Problem 1 derivation

1. $U_{J_2} = -\mu J_2 \frac{R_\oplus^2}{r^3}\left(\frac{3}{2}\sin^2\phi - \frac{1}{2}\right) = \mu J_2 \frac{R_\oplus^2}{r^3}\left(\frac{3}{2}\frac{z^2}{r^2} - \frac{1}{2}\right)$

$\ddot{\vec{r}}_{J_2} = \nabla U_{J_2}$

$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}\left(\mu J_2 R_\oplus^2\left(\frac{3}{2}\frac{z^2}{r^5} - \frac{1}{2}\cdot\frac{1}{r^3}\right)\right) = -\mu J_2 R_\oplus^2\left(\frac{3}{2}z^2\frac{\partial}{\partial x}\left((x^2+y^2+z^2)^{-5/2}\right) - \frac{1}{2}\frac{\partial}{\partial x}\left((x^2+y^2+z^2)^{-3/2}\right)\right)$

$= \mu J_2 R_\oplus^2\left(\frac{3}{2}z^2\cdot\frac{-5}{2}\cdot(x^2+y^2+z^2)^{\frac{-7}{2}}\cdot 2x + \frac{1}{2}\cdot\frac{3}{2}(x^2+y^2+z^2)^{-5/2}\cdot 2x\right)$

$= \mu J_2 R_\oplus^2\left(-\frac{15}{2}\frac{z^2 x}{r^7} + \frac{3}{2}\frac{x}{r^5}\right) = +\frac{3}{2}\mu J_2 \frac{R_\oplus^2}{r^5}\left(5\frac{z^2}{r^2} - 1\right)x$

$\frac{\partial U}{\partial y} = +\frac{3}{2}\mu J_2 \frac{R_\oplus^2}{r^5}\left(5\frac{z^2}{r^2} - 1\right)y$

$\frac{\partial U}{\partial z} = -\mu J_2 R_\oplus\left(-\frac{15}{2}\frac{z^2 z}{r^7} + \frac{3}{2}\cdot\frac{2z}{r^5} + \frac{3}{2}\frac{z}{r^5}\right) = +\frac{3}{2}\mu J_2 R_\oplus\left(5\frac{z^2}{r^2} - 3\right)z$