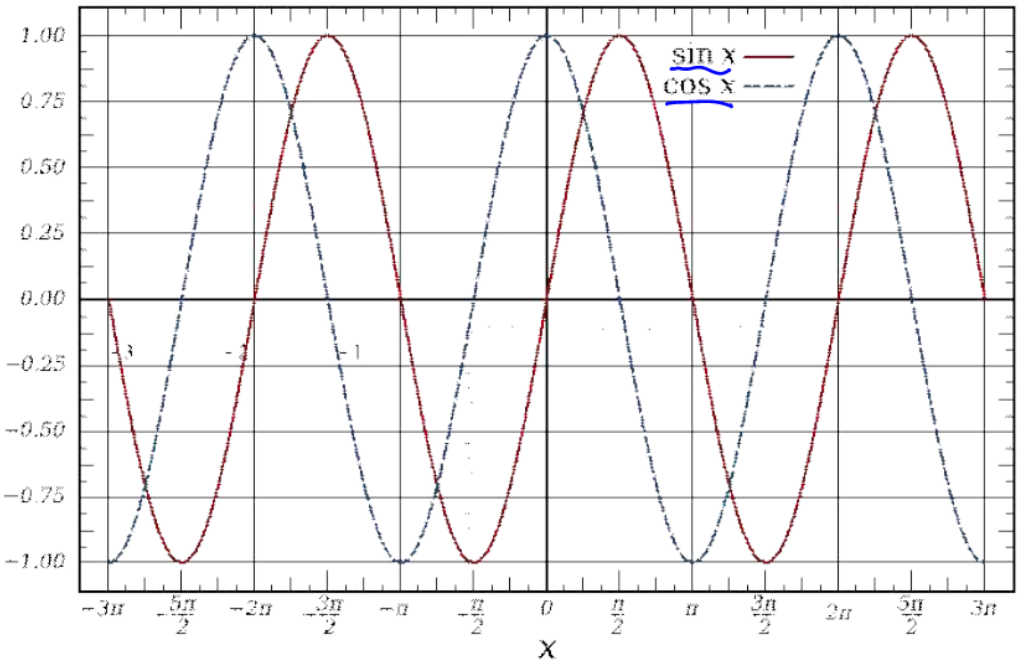
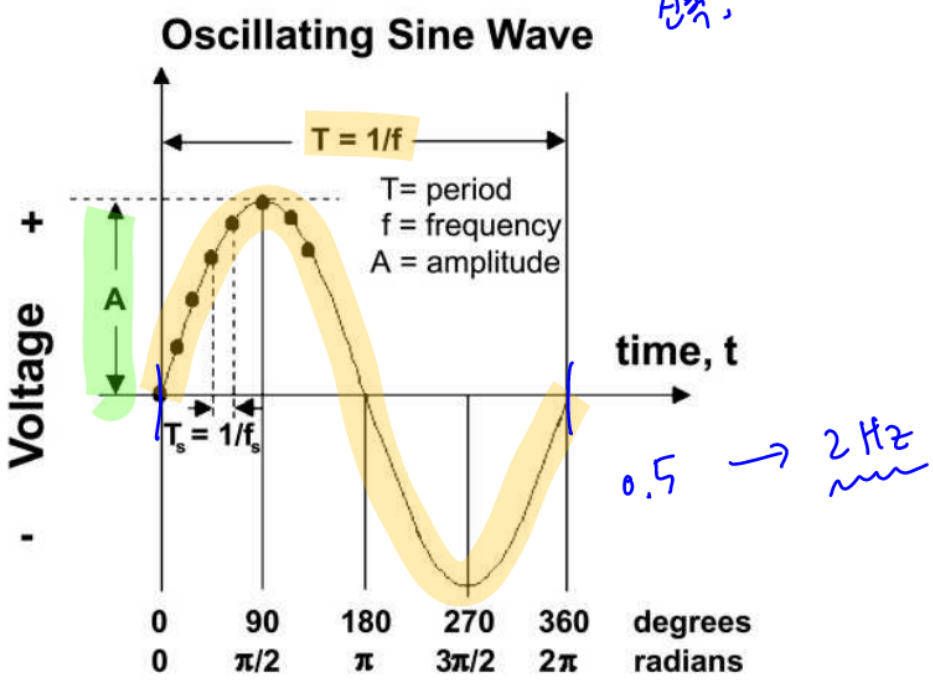
Fourier Transform

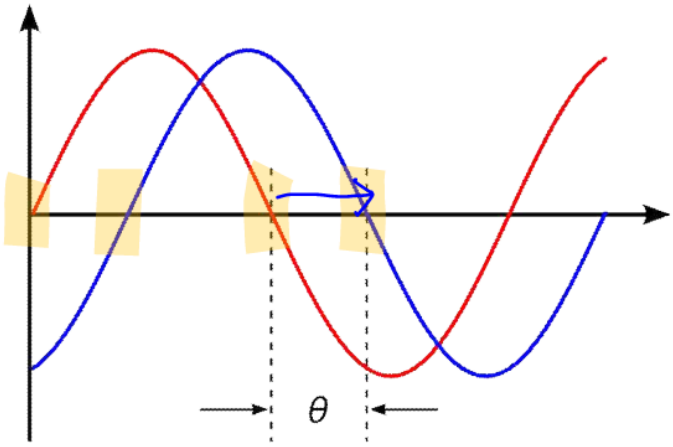
Sinusoid Curve



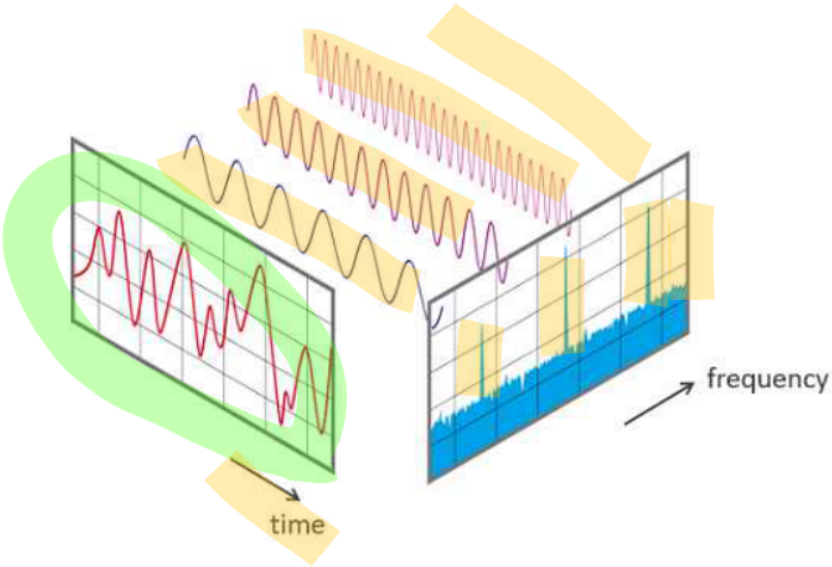
Period, Frequency and Amplitude

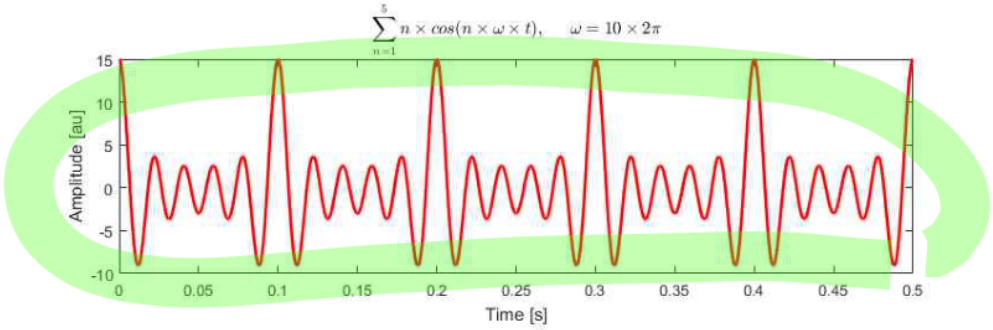


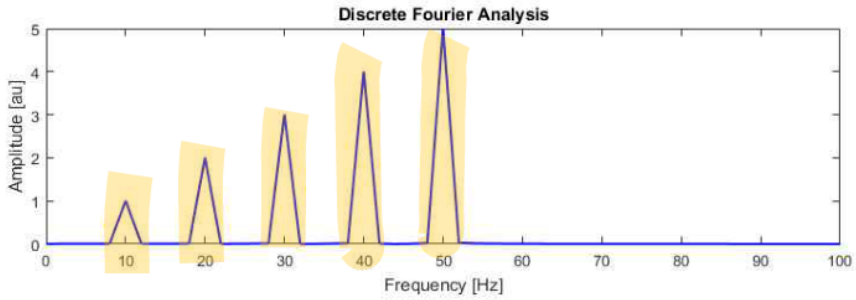
Phase 위상.



Observation



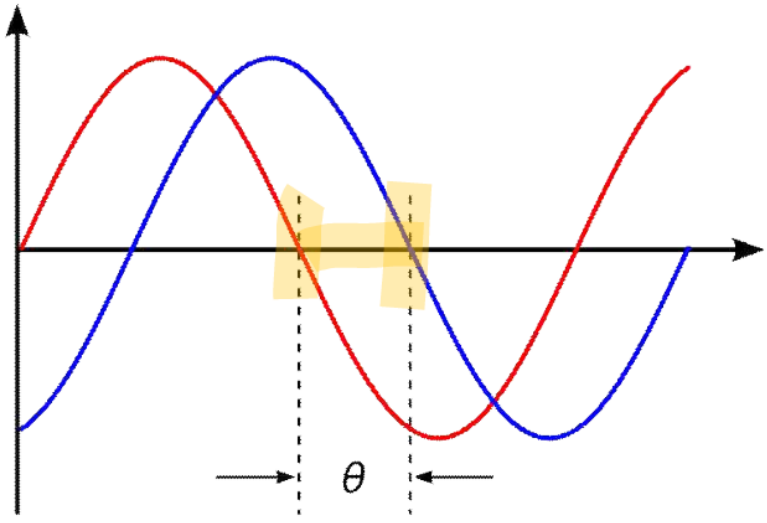




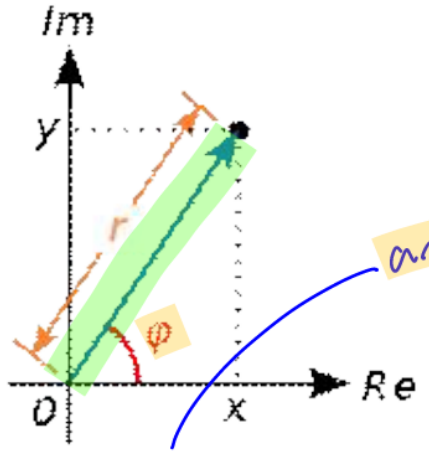
$$f(t) = \cos(2\pi t) + 0.5\cos(2\pi 4t) + \ldots$$

$$f(t) = \sum_{v=-\infty}^{\infty} A(v)\cos(2\pi v t)$$

Phase shift



$$f(v) = \cos(2\pi t) + 0.5\cos(2\pi 4t + \pi/4) + \ldots$$



amplitude

phase

frequency $\equiv \nu$

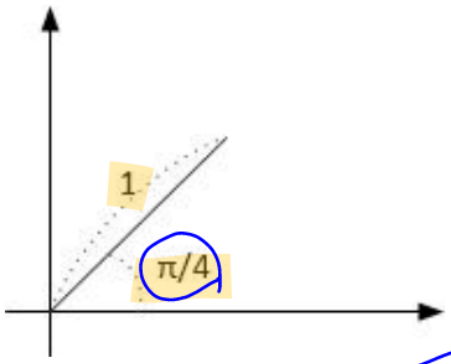
$$F(v) = r(\cos(\theta) + i\sin(\theta))$$

$$e^{ix} = \cos(x) + i\sin(x)$$

$$f(t) = \sum_{v=-\infty}^{\infty} F(v)e^{(2\pi ivt)}$$

$$f(t) = \int_{v=-\infty}^{\infty} F(v)e^{(2\pi ivt)}dv$$

Example

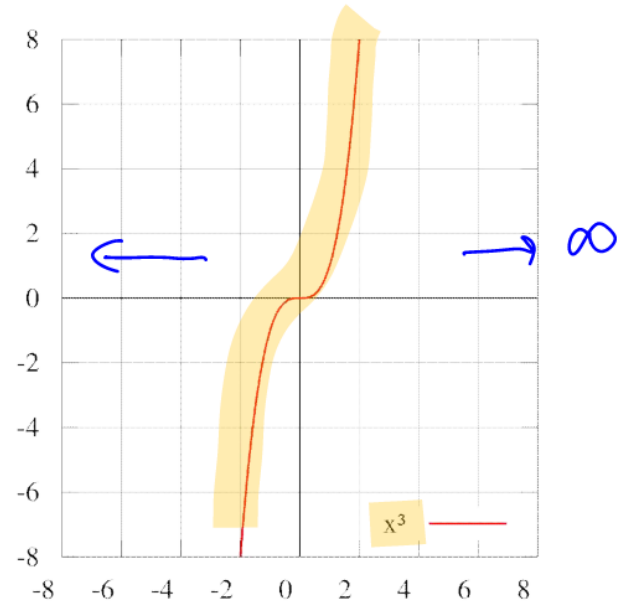


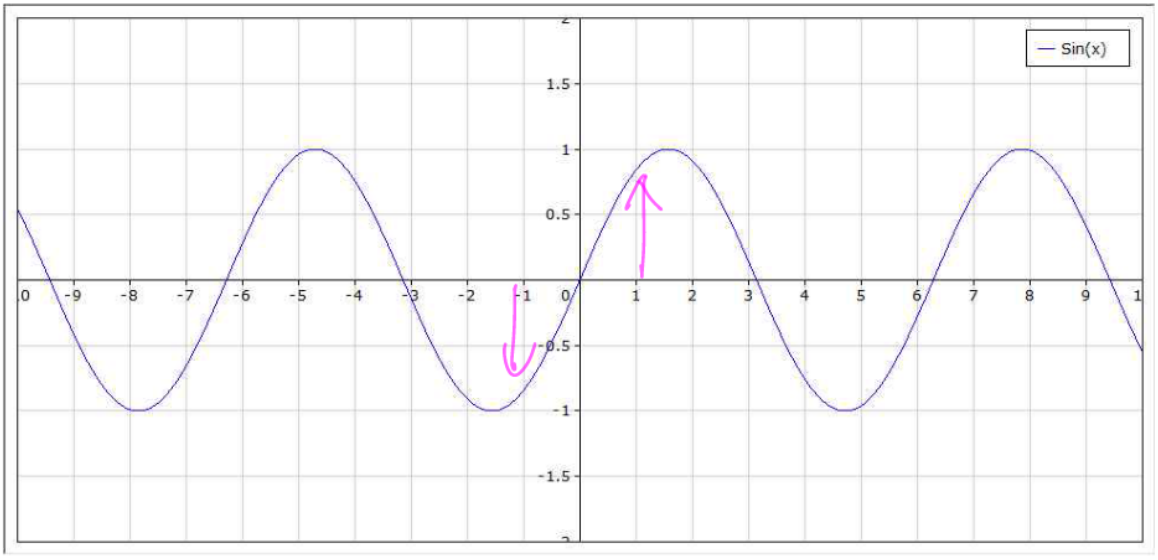
$i^2 = -1$

$$(\cos(\pi/4) + i\sin(\pi/4))e^{2\pi ivt}$$
$$= (\cos(\pi/4) + i\sin(\pi/4))(\cos(2\pi vt) + i\sin(2\pi vt))$$
$$= \cos(\pi/4)\cos(2\pi vt) + \cos(\pi/4)\sin(2\pi vt)i$$
$$+ \sin(\pi/4)\cos(2\pi vt)i - \sin(\pi/4)\sin(2\pi vt)$$
$$= \cos(2\pi vt + \frac{\pi}{4}) + \sin(\frac{\pi}{4} + 2\pi vt)i$$

Odd function

$-f(x) = f(-x)$





$\sin(x)$

Inverse Fourier Transform

$$f(t) = \sum_{v=-\infty}^{\infty} F(v) e^{2\pi i v t}$$

$$f(t) = \int_{v=-\infty}^{\infty} F(v) e^{2\pi i v t} dv$$

Forward Fourier Transform

$$F(v) = \sum_{t=-\infty}^{\infty} f(t) e^{-2\pi i v t}$$

$$F(v) = \int_{t=-\infty}^{\infty} f(t) e^{-2\pi i v t} dt$$

Hint

$$\frac{f(t)}{e^{2\pi i v t}} = F(v)$$

Implementation

```
using ComplexArray = std::valarray<std::complex<double> >;
```

Preparing Signal

```
std::complex<double> test[BIN_SIZE];
// fill test signal data
double x = 0;
double y;
for(int i = 0; i < BIN_SIZE; ++i)
{
    test[i] = std::complex<double>( SignalFunction( x )
```

(next video clip)