고등학교 수학 정리
Highschool Mathematics

[2019.10.9, Wed]

@ elementary number : $\emptyset, 1[, i, e, \pi]$ (Step1)

number as a vector

@ Elementary Functions

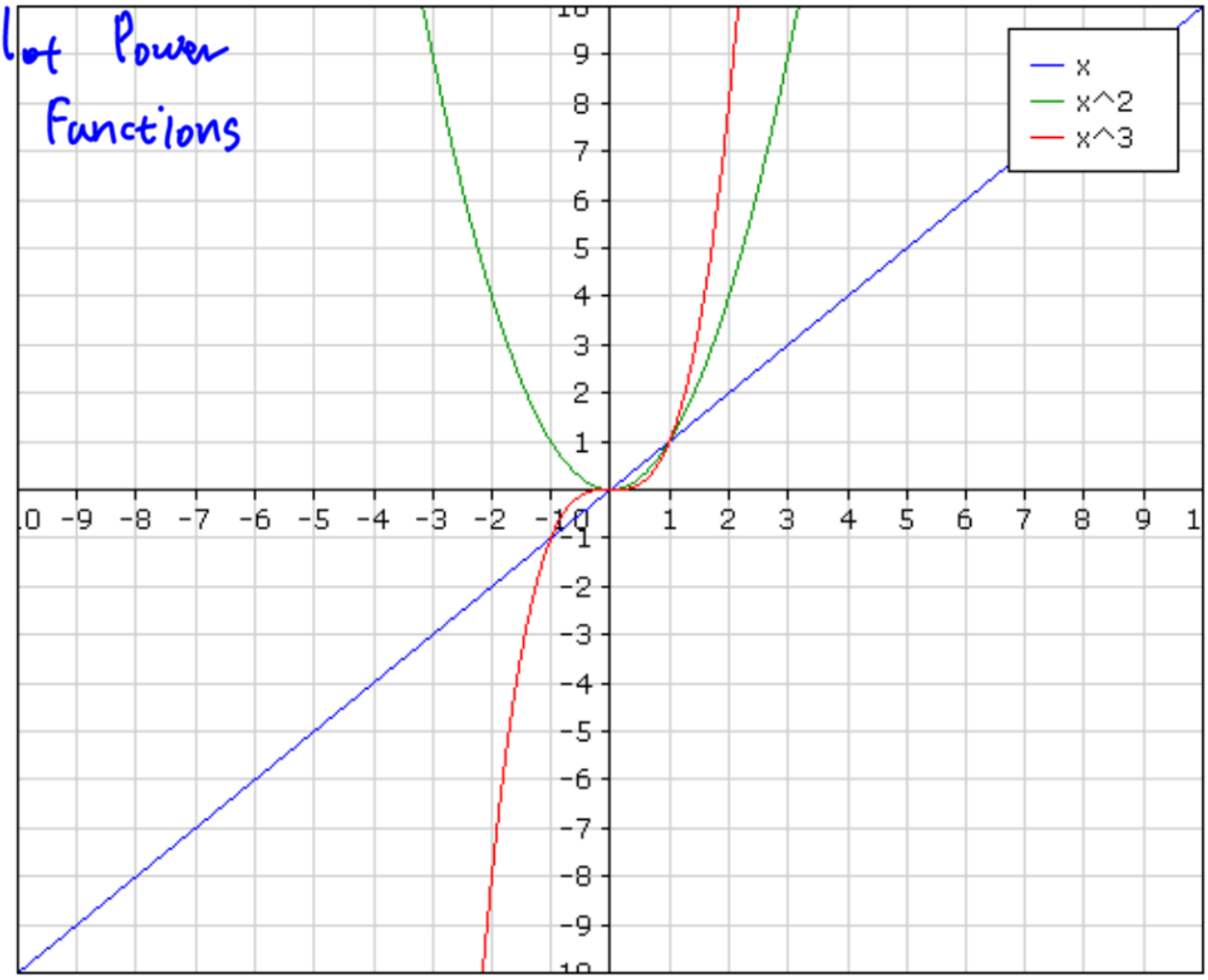
x^2, x^3	${}^2x \quad {}^3x$
$2^x, 3^x$	${}_2x \quad {}_3x$
$\rightarrow x \quad \uparrow x$	$\rightarrow^{-1} x \quad \uparrow^{-1} x$

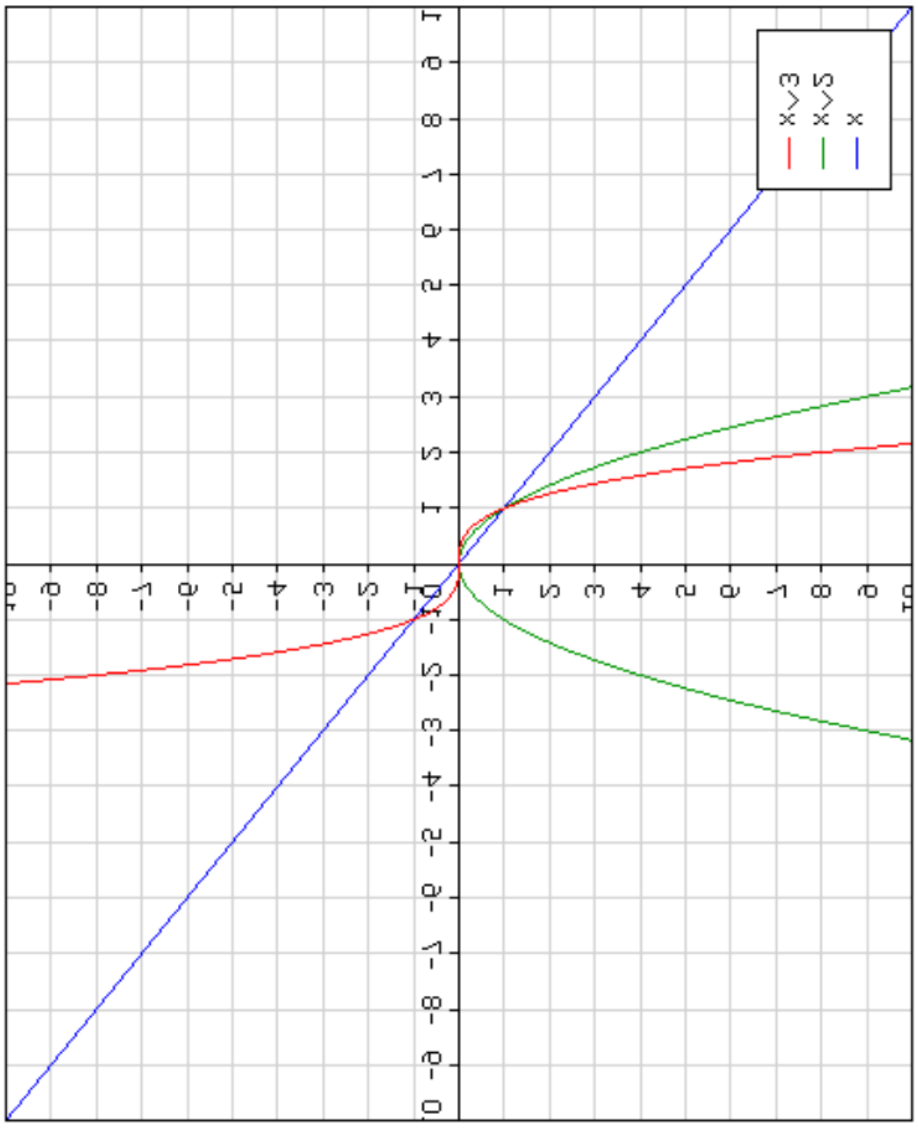
@ Power Functions

Plotting a function

Function as a graph

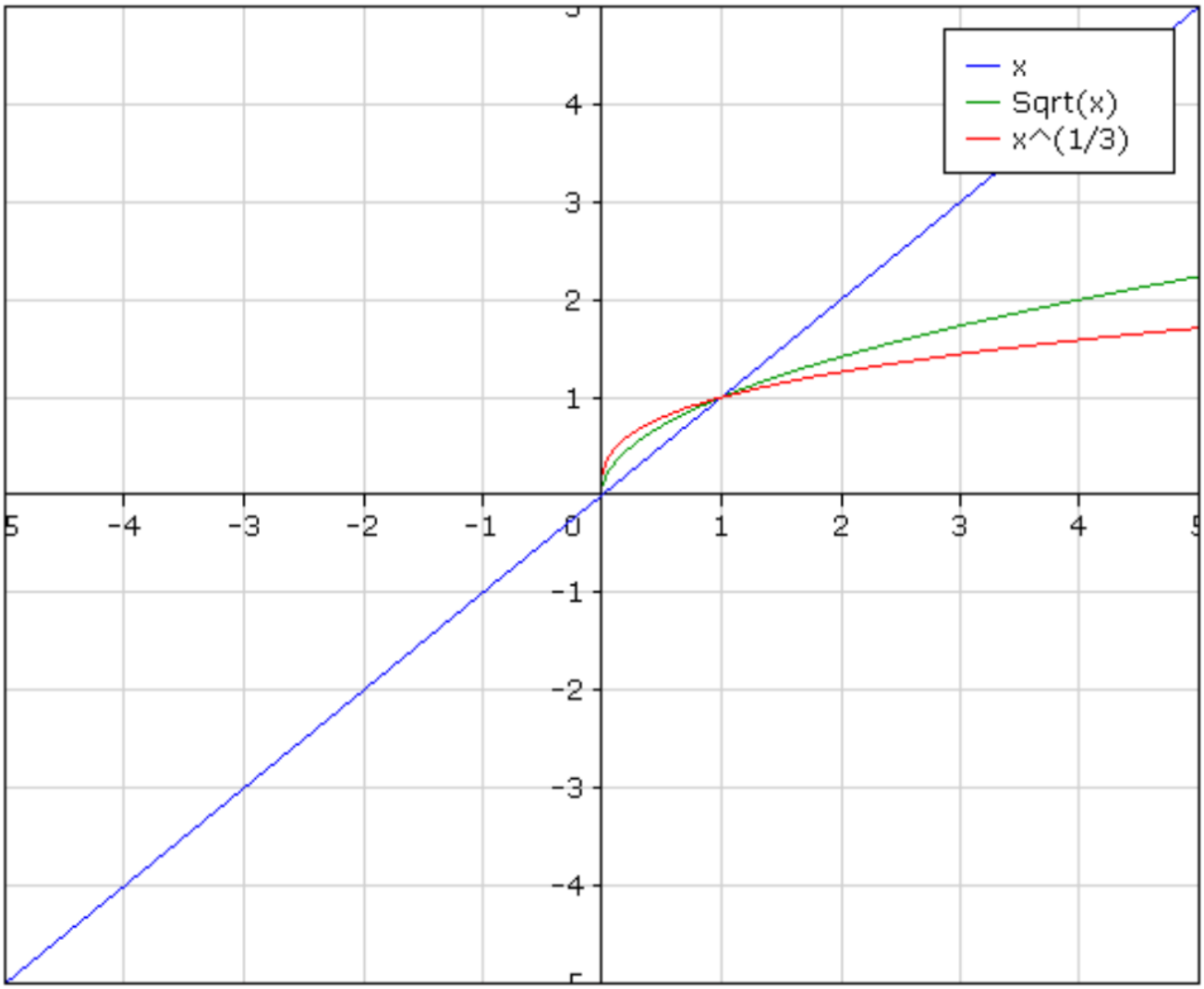
Plot Power Functions





Rotate CCW and Horizontal Mirroring

@ Inverse of a Power function



$\Rightarrow \sqrt[2]{x}, \sqrt[3]{x}, \cdots$ ⇒ irrational numbers.

ex) $\sqrt{2}, \sqrt{3}$

Q. No plotting on 2nd and 3rd Quadrants. why?

@ Elementary Number

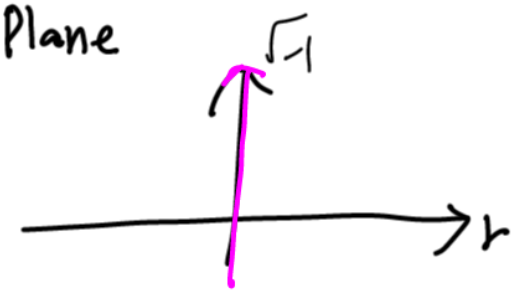
number as a vector

↳ 1dimensional vector

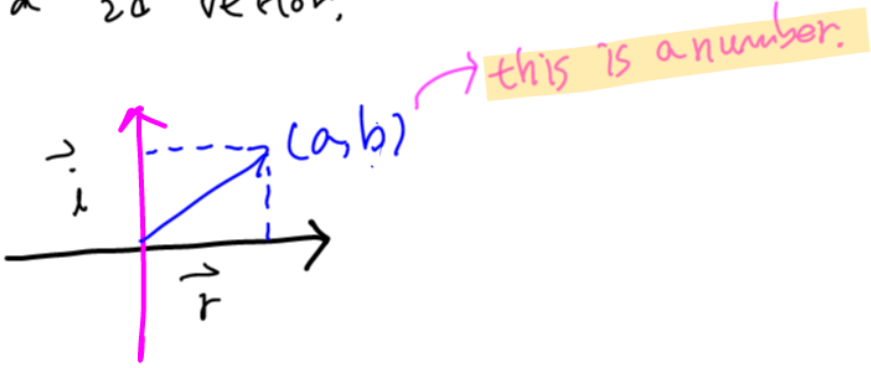
lateral number (positive number
negative number
lateral number

$\sqrt{-1}$

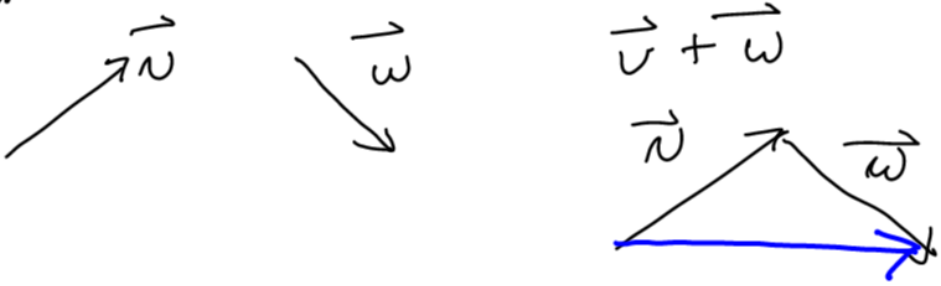
Complex Plane

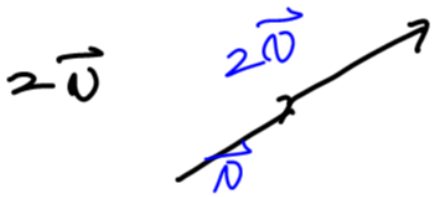


number as a 2d vector.



@ Vector





Complex Number

$$a\vec{1} + b\vec{i} = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}\sqrt{-1}$$

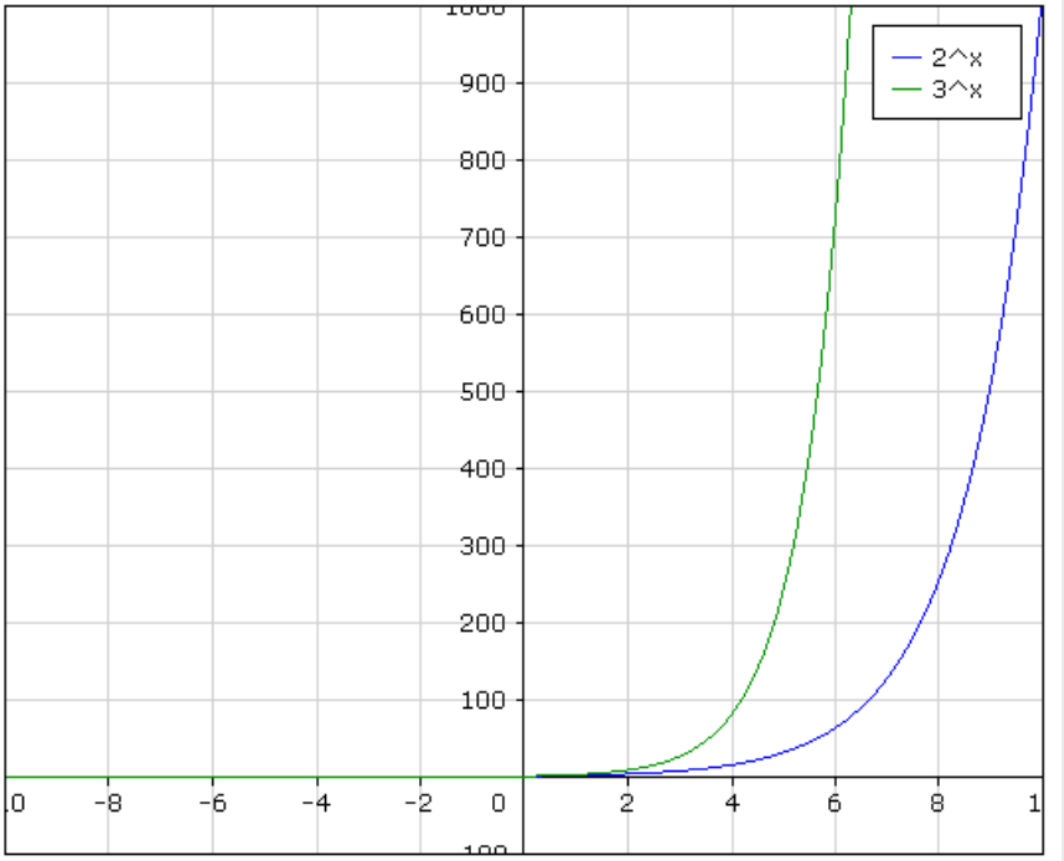
$$= a + b\sqrt{-1}$$

$$= a + ib$$

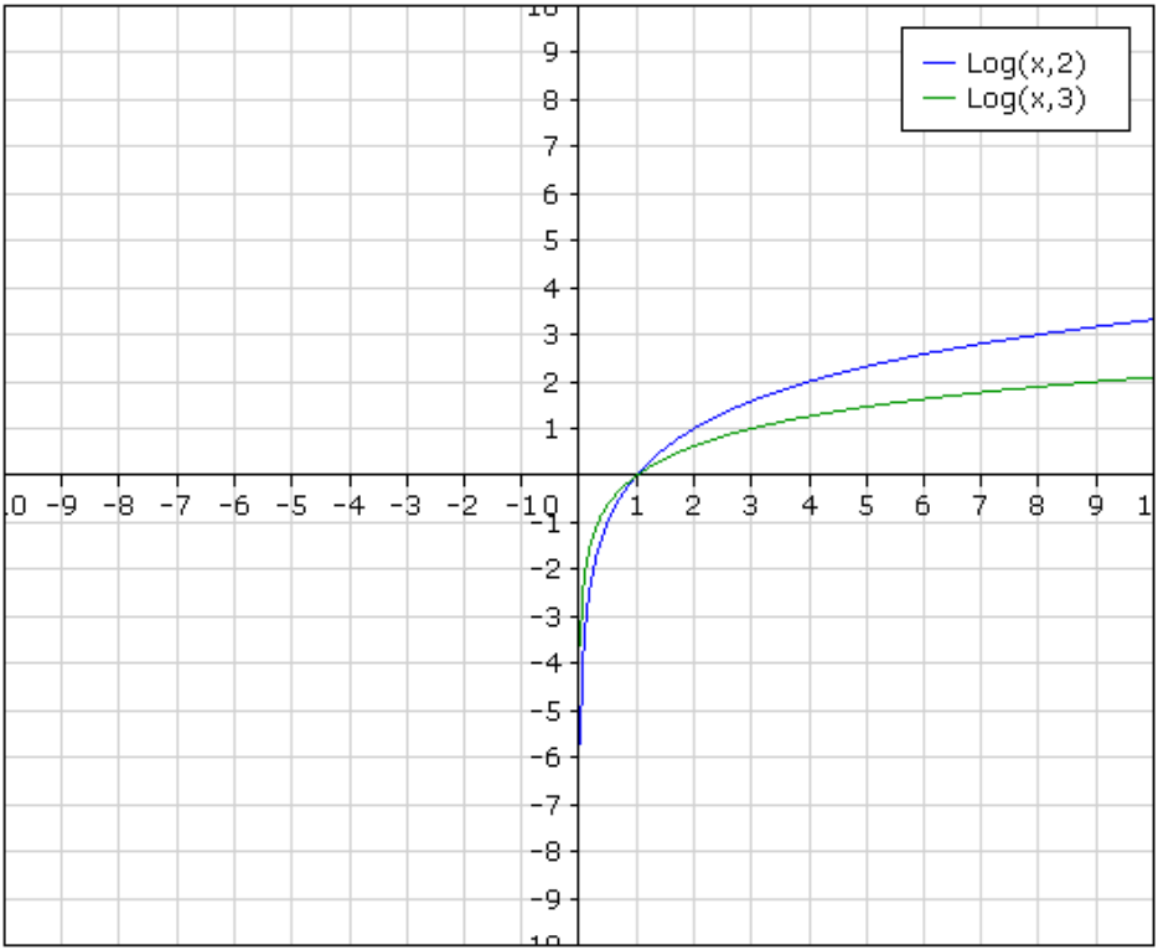
@ Elementary Number (Step 2)

$\phi, 1, i$

Exponential Functions



Log Functions (inverse of a exponential functions)



$\Rightarrow \log_2 x,\ \log_3 x$

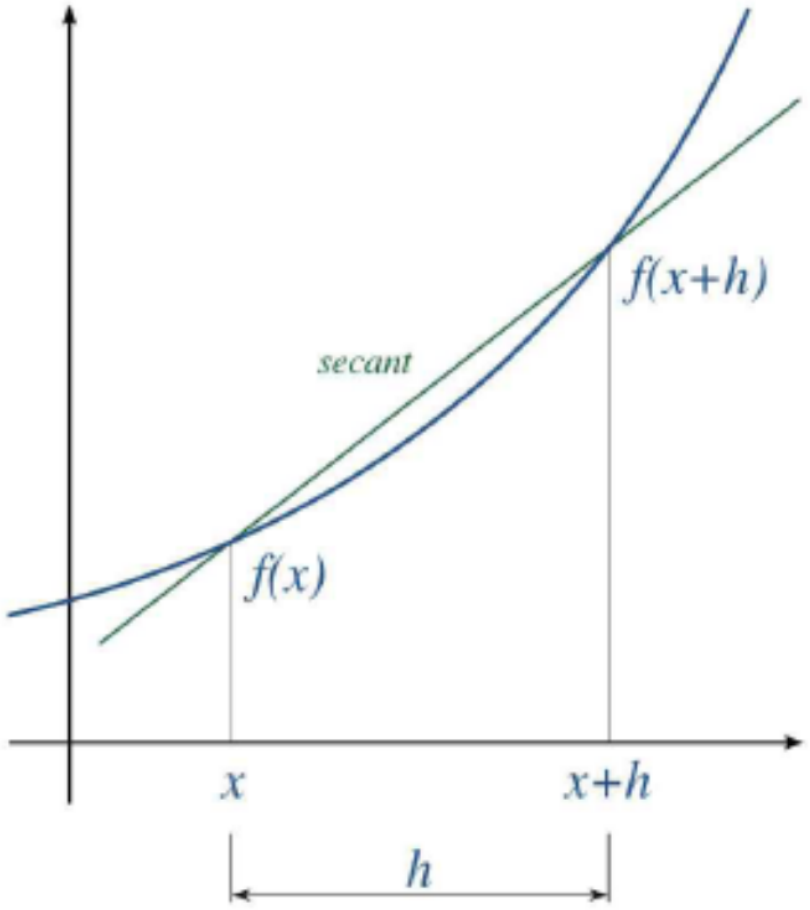
@Intermediate Summarize

Elementary Number : $\phi, 1, i$

Elementary Function

power	x^2, x^3	$\sqrt{x}, \sqrt[3]{x}$	root
exp	$2^x, 3^x$	$\log_2 x, \log_3 x$	log

@ Differentiation



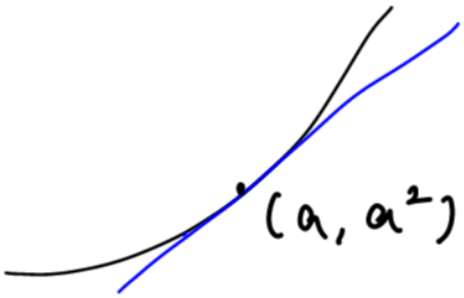
Newton's difference quotient(== first-order divided difference)

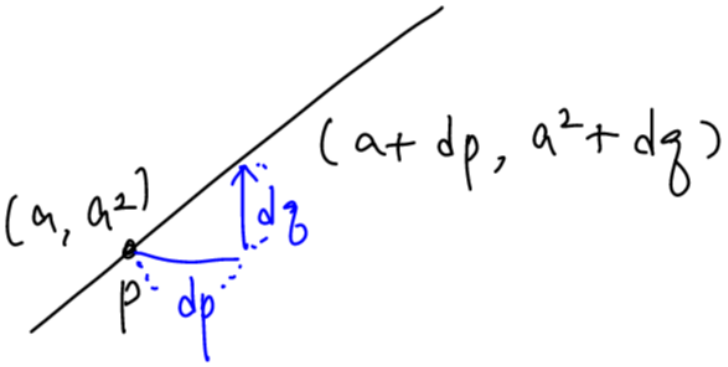
$$\frac{f(x+h)-f(x)}{h}$$

derivative of f at x

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

ex) Getting Slope of x^2





$$\text{slope} = \frac{dq}{dp} = \frac{q}{p}$$

for $y = x^2$

$$(a^2 + q) = (a+p)^2$$

$$= a^2 + 2ap + p^2$$

$$\frac{q}{p} = \frac{2ap + p^2}{p} \Rightarrow \frac{q}{p} = 2a + p$$

$\therefore \frac{q}{p} = 2a$

$\rightarrow$ slope $(x^2) = 2x \Rightarrow (x^2)' = 2x$

@ Differentiation for Elementary Functions.

$x^2 \to 2x$ $x^3 \to 3x^2$ $x^n \to nx^{n-1}$	$\sqrt{x} \to \frac{1}{2\sqrt{x}}$
$a^x \to b \cdot a^x$	

Is there a where b and c equals to 1?

$$(\square^x)' = \square^x$$

Differentiation of exponential function

$$\frac{d}{dx}a^x = \lim_{h\to 0}\frac{a^{x+h}-a^x}{h}$$

$$= \lim_{h\to 0}\frac{a^x a^h - a^x}{h}$$

$$= a^x\left(\lim_{h\to 0}\frac{a^h-1}{h}\right)$$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

$a^x = e^{cx}$

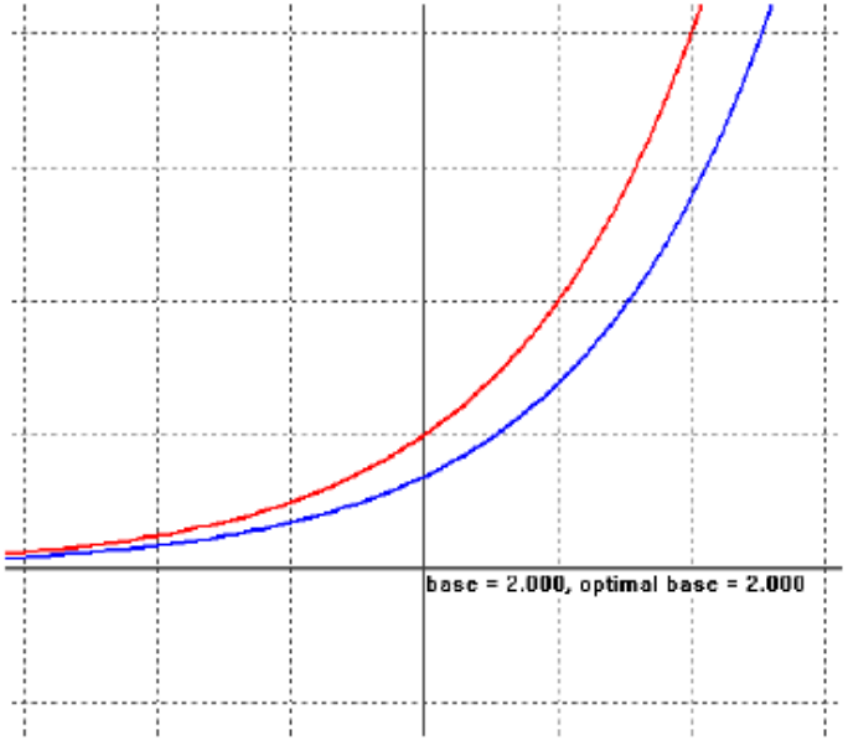
$2^x = e^{x\ln(2)}$

$3^x = e^{x\ln(3)}$

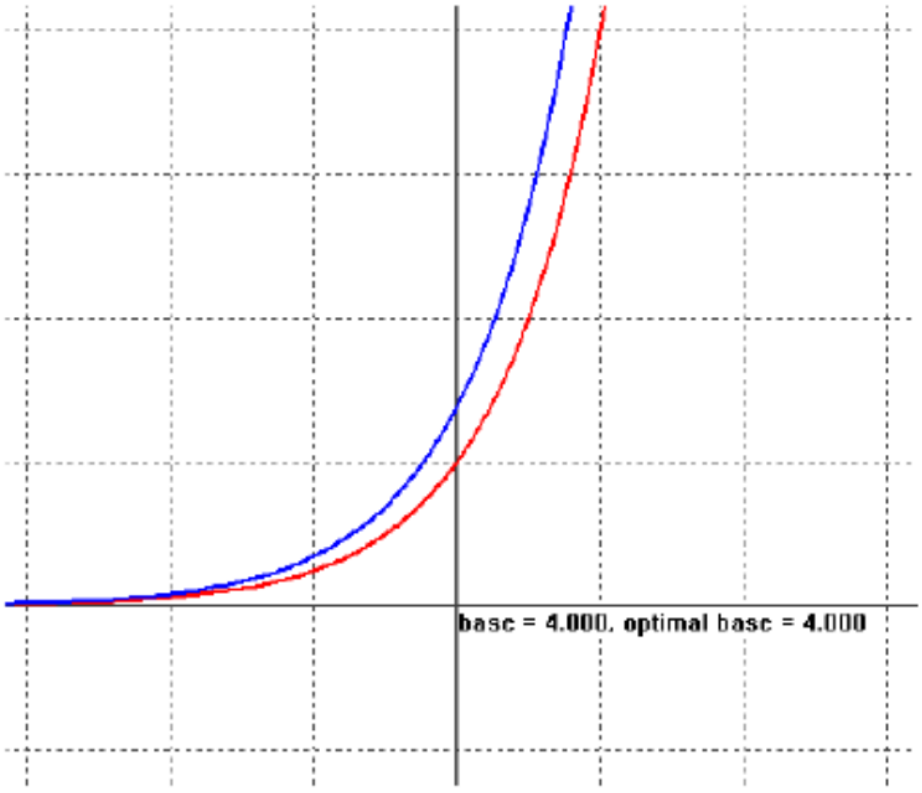
$$\left(e^{cx}\right)' = ce^{cx}$$

@ Natural Logarithm $\quad \log_e(x) \Rightarrow \ln(x)$

Demo)



[Fig] $2^x, \frac{d}{dx}2^x$



[Fig] $4^x, \frac{d}{dx}4^x$

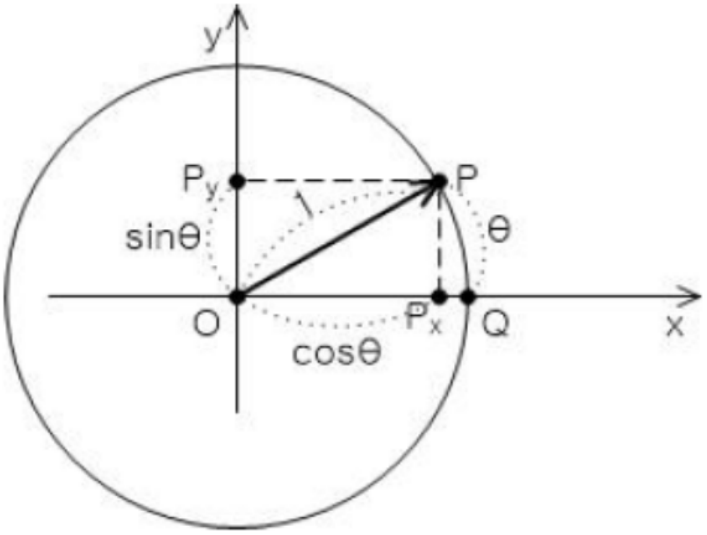
@ Intermediate Summarize (step3)

Elementary Number : $\emptyset, 1, i, e$

Elementary Function

x^2	$2x$	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
e^x	e^x	$\log_e x$	$\frac{1}{x}$

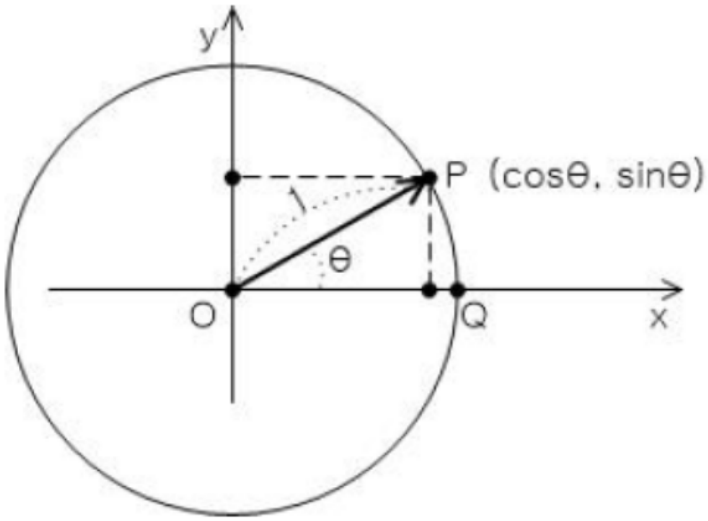
@ Trigonometric Functions



[Fig] Length of the projected line segment

Sine

Cosine



[Fig] Coordiante of P

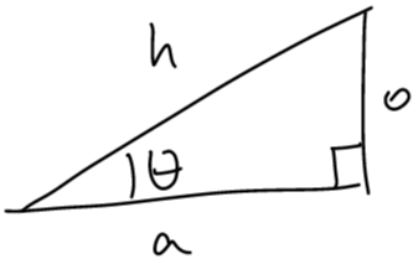
$\pi = 3.141592\cdots$

↓

additional elementary number.

(cos(θ), sin(θ))

Pythagorean Theorem



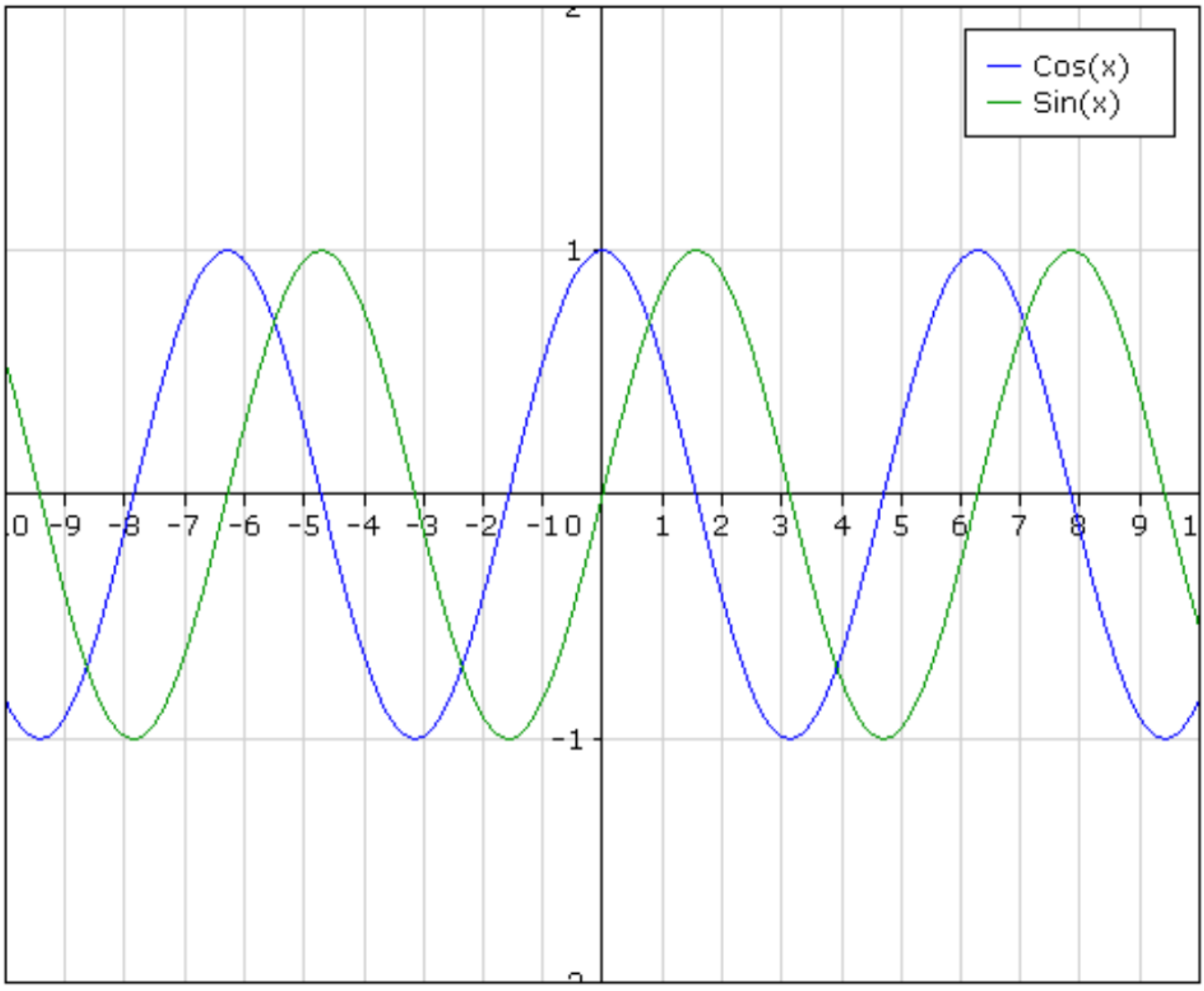
hypotenus
opposite
adjacent

$h^2 = a^2 + o^2$

$\cos\theta = \frac{a}{h}$

$\sin\theta = \frac{o}{h}$

$1^2 = \cos^2\theta + \sin^2\theta$



Inverse of cos, sin

$\cos^{-1}(x), \sin^{-1}(x) \Rightarrow \text{acos}(x), \text{asin}(x)$

@ Intermediate Summarize (Step 4)

Elementary Number : $0, 1, i, e, \pi$

Elementary Function

x^2	$2x$	$\sqrt{x}$	$1/2\sqrt{x}$
e^x	e^x	$\ln x$	$1/x$
$\cos(x), \sin(x)$	$-\sin(x)$ $\cos(x)$	$\arccos(x), \arcsin(x)$	...

Differentiation Rules

Product Rule $(f \cdot g)' = f' \cdot g + f g'$

Quotient Rule $(f/g)' = (f'g - fg')/g^2$

Chain Rule

$$dy/dx = dy/du \; du/dx$$

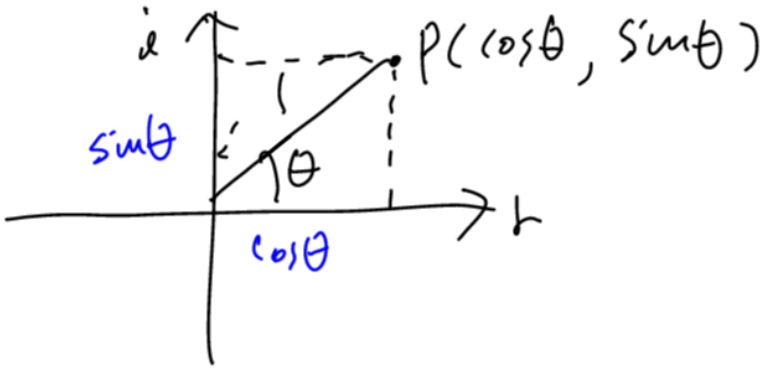
Q. Inverse of Differentiation?

↳ Integration

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

@ Numbers on Complex Plane

↳ Complex Number



$$P = \cos\theta + i\sin\theta = e^{i\theta}$$

$$e^{i\pi} = \cos\pi + i\sin\pi$$

$$e^{i\pi} = -1 + 0$$

$$e^{i\pi} + 1 = \phi$$ (Euler Formula)

* Property of Exponential

$$a^x a^y = a^{(x+y)}$$

* Complex Number as a Rotation.

↳ More Higher Dimension ⇒ Quaternion.

@ Remained Topics

Vectors and Matrices

Practice)

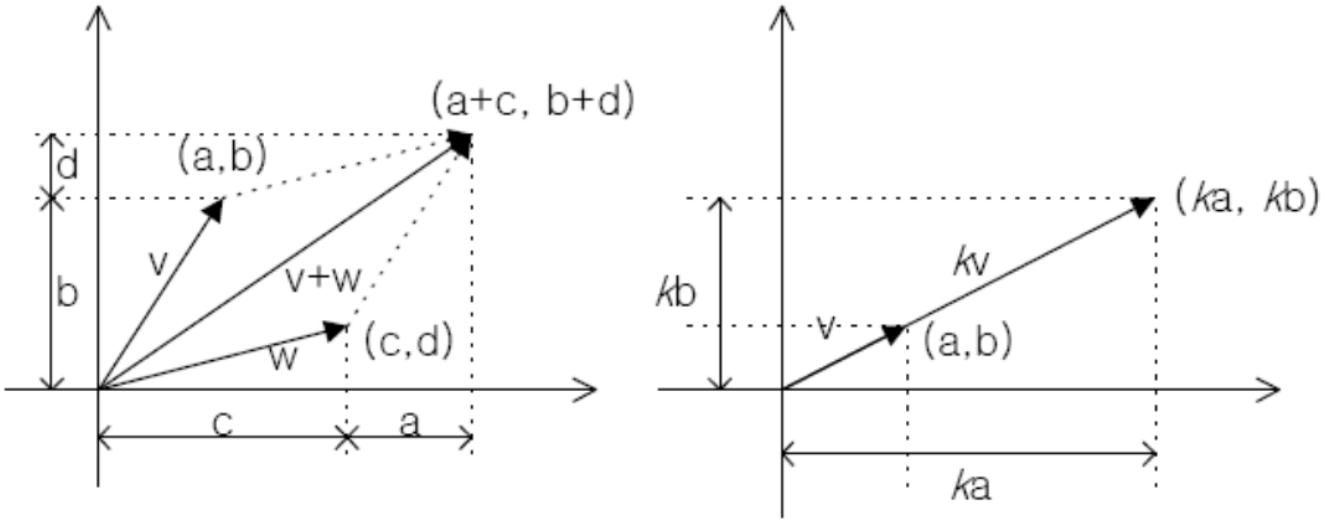
$$v = at \xrightarrow{①} v' = v_0 + at$$

$$\downarrow ②$$

$$S = v_0 t + \frac{1}{2}at^2 + S_0$$

Ch Vectors and Matrices

Vector Operations



[Fig] Vector Addition and Scalar Multiplication

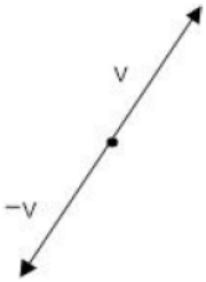
$\vec{v} = (a,b),\ \vec{w} = (c,d)$

$$\vec{v} + \vec{w} = (a,b) + (c,d) = (a+c,\ b+d)$$

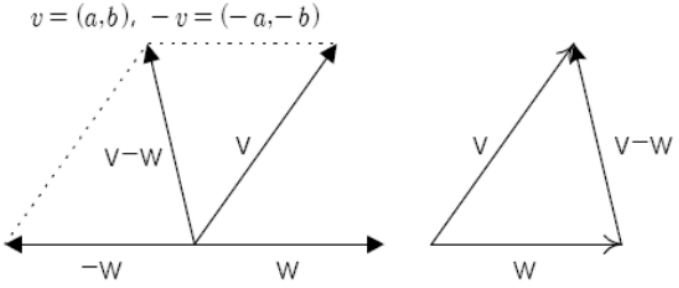
$\vec{v} = (a,b)$, real number k

$$k\vec{v} = k(a,b) = (ka,\ kb)$$

Negative Vector

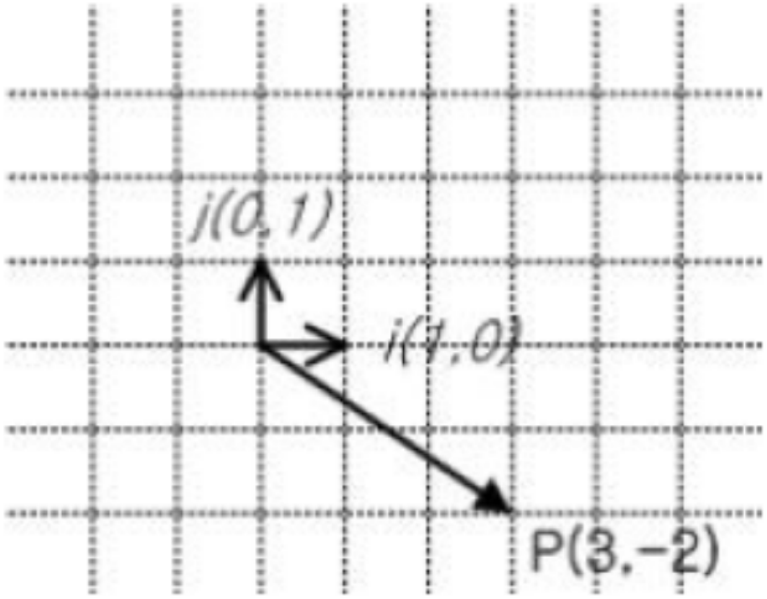


[Fig] Negative Vector



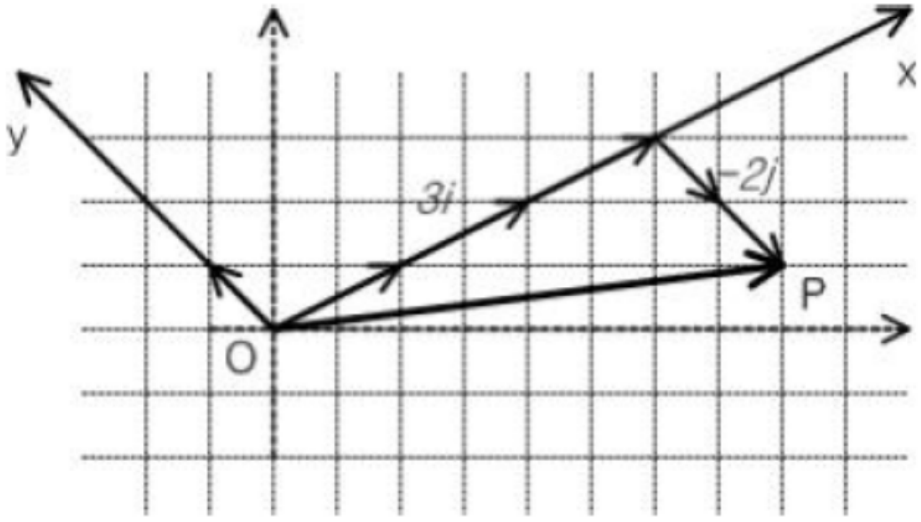
[Fig]Vector Subtraction

$$\vec{v} - \vec{w} = (a-c,\ b-d)$$



[Fig] Basis Vectors i(1,0) and j(0,1)

$$3i - 2j = 3\begin{bmatrix}1\\0\end{bmatrix} - 2\begin{bmatrix}0\\1\end{bmatrix}$$



[Fig] Meaning of (3,-2) in new Coordinate System

$$3i - 2j = 3\begin{bmatrix}2\\1\end{bmatrix} - 2\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}8\\1\end{bmatrix}$$

length or norm

$$|v| = \sqrt{a^2 + b^2}$$

normalization

@ Matrix

Idea: Simpler Notation

$$3i-2j=3\begin{bmatrix}2\\1\end{bmatrix}-2\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}8\\1\end{bmatrix}$$

$$[i\ \ j]\begin{bmatrix}3\\-2\end{bmatrix}=i3+j(-2)$$

$$\begin{bmatrix}2&-1\\1&1\end{bmatrix}\begin{bmatrix}3\\-2\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}3+\begin{bmatrix}-1\\1\end{bmatrix}(-2)=\begin{bmatrix}2\times3+(-1)\times(-2)\\1\times3+1\times(-2)\end{bmatrix}=\begin{bmatrix}8\\1\end{bmatrix}$$

@ Practice (Demo), Velocity and Acceleration.

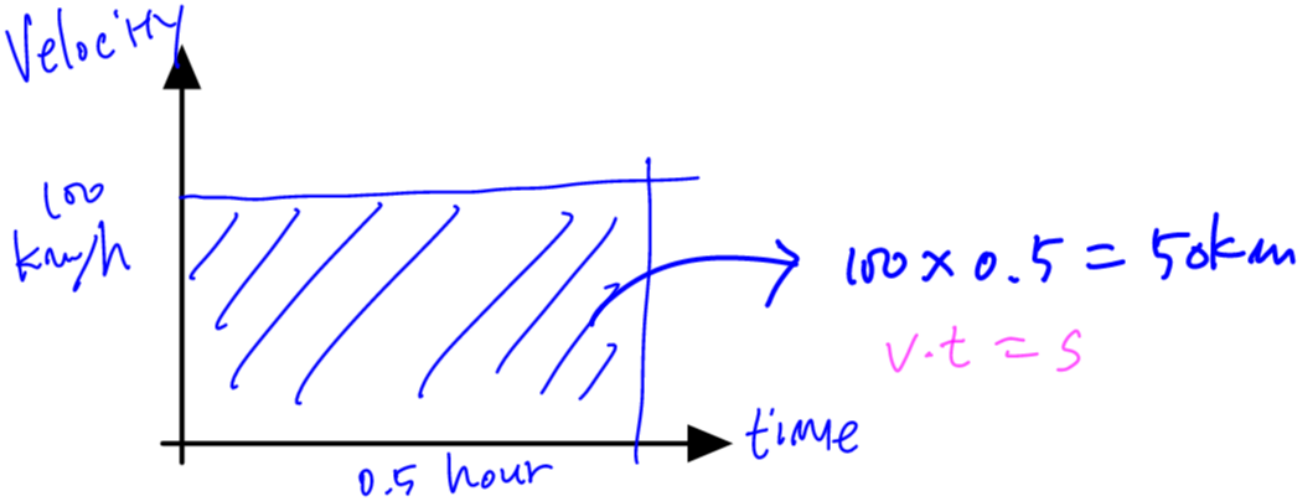
$s=vt$

$v=at$

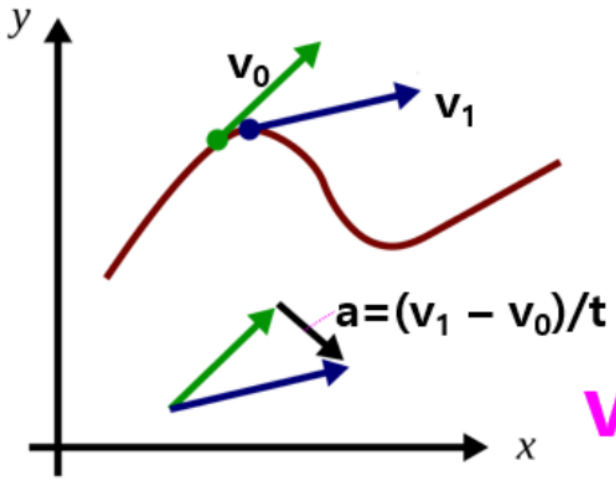
① → $v'=v_0+at$

②

$s'=s_0+v_0t+\frac{1}{2}at^2$



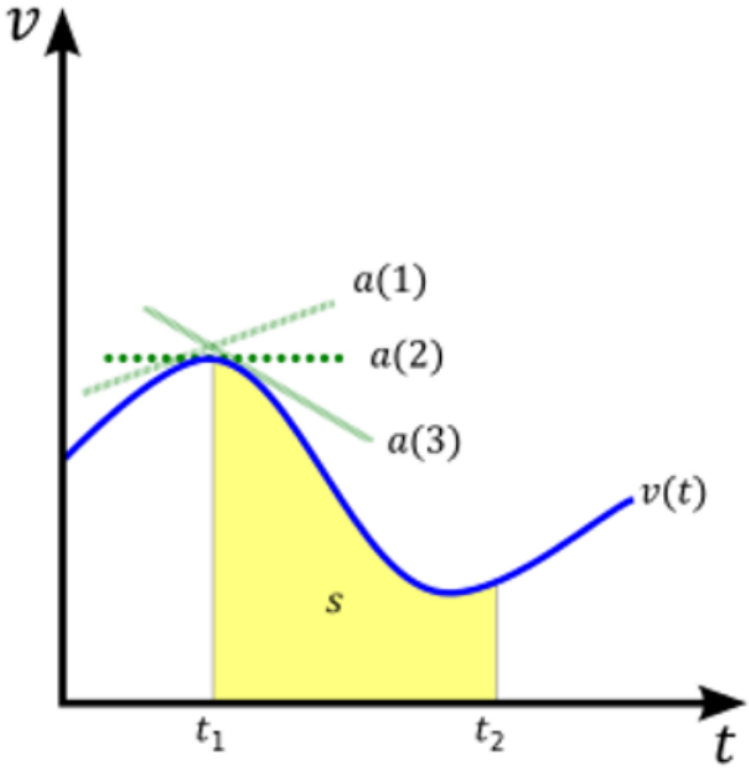
* Acceleration

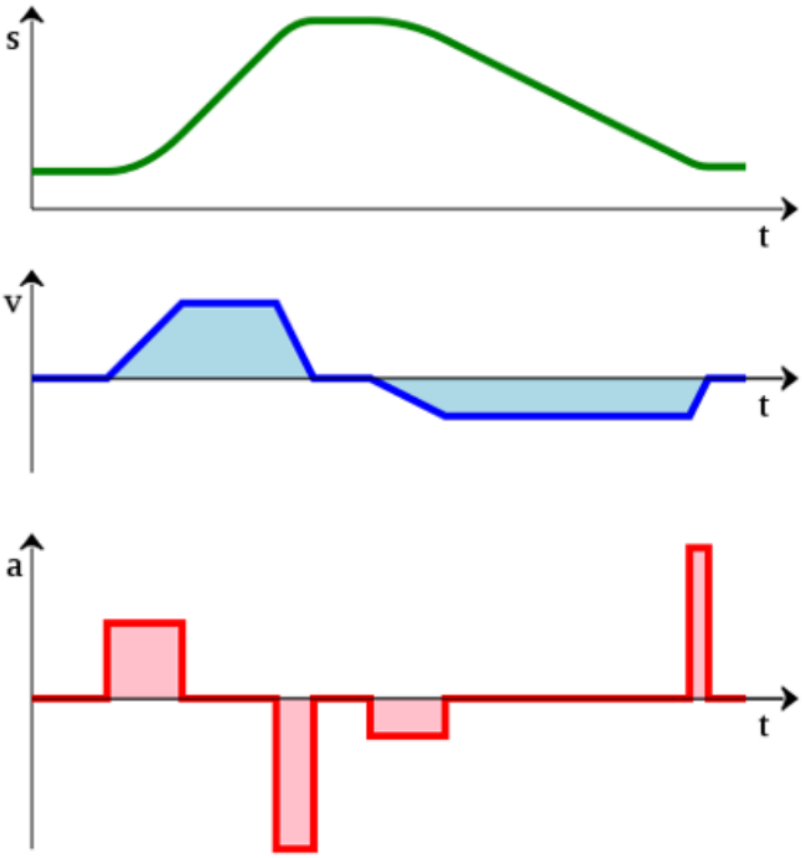


$$v(t) = v_0 + at$$

$$v = at$$

* Differentiation & Integration of v with respect t





✓ **From bottom to top**:

- an acceleration function $a(t)$;
- the integral of the acceleration is the velocity function $v(t)$;
- and the integral of the velocity is the distance function $s(t)$.

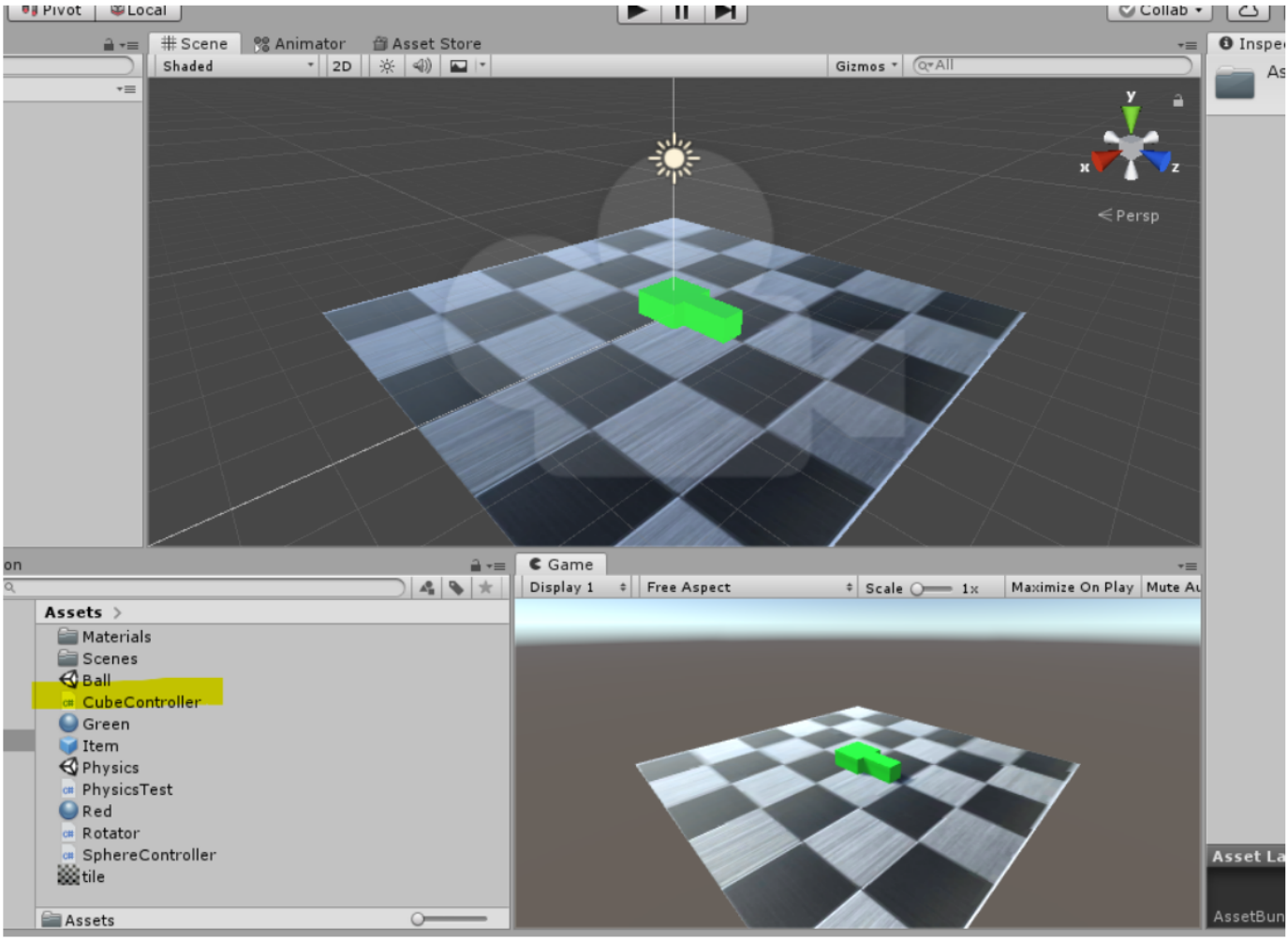
* Rotation → Quaternion. (will be covered in University)

$$Q = Q * Q'$$

@ Demo

* Unity Game Engine

unity




```
// Update is called once per frame
void Update()
{
    float iv = Input.GetAxis( "Vertical" );
    float ih = Input.GetAxis( "Horizontal" );
    float dt = Time.deltaTime;
    Vector3 pos = gameObject.transform.position;
    Vector3 a = Vector3.zero;
    a = gameObject.transform.forward * _accel * iv;
    _v = _v + a * dt;

    gameObject.transform.position
        = pos + _v * dt + 0.5f * a * dt * dt;

    float degree = Mathf.Rad2Deg * (dt * ih);
    Quaternion qprime = Quaternion.AngleAxis( degree, Vector3.up);
    gameObject.transform.rotation =
        gameObject.transform.rotation * qprime;

    _text.text = _v.ToString();
}
```

(2019.10.10)