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Dynamics of Mechanical Systems – Year Work 2020/21

Point 1

In order to define an appropriate FEM model for the harbour crane, I compute the maximum circular frequency for our case:

$$\Omega_{\mathrm{MAX}} = 10 \times 2\pi = 62.8\, rad/s\,;$$

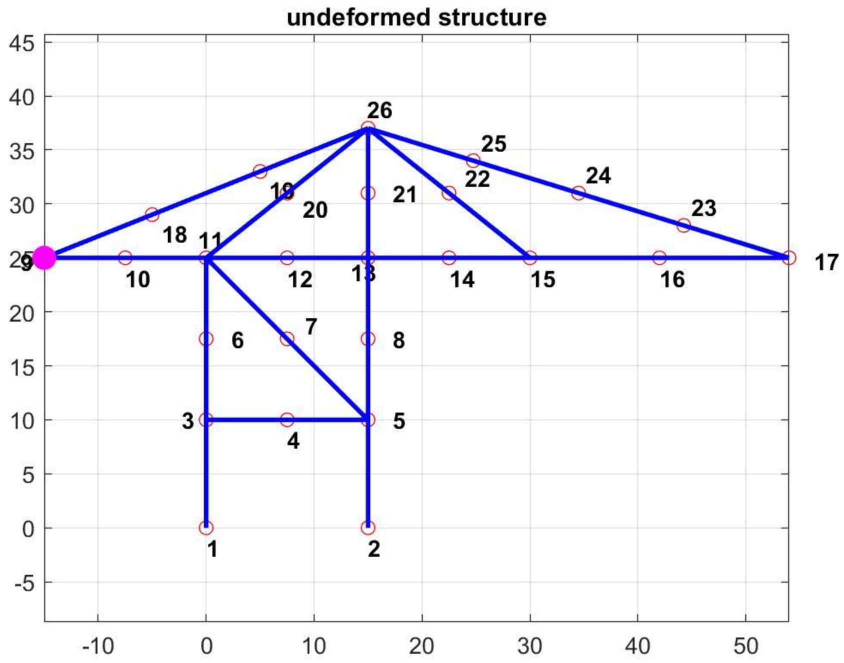
Now I can compute the maximum length of each finite element with the formula:

$$\Omega_{MAX} \ll \omega_k = \left(\frac{\pi}{L_k}\right)^2 \sqrt{\frac{EJ_k}{m_k}} \rightarrow L_k < L_{MAX};$$

which, applying a safety coefficient of 2, leads to:

$$L_{Blue,MAX} = 10.82m; \qquad L_{Green,MAX} = 10.85m; \qquad L_{Red,MAX} = 12.9m;$$

I have used 31 finite elements with 26 nodes in my model, whose undeformed structure plot and .inp file are shown below.



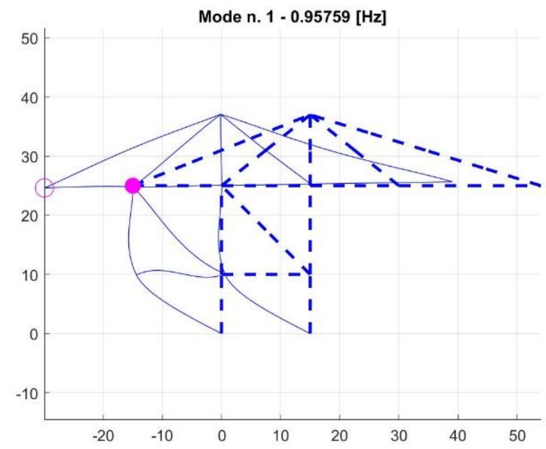
Harbour_Crane.inp

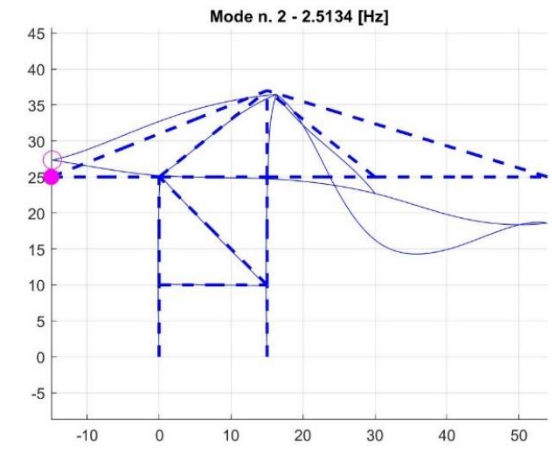
```
! node nr. - boundary conditions codes: x,y,theta    x        y
*NODES
1                              1 1 0     0.0       0.0
2                              1 1 0     15.0      0.0
3                              0 0 0     0.0       10.0
4                              0 0 0     7.5       10.0
```

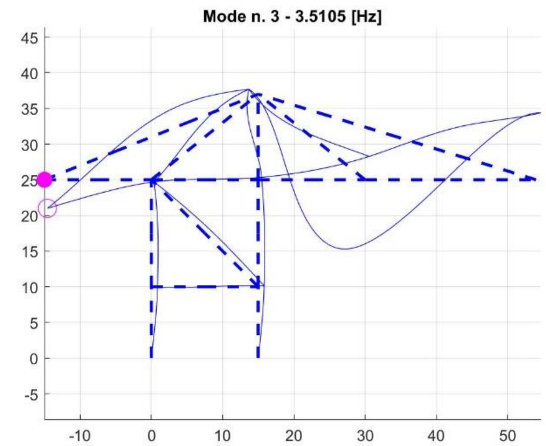
```
5                              0 0 0    15.0      10.0
6                              0 0 0    0.0       17.5
7                              0 0 0    7.5       17.5
8                              0 0 0    15.0      17.5
9                              0 0 0    -15.0     25.0
10                             0 0 0    -7.5      25.0
11                             0 0 0    0.0       25.0
12                             0 0 0    7.5       25.0
13                             0 0 0    15.0      25.0
14                             0 0 0    22.5      25.0
15                             0 0 0    30.0      25.0
16                             0 0 0    42.0      25.0
17                             0 0 0        54.0          25.0
18                             0 0 0    -5.0      29.0
19                             0 0 0    5.0       33.0
20                             0 0 0    7.5       31.0
21                             0 0 0    15.0      31.0
22                             0 0 0    22.5      31.0
23                             0 0 0    44.25     28.0
24                             0 0 0    34.5      31.0
25                             0 0 0    24.75     34.0
26                             0 0 0    15.0      37.0
*ENDNODES

! beams list :
! beam nr.   i-th node nr.  j-th node nr.      mass [kg/m]   EA [N]  EJ [Nm^2]
*BEAMS
1            1             3             200       5.4E9     4.5E8
2            2             5             200       5.4E9     4.5E8
3            3             4             200       5.4E9     4.5E8
4            4             5             200       5.4E9     4.5E8
5            3             6             200       5.4E9     4.5E8
6            6             11            200       5.4E9     4.5E8
7            5             8             200       5.4E9     4.5E8
8            8             13            200       5.4E9     4.5E8
9            5             7             200       5.4E9     4.5E8
10           7             11            200       5.4E9     4.5E8
11           9             10            312       8.2E9     1.40E9
12           10            11            312       8.2E9     1.40E9
13           11            12            312       8.2E9     1.40E9
14           12            13            312       8.2E9     1.40E9
15           13            14            312       8.2E9     1.40E9
16           14            15            312       8.2E9     1.40E9
17           15            16            312       8.2E9     1.40E9
18           16            17            312       8.2E9     1.40E9
19           9             18            90.0      2.4E9     2.0E8
20           18            19            90.0      2.4E9     2.0E8
21           19            26            90.0      2.4E9     2.0E8
22           11            20            90.0      2.4E9     2.0E8
23           20            26            90.0      2.4E9     2.0E8
24           13            21            90.0      2.4E9     2.0E8
25           21            26            90.0      2.4E9     2.0E8
26           15            22            90.0      2.4E9     2.0E8
27           22            26            90.0      2.4E9     2.0E8
28           17            23            90.0      2.4E9     2.0E8
29           23            24            90.0      2.4E9     2.0E8
30           24            25            90.0      2.4E9     2.0E8
31           25            26            90.0      2.4E9     2.0E8
*ENDBEAMS
! Mass list
! mass nr.   i-th node nr.mass[kg]            Moment of inertia[kgm^2]
*MASSES
1            9             2000          320
*ENDMASSES

! alpha and beta values to define the damping matrix
*DAMPING
0.1 2.0e-4
```

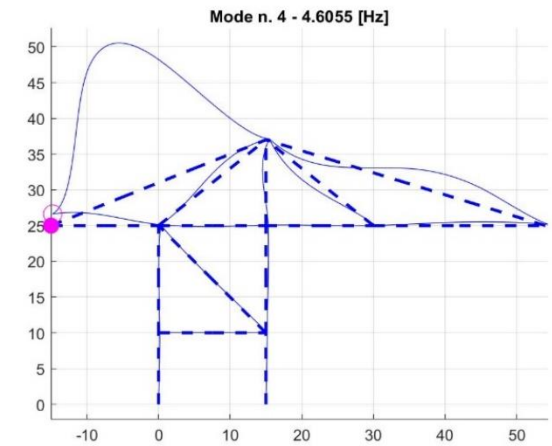
Point 2

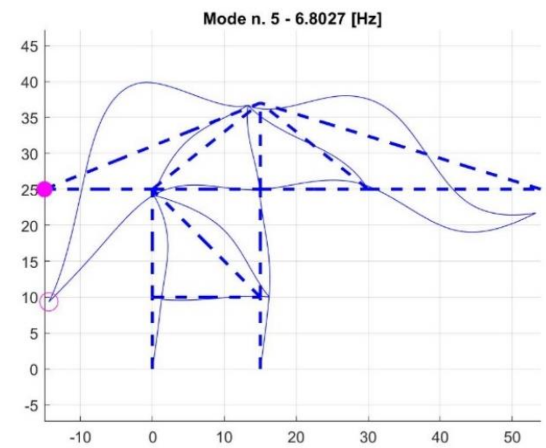
The natural frequencies lower than 10 Hz are computed using “dmb_fem2” function on the Harbour_Crane.inp file. The results are shown below.

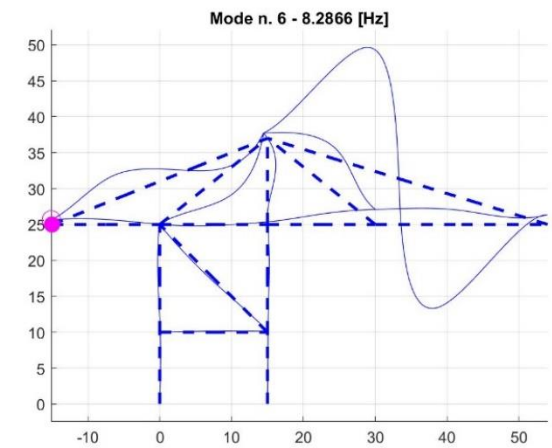






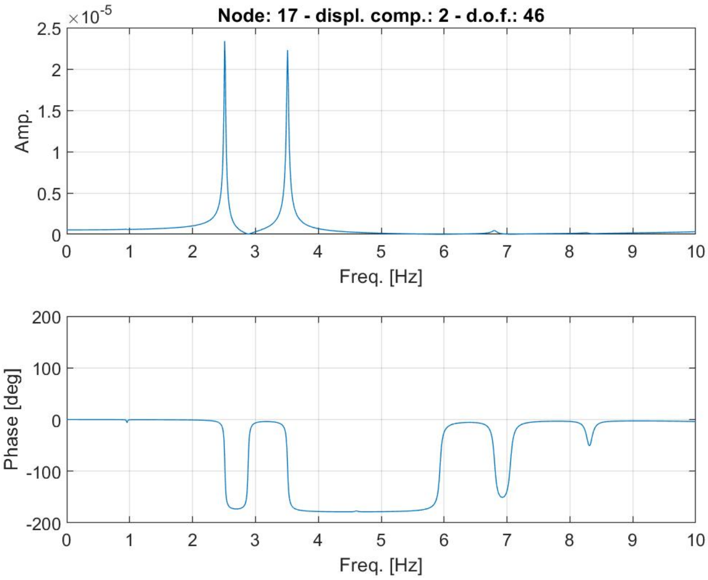






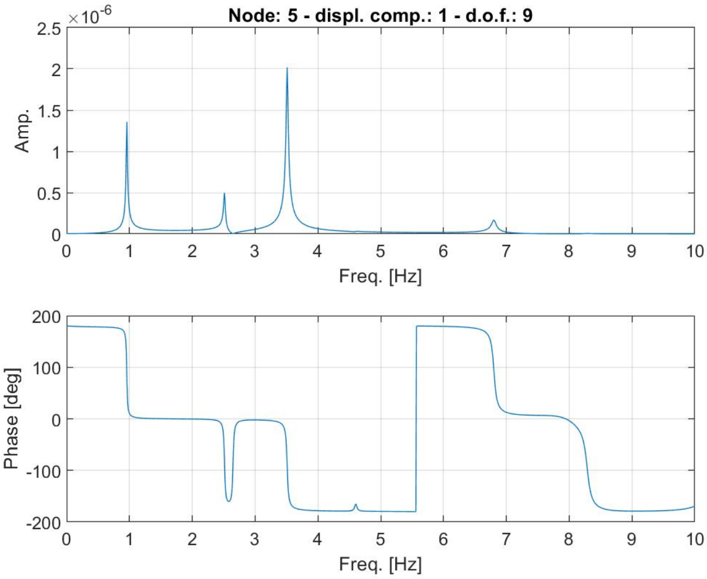
Point 3

By applying again MATLAB function “dmb_fem2”, I managed to compute the Frequency Response Functions of points 3.a) and 3.b).



3.a

Amplitude and Phase of FRF from vertical Force at point A (node 17) to vertical displacement of the same point



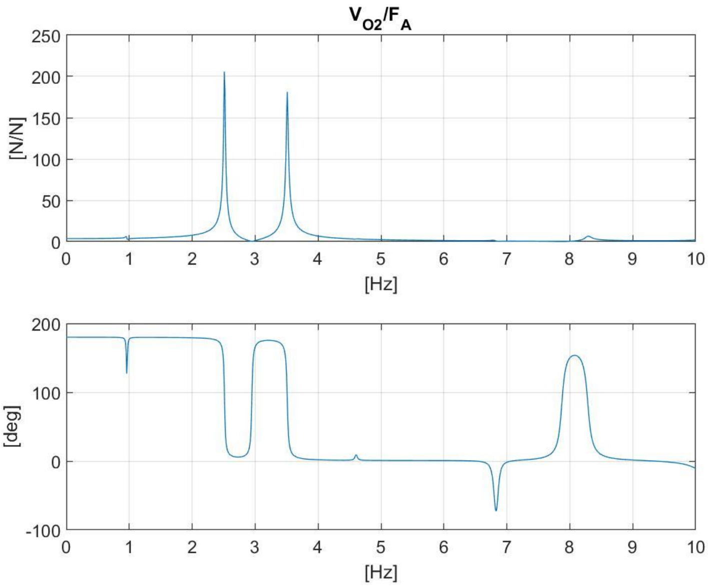
3.b

Amplitude and Phase of FRF from vertical Force at point A (node 17) to horizontal displacement of point B (node 5)

Nevertheless, in order to compute the Frequency Response Functions of point 3.c) and 3.d), the following MATLAB script has been used:

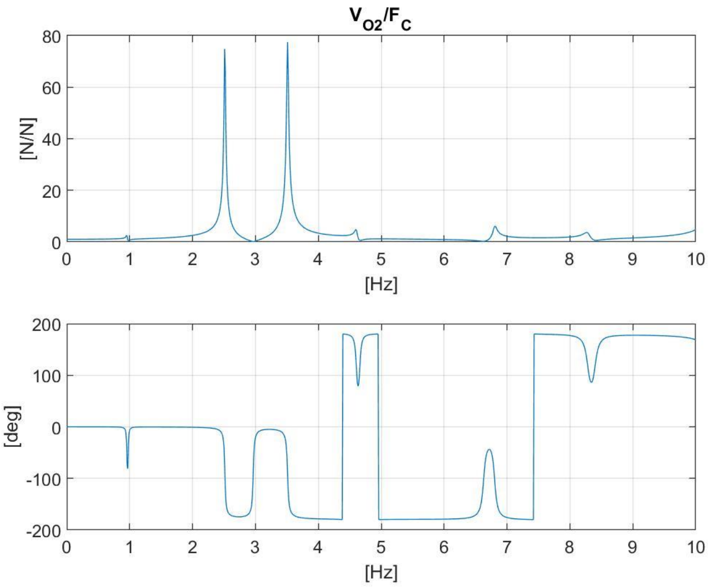
```
Point_3.m

close all
clear all
df = 0.01;

% Extraction & definition of matrices
load('Crane_mkr');

MFF = M(1:74,1:74);
MCF = M(75:78,1:74);
MFC = M(1:74,75:78);
```

```
MCC = M(75:78,75:78);

KFF = K(1:74,1:74);
KCF = K(75:78,1:74);
KFC = K(1:74,75:78);
KCC = K(75:78,75:78);

CFF = R(1:74,1:74);
CCF = R(75:78,1:74);
CFC = R(1:74,75:78);
CCC = R(75:78,75:78);

% FRF VO2 from input in C
Q0_F = zeros(74,1);
Q0_F(22,1) = 1;
vett_f = 0:df:10;
for k = 1:length(vett_f)
    ome=vett_f(k)*2*pi;
    A=-ome^2*MFF+i*ome*CFF+KFF;
    x0_F = A\Q0_F;
    Q0_C = (-ome^2*MCF+i*ome*CCF+KCF) * x0_F;
    VO2= Q0_C(4);
    mod(k)=abs(VO2);
    fas(k)=angle(VO2);
end

figure
subplot 211;plot(vett_f,mod);grid;xlabel('[Hz]');ylabel('[N/N]');title('V_{O2}/F_C');
subplot 212;plot(vett_f,180/pi*fas);grid;xlabel('[Hz]');ylabel('[rad/N]');


% FRF VO2 From input in A
i = sqrt(-1);
Q0_F = zeros(74,1);
Q0_F(46,1) = 1;
vett_f = 0:df:10;
for k = 1:length(vett_f)
    ome=vett_f(k)*2*pi;
    A=-ome^2*MFF+i*ome*CFF+KFF;
    x0_F = A\Q0_F;
    Q0_C = (-ome^2*MCF+i*ome*CCF+KCF) * x0_F;
    VO2= Q0_C(4);
    mod(k)=abs(VO2);
    fas(k)=angle(VO2);
end

figure
subplot 211;plot(vett_f,mod);grid;xlabel('[Hz]');ylabel('[N/N]');title('V_{O2}/F_A');
subplot 212;plot(vett_f,180/pi*fas);grid;xlabel('[Hz]');ylabel('[rad/N]');
```



3.c

Amplitude and Phase of FRF from vertical Force at point A (node 17) to vertical component of constraint force in O2 (node 2)



3.d

Amplitude and Phase of FRF from vertical Force at point C (node 9) to vertical component of constraint force in O2 (node 2)

Point 4

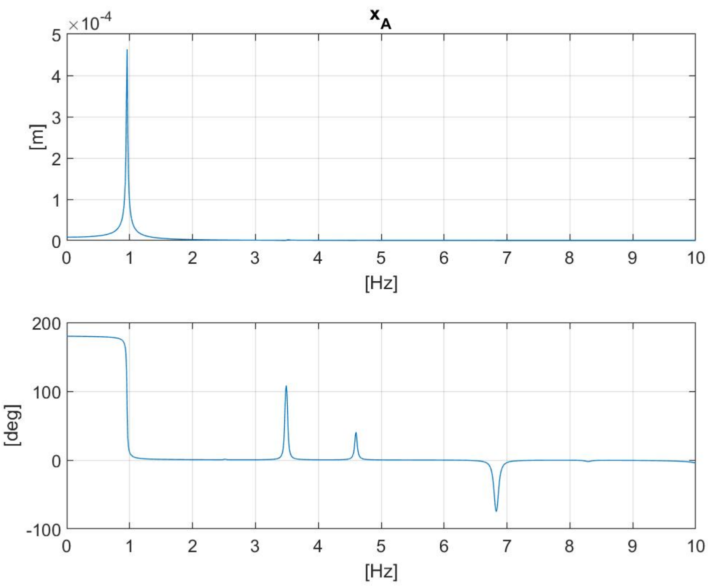
In order to compute the Frequency Response Functions with a distributed force as input, the following shape functions equations have been used:

$$\underline{f}_v(\xi) = \begin{Bmatrix} 0 \\ f_{v1}(\xi) \\ f_{v2}(\xi) \\ 0 \\ f_{v3}(\xi) \\ f_{v4}(\xi) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2\left(\frac{\xi}{L_k}\right)^3 - 3\left(\frac{\xi}{L_k}\right)^2 + 1 \\ L_k\left[\left(\frac{\xi}{L_k}\right)^3 - 2\left(\frac{\xi}{L_k}\right)^2 + \frac{\xi}{L_k}\right] \\ 0 \\ -2\left(\frac{\xi}{L_k}\right)^3 + 3\left(\frac{\xi}{L_k}\right)^2 \\ L_k\left[\left(\frac{\xi}{L_k}\right)^3 - \left(\frac{\xi}{L_k}\right)^2\right] \end{Bmatrix}$$

By integrating along the green Finite Elements' length the equations above, it is possible to apply the same procedure as the one used for concentrated forces.

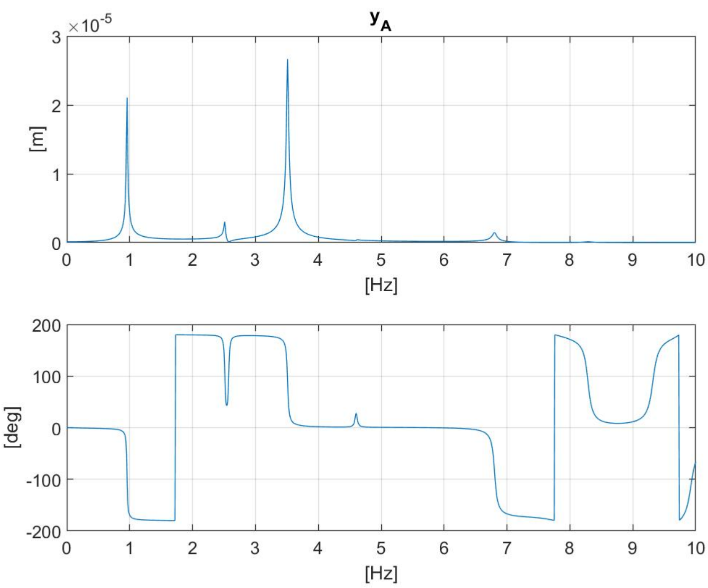
```
Point_4.m

close all
clear all
df = 0.01;
p = 1;
H2 = 15;

% Extraction & definition of matrices
load('Crane_mkr');

MFF = M(1:74,1:74);
MCF = M(75:78,1:74);
MFC = M(1:74,75:78);
MCC = M(75:78,75:78);

KFF = K(1:74,1:74);
KCF = K(75:78,1:74);
KFC = K(1:74,75:78);
KCC = K(75:78,75:78);

CFF = R(1:74,1:74);
CCF = R(75:78,1:74);
CFC = R(1:74,75:78);
CCC = R(75:78,75:78);

% FRFs of x_A, y_A & y_B from input distributed force p = 1 N/m

Q0_F = zeros(74,1);
Q0_F(9,1) = -p*(H2/2)/2; %-3.75
Q0_F(11,1) = -p*((H2/2)^2)/12; %-4.6875
Q0_F(18,1) = -p*H2/2;  %-7.5
Q0_F(33,1) = -p*(H2/2)/2;  %-3.75
Q0_F(35,1) = p*((H2/2)^2)/12; %4.6875
vett_f = 0:df:10;
for k = 1:length(vett_f)
    ome=vett_f(k)*2*pi;
    A=-ome^2*MFF+i*ome*CFF+KFF;
    x0_F = A\Q0_F;
```

```
    x_A = x0_F(45);
    y_A = x0_F(46);
    y_C = x0_F(22);
    mod1(k)=abs(x_A);
    fas1(k)=angle(x_A);
    mod2(k)=abs(y_A)
    fas2(k)=angle(y_A)
    mod3(k)=abs(y_C)
    fas3(k)=angle(y_C)
end

figure
subplot 211;plot(vett_f,mod1);grid;xlabel('[Hz]');ylabel('[m]');title('x_{A}');
subplot 212;plot(vett_f,180/pi*fas1);grid;xlabel('[Hz]');ylabel('[deg]');

figure
subplot 211;plot(vett_f,mod2);grid;xlabel('[Hz]');ylabel('[m]');title('y_{A}');
subplot 212;plot(vett_f,180/pi*fas2);grid;xlabel('[Hz]');ylabel('[deg]');

figure
subplot 211;plot(vett_f,mod3);grid;xlabel('[Hz]');ylabel('[m]');title('y_{C}');
subplot 212;plot(vett_f,180/pi*fas3);grid;xlabel('[Hz]');ylabel('[deg]');
```



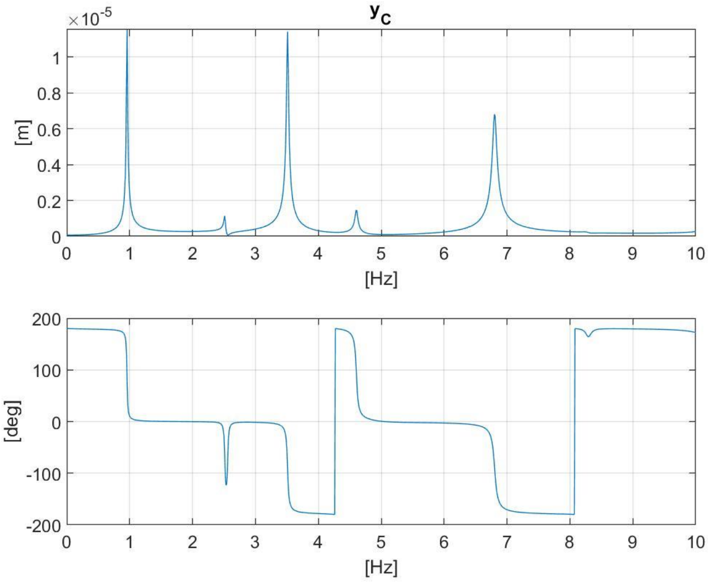
4.a

Amplitude and Phase of FRF from a Distributed Force between point B (node 5) and point D (node 13) to horizontal displacement of point A (node 17)



4.b

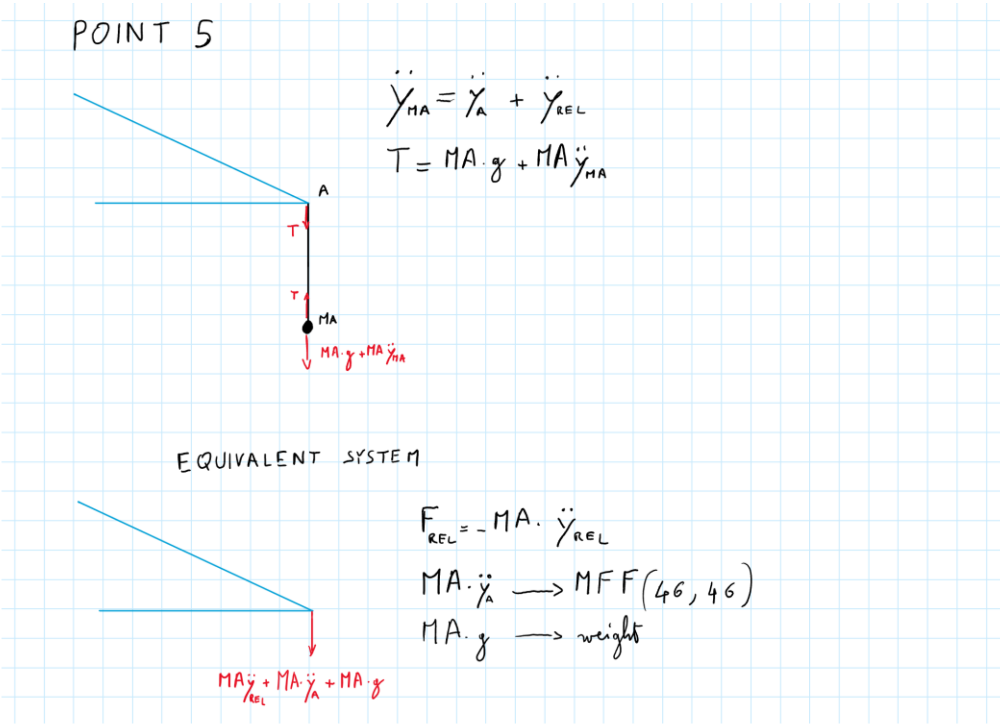
Amplitude and Phase of FRF from a Distributed Force between point B (node 5) and point D (node 13) to vertical displacement of point A (node 17)



4.c

Amplitude and Phase of FRF from a Distributed Force between point B (node 5) and point D (node 13) to vertical displacement of point C (node 9)

Point 5

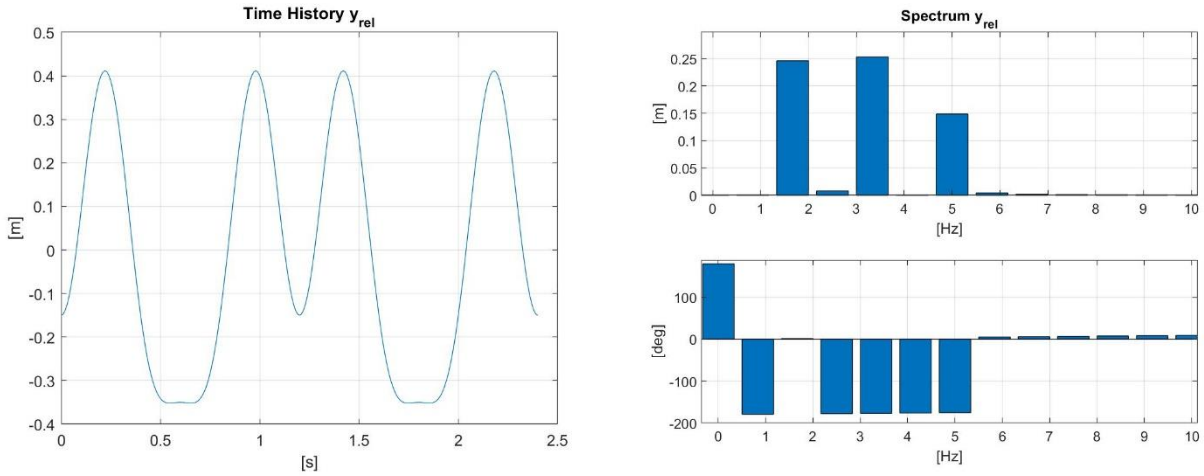


In order to solve point 5 I have used the following MATLAB script:

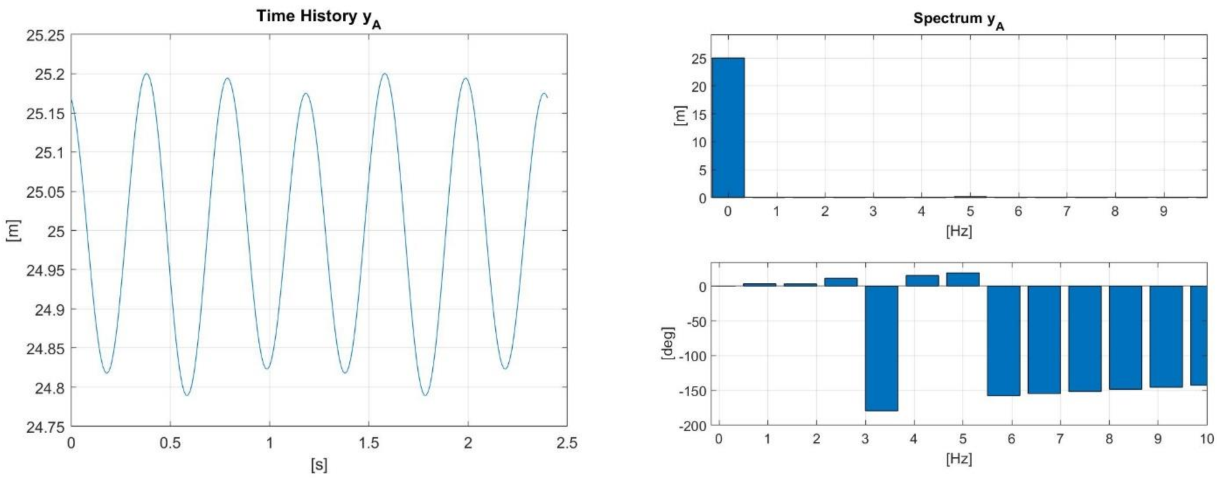
```
Point_5.m

close all
clear all
T = 1.2;
MA = 800;
A_th = [0.25,0.25,0.15];
phi_th = [0, pi ,pi];
ome_th = [1/T,2/T,3/T];

% Extraction & definition of matrices
load('Crane_mkr');

MFF = M(1:74,1:74);
MCF = M(75:78,1:74);
MFC = M(1:74,75:78);
MCC = M(75:78,75:78);
MFF(46,46)  = MFF(46,46)  + MA %additonal term due to inertia of MA

KFF = K(1:74,1:74);
KCF = K(75:78,1:74);
KFC = K(1:74,75:78);
KCC = K(75:78,75:78);

CFF = R(1:74,1:74);
CCF = R(75:78,1:74);
CFC = R(1:74,75:78);
CCC = R(75:78,75:78);

% compute y_rel & y_A

i = sqrt(-1);
vett_t = 0:0.01:2*T
vett_y_A=zeros(1,length(vett_t));
```

```
vett_y_rel=zeros(1,length(vett_t));

for iarm=1:3
    ome=2*pi*ome_th(iarm);
    A=-ome^2*MFF+i*ome*CFF+KFF;
    y_rel = A_th(iarm)*exp(i*phi_th(iarm));
    F_rel = -MA*(-ome^2*y_rel);
    Q0_F = zeros(74,1);
    Q0_F(46) = F_rel;
    x0_F = A\Q0_F;
    y_A = x0_F(46);
    vett_y_A=vett_y_A+abs(y_A)*cos(ome*vett_t+angle(y_A));
    vett_y_rel=vett_y_rel+abs(y_rel)*cos(ome*vett_t+angle(y_rel));
end
vett_y_Atot = 25 + vett_y_A;

% Spectrum & Time History y_A total

fft_yA=fft(vett_y_Atot);
N=length(vett_y_Atot);
df=1/T;
fmax=(N/2-1)*df;
vett_freq=0:df:fmax;
modf(1)=1/N*abs(fft_yA(1));
modf(2:N/2)=2/N*abs(fft_yA(2:N/2));
fasf(1:N/2)=angle(fft_yA(1:N/2));

figure
subplot 211;bar(vett_freq,modf);grid;xlabel('[Hz]');ylabel('[m]');title('Spectrum y_A');
subplot 212;bar(vett_freq,180/pi*fasf);grid;xlabel('[Hz]');ylabel('[deg]');

figure
plot(vett_t,vett_y_Atot);grid;xlabel('[s]');ylabel('[m]');title('Time History y_A');

% Spectrum & Time History y_rel

fft_yrel=fft(vett_y_rel);
N=length(vett_y_rel);
df=1/T;
fmax=(N/2-1)*df;
vett_freq=0:df:fmax;
modf(1)=1/N*abs(fft_yrel(1));
modf(2:N/2)=2/N*abs(fft_yrel(2:N/2));
fasf(1:N/2)=angle(fft_yrel(1:N/2));

figure
subplot 211;bar(vett_freq,modf);grid;xlabel('[Hz]');ylabel('[m]');title('Spectrum
y_{rel}');
subplot 212;bar(vett_freq,180/pi*fasf);grid;xlabel('[Hz]');ylabel('[deg]');

figure
plot(vett_t,vett_y_rel);grid;xlabel('[s]');ylabel('[m]');title('Time History y_{rel}');
```

Time History and Spectrum of relative vertical displacement y_rel.

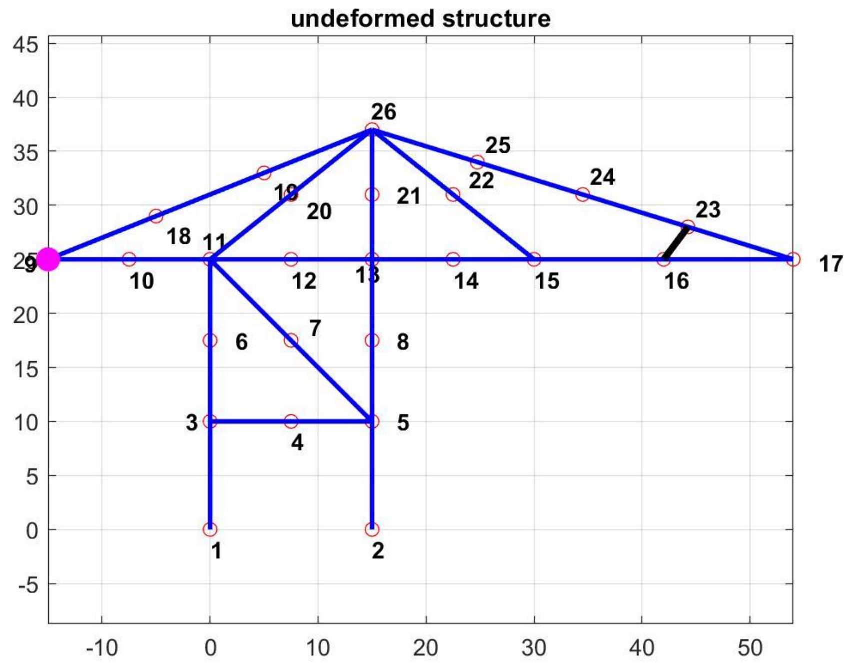


Time History and Spectrum of absolute vertical displacement of point A (node 17).



Point 6

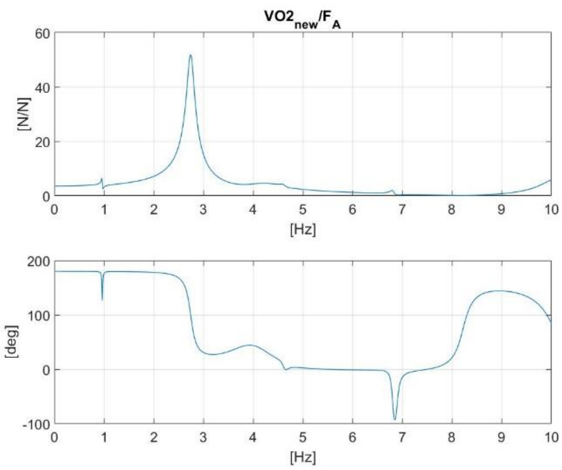
For the solution of point 6, I have chosen to add a crane pile damper, with k = 1.0e6 and c = 1.0e5, between node 16 and node 23, owing that their displacements are great for the modes excited. The “new” structure has been obtained with Harbour_Crane_P6.inp file and it can be seen below.

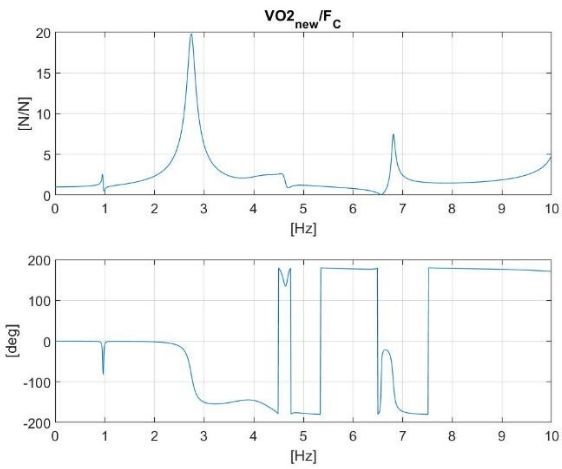



```
Harbour_Crane_P6.inp

*NODES
1                              1 1 0    0.0       0.0
2                              1 1 0    15.0      0.0
3                              0 0 0    0.0       10.0
4                              0 0 0    7.5       10.0
5                              0 0 0    15.0      10.0
6                              0 0 0    0.0       17.5
7                              0 0 0    7.5       17.5
8                              0 0 0    15.0      17.5
9                              0 0 0    -15.0     25.0
10                             0 0 0    -7.5      25.0
11                             0 0 0    0.0       25.0
12                             0 0 0    7.5       25.0
13                             0 0 0    15.0      25.0
14                             0 0 0    22.5      25.0
15                             0 0 0    30.0      25.0
16                             0 0 0    42.0      25.0
17                             0 0 0    54.0      25.0
18                             0 0 0    -5.0      29.0
19                             0 0 0    5.0       33.0
20                             0 0 0    7.5       31.0
21                             0 0 0    15.0      31.0
22                             0 0 0    22.5      31.0
23                             0 0 0    44.25     28.0
24                             0 0 0    34.5      31.0
25                             0 0 0    24.75     34.0
26                             0 0 0    15.0      37.0
*ENDNODES

*BEAMS
1             1             3             200       5.4E9     4.5E8
2             2             5             200       5.4E9     4.5E8
3             3             4             200       5.4E9     4.5E8
```

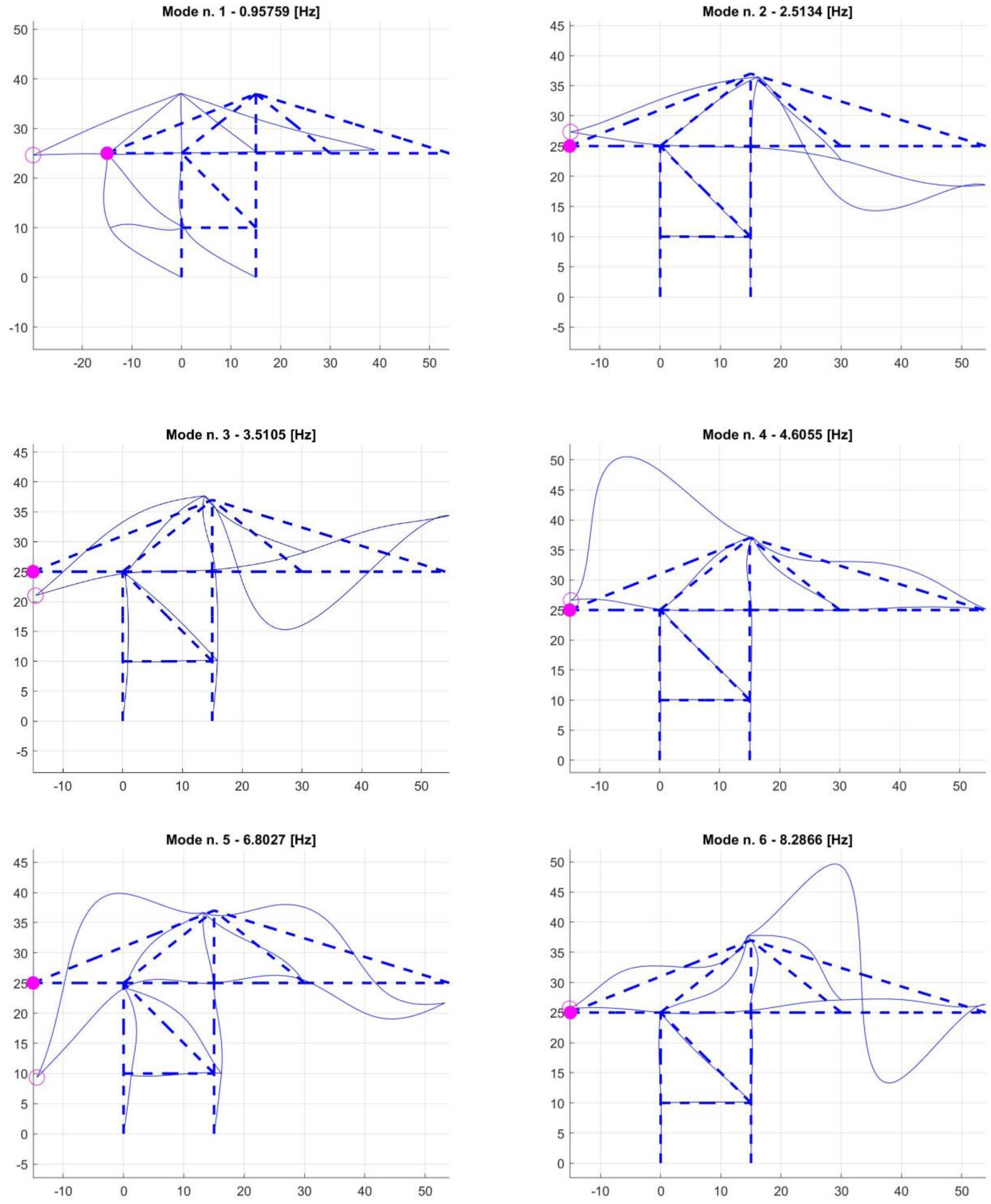
```
4             4             5             200       5.4E9      4.5E8
5             3             6             200       5.4E9      4.5E8
6             6             11            200       5.4E9      4.5E8
7             5             8             200       5.4E9      4.5E8
8             8             13            200       5.4E9      4.5E8
9             5             7             200       5.4E9      4.5E8
10            7             11            200       5.4E9      4.5E8
11            9             10            312       8.2E9      1.40E9
12            10            11            312       8.2E9      1.40E9
13            11            12            312       8.2E9      1.40E9
14            12            13            312       8.2E9      1.40E9
15            13            14            312       8.2E9      1.40E9
16            14            15            312       8.2E9      1.40E9
17            15            16            312       8.2E9      1.40E9
18            16            17            312       8.2E9      1.40E9
19            9             18            90.0      2.4E9      2.0E8
20            18            19            90.0      2.4E9      2.0E8
21            19            26            90.0      2.4E9      2.0E8
22            11            20            90.0      2.4E9      2.0E8
23            20            26            90.0      2.4E9      2.0E8
24            13            21            90.0      2.4E9      2.0E8
25            21            26            90.0      2.4E9      2.0E8
26            15            22            90.0      2.4E9      2.0E8
27            22            26            90.0      2.4E9      2.0E8
28            17            23            90.0      2.4E9      2.0E8
29            23            24            90.0      2.4E9      2.0E8
30            24            25            90.0      2.4E9      2.0E8
31            25            26            90.0      2.4E9      2.0E8
*ENDBEAMS

*MASSES
1             9             2000          320
*ENDMASSES

*SPRINGS
1      16     23     0      1e7    0      0      1e5    0
*ENDSPRINGS

*DAMPING
0.1 2.0e-4
```

The figures below represent the Frequency Response Functions of the vertical component of constraint force in O2 (node 2) with respect to an input vertical force in A (node 17) and an input vertical force in C (node 9) respectively. It can be noticed that the peaks of both diagrams are at about 25% of the value seen in the previous non-modified structure.





And here are shown the “new” modal shapes of the system after the implementation of the damper:



Other possible solution for Point 6 are shown below, even if they are less efficient (due to dimensional & economical reasons) than the one exposed above.