Libraries for Scientific Computing

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Including and adapting material provided by
Dr. William Sawyer, Dr. Karl Rupp, Dr. Michael Heroux, Dr. Dimitar Lukarski, Prof. Stan Tomov, Dr. Peter Messmer

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Università della Svizzera italiana | Faculty of Informatics | Institute of Computational Science ICS

ETH zürich

Platform for Advanced Scientific Computing

CSCS
Centro Svizzero di Calcolo Scientifico
Swiss National Supercomputing Centre

This Tutorial

- Some high-level discussion of libraries for scientific computing
- Some more specific discussion of numerical libraries, focusing on GPU/MIC/accelerator/coprocessor-enabled libraries
- A quick tour of some available libraries
- We will aim to finish a bit early, and will begin the next tutorial. This will give some extra flexibility during the break if anyone has issues compiling the demo code.

What is a software library?

- A set of related functions to accomplish tasks required by more than one application
- Written and used by different people
- Relies on an application public interface (API)
- Typically versioned, documented, distributed, licensed

Libraries for Scientific Computing: Pros

```
#include <wheellib.h>
```

- Don't reinvent the wheel
- Don't reimplement the wheel
- Use the wheel even if you don't understand it or know how to optimize it!
- Leverage the work of experts
- Focus on your part of the "stack" to do science
- Experiment quickly
- Avoid "lock in" with respect to data structures and algorithms (maybe a wheel wasn't the right choice)
- Open source or community projects allow
  - Consolidation of efforts from many people
  - Continuity on time scales longer than projects/PhDs/grants/careers
  - Collaborative efforts good for science

Libraries for Scientific Computing: Cons

```
*** WHEELLIB ERROR ***
Fatal: Moving wheels only supported in Fortran 66.
To file a bug report, email wheellib4ever@aol.com
**********************
```

- Learning curves
- Versioning, changing APIs
- Bugs that someone else must fix
- Syntax, design choices
- Lack of documentation (or local experts)
- Oversold software, vaporware
- The scientific risks of using algorithms (or hardware) that you don't understand
- "Hell is other people['s code]"

Disclaimers

- I will not cover all libraries available, even within the subfields I discuss here.
- The inclusion or exclusion of a library (including PETSc, used as the basis for the extended tutorial later) is as much a function of my familiarity with it than its absolute quality. I will have missed some important ones, so let me know for the next time this lecture is given!

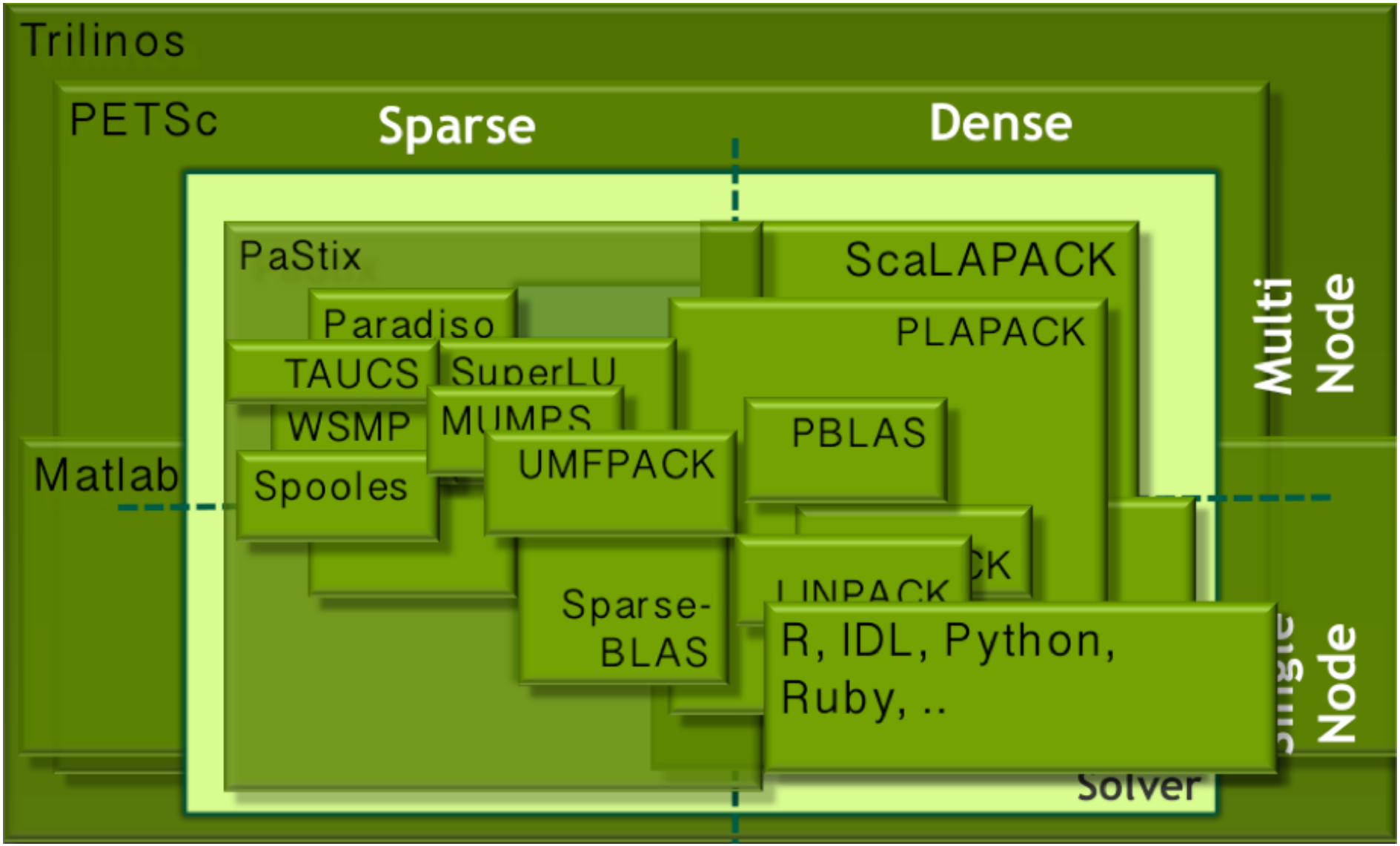
Section 1

Linear Algebra Libraries

Linear Algebra Libraries

- Operations involving vector spaces and linear operators are involved in most, if not all, scientific codes
- As such, for decades [1], libraries have existed to provide efficient, reusable abstractions and implementations to perform these operations
- Functionality includes
  - Vector operations
  - Dense matrix operations
  - Sparse matrix operations
  - Linear solvers: dense/sparse, exact/approximate, direct/iterative
  - Eigenanalysis
- As ubiquitious low-level operations that often form the majority of the computational effort, adaptation and optimization for different environments is important
  - Shared memory
  - Distributed memory
  - Accelerators / Coprocessors/ Throughput-oriented devices

[1] BLAS originated in 1979, for example


Trilinos
PETSc
Sparse
Dense
PaStix
ScaLAPACK
Paradiso
PLAPACK
TAUCS
SuperLU
WSMP
MUMPS
PBLAS
Matlab
Spooles
UMFPACK
Sparse-
BLAS
R, IDL, Python,
Ruby, ..
Solver
Multi
Node
Node

Dense Linear Algebra

- You almost certainly use these operations already
- You likely leverage (perhaps indirectly) libraries to do so
- Typical Operations include
  - Elementary elementwise operations on matrices and vectors : $A + B$,etc.
  - Norms, inner products, matrix-matrix multiplies, matrix-vector multiplies : $||x||_2, \langle x, y \rangle, AB, Ax$, etc.
  - Cholesky factorization: $A = LL^T$,L lower triangular
  - QR decomposition: $A = QR$, $Q^H Q = I$, R upper triangular
  - LU factorization: $A = P^T LU$, P permutation, L lower tri., R upper tri.
  - Triangular solves $y = L^{-1}x$
  - Eigenvalue decomposition : $Ax = \lambda x \iff A = Q\Lambda Q^T$, $Q^H Q = I$
  - Singular value decomposition $A = U\Sigma V^H$, $U^H H = I$, $V^H V = I$

BLAS and LAPACK

- Fundamental numerical libraries
- Many implementations, optimized for different architectures
- BLAS
  - vector operations (BLAS-1)
  - matrix-vector operations (BLAS-2)
  - matrix-matrix operations (BLAS-3)
- LAPACK
  - Matrix factorization and linear system solution
  - Least squares
- SCALAPACK : distributed memory LAPACK (includes BLACS as a communication layer)
- Available implementations at CSCS include the following.
  - Intel's math kernel library (MKL) includes BLAS and LAPACK, available with `PrgEnv-intel` [2]
  - Cray's `libsci` : heavily optimized BLAS, LAPACK, SCALAPACK within the Cray, PGI, and GNU environments.

[2] if you are an advanced MKL user and want the raw path, note that `MKLROOT` will be set in your environment

Elemental

- C++11 linear algebra library
- libelemental.org
- Depends on BLAS, LAPACK, and MPI
- Includes libFlame (Multithreaded dense linear algebra, independent of LAPACK)
- Competitive with other distributed-memory packages[3]

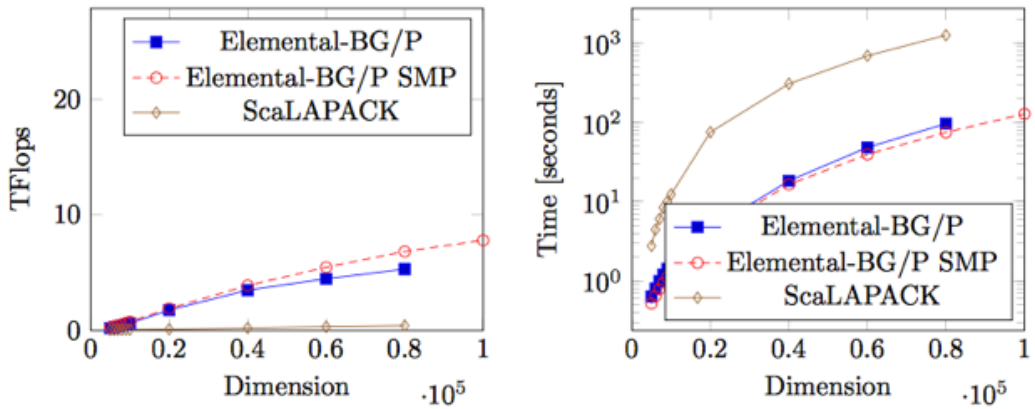


Figure 5: Real double-precision reduction of generalized eigenvalue problem to symmetric standard form on 8192 cores.

[3] Jack Poulson, Bryan Marker, Robert A. van de Geijn, Jeff R. Hammond, and Nichols A. Romero. "Elemental: A New Framework for Distributed Memory Dense Matrix Computations". In: *ACM Trans. Math. Softw.* 39.2 (Feb. 2013), 13:1–13:24. ISSN: 0098-3500. DOI: 10.1145/2427023.2427030. URL: http://doi.acm.org/10.1145/2427023.2427030.

Eigen

- eigen.tuxfamily.org
- A template-only C++ dense and sparse linear algebra library (not distributed-memory)
  - Pro: efficent, natural syntax
  - Con: can be bewildering to debug, source code opaque
- Interfaces with ViennaCL, MKL, and other libraries discussed here

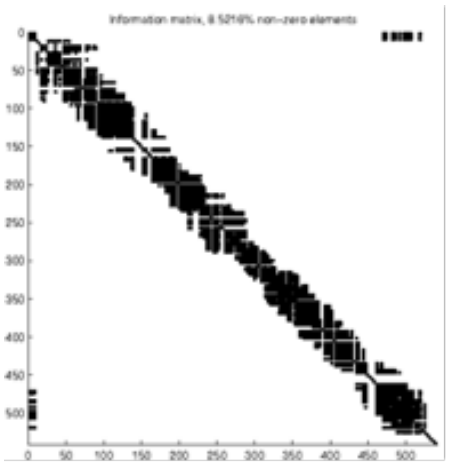
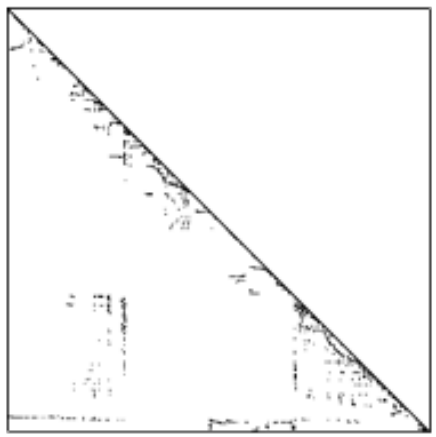
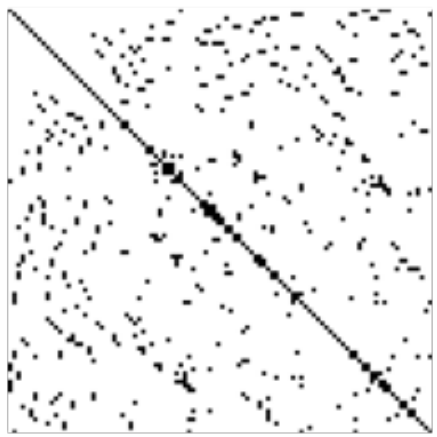
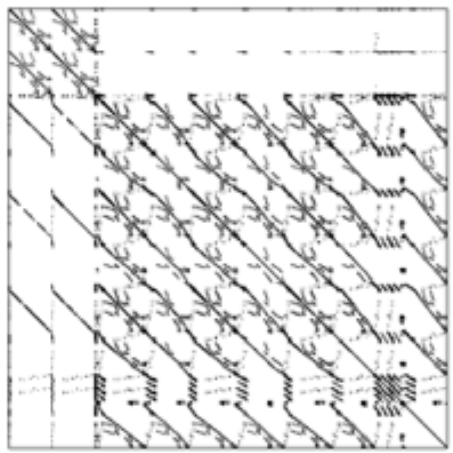
```
Matrix2d mat;
mat << 1, 2,
       3, 4;
Vector2d u(-1,1), v(2,0);
std::cout << "mat*mat:\n" << mat*mat << std::endl;
std::cout << "mat*u:\n" << mat*u << std::endl;
std::cout << "u^T*mat:\n" << u.transpose()*mat << std::
    endl;
std::cout << "u^T*v:\n" << u.transpose()*v << std::endl;
std::cout << "u*v^T:\n" << u*v.transpose() << std::endl;
```

Section 2

Sparse Linear Algebra

Sparse Linear Algebra

- Use cases: sparse PDE, big sparse data
- Fundamentally very different from dense linear algebra
  - Operations are difficult to vectorize
  - Typically limited by data movement (memory bandwidth), not floating-point performance
  - Any operator which can be applied (hence potentially inverted) in linear time must be sparse, and most sparse linear algebra libraries are aimed at large systems

http://math.nist.gov/MatrixMarket/

Sparse Direct Solvers

$$A = LU, \quad A = LDL^T$$

- Factor a matrix into a produce of easy-to-invert matrices
- Modern libraries apply to a wide range of matrices, approaching true "black box" status
- Efficent for repeated solves
- Suboptimal scaling and entry-dependent[4] factorization time and storage
- Challenging to parallelize
- For large-enough systems, eventually beaten by optimally-scaling methods (iterative and/or multilevel algorithms[5])

[4] and implementation-dependent

[5] Though those methods often involve sparse direct solvers, and multi-level sparse direct methods exist

Popular Distributed-Memory Sparse Direct Solver Packages

- MUMPS
- SuperLU
- PASTIX
- PARDISO (and MKL PARDISO)
- SuiteSparse UMFPACK
- SPOOLES
- WSMP

- Amongst optimized sparse direct solvers, there is typically a tradeoff between robustness (more elaborate pivoting and iterative refinement) and speed [6]

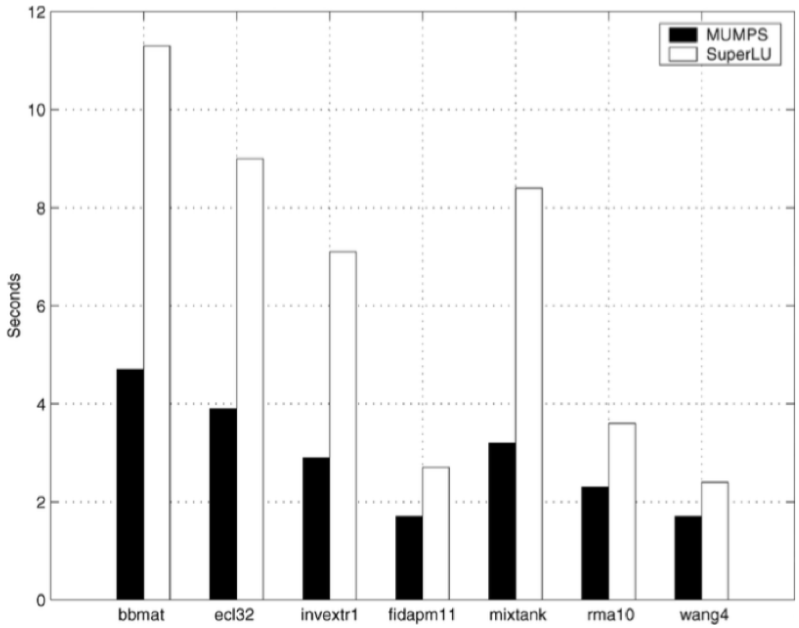
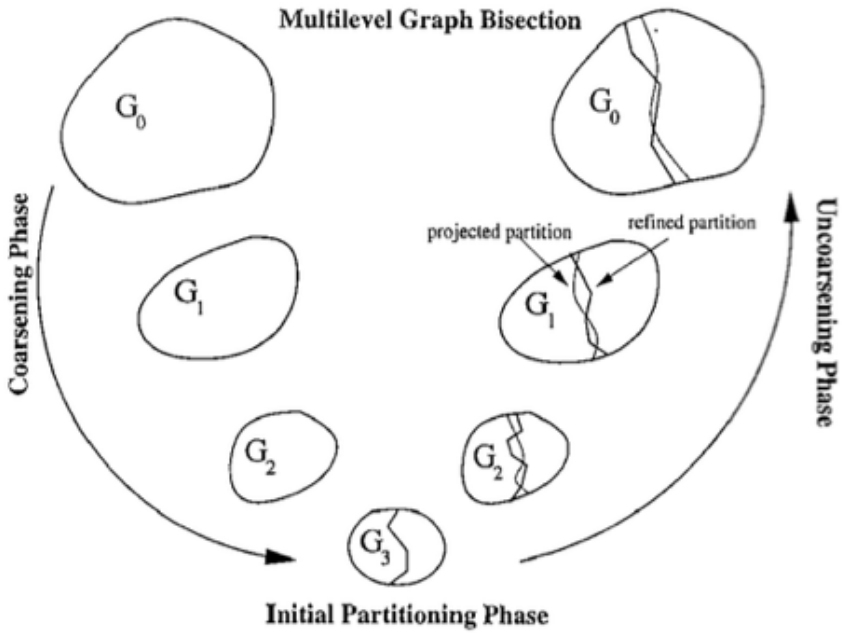


Fig. 5. Time comparison of the analysis phases of MUMPS and SuperLU. MC64 preprocessing is *not* used and AMD ordering *is* used.

[6] Patrick R. Amestoy, Iain S. Duff, Jean-Yves L'excellent, and Xiaoye S. Li. "Analysis and Comparison of Two General Sparse Solvers for Distributed Memory Computers". In: *ACM Trans. Math. Softw.* 27.4 (Dec. 2001), pp. 388–421. ISSN: 0098-3500. DOI: 10.1145/504210.504212. URL: http://doi.acm.org/10.1145/504210.504212

PARDISO and MKL PARDISO

- pardiso-project.org
- Fortran, using OpenMP and OpenMPI
- Sparse direct solvers and related algorithms such as efficient evaluation of Schur complements
- Developed locally by Olaf Schenk and his group at USI.



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[7] Olaf Schenk, Klaus Gärtner, and Wolfgang Fichtner. "Efficient sparse LU factorization with left-right looking strategy on shared memory multiprocessors". In: *BIT Numerical Mathematics* 40.1 (2000), pp. 158–176

Section 3

Iterative Solvers

Iterative Solvers

- Known for very long time (first methods with Gauss/Jacobi, CG invented in the 1950s)
- For an important class of discretized PDEs, Krylov methods and/or multilevel methods are the only know scalable ($O(N)$ time to solution) methods of solution

1: **function** PCG(A, M^{-1}, b, x_0)
2: $r_0 \leftarrow b - Ax_0$
3: $u_0 \leftarrow M^{-1}r_0$
4: $p_0 \leftarrow u_0$
5: $s_0 \leftarrow Ap_0$
6: $\gamma_0 \leftarrow \langle u_0, r_0 \rangle$
7: $\eta_0 \leftarrow \langle s_0, p_0 \rangle$
8: $\alpha_0 \leftarrow \gamma_0/\eta_0$
9: **for** $i = 1, 2, \ldots$ **do**
10: $x_i \leftarrow x_{i-1} + \alpha_{i-1}p_{i-1}$
11: $r_i \leftarrow r_{i-1} - \alpha_{i-1}s_{i-1}$
12: $u_i \leftarrow B(r_i)$
13: $\gamma_i \leftarrow \langle u_i, r_i \rangle$
14: $\beta_i \leftarrow \gamma_i/\gamma_{i-1}$
15: $p_i \leftarrow u_i + \beta_i p_{i-1}$
16: $s_i \leftarrow Ap_i$
17: $\eta_i \leftarrow \langle s_i, p_i \rangle$
18: $\alpha_i \leftarrow \gamma_i/\eta_i$

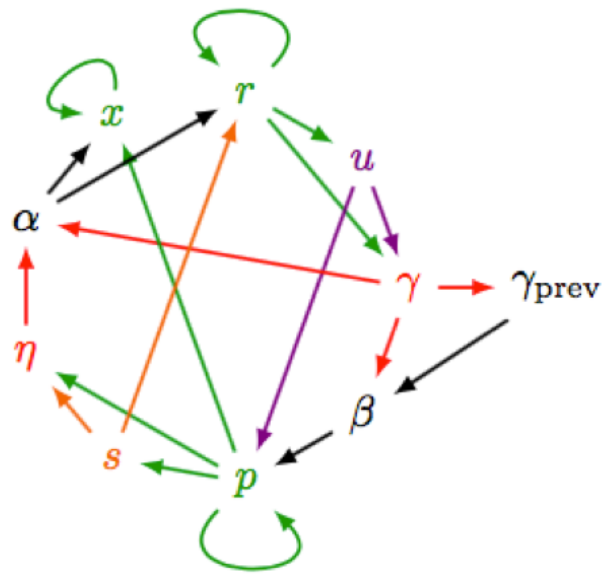


Figure 2: Schematic of the main loop of the preconditioned Conjugate Gradient (PCG) method, as described in Algorithm 3.

Iterative Solver Libraries

- Krylov methods can be written by hand with a lower-level library
- Some packages (Trilinos, PETSc,..) also include Krylov methods
- Multigrid methods [8], especially algebraic multigrid (AMG) methods, are rarely written by hand. Some AMG packages include
  - BoomerAMG (in Hypre)
  - ML
  - GAMG (in PETSc)
  - ILUPACK (multi-level ILU)

[8] To be pedantic, FMG can be described as a direct solver

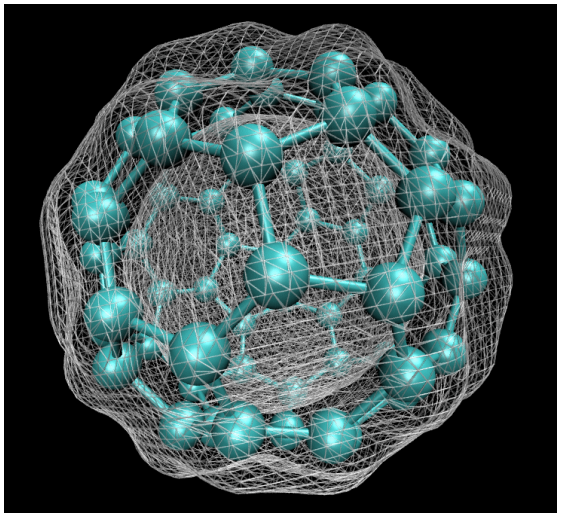
Section 4

Eigensolvers

Eigensolvers

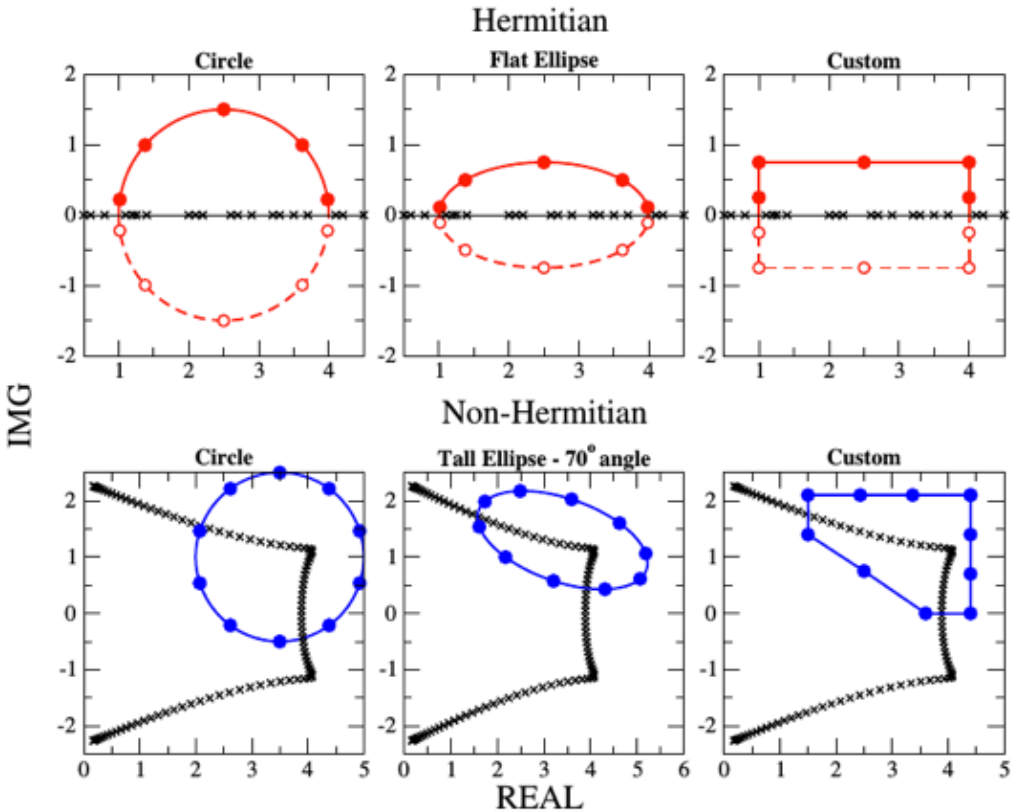
$$Ax = \lambda x$$

- The bottleneck in many physics computations is the computation of a large number of eigenvalues of a large system
- Specialized methods are required, beyond those available in the classic libraries discussed above



FEAST

- www.ecs.umass.edu/~polizzi/feast/
- Uses the FEAST algorithm, based on contour integration
- Implemented for shared- and distributed-memory environments



From FEAST documentation

SLEPc

- slepc.upv.es
- Library for eigenanalysis of large, sparse, distributed linear systems
- Built very closely on top of PETSc
- Well-documented
- Sophisticated algorithms

Problem class	Model equation	Module
Linear eigenvalue problem	$Ax = \lambda x, \quad Ax = \lambda Bx$	EPS
Quadratic eigenvalue problem	$(K + \lambda C + \lambda^2 M)x = 0$	–
Polynomial eigenvalue problem	$(A_0 + \lambda A_1 + \cdots + \lambda^d A_d)x = 0$	PEP
Nonlinear eigenvalue problem	$T(\lambda)x = 0$	NEP
Singular value decomposition	$Av = \sigma u$	SVD
Matrix function (action of)	$y = f(A)v$	MFN

From SLEPc documentation

Section 5

GPU-enabled Linear Algebra Libraries

GPU-enabled Linear Algebra Libraries

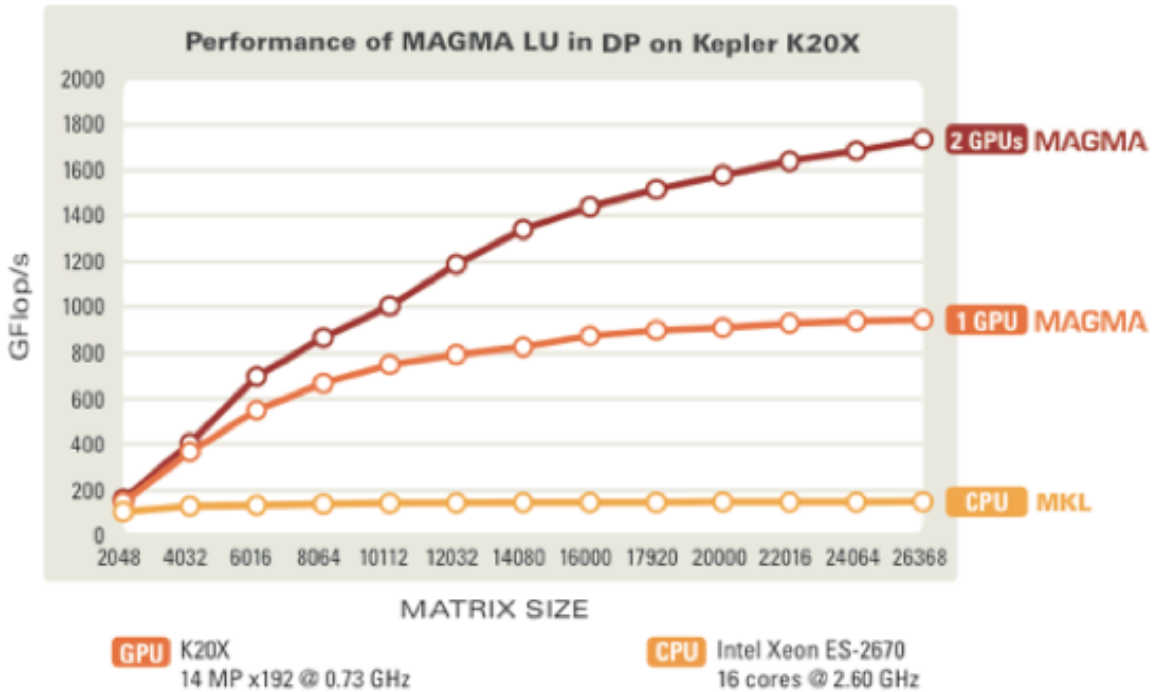
- Well-designed libraries are in high demand to remove the burden of porting common operations to GPU and other accelerators
- Transparent performance portability is very difficult
- For a overview of more then-current libraries, see the material from Will Sawyer from the 2014 Summer School.

MAGMA

HYBRID ALGORITHMS

MAGMA uses a hybridization methodology where algorithms of interest are split into tasks of varying granularity and their execution scheduled over the available hardware components. Scheduling can be static or dynamic. In either case, small non-parallelizable tasks, often on the critical path, are scheduled on the CPU, and larger more parallelizable ones, often Level 3 BLAS, are scheduled on the GPU.

PERFORMANCE

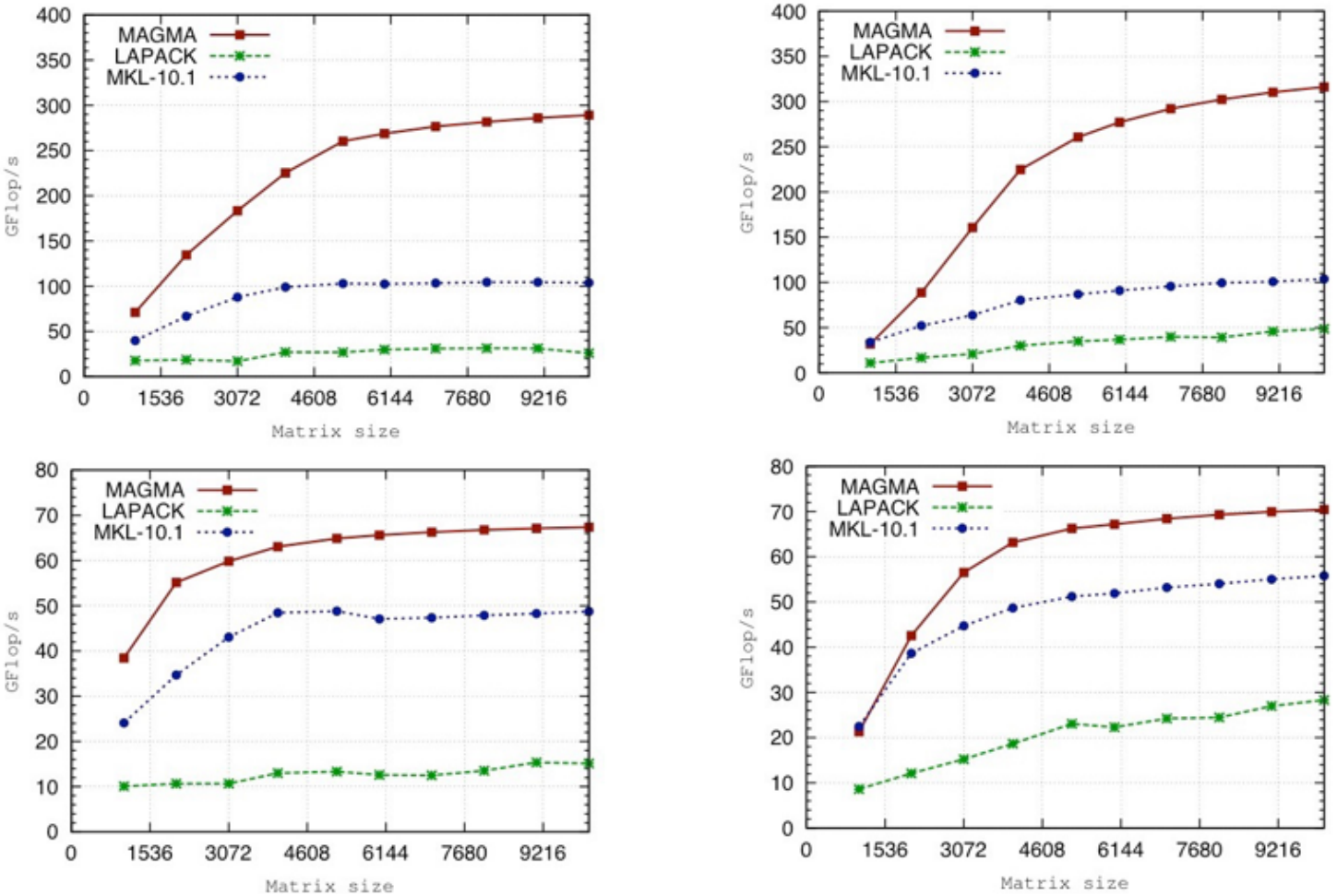


FEATURES AND SUPPORT

- **MAGMA 1.3** FOR **CUDA**
- **clMAGMA 1.0** FOR **OpenCL**
- **MAGMA MIC 0.3** FOR **Intel Xeon Phi**

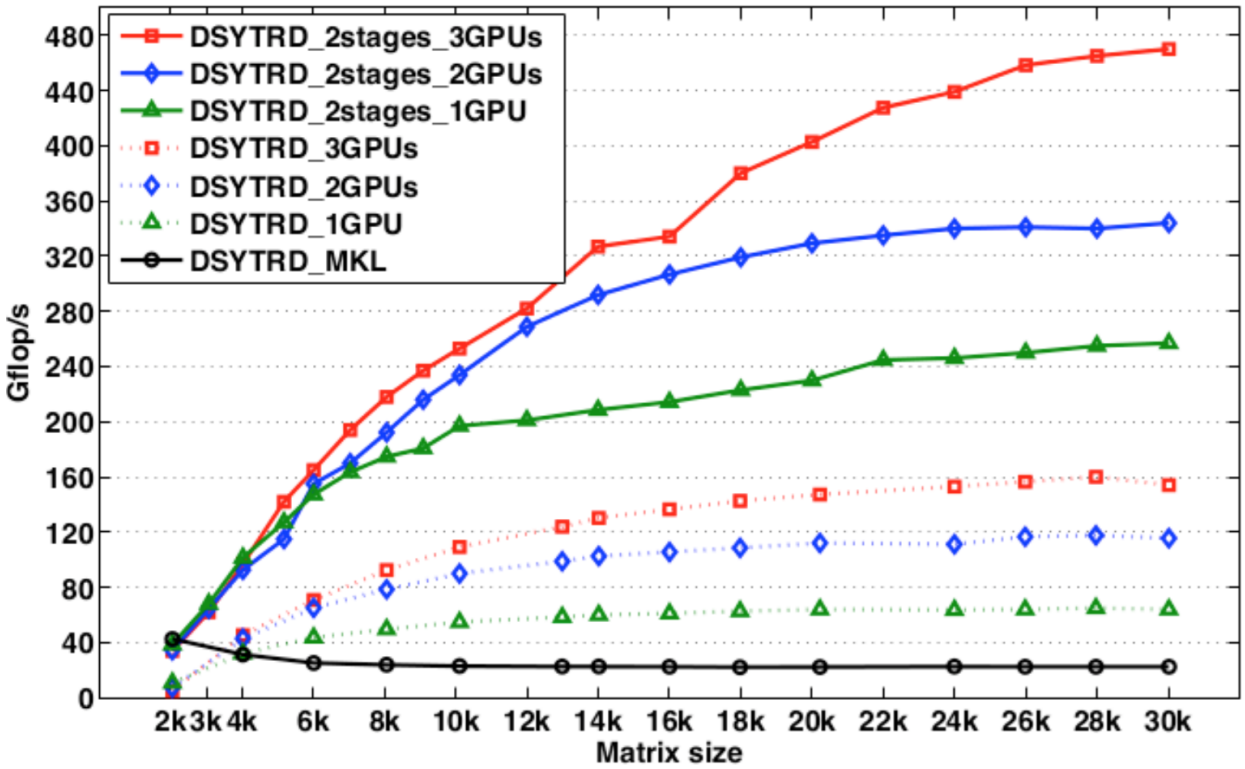
CUDA	OpenCL	Intel Xeon Phi	
●	●	●	Linear system solvers
●	●		Eigenvalue problem solvers
●			MAGMA BLAS
●			CPU Interface
●	●	●	GPU Interface
●	●	●	Multiple precision support
●			Non-GPU-resident factorizations
●			Multicore and multi-GPU support
●			Tile factorizations with StarPU dynamic scheduling
●	●	●	LAPACK testing
●	●	●	Linux
●			Windows
●			Mac OS

MAGMA Performance



MAGMA on GTX280 vs. Xeon quad core Left: QR decomp. SP/DP Right: LU decomp. SP/DP

MAGMA Gen EVP



MKL Implementation

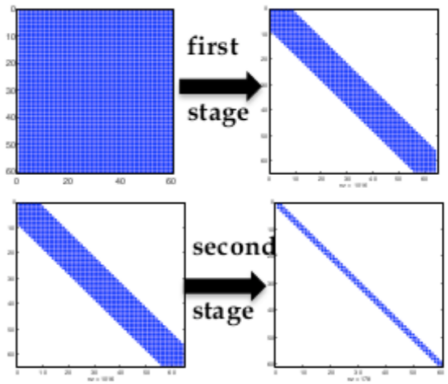
- Too many Blas-2 op,
- Relies on panel factorization,
- ➔Bulk sync phases,
- ➔Memory bound algorithm.

GPU 1-Stage

- Blas-2 GEMV moved to the GPU,
- Accelerate the algorithm by doing all BLAS-3 on GPU,
- ➔Bulk sync phases,
- ➔Memory bound algorithm.

GPU 2-Stage

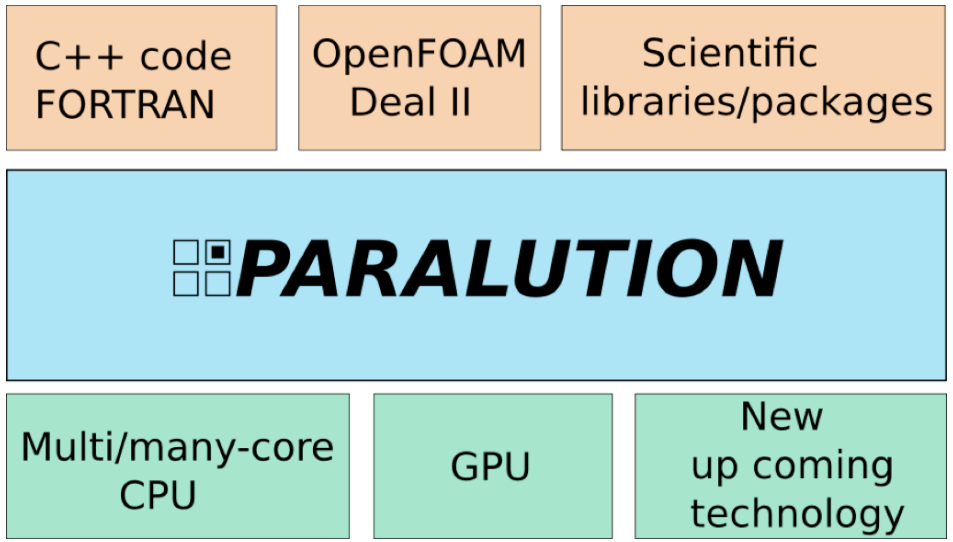
- **Stage 1:** BLAS-3, increasing computational intensity,
- **Stage 2:** BLAS-1.5, new cache friendly kernel,
- 4X/12X faster than standard approach,
- Bottelneck: if all Eigenvectors are required, it has 1 back transformation extra cost.



A. Haidar, S. Tomov, J. Dongarra, T. Schulthess, and R. Solca, *A novel hybrid CPU-GPU generalized eigensolver for electronic structure calculations based on fine grained memory aware tasks*, ICL Technical report, 03/2012.

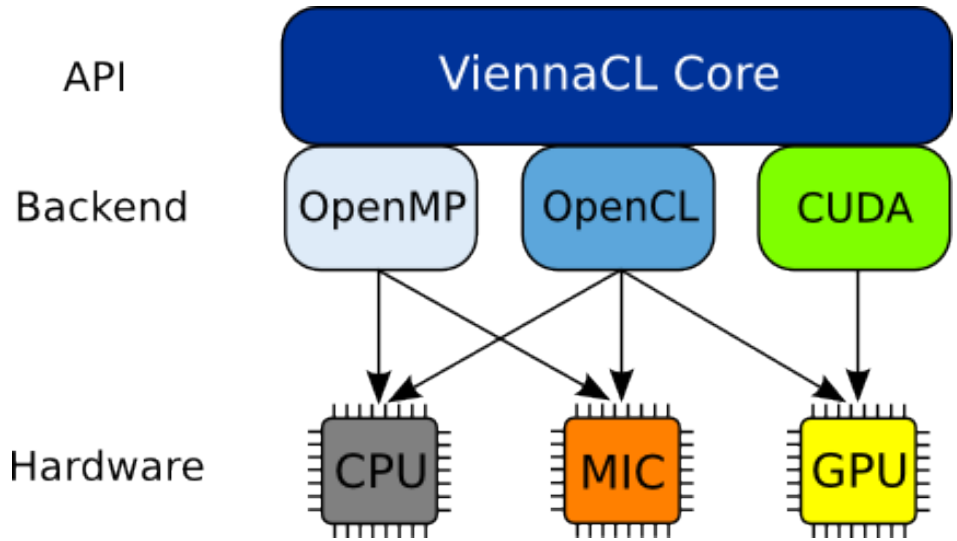
Paralution

- paralution.com
- Sparse iterative solvers and (numerous) preconditioners
- Targeted: CPUs + accelerators
- Hardware abstraction
- OpenMP/CUDA/OpenMP opaque to user
- Code portable
- GPL v3

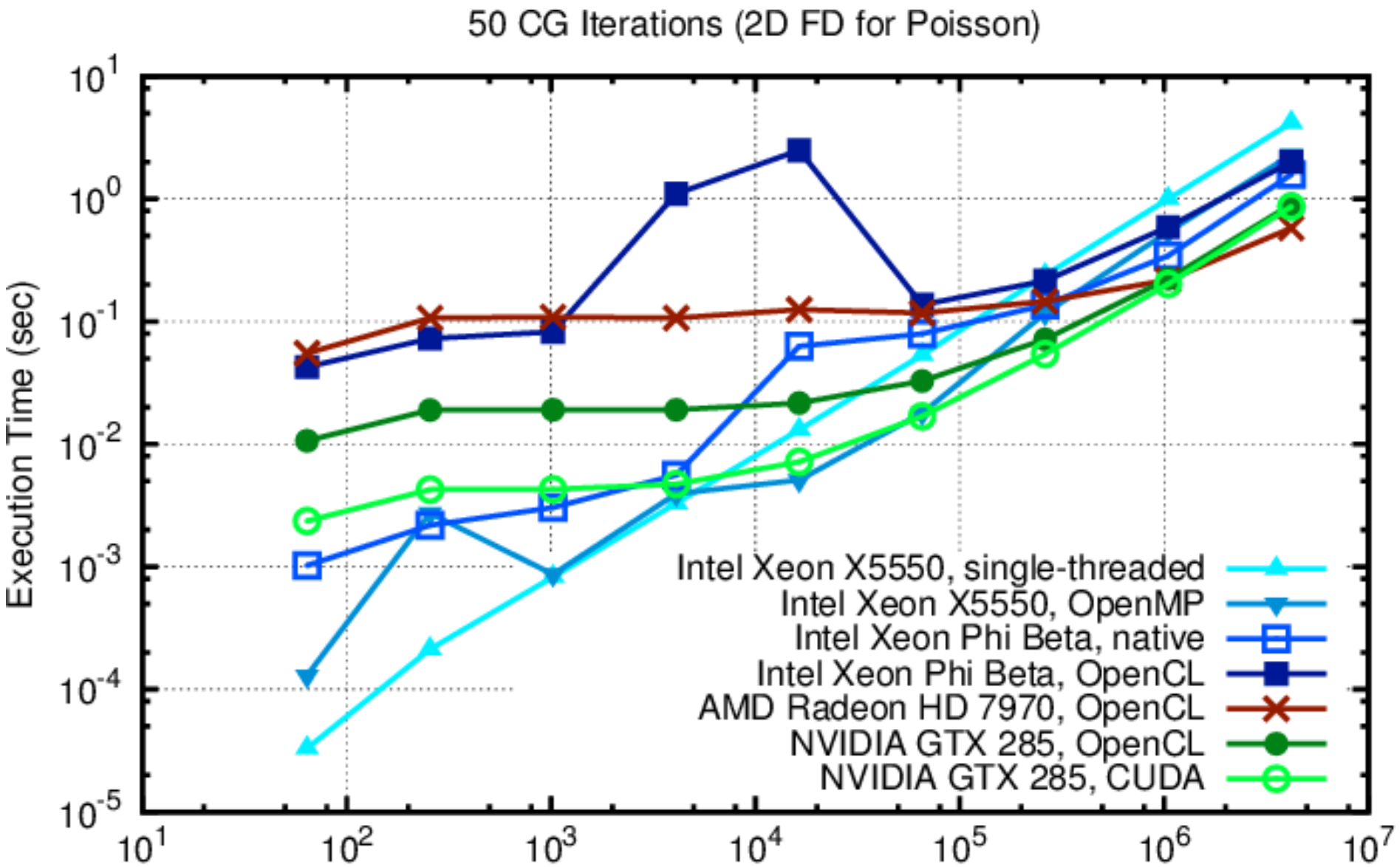


ViennaCL

- C++ Linear algebra library for many core architectures (GPUs, CPUs, Intel Xeon Phi)
- Supports BLAS 1-3
- Iterative solvers and preconditioners
- Sparse row matrix-vector multiplication, solvers
- Goals: Simplicity, minimal dependencies
- Compatible with Boost.uBLAS, Eigen,...
- Open source, header-only library



ViennaCL Benchmark



GPU-enabled Linear Algebra Libraries: Words of Caution

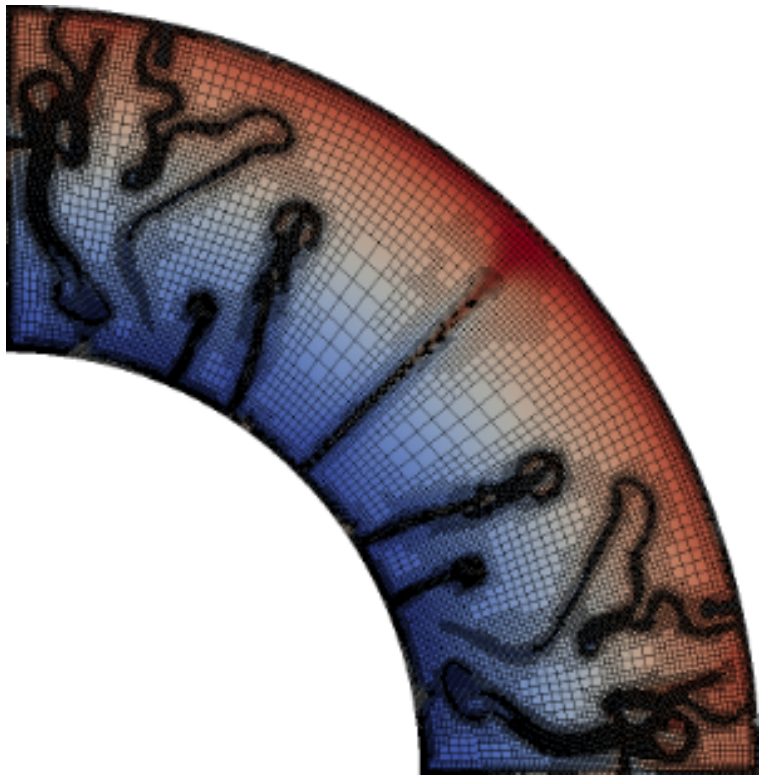
- This is an exciting and dynamic field, prone to sensational claims
- Software and hardware are in flux: "Don't believe it unless you can run it"
- **Be aware of the fundamental limitations of the hardware and current software, especially with sparse linear algebra**
- If you expect to be memory bandwidth limited, look up the values for your CPU and GPU. This (not peak floating point performance) will likely bound your speedup.
- For example, on Piz Daint (before autumn 2016 upgrade):
- The theoretical peak memory bandwidth of the Ivy Bridgenode is 51.2 GB/s
- The peak memory bandwidth of the Tesla K20x is about 250 GB/s
- Note that one should also consider these numbers weighted by power consumption, hardware cost, ..
- If you plan to solve linear systems iteratively, be aware that there are typically fewer and less-optimal preconditioners available on a GPU

Section 6

Higher-level and Expansive Libraries/Frameworks

Finite Element Libraries

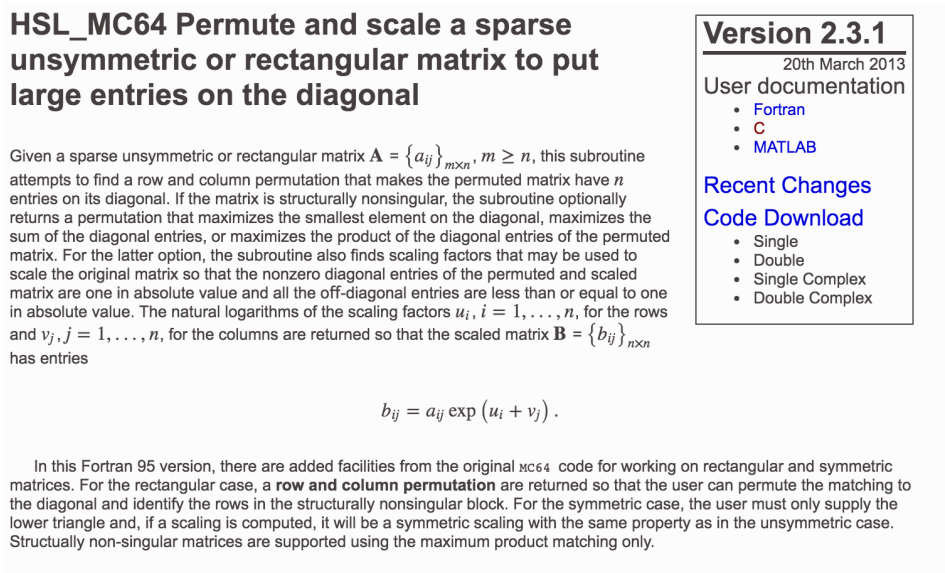
- Libraries are increasingly allowing for flexibility at higher and higher levels of abstraction, and can benefit from tight coupling between discretizations and linear algebra / solver software
- Fenics (fenicsproject.org)
- Firedrake (firedrakeproject.org)
- Deal.II (dealii.org)
- Libmesh (libmesh.github.io)
- These libraries all offer excellent documentation and high-level interfaces in the case of Fenics and Firedrake



aspect.dealii.org

HSL

- hsl.rl.ac.uk
- Formerly "The Harwell Subroutine Library", originally in Fortran with a long history
- Fortran, C, MATLAB interfaces
- Eigenanalysis, general linear algebra, ordering routines, and Krylov methods,
- Including some algorithms not commonly found elsewhere.



HSL_MC64 Permute and scale a sparse unsymmetric or rectangular matrix to put large entries on the diagonal

Version 2.3.1
20th March 2013
User documentation
- Fortran
- C
- MATLAB

Recent Changes
Code Download
- Single
- Double
- Single Complex
- Double Complex

Given a sparse unsymmetric or rectangular matrix $\mathbf{A} = \{a_{ij}\}_{m\times n}$, $m \geq n$, this subroutine attempts to find a row and column permutation that makes the permuted matrix have n entries on its diagonal. If the matrix is structurally nonsingular, the subroutine optionally returns a permutation that maximizes the smallest element on the diagonal, maximizes the sum of the diagonal entries, or maximizes the product of the diagonal entries of the permuted matrix. For the latter option, the subroutine also finds scaling factors that may be used to scale the original matrix so that the nonzero diagonal entries of the permuted and scaled matrix are one in absolute value and all the off-diagonal entries are less than or equal to one in absolute value. The natural logarithms of the scaling factors $u_i, i = 1, \ldots, n$, for the rows and $v_j, j = 1, \ldots, n$, for the columns are returned so that the scaled matrix $\mathbf{B} = \{b_{ij}\}_{n\times n}$ has entries

$$b_{ij} = a_{ij} \exp(u_i + v_j).$$

In this Fortran 95 version, there are added facilities from the original MC64 code for working on rectangular and symmetric matrices. For the rectangular case, a **row and column permutation** are returned so that the user can permute the matching to the diagonal and identify the rows in the structurally nonsingular block. For the symmetric case, the user must only supply the lower triangle and, if a scaling is computed, it will be a symmetric scaling with the same property as in the unsymmetric case. Structually non-singular matrices are supported using the maximum product matching only.

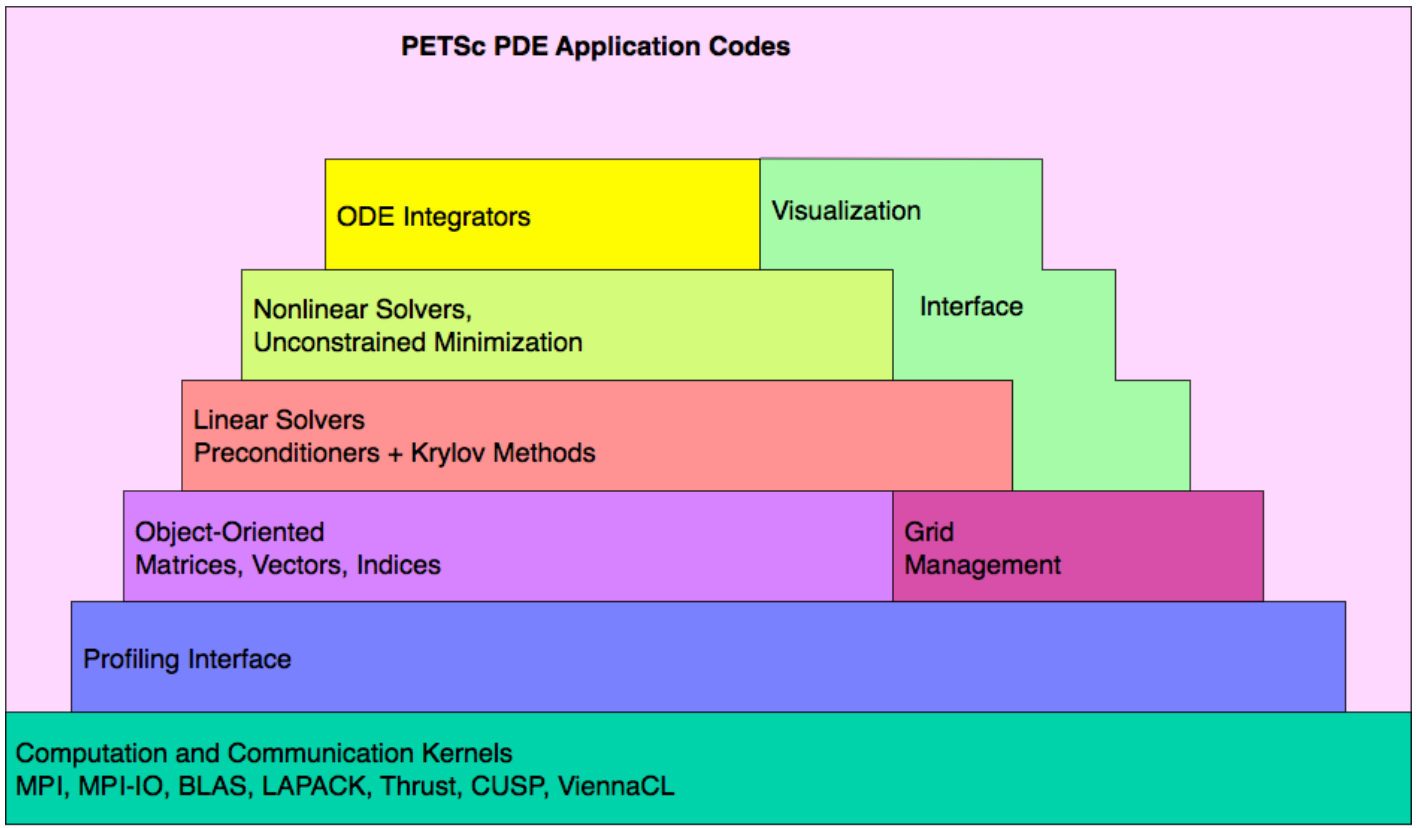
www.hsl.rl.ac.uk/catalogue/hsl_mc64.html

Trilinos

- trilinos.sandia.gov
- A collection of several somewhat-independent packages for scientific computation, covering essentially veery topic discussed so far
  - Basic linear algebra: Epetra/EpetraExt (C++), Tpetra (C++ templates)
  - Preconditioners: AztecOO, Ifpack2, ML, Meros
  - Iterative linear solvers: AztecOO, Belos
  - Direct linear solvers: Amesos (SuperLU, UMFPACK, MUMPS, ScaLAPACK,)
  - Non-linear / optimization solvers: NOX, MOOCHO
  - Eigensolvers: Anasazi
  - Mesh generation / adaptivity: Mesquite, PAMGEN
  - Domain decomposition: Claps
  - Partitioning / load balance: Isorropia, Zoltan2

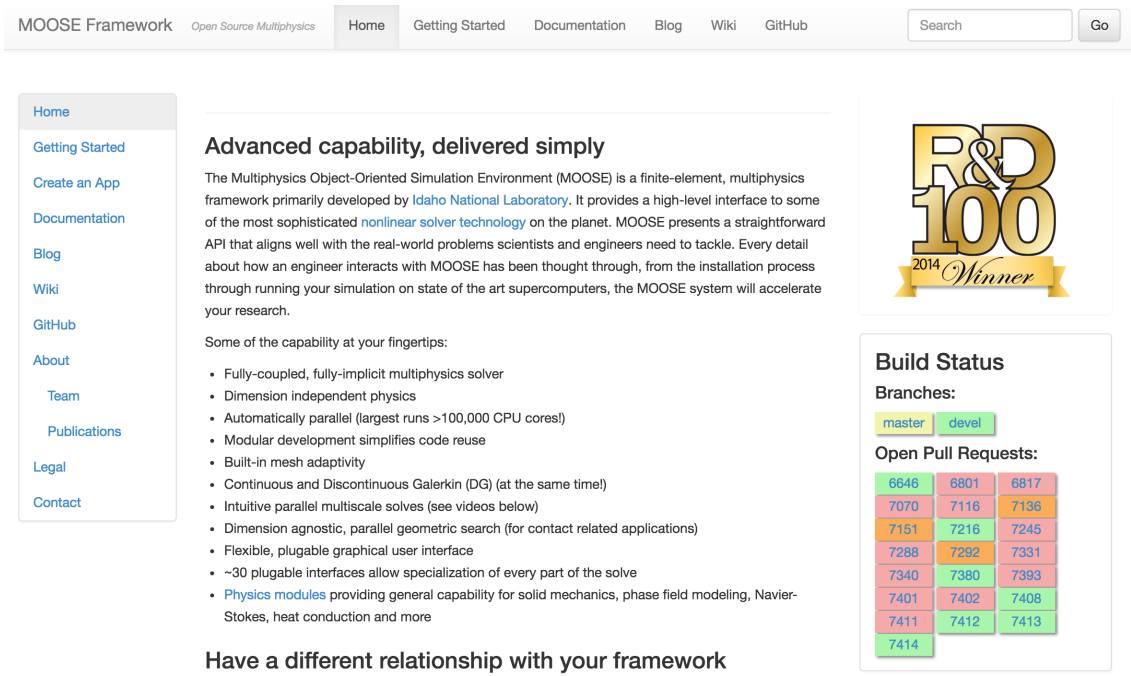
PETSc

- Toolkit centered around tools for large-scale discretized PDE
- Distributed sparse linear solvers, nonlinear solvers, ODE/DAE solver, distributed data structurs, ..
- More in the next lecture



MOOSE

- mooseframework.org
- Framework for multiphysics problems
- Developed for nuclear reactor simulation
- On top of PETSc and libmesh



Section 7

Other Topics and Final Thoughts

Other Topics

We've focused on solver libraries here, due to time and my expertise, but there are entire classes of libraries we've left out:

- Optimization Libraries: TAO, IPOPT, Dolphin-adjoint, ...
- ODE/Timestepper libraries : SUNDIALS, ...
- Meshing and Partitioning Libraries: METIS/PARMETIS, TetGen ...
- I/O and database libraries (see other Summer School lectures)
- Communication Libraries (see other Summer School lectures)
- Domain/physics-specific libraries
- Many more... so many more that it can be daunting!

Final Thoughts

- Libraries are, ultimately, supposed to save time
- How do you know if a library is worth your time? "Measure twice; cut once"
  - Make sure you understand what the needs of your code actually are (Not as easy as it sounds, as you may want your library to keep up as your code scales and changes)
  - Make sure that you understand what each library actually does (Read documentation and publications. If you can't tell what the library does, that's a bad sign)
  - Ask everyone that you can
  - Look for real, working examples
  - Look for active communities and help streams
  - Is the library depended upon by other libraries?
- Other time-saving tips
  - Do what you can to use the most recent version of a library. Practically, that is the one which will be supported.
  - Practice your question-asking skills (mailing lists, StackExchange, etc.)