ASEN 5070 Final Exam

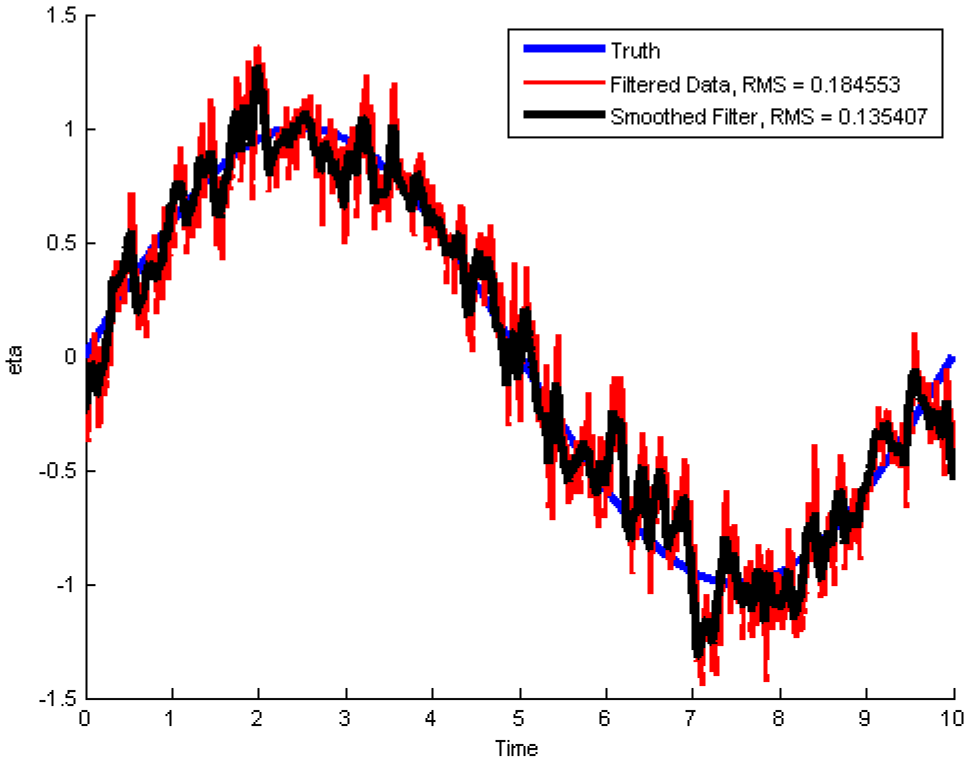
John Clouse

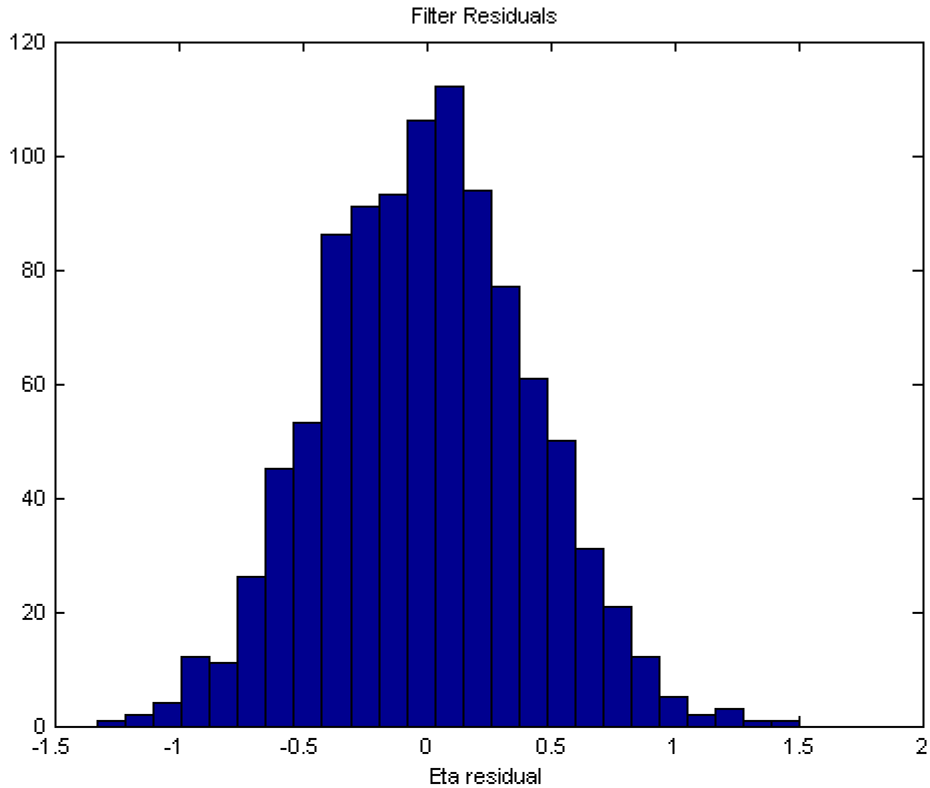
Problem 1

RMS non-smoothed: 0.184553

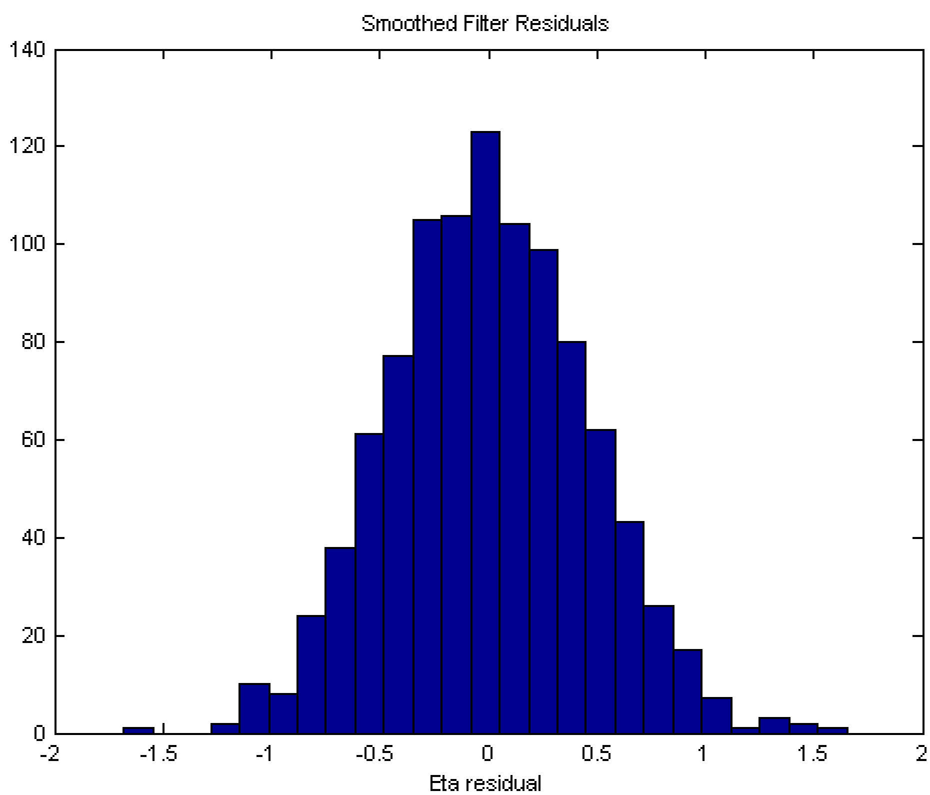
RMS smoothed: 0.135407

The histogram of the smoothed results is more Gaussian.





Filter Residuals
120
100
80
60
40
20
0
-1.5
-1
-0.5
0
0.5
1
1.5
2
Eta residual



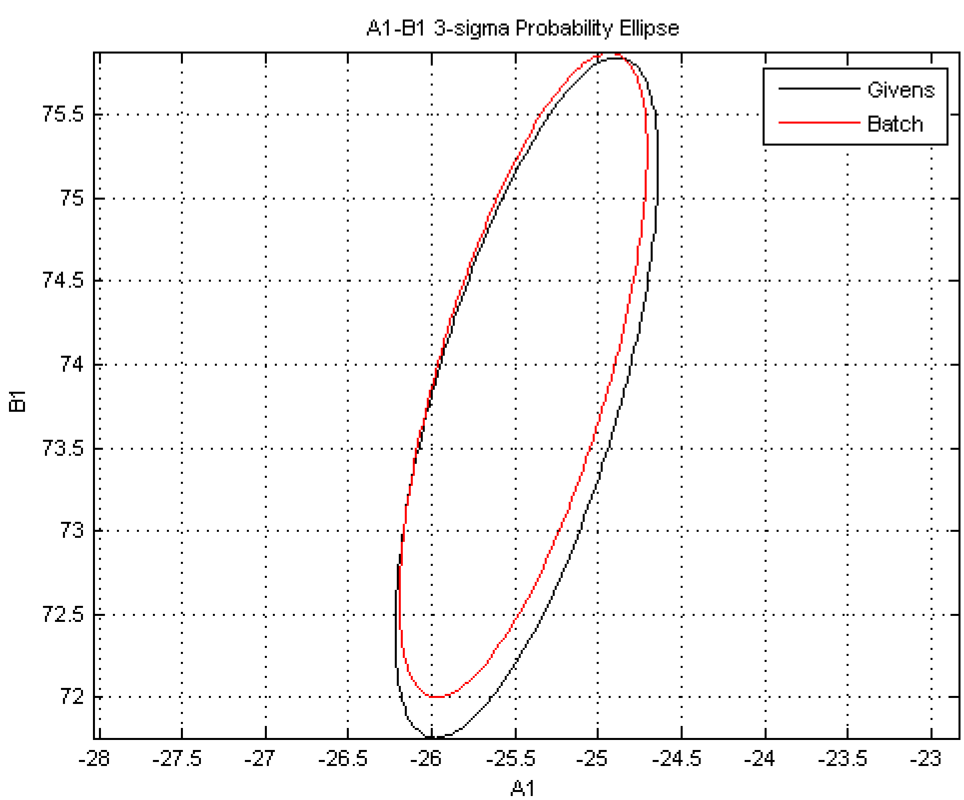
Smoothed Filter Residuals
140
120
100
80
60
40
20
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Eta residual

Problem 2

a)

X_est_Givens =	std_Givens =
-29.813422329056046	0.392536741587372
0.675708079081207	0.015505826421920
-0.000753693605452	0.000085199406076
0.000000159940177	0.000000139435309
0.000000000067455	0.000000000069609
-25.430570609626745	0.263745050875215
8.003182482083073	0.076466468594406
-13.012860522773781	0.048674557278335
7.948520928074068	0.049855582263566
73.799764999857416	0.680434501336939
-0.362037944501615	0.107102189890218
-12.998727514670593	0.050489587657782
-1.068726601580486	0.051415507758401

b)



c)

X_est_batch =	std_batch =
-29.699450238323188	0.364170480004120
0.671710556400109	0.014661467935057
-0.000735334925939	0.000080686119601
0.000000133026177	0.000000132113834
0.000000000079780	0.000000000065971
-25.451430452206690	0.247904425806435
7.977723424969040	0.071328460758129
-13.018007034361684	0.045305198145460
7.937756185907434	0.044699935025174
73.933962255828277	0.644020992993889
-0.351550158819573	0.101162202187059
-12.999805774111280	0.045918251376173
-1.064354324586364	0.044717730442813

```
>> rel_diff = (X_est_Givens - X_est_batch)./X_est_batch

rel_diff =

   0.003837515166722
   0.005951257789549
   0.024966418519794
   0.202321078640794
  -0.154485619724847
  -0.000819594113546
   0.003191268455654
  -0.000395337901901
   0.001356144219414
  -0.001815096227448
   0.029832970968518
  -0.000082944273124
   0.004107914904956
```

The solutions differ slightly due to the condition of P. The Givens algorithm works on orthogonal transformations of the square root of P (with half of P's condition), while batch works on P itself. Even while batch uses Cholesky decomposition, the Givens algorithm will suffer from less error due to machine precision.

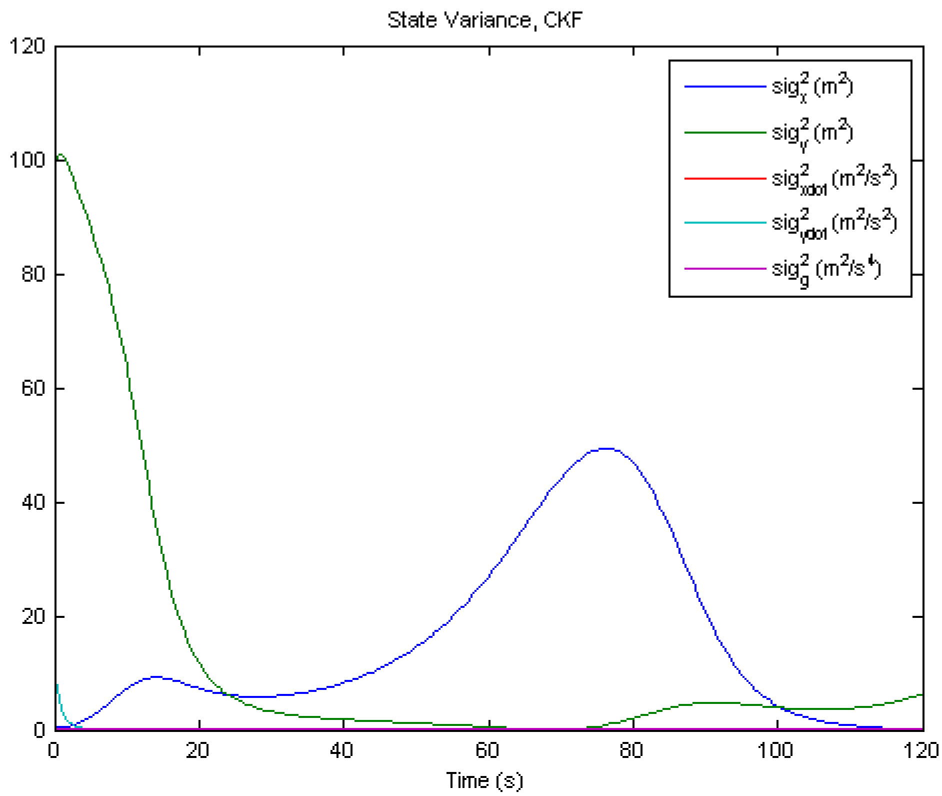
Problem 3

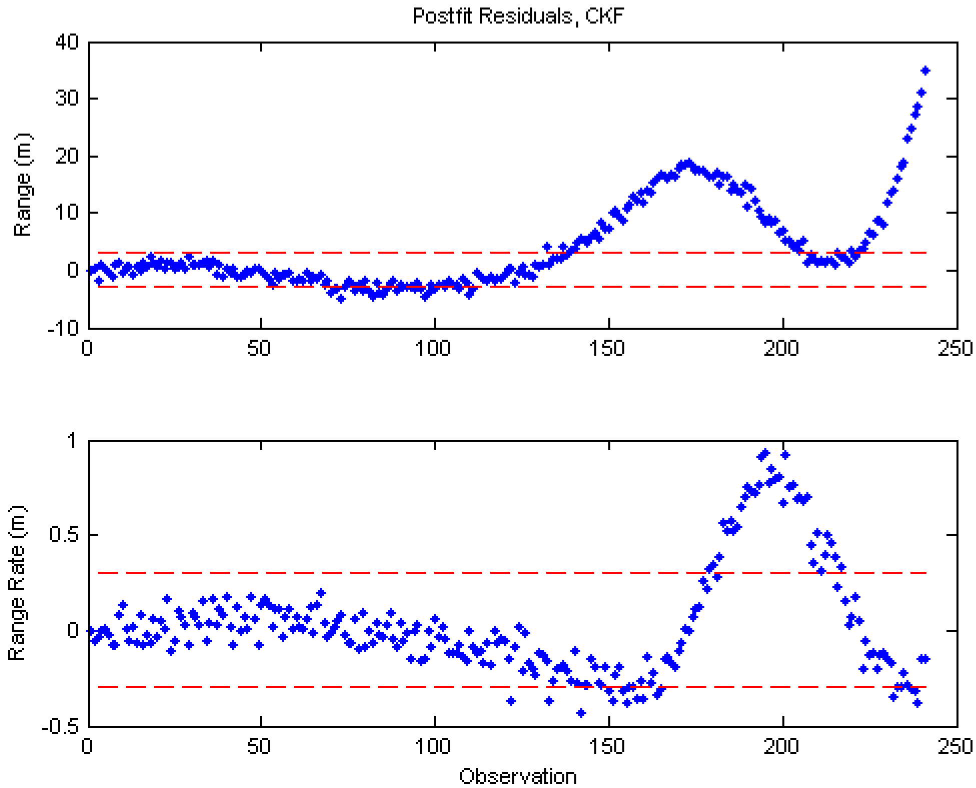
a)

```
final_est_state_CKF =

   1.0e+04 *

   3.583741841897519
   0.177509360885704
   0.030798171501356
  -0.056121375377933
   0.000974015522315
```



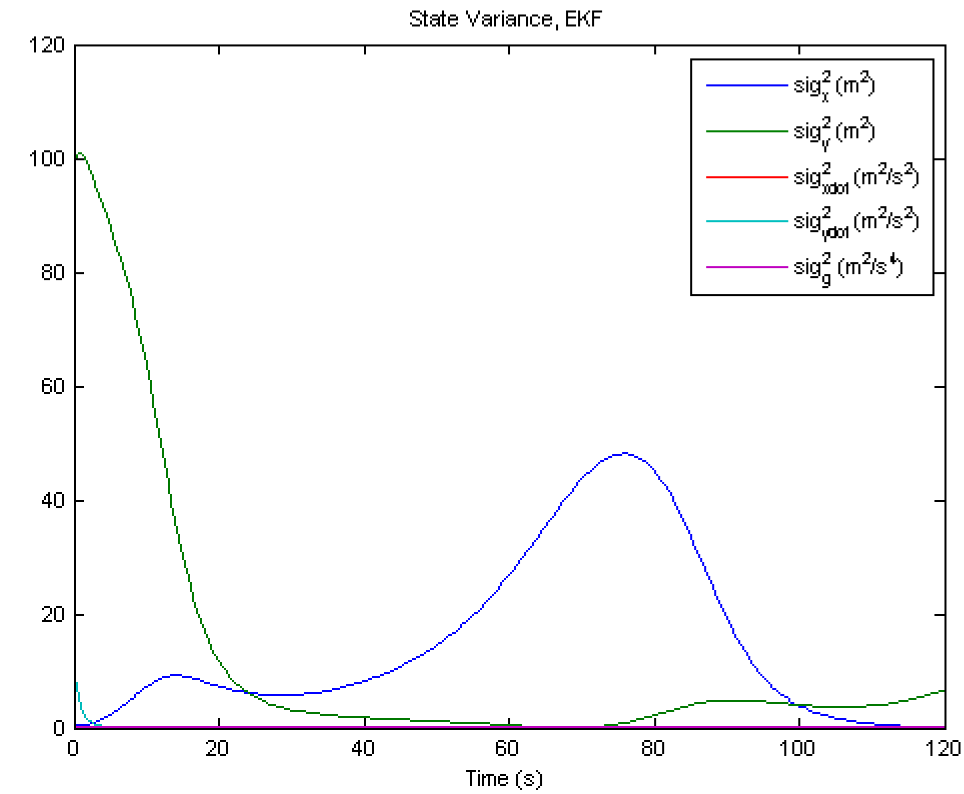


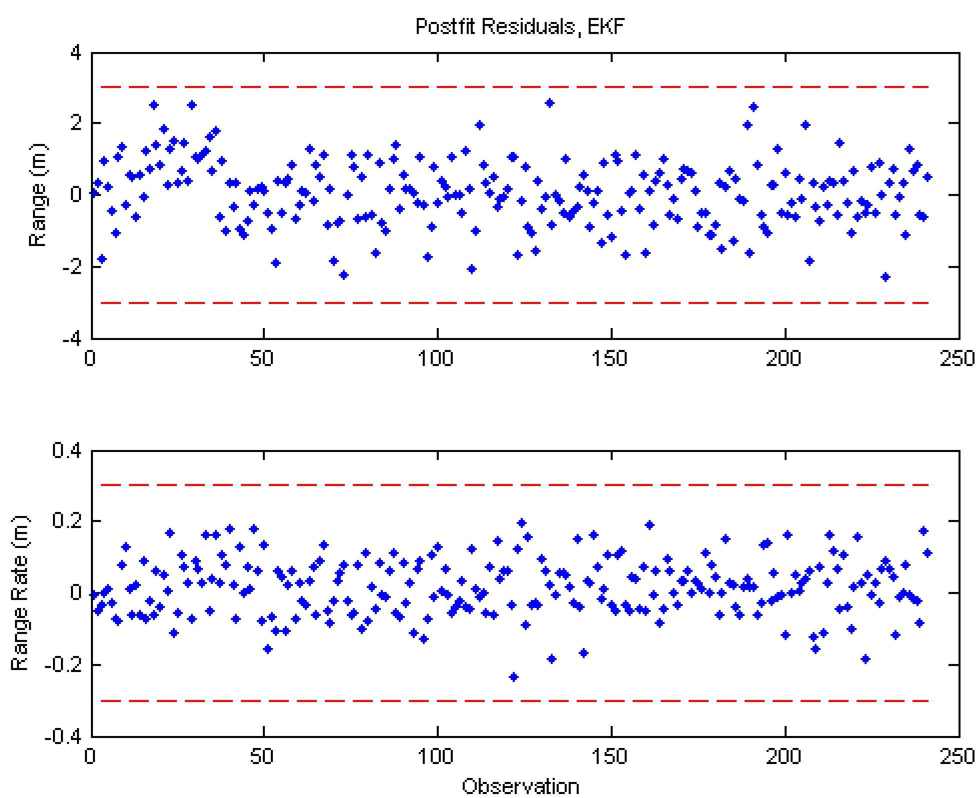
b)
I chose to switch to EKF after 50 observations. The postfit residuals looked converged up to that point in the CKF, and started diverging after that. The covariance was also relatively low at that point (25 seconds).

```
final_est_state_EKF =

   1.0e+04 *

   3.592236384436523
   0.122517117640428
   0.029696318022659
  -0.057787927848967
   0.000980000489809
```


State Variance, EKF
sig^2_x (m^2)
sig^2_y (m^2)
sig$^2_{xdot}$ (m^2/s^2)
sig$^2_{ydot}$ (m^2/s^2)
sig^2_g (m^2/s^4)
Time (s)


Postfit Residuals, EKF
Range (m)
Range Rate (m)
Observation

c)
The estimated states differ because the EKF's reference trajectory stays closer to the true trajectory, keeping linearity errors low. CKF's reference trajectory resembles truth less and less as time goes on , as seen in the postfit residuals. Its deviation gets large because the linearity assumption between reference and truth breaks down.

The covariances are nearly the same. Their (small) differences arise from the H matrix being calculated by reference-trajectory components. They are small differences because the range and range-rate measurements aren't affected much over the course of the observations:

```
>> (CKF_final_calc_range - EKF_final_calc_range)/EKF_final_calc_range

ans =

    -4.540464556983306e-06
```

Finally the postfit residuals for EKF are better than CKF (3-sigma of measurement error). This is because the *a priori* state deviation is zero during the processing of each observation for the EKF. The CKF *a priori* state deviation gets farther from the reference trajectory as time goes on.