EE2-08C Numerical Analysis

Group 9

Exercise 3

From Kirchoff's Voltage Law (KVL) the sum of all the voltages in a closed loop must sum to 0 V. Therefore from this we can derive the equation $V_{in}(t) = v_L(t) + v_R(t) + v_C(t)$. All of the voltages in the closed loop current which flows through them, all being equal to the inductor current. The voltage across a conductor is the inductance multiplied by the derivative of it's current, the voltage across the resistor is equal to it's resistance multiplied by the current travelling through it and the voltage across the capacitor is the charge held within the capacitor divided by the capacitance of the capacitor. Substituting these characteristics into the equation above gives us the equation:

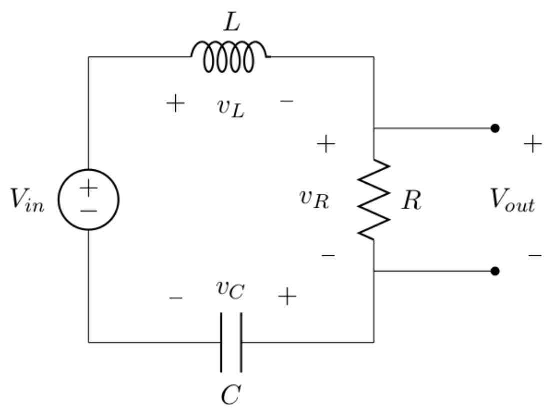

Figure 1: RLC Circuit

$$L\frac{d}{dt}i_L(t) + Ri_L(t) + \frac{1}{C}\int_0^t i_L(t)\ dt = V_{in}(t)$$

Substituting $i_L(t)$ for $\frac{d}{dt}q_C(t)$ you make the equation:

$$L\frac{d^2}{dt^2}q_C(t) + R\frac{d}{dt}q_C(t) + \frac{1}{C}q_C(t) = V_{in}(t)$$

We were then able to turn this second order ODE into a set of 2 coupled first order ODEs through the use of equating $q'_C(t) = y(t)$ and substituting $q_C(t)$ for $x(t)$ we have the coupled equations:

$$y(t) = x'(t) \quad \text{and} \quad y'(t) = \frac{1}{L}[V_{in}(t) - Ry(t) - \frac{1}{C}x(t)]$$

For our project we have values:

$$R = 250\Omega, C = 3\mu F, L = 650mH, i_L(0) = 0\ \ (y_0)\ \text{and}\ \ q_C(0) = 500nC\ \ (x_0).$$

Now that we have these two equations we are able to implement them into MATLAB scripf called 'RLC_script.m' by defining them as:

```
R = 250; L = 650*10^-3; C = 3*10^-6; %Impedance values for the components
w1 = 2*pi*500; %frequency for the 500 Hz sinusoid
w2 = 2*pi*100; %frequency for the sinusoid
w3 = 2*pi*5; %frequency for the sinusoid
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tc = 3*10^-6; %tau for the exponential decay input
y0 = 0; x0 = 500*10^-9; t0 = 0; %Initial conditions y = iL, x =qC and t = time
h = 0.00001; %step size
tf = 0.03; %final condition
func1 = @(x, y, t) y; %y = q'
func2 = @(x, y, t) (Vin - R*y - x/C)/L; %the second coupled equation
```

To now evaluate these coupled equations, we use the Runge Kutta 4th order 3/8 algorithm to use these two coupled equations to estimate the next point of the current i_L and the charge q_C. The algorithm is:

$$f_1(x,y,t) = y \quad f_2(x,y,z) = \frac{1}{L}[V_{in}(t) - Ry - \frac{x}{C}]$$
$$k1_x = hf_1(x(i), y(i), t(i)) \quad k1_y = hf_2(x(i), y(i), t(i))$$
$$k2_x = hf_1(x(i) + \frac{k1_x}{3}, y(i) + \frac{k1_y}{3}, t(i) + \frac{h}{3})$$
$$k2_y = hf_2(x(i) + \frac{k1_x}{3}, y(i) + \frac{k1_y}{3}, t(i) + \frac{h}{3})$$
$$k3_x = hf_1(x(i) - \frac{k1_x}{3} + k2_x, y(i) - \frac{k1_y}{3} + k2_y, t(i) + \frac{2h}{3})$$
$$k3_y = hf_2(x(i) - \frac{k1_x}{3} + k2_x, y(i) - \frac{k1_y}{3} + k2_y, t(i) + \frac{2h}{3})$$
$$k4_x = hf_1(x(i) + k1_x - k2_x + k3_x, y(i)k1_y - k2_y + k3_y, t(i) + h)$$
$$k4_y = hf_2(x(i) + k1_x - k2_x + k3_x, y(i)k1_y - k2_y + k3_y, t(i) + h)$$
$$x(i+1) = x(i) + \frac{k1_x + 3k2_x + 3k3_x + k4_x}{8} \quad y(i+1) = y(i) + \frac{k1_y + 3k2_y + 3k3_y + k4_y}{8}$$
$$t(i+1) = t(i) + h \quad \text{where h is time step}$$

Exercise 4

The Problem