Clouse
ASEN 5044
HW4

Problem 1: $x_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$ $x_2 = \begin{bmatrix}1\\-2\\3\end{bmatrix}$ $x_3 = \begin{bmatrix}0\\1\\1\end{bmatrix}$

$$V_1 = X_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$V_2 = X_2 - \frac{V_1^T X_2}{V_1 V_1} V_1 = \begin{pmatrix}1\\-2\\3\end{pmatrix} - \frac{\overset{6}{(1-4+9)}}{\underset{14}{1+4+9}} V_1 = \begin{matrix}1\\-2\\3\end{matrix} - \frac{3}{7}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}4/7\\-20/7\\12/7\end{pmatrix} = \begin{pmatrix}1\\-5\\3\end{pmatrix}\cancel{4/7}$$

$$V_3 = X_3 - \frac{V_1^T X_3}{V_1^T V_1} V_1 - \frac{V_2^T X_3}{V_2^T V_2} V_2 = \begin{pmatrix}0\\1\\1\end{pmatrix} - \frac{2+3}{14} V_1 - \frac{-2+3}{14} V_2 = \begin{matrix}0\\1\\1\end{matrix} - \frac{5}{14}\begin{pmatrix}1\\2\\3\end{pmatrix} - \frac{1}{14}\cdot\begin{pmatrix}1\\-5\\3\end{pmatrix}$$

$$= \begin{pmatrix}-6\\9\\-4\end{pmatrix}\cancel{1/14}$$

$$\boxed{\hat{V}_1 = \begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\frac{1}{\sqrt{14}} \quad \hat{V}_2 = \begin{pmatrix}1\\-5\\3\end{pmatrix}\cdot\frac{1}{\sqrt{35}} \quad \hat{V}_3 = \begin{pmatrix}-6\\9\\-4\end{pmatrix}\cdot\frac{1}{\sqrt{133}}}$$

Problem 2: $Z = \begin{pmatrix}6\\4\\-3\end{pmatrix} = a_1\hat{V}_1 + a_2\hat{V}_2 + a_3\hat{V}_3$

$$a_1 = \hat{V}_1 \cdot Z = \frac{6+8-9}{\sqrt{14}} = \frac{5}{\sqrt{14}}$$

$$a_2 = \hat{V}_2 \cdot Z = \frac{6-20-9}{\sqrt{35}} = \frac{-23}{\sqrt{35}} \implies \boxed{Z = \frac{5}{\sqrt{14}}\hat{V}_1 + \frac{-23}{\sqrt{35}}\hat{V}_2 + \frac{12}{\sqrt{133}}\hat{V}_3}$$

$$a_3 = \hat{V}_3 \cdot Z = \frac{-36+36+12}{\sqrt{133}} = \frac{12}{\sqrt{133}}$$

Problem 3: for $\vec{x} \in \mathbb{V}$, $\vec{x} = \underline{\sum_{i=1}^{n} a_i \vec{v}_i}$

subspace:

$\vec{x} = \vec{y} + \vec{u}$, where $\vec{y} \in \mathbb{V}_{sub}$, $\vec{u} \in \mathbb{U}$

if $\vec{u} = \vec{0}$, $\vec{x} = \vec{y}$, $\mathbb{V}_{sub}$ is spanned by $\{\vec{v}_i\}$, $\therefore$ $\underline{\mathbb{V}_{sub} \text{ is a subspace of } \mathbb{V}}$

Problem 4:

Orth. compliment if $\mathbb{R}^5 = S \oplus C$, so $\dim(C) = 3$, $\therefore$ 3 basis vectors need to span it.

$\perp$ if $\langle V_c, V_1 \rangle = \langle V_c, V_2 \rangle = 0$

for $y \in S$, $u \in C$:

$y = \langle a_y, V_s \rangle$, $u = \langle a_u, V_c \rangle$

$\langle V_{ci}, V_1 \rangle = 2v_{ci1} + 0v_{ci2} + 1v_{ci3} + 2v_{ci4} + 2v_{ci5} = 0$

$\langle V_{ci}, V_2 \rangle = 1v_{ci1} + -1v_{ci2} + 1v_{ci3} + 2v_{ci4} + 1v_{ci5} = 0$

Let $V_{c1} = \begin{pmatrix} 1 \\ V_{c12} \\ 1 \\ 1 \\ V_{c15} \end{pmatrix} \Rightarrow V_{c15} = -\frac{5}{2} \Rightarrow V_{c12} = -4 + \frac{5}{2} = -\frac{3}{2}$

$\Rightarrow \boxed{V_{c1} = (1 \ -3/2 \ 1 \ 1 \ -5/2)^T}$

Let $V_{c2} = (V_{c21} \ V_{c22} \ 1 \ 1 \ -1)^T \Rightarrow V_{c21} = -\frac{1}{2} \Rightarrow V_{c22} = \frac{3}{2}$

$\Rightarrow \boxed{V_{c2} = (-\frac{1}{2} \ \frac{3}{2} \ 1 \ 1 \ -1)^T}$ & is LI to $\underline{V_{c1}}$

Let $V_{c3} = (1 \ V_{c32} \ V_{c33} \ 1 \ 1)^T \Rightarrow V_{c33} = -6 \Rightarrow V_{c32} = -2$

$\Rightarrow \boxed{V_{c3} = (1 \ -6 \ -2 \ 1 \ 1)^T}$ & is LI to $\underline{V_{c1} \text{ \& } V_{c2}}$

Challenge 1 for $\vec{x}$ in $\mathbb{R}^n$ w/ orthonormal basis $\{\vec{v}_i\}$ & for any subset of m basis vectors:

$$\|\vec{x}\|^2 \geq \sum_{i=1}^{m} |\langle \vec{v}_i, \vec{x} \rangle|^2$$

$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$, $\langle \vec{v}_i, \vec{x} \rangle = a_i$ = projection of $\vec{x}$ on $\vec{v}_i$

$\langle \vec{v}_i, \vec{v}_j \rangle = \delta_{ij}$

$\|\vec{v}_i\| = 1$

$\vec{x} = a_1 \vec{v}_1 + a_2 \vec{v}_2 \dots a_n \vec{v}_n = \sum_{i=1}^{n} a_i \vec{v}_i$ if $\{\vec{v}_i\}$ is orthonormal

$x_i = a_1 v_{1i} + a_2 v_{2i} \dots a_n v_{ni}$

$$\|\vec{x}\|^2 = \langle \vec{x}, x \rangle = \langle \sum_{i=1}^{n} a_i \vec{v}_i, \sum_{i=1}^{n} a_i \vec{v}_i \rangle = \sum_{i=1}^{n} a_i^2$$

$$\Rightarrow \sum_{i=1}^{n} a_i^2 \geq \sum_{i=1}^{m} |a_i|^2$$, since $n \geq m$ this is proven

Challenge 2: A & B are $n \times n$

a) if $AB = O_{n\times n}$, $A \neq O_{n\times n} \neq B$, show neither A nor B is invertable

If A is invertable,

$$A^{-1}AB = O_{n\times n} = B$$, which cannot be

If B is invertable,

$$ABB^{-1} = O_{n\times n} = A$$, which cannot be

$\therefore$ neither A nor B is invertable

b) See above. for one to be invertable, the other must equal $O_{n\times n}$