Number

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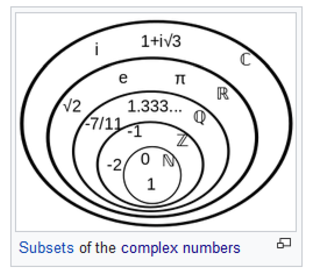
For other uses, see Number (disambiguation).

A **number** is a mathematical object used to count, measure, and label. The original examples are the natural numbers 1, 2, 3, 4, and so forth.[1] Numbers can be represented in language with number words. More universally, individual numbers can be represented by symbols, called *numerals*; for example, "5" is a numeral that represents the number five. As only a relatively small number of symbols can be memorized, basic numerals are commonly organized in a numeral system, which is an organized way to represent any number. The most common numeral system is the Hindu–Arabic numeral system, which allows for the representation



Subsets of the complex numbers

Main classification [edit]

"Number system" redirects here. For systems for expressing numbers, see Numeral system.

See also: List of types of numbers

Numbers can be classified into sets, called **number systems**, such as the natural numbers and the real numbers.[31] The major categories of numbers are as follows:

Main number systems

$\mathbb{N}$	**Natural**	0, 1, 2, 3, 4, 5, ... or 1, 2, 3, 4, 5, ... $\mathbb{N}_0$ or $\mathbb{N}_1$ are sometimes used.
$\mathbb{Z}$	**Integer**	..., −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, ...
$\mathbb{Q}$	**Rational**	$\frac{a}{b}$ where *a* and *b* are integers and *b* is not 0
$\mathbb{R}$	**Real**	The limit of a convergent sequence of rational numbers
$\mathbb{C}$	**Complex**	*a* + *bi* where *a* and *b* are real numbers and *i* is a formal square root of −1

There is generally no problem in identifying each number system with a proper subset of the next one (by abuse of notation), because each of these number systems is canonically isomorphic to a proper subset of the next one.[citation needed] The resulting hierarchy allows, for example, to talk, formally correctly, about real numbers that are rational numbers, and is expressed symbolically by writing

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}.$$

Quaternion

From Wikipedia, the free encyclopedia

This article is about quaternions in mathematics. For other uses, see Quaternion (disambiguation).

In mathematics, the **quaternions** are a number system that extends the complex numbers. They were first described by Irish mathematician William Rowan Hamilton in 1843[1][2] and applied to mechanics in three-dimensional space. A feature of quaternions is that multiplication of two quaternions is noncommutative. Hamilton defined a quaternion as the quotient of two directed lines in a three-dimensional space[3] or equivalently as the quotient of two vectors.[4]

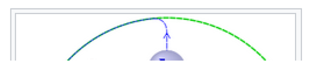
Quaternion multiplication

↓ × →	$\mathbf{1}$	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$
$\mathbf{i}$	$\mathbf{i}$	-1	$\mathbf{k}$	$-\mathbf{j}$
$\mathbf{j}$	$\mathbf{j}$	$-\mathbf{k}$	-1	$\mathbf{i}$
$\mathbf{k}$	$\mathbf{k}$	$\mathbf{j}$	$-\mathbf{i}$	-1

Quaternions are generally represented in the form:

$$a + b\,\mathbf{i} + c\,\mathbf{j} + d\,\mathbf{k}$$

where a, b, c, and d are real numbers, and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the fundamental *quaternion units*.



Elementary function

From Wikipedia, the free encyclopedia

For the complexity class, see ELEMENTARY. For the logical system, see Elementary function arithmetic.

In mathematics, an **elementary function** is a function of a single variable composed of particular simple functions.

Elementary functions are typically defined as a sum, product, and/or composition of finitely many polynomials, rational functions, trigonometric and exponential functions, and their inverse functions (including arcsin, log, $x^{1/n}$).[1]

Elementary functions were introduced by Joseph Liouville in a series of papers from 1833 to 1841.[2][3][4] An algebraic treatment of elementary functions was started by Joseph Fels Ritt in the 1930s.[5]

Basic examples [edit]

The elementary functions (of x) include:

- Constant functions: $2,\ \pi,\ e$, etc.
- Powers of x: $x,\ x^2,\ x^3$, etc.
- Roots of x: $\sqrt{x},\ \sqrt[3]{x}$, etc.
- Exponential functions: e^x
- Logarithms: $\log x$
- Trigonometric functions: $\sin x,\ \cos x,\ \tan x$, etc.
- Inverse trigonometric functions: $\arcsin x,\ \arccos x$, etc.
- Hyperbolic functions: $\sinh x,\ \cosh x$, etc.
- Inverse hyperbolic functions: $\operatorname{arsinh} x,\ \operatorname{arcosh} x$, etc.
- All functions obtained by adding, subtracting, multiplying or dividing any of the previous functions[6]
- All functions obtained by composing previously listed functions

Some elementary functions, such as roots, logarithms, or inverse trigonometric functions, are not entire functions and may be multivalued.

Composite examples [edit]

Examples of elementary functions include:

- Addition, e.g. (*x*+1)
- Multiplication, e.g. (2*x*)
- Polynomial functions
- $\frac{e^{\tan x}}{1+x^2}\sin\left(\sqrt{1+(\ln x)^2}\right)$
- $-i\ln(x+i\sqrt{1-x^2})$

The last function is equal to $\arccos x$, the inverse cosine, in the entire complex plane.

All monomials, polynomials and rational functions are elementary. Also, the absolute value function, for real x, is also elementary as it can be expressed as the composition of a power and root of x: $|x| = \sqrt{x^2}$.

Powers of x

$$x \times x = x^2$$
$$x \times x \times x = x^3$$

$$x^2 \times x^3 = x^5$$
$$(x^2)^3 = x^2 \times x^2 \times x^2 = x^6$$

Inverse of Powers of x

$${}^2x$$
$${}^3x$$

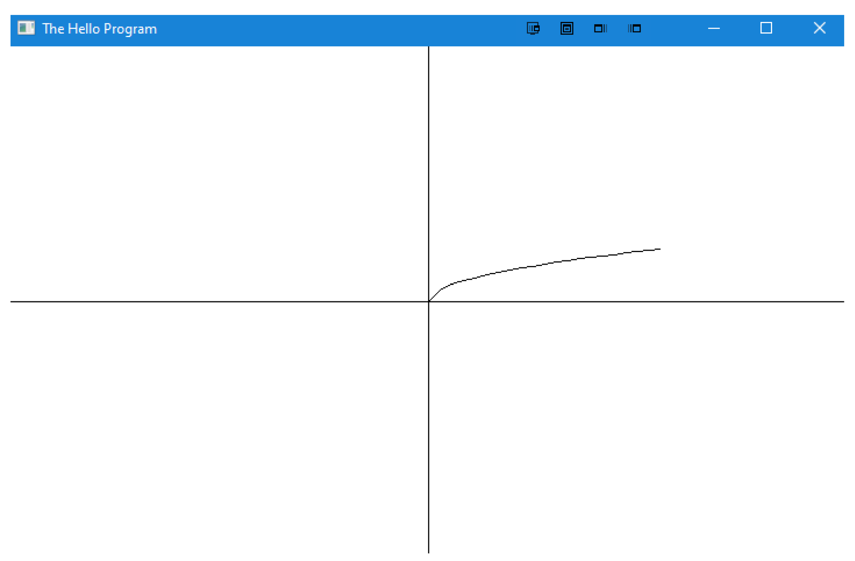
$${}^24=2$$
$${}^38=2, \quad \leftarrow \quad 2\times2\times2=2^3=8$$
$${}^416=2$$
$${}^29=3$$

$$\sqrt[2]{4}=2$$
$$\sqrt[3]{8}=2$$
$$\sqrt[4]{16}=2$$
$$\sqrt{9}=3$$

$$(\sqrt[2]{x})^2 = x$$
$$(x^n)^2 = x^{2n} = x^1, \quad n \equiv \frac{1}{2}$$
$$\sqrt[2]{x} = x^{1/2} = \sqrt{x}$$

$$\frac{1}{x} \times x = 1$$
$$x^n \times x^1 = 1$$
$$x^{n+1} = 1, \quad n \equiv -1$$
$$\frac{1}{x} = x^{-1}$$
$$\frac{1}{x^n} = x^{-n}$$

Practice: Draw a function in Windows

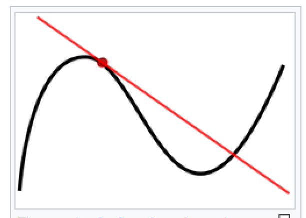


Derivative

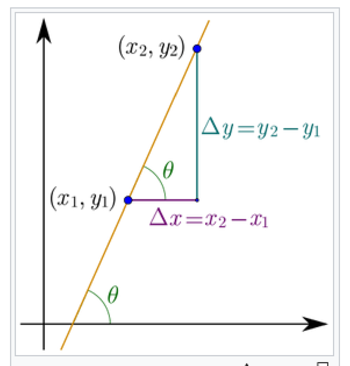
From Wikipedia, the free encyclopedia
(Redirected from Differentiation (mathematics))

This article is about the term as used in calculus. For a less technical overview of the subject, see differential calculus. For other uses, see Derivative (disambiguation).

The **derivative** of a function of a real variable measures the sensitivity to change of the function value (output value) with respect to a change in its argument (input value). Derivatives are a fundamental tool of calculus. For example, the derivative of the position of a moving object with respect to time is the object's velocity: this measures how quickly the position of the object changes when time advances.


The graph of a function, drawn in black, and a tangent line to that function, drawn in red. The slope of the tangent line is equal to the derivative of the function at the marked point.

The derivative of a function of a single variable at a chosen input value, when it exists, is the slope of the tangent line to the graph of the function at that point. The tangent line is the best linear approximation of the function near that input value. For this reason, the derivative is often described as the "instantaneous rate of



Slope of a linear function: $m = \frac{\Delta y}{\Delta x}$

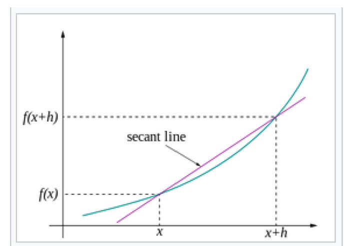


Figure 2. The secant to curve $y = f(x)$ determined by points $(x, f(x))$ and $(x + h, f(x + h))$

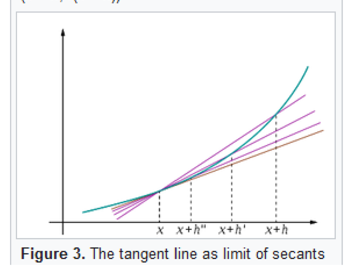


Figure 3. The tangent line as limit of secants

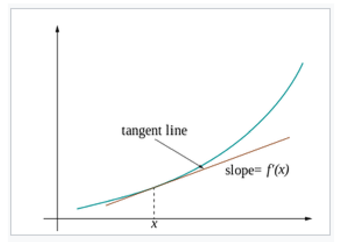


Figure 1. The tangent line at $(x, f(x))$

$$m = \frac{\Delta f(a)}{\Delta a} = \frac{f(a+h)-f(a)}{(a+h)-(a)} = \frac{f(a+h)-f(a)}{h}.$$

This expression is Newton's difference quotient. Passing from an approximation to an exact answer is done using a limit. Geometrically, the limit of the secant lines is the tangent line. Therefore, the limit of the difference quotient as h approaches zero, if it exists, should represent the slope of the tangent line to $(a, f(a))$. This limit is defined to be the derivative of the function f at a:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}.$$

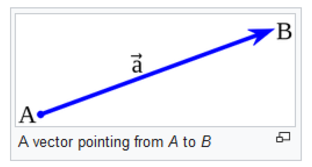
```
double NewtonsDifference(FUNCTION f, double base_, double x, double dx = 0.0001)
{
    const double y0 = f(x);
    const double y1 = f(x + dx);
    return (y1 - y0) / dx;
}
```

Euclidean vector

From Wikipedia, the free encyclopedia

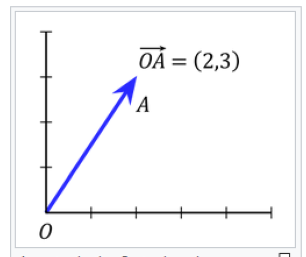
This article is about the vectors mainly used in physics and engineering to represent directed quantities. For mathematical vectors in general, see Vector (mathematics and physics). For other uses, see Vector (disambiguation).

In mathematics, physics and engineering, a **Euclidean vector** (sometimes called a **geometric**[1] or **spatial vector**,[2] or—as in here—simply a **vector**) is a geometric object that has magnitude (or length) and direction. Vectors can be added to other vectors according to vector algebra. A Euclidean vector is frequently represented by a ray (a line segment with a definite direction), or graphically as an arrow connecting an *initial point A* with a *terminal point B*,[3] and denoted by $\overrightarrow{AB}$.[4]



A vector pointing from *A* to *B*

In order to calculate with vectors, the graphical representation may be too cumbersome. Vectors in an *n*-dimensional Euclidean space can be represented as coordinate vectors in a Cartesian coordinate system. The endpoint of a vector can be identified with an ordered list of *n* real numbers (*n*-tuple). These numbers are the coordinates of the endpoint of the vector, with respect to a given Cartesian coordinate system, and are typically called the **scalar components** (or **scalar projections**) of the vector on the axes of the coordinate system.



A vector in the Cartesian plane, showing the position of a point *A* with coordinates (2, 3).

As an example in two dimensions (see figure), the vector from the origin *O* = (0, 0) to the point *A* = (2, 3) is simply written as

$$\mathbf{a} = (2,3).$$

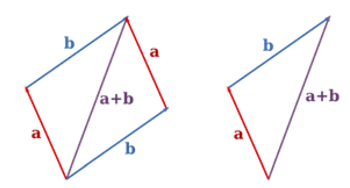
Addition and subtraction [edit]

Further information: Vector space

Assume now that **a** and **b** are not necessarily equal vectors, but that they may have different magnitudes and directions. The sum of **a** and **b** is

$$\mathbf{a} + \mathbf{b} = (a_1 + b_1)\mathbf{e}_1 + (a_2 + b_2)\mathbf{e}_2 + (a_3 + b_3)\mathbf{e}_3.$$

The addition may be represented graphically by placing the tail of the arrow **b** at the head of the arrow **a**, and then drawing an arrow from the tail of **a** to the head of **b**. The new arrow drawn represents the vector **a** + **b**, as illustrated below:[8]

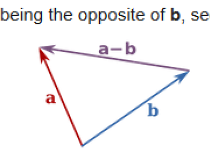


This addition method is sometimes called the *parallelogram rule* because **a** and **b** form the sides of a parallelogram and **a** + **b** is one of the diagonals. If **a** and **b** are bound vectors that have the same base point, this point will also be the base point of **a** + **b**. One can check geometrically that **a** + **b** = **b** + **a** and (**a** + **b**) + **c** = **a** + (**b** + **c**).

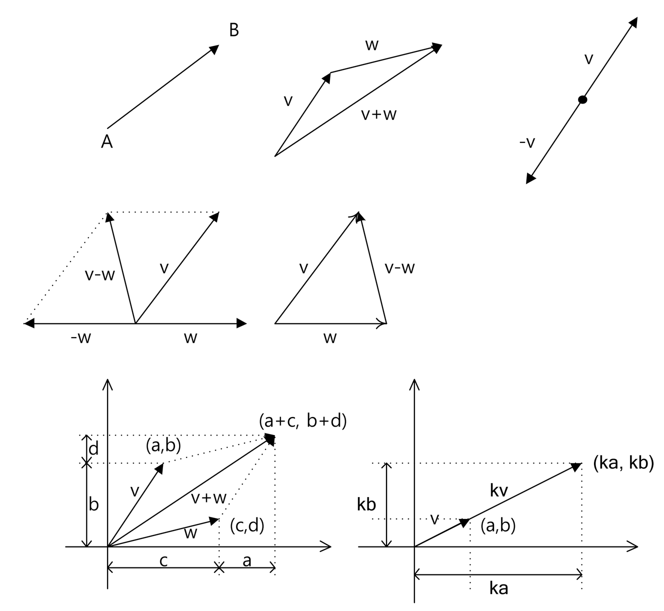
The difference of **a** and **b** is

$$\mathbf{a} - \mathbf{b} = (a_1 - b_1)\mathbf{e}_1 + (a_2 - b_2)\mathbf{e}_2 + (a_3 - b_3)\mathbf{e}_3.$$

Subtraction of two vectors can be geometrically illustrated as follows: to subtract **b** from **a**, place the tails of **a** and **b** at the same point, and then draw an arrow from the head of **b** to the head of **a**. This new arrow represents the vector **(-b)** + **a**, with **(-b)** being the opposite of **b**, see drawing. And **(-b)** + **a** = **a** − **b**.



Practice: Define a Vector2



Practice: Draw a tangent line of a function at x==2 for x^2

```
double NewtonsDifference(FUNCTION f, double base_, double x, double dx = 0.0001)
{
   const double y0 = f(x);
   const double y1 = f(x + dx);
   return (y1 - y0) / dx;
}
```