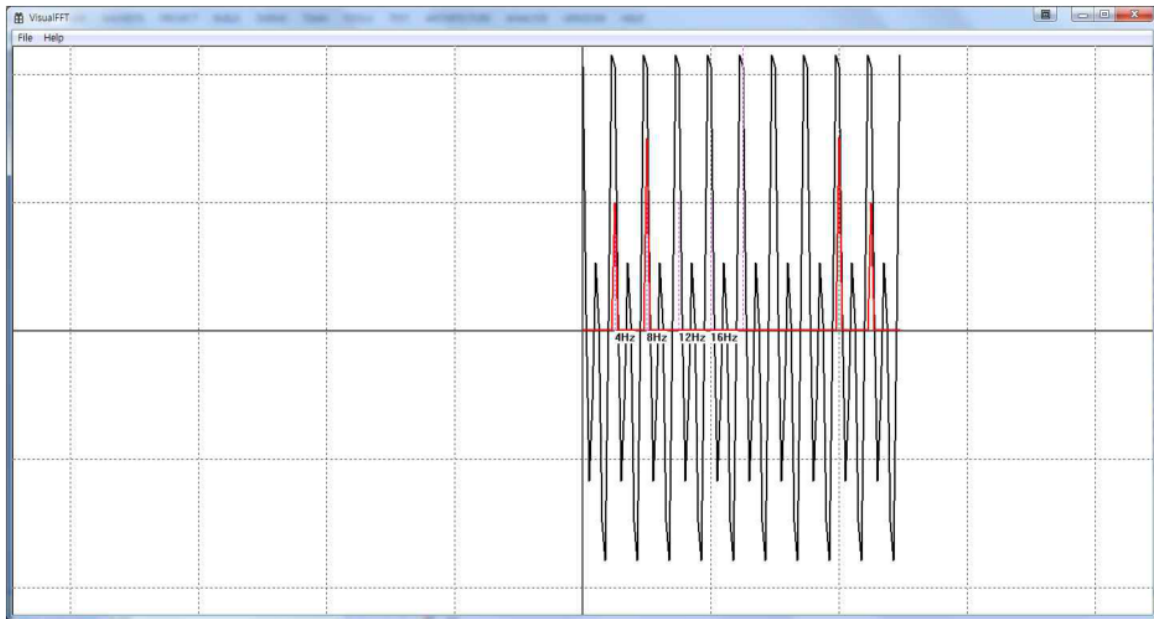
```

Interpretation of the result

```

// display Hz for every 10 steps
if(i % 10 == 0)
{
    KVectorUtil::DrawLine( g_hdc, KVector2(
(double)x, (double)1 ), KVector2( (double)x, (double)0 ), 1,
PS_DOT, RGB( 255, 0, 255 ) );
    KVector2 screenPos =
KVectorUtil::GetScreenPoint( KVector2( x, 0.0 ) );
    char buffer[80];
    double ratio = (double)i / double(BIN_SIZE);
    sprintf_s( buffer, "%gHz", ratio /
SAMPLING_STEP );
    ::TextOutA( hdc, (int)screenPos.x,
(int)screenPos.y, buffer, strlen( buffer ) );
}

```



Fast Fourier Transform

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