Exercise 2

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1 Introduction

In Exercise1,we have already use three numerical methods(heun,midpoint and ralston) to estimate the value of the function.However,the numerical methods will cause errors.In Exercise2, we are going to work out exact values by using Matlab.Then we work out errors by subtracting estimated values from exact values.

2 Exact Value

For this RL circuit,we use linear first order ODE to get the exact equation.

2.1 Linear first order ODE

$$\frac{dy}{dx} + p(x)y = Q(x) \tag{1}$$

$$y(x) = Ce^{-\int P(x)dx} + e^{-\int P(x)dx} \int Q(x)e^{\int P(x)dx} dx \tag{2}$$

2.2 Linear first order ODE of RL circuit

$$v_L(t) + v_R(t) = V_{in}(t) \tag{3}$$

$$L\frac{di_L(t)}{dt} + Ri_L(t) = V_{in}(t) \tag{4}$$

R=0.5Ω
L=1.5mH
Input is given as

$$V_{in} = 6cos(\frac{2\pi t}{150 * 10^{-6}}) \tag{5}$$

According to linear first order ODE,we get:

$$1.5 * 10^{-3}\frac{di_L(t)}{dt} + 0.5i_L(t) = 6cos(\frac{2\pi t}{150 * 10^{-6}}) \tag{6}$$

$$\frac{di_L(t)}{dt} + \frac{1}{3} * 10^3 i_L(t) = 4 * 10^3 cos(\frac{2\pi t}{150 * 10^{-6}}) \tag{7}$$

$$e^{\frac{1}{3}*10^3 t}\frac{di_L(t)}{dt} + e^{\frac{1}{3}*10^3 t} * \frac{1}{3} * 10^3 i_L(t) = e^{\frac{1}{3}*10^3 t} * 4 * 10^3 cos(\frac{2\pi t}{150 * 10^{-6}}) \tag{8}$$

$$\frac{d(e^{\frac{1}{3}*10^3 t} i_L(t))}{dt} = e^{\frac{1}{3}*10^3 t} * 4 * 10^3 cos(\frac{2\pi t}{150 * 10^{-6}}) \tag{9}$$

Integrate both sides at the same:

$$exact = -\frac{12}{1600\pi^2+1} exp(-1000/3t) + \frac{480\pi}{1600\pi^2+1} sin(\frac{40000t\pi}{3}) + \frac{1}{1600\pi^2+1} cos(\frac{40000t\pi}{3}) \tag{10}$$

3 Error Function for different numerical method

We have already worked out the estimated value in Exercise 1.We can use them directly,by calling heun.m,midpoint.m and ralston.m.

3.1 Heun method

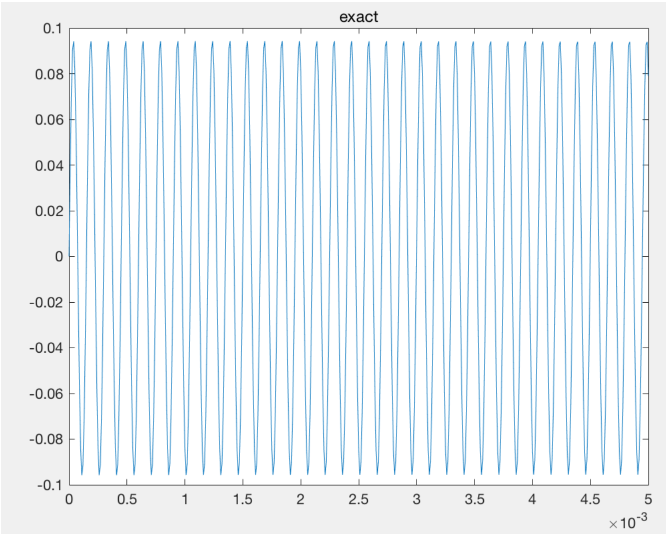

Figure 1: Exact Value

By using heun method,we get the estimated value below and then we get the error function by subtracting estimated values from exact values:

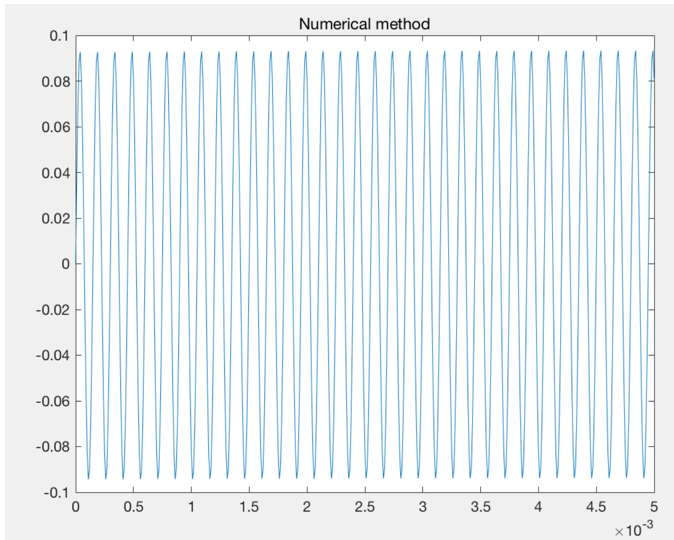

Figure 2: Heun Method

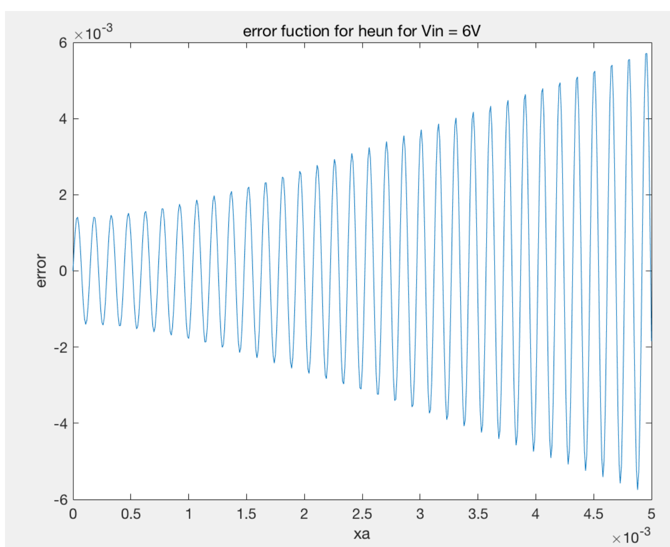

Figure 3: Error function of Heun Method

3.2 Midpoint method

By using midpoint method,we get the estimated value below and then we get the error function by subtracting estimated values from exact values:

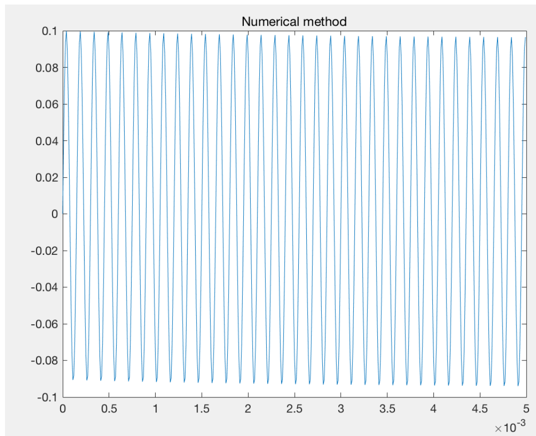

Figure 4: Midpoint Method

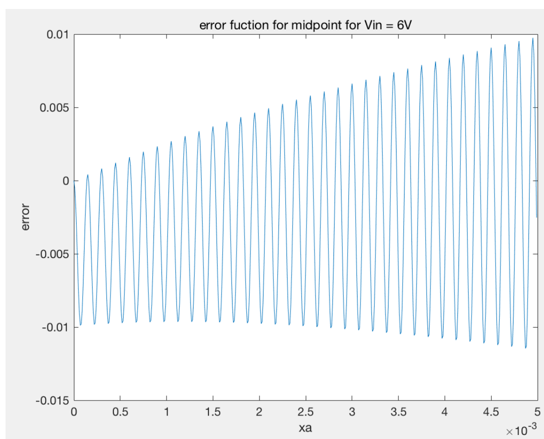

Figure 5: Error function of Midpoint Method

3.3 Ralston method

By using midpoint method,we get the estimated value below and then we get the error function by subtracting estimated values from exact values:

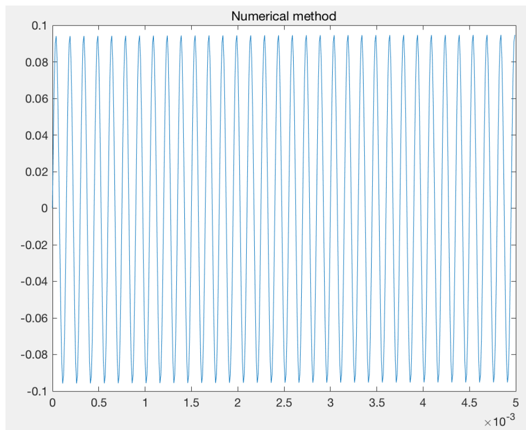

Figure 6: Ralston Method

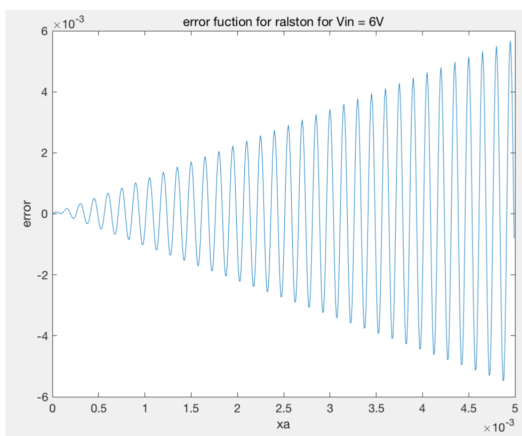

Figure 7: Error function of Ralston method

3.4 Comparison

For **heun** function, the error is increasing from $\pm 1.1 * 10^{-3}$ at 0 to $\pm 5.9 * 10^{-3}$ at $5 * 10^{-3}$. For **midpoint** function, the error is increasing from [-0.01,0] at 0 to [-0.011,0.01] at $5 * 10^{-3}$. For **ralston** function, the error is increasing from 0 at 0 to $\pm 5.9 * 10^{-3}$ at $5 * 10^{-3}$.

Therefore, the ralston method causes less error.

4 log-log plot

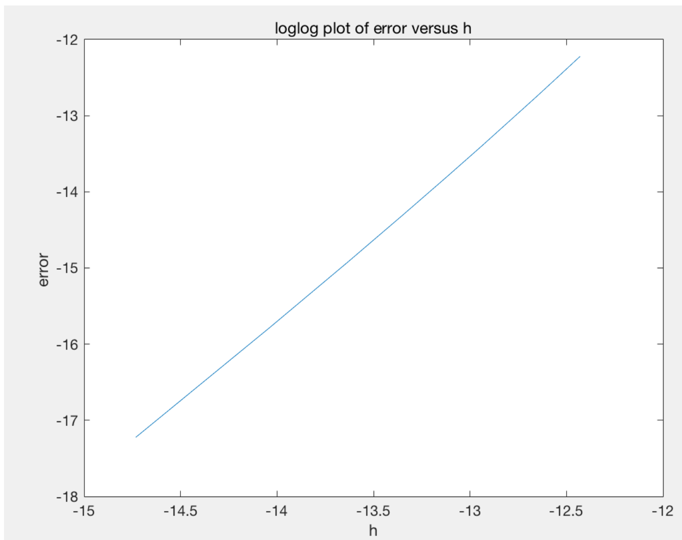

Figure 8: Log-Log plot of ralston method

From the log-log graph,as h increases,error increases.Error is in positive linear relationship with h.Gradient of the line is about 2.It can be expressed as:

$$logE = 2log(h) + constant \tag{11}$$

Thus,it can be deduced that

$$E(y, x) = O(h^2); \tag{12}$$