

Problem 4: S&J 8.13

assume CLO of $\Sigma I \delta \ddot{\theta} = -[P] \delta \dot{\theta} - K \delta \theta$ using $\ddot{\theta} = \ddot{\omega}/\omega$, use linear control theory to find critically damped response about each B axis. Tracking errors σ_i, ω_i have $T_i = 100s$

$$I_{11} = 10 \text{ kg}\cdot\text{m}^2$$

$$P_1 = 2I_{11}/T_i = \frac{20}{100} = 0.2$$

$$\xi_1 = 1 = \frac{P_1}{\sqrt{KI_{11}}} \Rightarrow K = \frac{P_1^2}{I_{11}} = \frac{0.04}{10} = \underline{0.004}$$

$$\xi_2 = 1 = \frac{P_2}{\sqrt{KI_{22}}} \Rightarrow P_2 = \sqrt{KI_{22}} = \sqrt{0.004 \cdot 20} = \underline{\sqrt{0.08}}$$

$$\xi_3 = 1 = \frac{P_3}{\sqrt{KI_{33}}} \Rightarrow P_3 = \sqrt{KI_{33}} = \sqrt{0.004 \cdot 30} = \underline{\sqrt{0.12}}$$

$$[P] = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & \sqrt{0.08} & 0 \\ 0 & 0 & \sqrt{0.12} \end{pmatrix}$$

The Linearized Model seems to be underdamped

Nonlinear Model looks overdamped. Linearization about $\ddot{\theta} = 0$, and decoupling, would cause this.