Cross product

From Wikipedia, the free encyclopedia

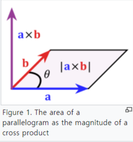
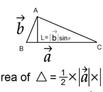


Figure 1. The area of a parallelogram as the magnitude of a cross product

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\|\,\|\mathbf{b}\|\sin(\theta)\,\mathbf{n}$$



$$\text{area of } \triangle = \frac{1}{2}\times\left|\vec{a}\right|\times\left|\vec{b}\right|\times sin\theta$$

$$= \frac{1}{2}\times[\vec{a}\times\vec{b}\,]$$

Example 5: Finding the Area of a Triangle Given Its Three Vertices

Find the area of a triangle ABC, where $A(-8,-9)$, $B(-7,-8)$, and $C(9,-2)$.

Answer

The magnitude of the cross product of two vectors is equal to the area of the parallelogram spanned by them. The area of the triangle ABC is equal to half the area of the parallelogram spanned by two vectors defined by its vertices:

$$\text{the area of } ABC = \frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\| = \frac{1}{2}\left\|\overrightarrow{BA}\times\overrightarrow{BC}\right\| = \frac{1}{2}\left\|\overrightarrow{CB}\times\overrightarrow{CA}\right\|.$$

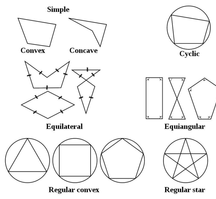
Polygon

From Wikipedia, the free encyclopedia

Convexity and non-convexity

Polygons may be characterized by their convexity or type of non-convexity:

- Convex: any line drawn through the polygon (and not tangent to an edge or corner) meets its boundary exactly twice. As a consequence, all its interior angles are less than 180°. Equivalently, any line segment with endpoints on the boundary passes through only interior points between its endpoints.
- Non-convex: a line may be found which meets its boundary more than twice. Equivalently, there exists a line segment between two boundary points that passes outside the polygon.
- Simple: the boundary of the polygon does not cross itself. All convex polygons are simple.
- Concave: Non-convex and simple. There is at least one interior angle greater than 180°.



Equality and symmetry

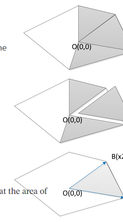
- Regular: the polygon is both *isogonal* and *isotoxal*. Equivalently, it is both *cyclic* and *equilateral*, or both *equilateral* and *equiangular*. A non-convex regular polygon is called a *regular star polygon*.
- Isogonal or vertex-transitive: all corners lie within the same symmetry orbit. The polygon is also cyclic and equiangular.
- Isotoxal or edge-transitive: all sides lie within the same symmetry orbit. The polygon is also equilateral and tangential.
- Cyclic: all corners lie on a single circle, called the circumcircle.
- Equilateral: all edges are of the same length. The polygon need not be convex.
- Equiangular: all corner angles are equal.
- Tangential: all sides are tangent to an inscribed circle.

Area

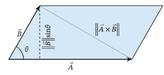
In this section, the vertices of the polygon under consideration are taken to be $(x_0, y_0), (x_1, y_1), \ldots, (x_{n-1}, y_{n-1})$ in order. For convenience in some formulas, the notation $(x_n, y_n) = (x_0, y_0)$ will also be used.

If the polygon is non-self-intersecting (that is, simple), the signed area is

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i) \quad \text{where } x_n = x_0 \text{ and } y_n = y_0,$$



- The cross product is *distributive*: $(\vec{A}+\vec{B})\times\vec{C} = \vec{A}\times\vec{C} + \vec{B}\times\vec{C}$.
- The cross product is *anticommutative*: $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$.
- The cross product of two collinear vectors is zero, and so $\vec{A}\times\vec{A} = 0$.
- The area of the parallelogram spanned by $\vec{A}$ and $\vec{B}$ is given by $\left\|\vec{A}\times\vec{B}\right\|$. It follows that the area of the triangle with $\vec{A}$ and $\vec{B}$ defining two of its sides is given by $\frac{1}{2}\left\|\vec{A}\times\vec{B}\right\|$.



$$\mathbf{c} = \frac{1}{A}\sum_{i=0}^{N-1}\frac{1}{2}(x_i y_{i+1} - x_{i+1} y_i)\mathbf{c}_i. \qquad (1)$$

The centroid of a triangle with vertices $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ is just $\frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})$. Substituting this into (1) yields the formula in the question.

Centroid

Using the same convention for vertex coordinates as in the previous section, the coordinates of the centroid of a solid simple polygon are

$$C_x = \frac{1}{6A}\sum_{i=0}^{n-1}(x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i),$$

$$C_y = \frac{1}{6A}\sum_{i=0}^{n-1}(y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i).$$

In these formulas, the signed value of area A must be used.

For triangles ($n = 3$), the centroids of the vertices and of the solid shape are the same, but, in general, this is not true for $n > 3$. The centroid of the vertex set of a polygon with n vertices has the coordinates

$$c_x = \frac{1}{n}\sum_{i=0}^{n-1} x_i,$$

$$c_y = \frac{1}{n}\sum_{i=0}^{n-1} y_i.$$

```
//std::vector<KVector2>   g_vertices;
KVector2 geoCenter = KVector2::zero;
for (auto v : g_vertices)
{
    geoCenter += v;
}
geoCenter.x /= g_vertices.size();
geoCenter.y /= g_vertices.size();
for (auto& v : g_vertices)
{
    v -= geoCenter;
}
```

Convex hull

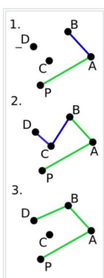
From Wikipedia, the free encyclopedia

In geometry, the **convex hull** or **convex envelope** or **convex closure** of a shape is the smallest convex set that contains it. The convex hull may be defined either as the intersection of all convex sets containing a given subset of a Euclidean space, or equivalently as the set of all convex combinations of points in the subset. For a bounded subset of the plane, the convex hull may be visualized as the shape enclosed by a rubber band stretched around the subset.

Graham scan

From Wikipedia, the free encyclopedia

Graham's scan is a method of finding the convex hull of a finite set of points in the plane with time complexity $O(n \log n)$. It is named after Ronald Graham, who published the original algorithm in 1972.[1] The algorithm finds all vertices of the convex hull ordered along its boundary. It uses a stack to detect and remove concavities in the boundary efficiently.




```
let points be the list of points
let stack = empty_stack()

find the lowest y-coordinate and leftmost point, called P0
sort points by polar angle with P0, if several points have the same polar angle then only keep the farthest

for point in points:
    # pop the last point from the stack if we turn clockwise to reach this point
    while count stack > 1 and ccw(next_to_top(stack), top(stack), point) <= 0:
        pop stack
    push point to stack
end
```