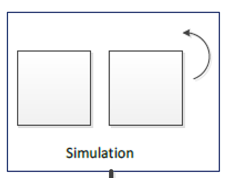


Impulse (physics)

From Newton's second law, force is related to momentum **p** by

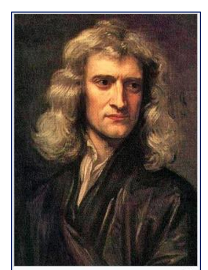
$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$

Therefore,

$$\mathbf{J} = \int_{t_1}^{t_2} \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}\,\mathrm{d}t = \int_{\mathbf{p}_1}^{\mathbf{p}_2} \mathrm{d}\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1 = \Delta\mathbf{p}$$

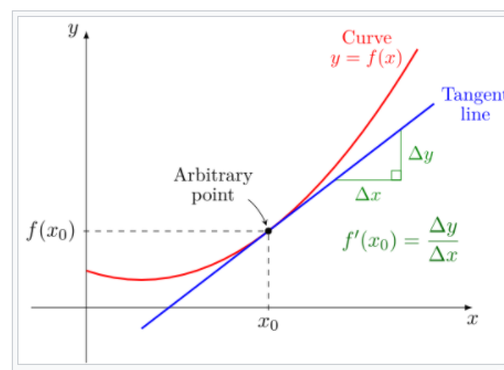
Calculus

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$



Isaac Newton developed the use of calculus in his laws of motion and gravitation.

Differential calculus



Tangent line at $(x_0, f(x_0))$. The derivative $f'(x)$ of a curve at a point is the slope (rise over run) of the line tangent to that curve at that point.

The polynomial or elementary power rule [edit]

Main article: Power rule

If $f(x) = x^r$, for any real number $r \neq 0$, then

$$f'(x) = rx^{r-1}.$$

When $r = 1$, this becomes the special case that if $f(x) = x$, then $f'(x) = 1$.

The quotient rule [edit]

Main article: Quotient rule

If f and g are functions, then:

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2} \quad \text{wherever } g \text{ is nonzero.}$$

The product rule [edit]

Main article: Product rule

For the functions f and g, the derivative of the function $h(x) = f(x)\,g(x)$ with respect to x is

$$h'(x) = (fg)'(x) = f'(x)g(x) + f(x)g'(x).$$

In Leibniz's notation this is written

$$\frac{d(fg)}{dx} = \frac{df}{dx}g + f\frac{dg}{dx}.$$

The chain rule [edit]

Main article: Chain rule

The derivative of the function $h(x) = f(g(x))$ is

$$h'(x) = f'(g(x)) \cdot g'(x).$$

Derivatives of exponential and logarithmic functions [edit]

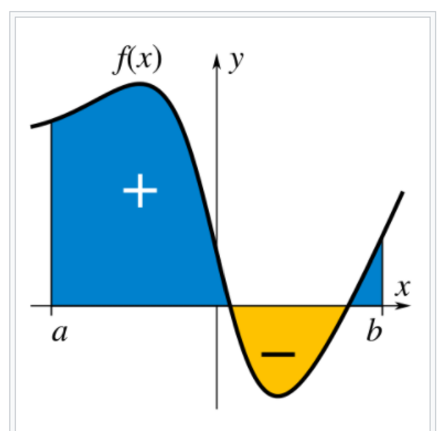
$$\frac{d}{dx}\left(c^{ax}\right) = ac^{ax}\ln c, \qquad c > 0$$

the equation above is true for all c, but the derivative for $c < 0$ yields a complex number.

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_c x\right) = \frac{1}{x\ln c}, \qquad c > 0, c \neq 1$$

Integral

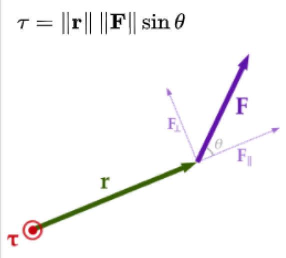


A definite integral of a function can be represented as the signed area of the region bounded by its graph.

Torque

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\tau = \|\mathbf{r}\|\,\|\mathbf{F}\| \sin\theta$$



The net torque on a body determines the rate of change of the body's angular momentum,

$$\boldsymbol{\tau} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}$$

where **L** is the angular momentum vector and t is time.

For the motion of a point particle,

$$\mathbf{L} = I\boldsymbol{\omega},$$

where I is the moment of inertia and $\boldsymbol{\omega}$ is the orbital angular velocity pseudovector. It follows that

$$\boldsymbol{\tau}_{\mathrm{net}} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}(I\boldsymbol{\omega})}{\mathrm{d}t} = I\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} + \frac{\mathrm{d}I}{\mathrm{d}t}\boldsymbol{\omega} = I\boldsymbol{\alpha} + \frac{\mathrm{d}(mr^2)}{\mathrm{d}t}\boldsymbol{\omega} = I\boldsymbol{\alpha} + 2rp_{||}\boldsymbol{\omega},$$

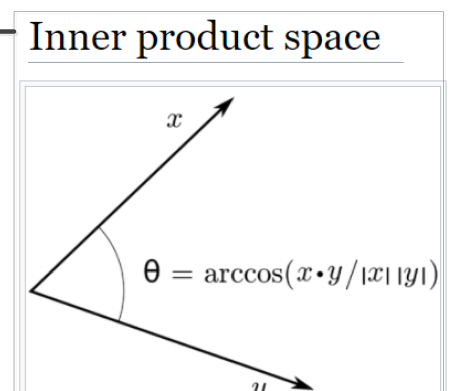
Linear algebra

From Wikipedia, the free encyclopedia

Linear algebra is the branch of mathematics concerning linear equations such as:

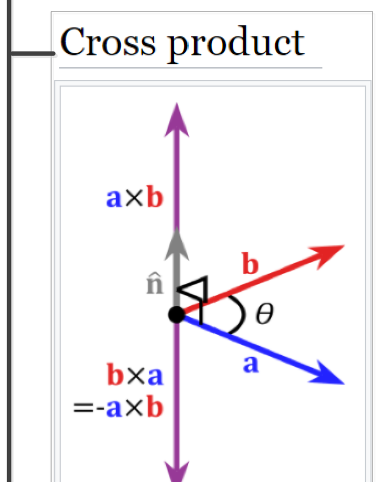
$$a_1x_1 + \cdots + a_nx_n = b,$$

Inner product space



Geometric interpretation of the angle between two vectors defined using an inner product

Cross product



The cross product with respect to a right-handed coordinate system

Matrix (mathematics)

From Wikipedia, the free encyclopedia

For other uses, see Matrix.
"Matrix theory" redirects here. For the physic

In mathematics, a **matrix** (plural **matrices**) is a r arranged in *rows* and *columns*.[1][2] For example because there are two rows and three columns:

$$\begin{bmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{bmatrix}.$$

Eigenvalues and eigenvectors

Eigenvalues and the characteristic polynomial [edit]

Main article: Characteristic polynomial

Equation (2) has a nonzero solution v if and only if the determinant the equation

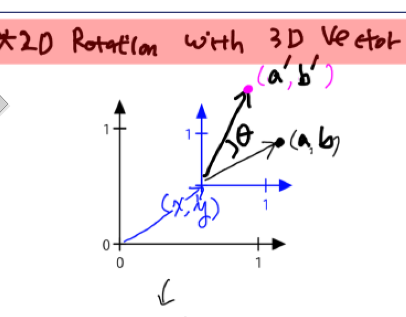
$$|A - \lambda I| = 0$$

Quaternion

Quaternion multiplication table

↓ × →	**1**	**i**	**j**	**k**
1	1	i	j	k
i	i	−1	k	−j
j	j	−k	−1	i
k	k	j	−i	−1

***2D Rotation with 3D Vector** (42)



quaternion idea

$$(a', b') = (x, y, \theta) \otimes (a, b, \phi)$$

$$\Rightarrow (a - x,\ b - y,\ -\theta)$$

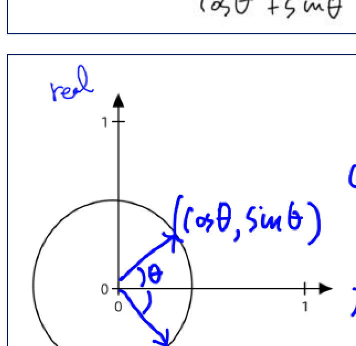
$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a - x \\ b - y \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \big((a-x)\cos\theta - (b-y)\sin\theta,\ (a-x)\sin\theta + (b-y)\cos\theta,\ 1 \big)$$

$$= \begin{pmatrix} (a-x)\cos\theta - (b-y)\sin\theta + x, \\ (b-y)\cos\theta + (a-x)\sin\theta + y, \\ 0 \end{pmatrix}$$

$\cos\theta - \sin\theta\, i$ when $x = \phi$, $y = \phi$

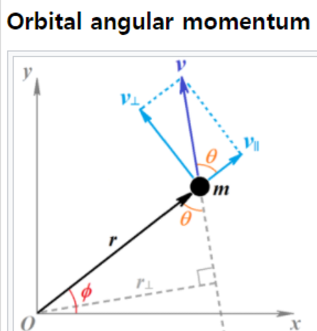
$\cos\theta + \sin\theta\, i$



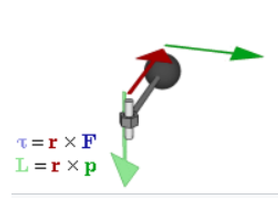
Conjugate.

Angular momentum

Orbital angular momentum in two dimensions



Velocity of the particle m with respect to the origin O can be resolved into components parallel to ($v_{||}$) and perpendicular to ($v_\perp$) the radius vector r. The **angular momentum** of m is proportional to the perpendicular component $v_\perp$ of the velocity, or equivalently, to the perpendicular distance $r_\perp$ from the origin.



$$\mathbf{L} = (r^2 m)\left(\frac{\mathbf{r}\times\mathbf{v}}{r^2}\right) = m(\mathbf{r}\times\mathbf{v}) = \mathbf{r}\times m\mathbf{v} = \mathbf{r}\times\mathbf{p},$$

Relationship between force (**F**), torque (**τ**), momentum (**p**), and angular momentum (**L**) vectors in a rotating system. **r** is the position vector.

Common symbols	L
In SI base units	kg m^2 s^{-1}
Conserved?	yes
Derivations from other quantities	$\mathbf{L} = I\boldsymbol{\omega} = \mathbf{r} \times \mathbf{p}$
Dimension	M L^{2}T^{-1}

Moment of inertia

Moment of inertia is defined as the product of mass of section and the square of the distance between the reference axis and the centroid of the section .

Moment of inertia I is defined as the ratio of the net angular momentum L of a system to its angular velocity ω around a principal axis,[7][8] that is

$$I = \frac{L}{\omega}.$$

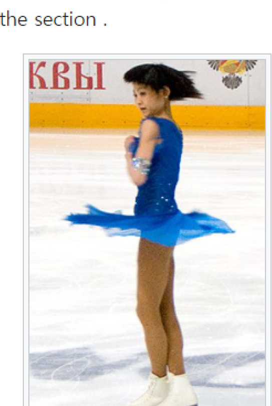
If the angular momentum of a system is constant, then as the moment of inertia gets smaller, the angular velocity must increase. This occurs when spinning figure skaters pull in their outstretched arms or divers curl their bodies into a tuck position during a dive, to spin faster.[7][8][9][10][11][12][13]

If the shape of the body does not change, then its moment of inertia appears in Newton's law of motion as the ratio of an applied torque τ on a body to the angular acceleration α around a principal axis, that is

$$\tau = I\alpha.$$

For a simple pendulum, this definition yields a formula for the moment of inertia I in terms of the mass m of the pendulum and its distance r from the pivot point as,

$$I = mr^2.$$



Spinning figure skaters can reduce their moment of inertia by pulling in their arms, allowing them to spin faster due to conservation of angular momentum.

Inertia tensor [edit]

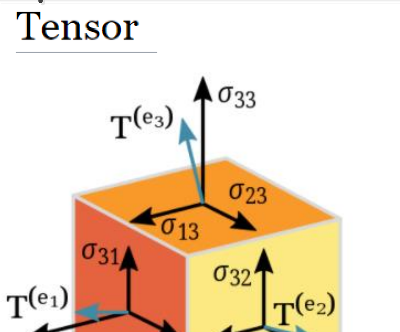
For the same object, different axes of rotation will have different moments of inertia about those axes. In general, the moments of inertia are not equal unless the object is symmetric about all axes. The **moment of inertia tensor** is a convenient way to summarize all moments of inertia of an object with one quantity. It may be calculated with respect to any point in space, although for practical purposes the center of mass is most commonly used.

Definition [edit]

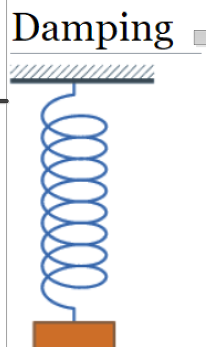
For a rigid object of N point masses m_k, the moment of inertia tensor is given by

$$\mathbf{I} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}.$$

Tensor



Damping



Ordinary differential equation

General definition [edit]

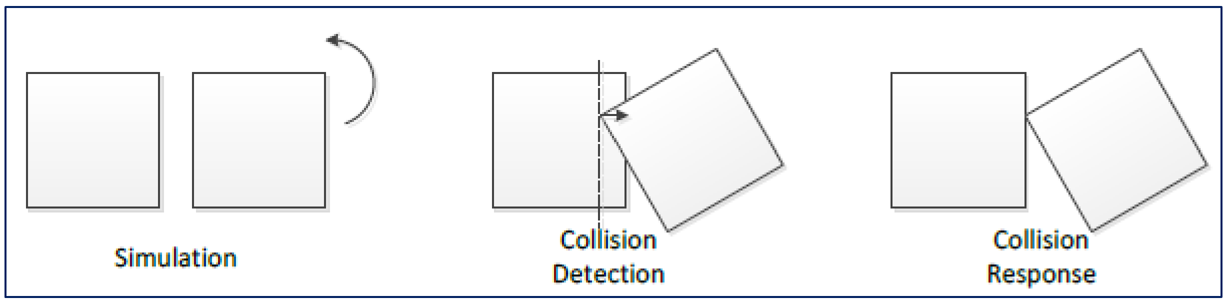
Given F, a function of x, y, and derivatives of y. Then an equation of the form

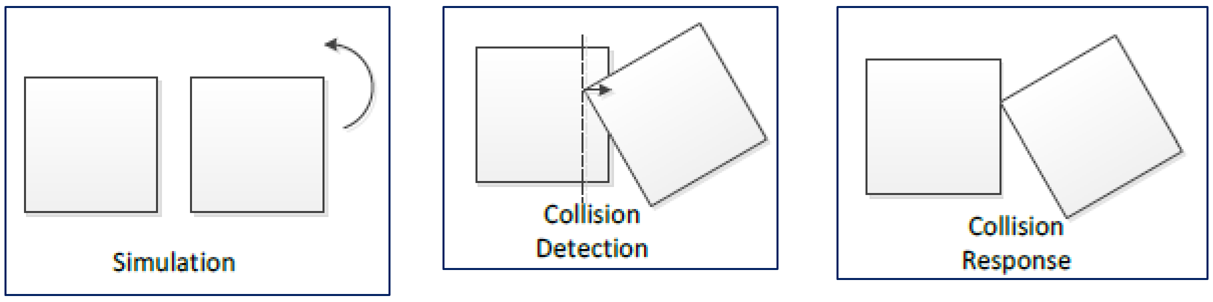
$$F\left(x, y, y', \ldots, y^{(n-1)}\right) = y^{(n)}$$

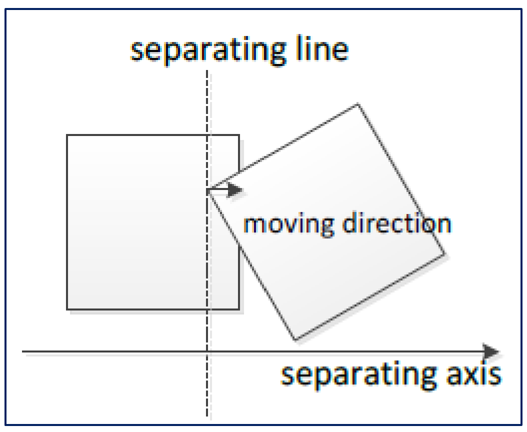
is called an *explicit ordinary differential equation of order n*.[8][9]

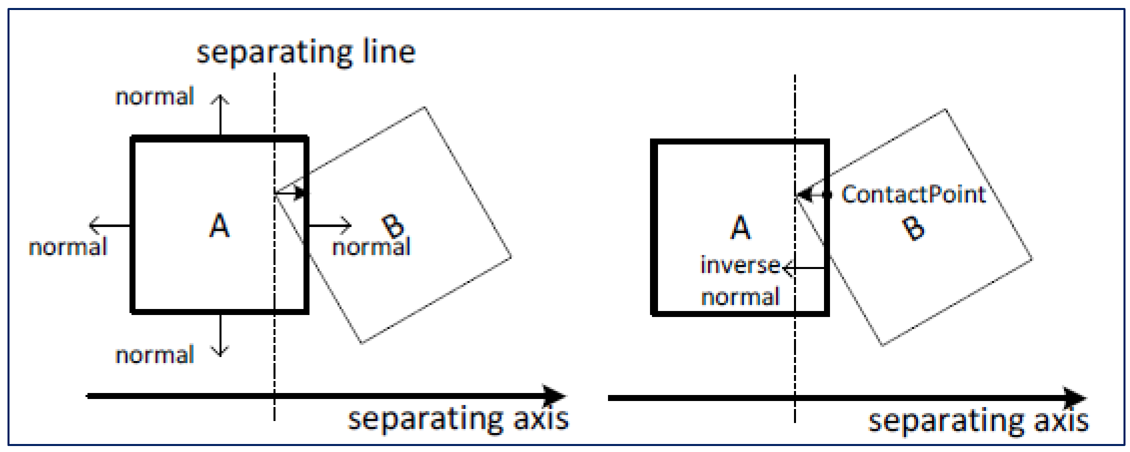
More generally, an *implicit* ordinary differential equation of order n takes the form:[10]

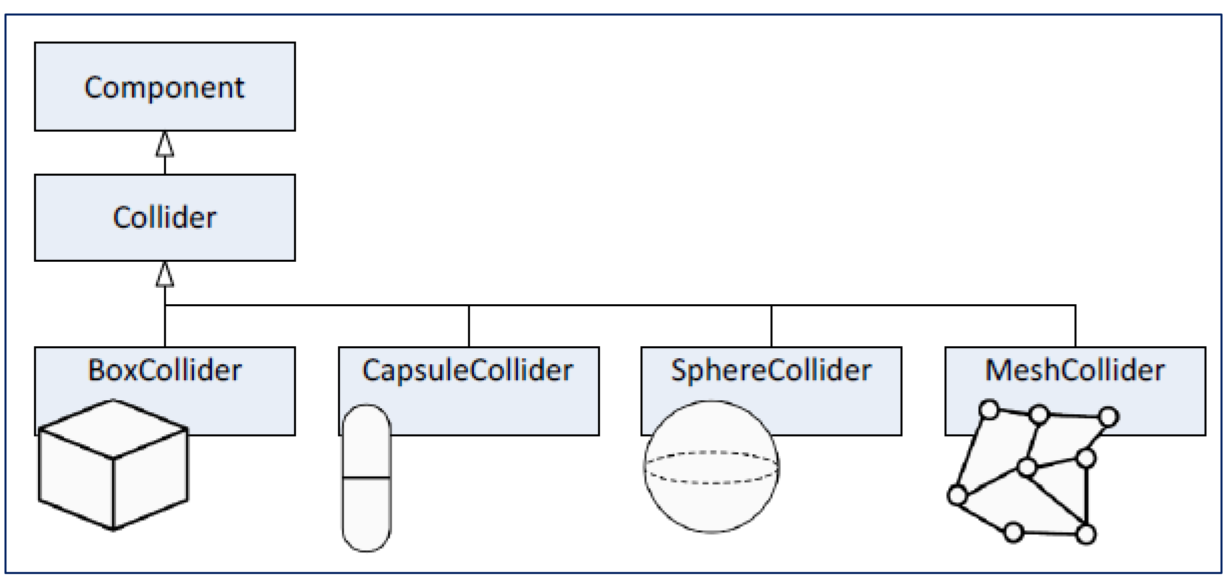
$$F\left(x, y, y', y'',\ \ldots,\ y^{(n)}\right) = 0$$


Simulation
Collision Detection
Collision Response


Simulation
Collision Detection
Collision Response


separating line
moving direction
separating axis


separating line
normal
normal
A
B
normal
normal
separating axis
A
ContactPoint
B
inverse normal
separating axis


Component
Collider
BoxCollider
CapsuleCollider
SphereCollider
MeshCollider