HW 6: Setting Up the Term Project

Table of Contents

Initialize

clearvars -except function_list pub_opt

```
global function_list;
function_list = {};
close all

stat_od_proj_init
ObsData = load('ObsData.txt');
```

Compute information matrix, normal matrix, x_hat

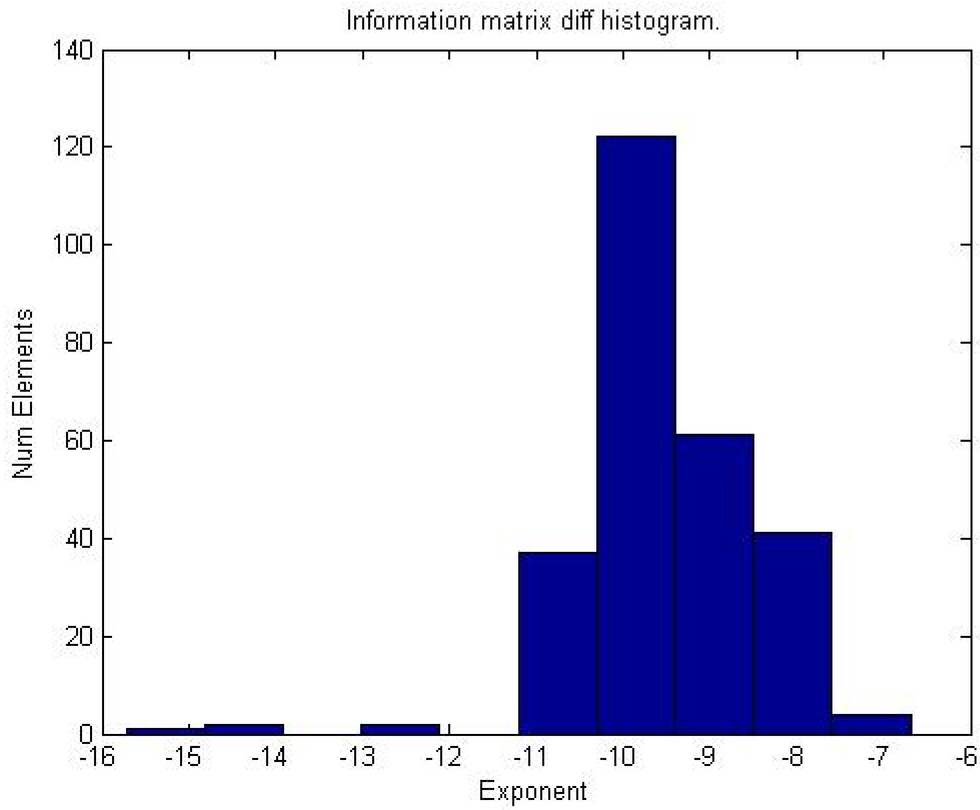
```
consts.Re = Re;
consts.area = drag.A;
consts.rho = compute_density(ri);
consts.theta_dot = theta_dot;
consts.m = drag.m;
consts.state_len = 18;

P0 = eye(consts.state_len)*1e6;
P0(7,7) = 1e20;
P0(10:12,10:12) = eye(3)*1e-10;

x0_ap = zeros(consts.state_len,1);

sig_range = 0.01; % m
sig_rangerate = 0.001; %m/s
W = [1/(sig_range*sig_range) 0; 0 1/(sig_rangerate*sig_rangerate)];


dt = 0.1;
times = 0:dt:18340;
ode_opts = odeset('RelTol', 1e-12, 'AbsTol', 1e-20);

% for iter = 1:3
[T,X] = ode45(@two_body_state_dot, times, state, ode_opts, propagator_opts);

% Store off every 20 seconds of data
X_store = X(mod(times,20) == 0,:);
T_store = T(mod(times,20) == 0);
```

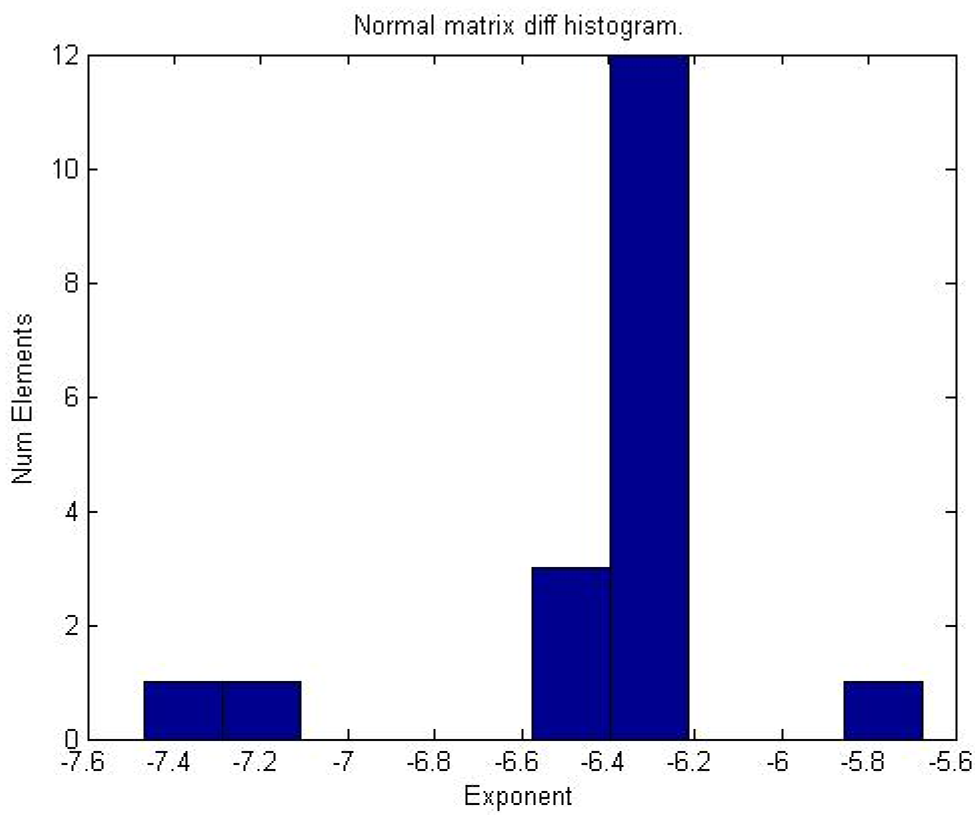
```
% Accumulate the information and normal matrices
[num_obs, ~] = size(ObsData);
% info_mat = zeros(consts.state_len);
chol_P0 = chol(P0,'lower');
P0_inv = chol_P0'\inv(chol_P0);
info_mat = P0_inv;
norm_mat = P0_inv*x0_ap;
% H_tilda_given = load('BatchHtilda.mat');
cntr =1 ;
% Obs. deviation
y1 = zeros(num_obs,1);
y2 = zeros(num_obs,1);
for ii = 1:num_obs
    site_num = 0;
    for jj = 1:3
        if ObsData(ii, 2) == site(jj).id
            site_num = jj;
            break
        end
    end
    t_obs = ObsData(ii,1);
    ostate = X(T(:,1)==t_obs,1:6);

    r_comp = compute_range_ECFsite(ostate(1:3),...
        site(site_num).r,theta_dot*t_obs);
    rr_comp = compute_range_rate_ECFsite(ostate(1:6),...
        site(site_num).r,theta_dot*t_obs, theta_dot);

    y1(ii) = (ObsData(ii,3)-r_comp);
    y2(ii) = (ObsData(ii,4)-rr_comp);

end
for ii = 1:num_obs
    obs_time = ObsData(ii,1);
    obs_site = ObsData(ii,2);
    % STM
    STM = eye(consts.state_len);
    STM(1:important_block(1),1:important_block(2)) = ...
        reshape(X_store(T_store == obs_time,consts.state_len+1:end), ...
        important_block(1), important_block(2));

    % H~
    consts.t = obs_time;
    for xx = 1:3
        if site(xx).id == obs_site
            consts.site = xx;
            break
        end
    end
    state_at_obs = X_store(T_store == obs_time,1:consts.state_len);
    H_tilda = stat_od_proj_H_tilda(state_at_obs, consts);

    %H
```

```
    H = H_tilda*STM;

    % Accumulate information matrix
    info_mat = info_mat + H'*W*H;

    % Accumulate normal matrix
    y = [y1(ii); y2(ii)];
    norm_mat = norm_mat + H'*W*y;

end
x_est = cholesky_linear_solver(info_mat,norm_mat);
% x0_ap = x0_ap-x_est;
% state(1:18) = state(1:18) + x_est;
% end

info_mat_given = load('BatchInfoMat.mat');
info_diff = abs((info_mat-info_mat_given.InfoMat)./info_mat_given.InfoMat);
hist(reshape(log10(info_diff),18*18,1))
title('Information matrix diff histogram.')
xlabel('Exponent')
ylabel('Num Elements')

figure
norm_mat_given = load('BatchNormMat.mat');
norm_diff = abs((norm_mat-norm_mat_given.NormMat)./norm_mat_given.NormMat);
hist(log10(norm_diff))
title('Normal matrix diff histogram.')
xlabel('Exponent')
ylabel('Num Elements')

x_hat_given = load('BatchXhat');
x_diff = abs((x_est - x_hat_given.xhat_pass1)./x_hat_given.xhat_pass1);
x_est
fprintf('Estimated State Deviation Error\n')
for ii = 1:consts.state_len
    fprintf('%.0e\n',x_diff(ii))
end
```

```
x_est =

   1.0e+06 *

  -0.000000036301876
  -0.000000274106867
  -0.000000180875321
   0.000000040934961
   0.000000032748395
  -0.000000014753039
  -9.463433517218595
  -0.000000000000657
   0.000000147561584
   0.000000000001863
   0.000000000001379
```

```
  -0.000000000000254
  -0.000010563629542
   0.000009983376706
   0.000005794325174
  -0.000005781912402
   0.000002344367740
   0.000001512457637

Estimated State Deviation Error
3e-03
2e-05
8e-04
2e-06
2e-07
3e-06
3e-04
4e-06
6e-03
6e-05
6e-05
7e-06
2e-06
8e-06
8e-06
9e-06
5e-05
7e-06
```


Information matrix diff histogram.
Num Elements
Exponent
140
120
100
80
60
40
20
0
-16
-15
-14
-13
-12
-11
-10
-9
-8
-7
-6


Normal matrix diff histogram.
Num Elements
Exponent
12
10
8
6
4
2
0
-7.6
-7.4
-7.2
-7
-6.8
-6.6
-6.4
-6.2
-6
-5.8
-5.6

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```
function x = cholesky_linear_solver(Y,N)
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

vlen = length(N);
z = zeros(vlen,1);
x = zeros(vlen,1);
U = chol(Y); % Upper
UT = U';
for ii = 1:vlen
    tmp = N(ii);
    for jj = 1:(ii-1)
        tmp = tmp - UT(ii,jj)*z(jj);
    end
    z(ii) = tmp/UT(ii,ii);
end

for ii = vlen:-1:1
    tmp = z(ii);
    for jj = (ii+1):vlen
        tmp = tmp - U(ii,jj)*x(jj);
    end
    x(ii) = tmp/U(ii,ii);
end
```

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```
function range = compute_range_ECFsite( inrtl_pos, ecf_site, theta )
%compute_range_ECFsite Summary of this function goes here
%   Detailed explanation goes here
fcnPrintQueue(mfilename('fullpath'))

x = inrtl_pos(1);
y = inrtl_pos(2);
z = inrtl_pos(3);
xs = ecf_site(1);
ys = ecf_site(2);
zs = ecf_site(3);

range = sqrt(...
    (x-(xs*cos(theta)-ys*sin(theta)))*(x-(xs*cos(theta)-ys*sin(theta))) + ...
    (y-(xs*sin(theta)+ys*cos(theta)))*(y-(xs*sin(theta)+ys*cos(theta))) + ...
    (z-zs)*(z-zs));

end
```

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```
function range_rate = compute_range_rate_ECFsite( inrtl_state, ...
    ecf_site, theta, theta_dot )
%UNTITLED2 Summary of this function goes here
%   Detailed explanation goes here
fcnPrintQueue(mfilename('fullpath'))

x = inrtl_state(1);
y = inrtl_state(2);
z = inrtl_state(3);
xdot = inrtl_state(4);
ydot = inrtl_state(5);
zdot = inrtl_state(6);
xs = ecf_site(1);
ys = ecf_site(2);
zs = ecf_site(3);

range_rate = (x*xdot+y*ydot+z*zdot...
    - (xdot*xs+ydot*ys)*cos(theta) + theta_dot*(x*xs + y*ys)*sin(theta) ...
    + (xdot*ys-ydot*xs)*sin(theta) + theta_dot*(x*ys-y*xs)*cos(theta) ...
    - zdot*zs)/compute_range_ECFsite(inrtl_state(1:3), ecf_site, theta);

end
```

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```matlab
function accel = drag_accel( state, drag_data )
%drag_accel calculate drag on spacecraft
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

Cd = drag_data.Cd;
A = drag_data.A;
m = drag_data.m;

rho0 = 4e-13; %kg/m3
r0 = 7298.145; %km
H = 200.0; %km
theta_dot = 7.29211585530066e-5; %rad/s

if isfield(drag_data, 'model_params')
    if isfield(drag_data.model_params, 'rho0')
        rho0 = drag_data.model_params.rho0;
    end
    if isfield(drag_data.model_params, 'r0')
        r0 = drag_data.model_params.r0;
    end
    if isfield(drag_data.model_params, 'H')
        H = drag_data.model_params.H;
    end
    if isfield(drag_data.model_params, 'theta_dot')
        theta_dot = drag_data.model_params.theta_dot;
    end
end

r = norm(state(1:3));

rho = rho0*exp(-(r-r0)/H);
rel_wind = [state(4) + theta_dot*state(2);
    state(5) - theta_dot*state(1);
    state(6)];

accel = -0.5*Cd*A/m*rho*rel_wind*norm(rel_wind);

end
```

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```
function fcnPrintQueue( filename )
global function_list;
if exist('function_list', 'var')
    file_in_list = 0;
    for idx = 1:length(function_list)
        if strcmp(function_list(idx), filename);
            file_in_list = 1;
            break
        end
    end
    if ~file_in_list
%          fprintf('%s\n', filename);
        function_list = [function_list; filename];
    end
end
end
```

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```
function accel = J2_accel( pos, params )
%J2_accel Acceleration due to J2
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

% Defaults (earth)
J2 = 0.00108248;
mu = 398600.4; %km3/s2
Re = 6378.145; %km

% Use input params if desired
if nargin > 1 % There are params
    if isfield(params, 'J2')
        J2 = params.J2;
    end
    if isfield(params, 'mu')
        mu = params.mu;
    end
    if isfield(params, 'Re')
        Re = params.Re;
    end
end

%Calculate accel
r = norm(pos);
z = pos(3);
const = 1.5*mu*J2*Re*Re/(r*r*r*r*r);
sin_sq_phi = z*z/(r*r);

accel = const*[5*sin_sq_phi - 1;
    5*sin_sq_phi - 1;
    5*sin_sq_phi - 3].*pos;
end
```

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```
function A = stat_od_proj_A(state, consts)
%stat_od_proj_A Calculate A matrix for Stat OD project
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

% Init A, set up local vars
A = zeros(consts.state_len);
x = state(1);
y = state(2);
z = state(3);
xdot = state(4);
ydot = state(5);
zdot = state(6);
mu = state(7);
J2 = state(8);
Cd = state(9);

Re = consts.Re;
area = consts.area;
rho = consts.rho;
theta_dot = consts.theta_dot;
m = consts.m;

H = 88667; %m

% vars to reduce computations
x2 = x*x;
y2 = y*y;
z2 = z*z;
r = sqrt(x2+y2+z2);
sqrt_r = sqrt(r);
v = sqrt(xdot*xdot+ydot*ydot+zdot*zdot);
rel_wind_x = (xdot + theta_dot*y);
rel_wind_y = (ydot - theta_dot*x);
zdot2 = zdot*zdot;
rel_wind_mag = sqrt(rel_wind_x*rel_wind_x + rel_wind_y*rel_wind_y + zdot2);
Re2 = Re*Re;

rho0 = 3.614e-13; %kg/m3
r0 = 700000+6378136.3; %km
H = 88667.0; %km

% Only a few elements are populated
A(1,4) = 1;
A(2,5) = 1;
A(3,6) = 1;

A(4,1) = (3*mu*x^2)/(r*r*r*r*r) - ...
    mu/(r*r*r) + ...
    (3*J2*Re2*mu*((5*z2)/(r*r) - 1))/(2*(r)^(5)) - ...
    (15*J2*Re2*mu*x2*z2)/(r)^(9) - ...
    (15*J2*Re2*mu*x2*((5*z2)/(r*r) - 1))/(2*(r)^(2)) + ...
    (Cd*area*theta_dot*rho*rel_wind_x*rel_wind_y)/(2*m*rel_wind_mag) + ...
```

```
    (Cd*area*x*rho*(xdot + theta_dot*y)*rel_wind_mag)/(2*H*m*r);
A1 = [
A2 = [
A3 = [
A4 = [            (3*mu*x^2)/(x^2 + y^2 + z^2)^(5/2) - mu/(x^2 + y^2 + z^2)^(3/2)
A5 = [ (3*mu*x*y)/(x^2 + y^2 + z^2)^(5/2) + (Cd*area*rho0*theta_dot*exp((r0 - (x^2
A6 = [
A(1,:) = A1;
A(2,:) = A2;
A(3,:) = A3;
A(4,:) = A4;
A(5,:) = A5;
A(6,:) = A6;
A4(4);
```

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```
function H_tilda = stat_od_proj_H_tilda(state, consts)
%stat_od_proj_H_tilda Calculate H_tilda matrix for Stat OD project
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

% Init H_tilda, set up local vars
x = state(1);
y = state(2);
z = state(3);
xdot = state(4);
ydot = state(5);
zdot = state(6);
% mu = state(7);
% J2 = state(8);
% Cd = state(9);

theta_dot = consts.theta_dot;
theta = consts.t*consts.theta_dot;

%Identify the site the observation was from:
xs = state(9+(consts.site-1)*3+1);
ys = state(9+(consts.site-1)*3+2);
zs = state(9+(consts.site-1)*3+3);

H_tilda = zeros(2,18);
H_tilda(1,:) = [
H_tilda(2,:) = [ (xdot + theta_dot*ys*cos(theta) + theta_dot*xs*sin(theta))/((ys*c

% Zero out the site terms  where there weren't observations.
for ii = 1:3
    if consts.site ~= ii
        H_tilda(:,9+(ii-1)*3+1:9+(ii-1)*3+3) = zeros(2,3);
    end
end
```

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```
fcnPrintQueue(mfilename('fullpath'))
```

Initial data for stat OD project

```
mu = 3.986004415e14; %m3/s2
J2 = 1.082626925638815e-3;
Re = 6378136.3; %m

theta_dot = 7.2921158553e-5; %rad/s

site(1).name = 'Pacific Ocean Ship Sensor';
site(1).id = 101;
site(1).r = [-5127510.0 -3794160.0 0.0]'; % m

site(2).name = 'Pirinclik, Turkey';
site(2).id = 337;
site(2).r = [3860910.0 3238490.0 3898094.0]'; % m

site(3).name = 'Thule, Greenland';
site(3).id = 394;
site(3).r = [549505.0 -1380872.0 6182197.0]'; % m

ri = [757700.0 5222607.0 4851500.0]';
vi = [2213.21 4678.34 -5371.30]';

drag.Cd = 2.0;
drag.A = 3.0; % m
drag.m = 970; %kg

% kilometerize everything
% mu = mu*1e-9;
% Re = Re*1e-3;
% site1.r = site1.r*1e-3; %km
% site2.r = site2.r*1e-3; %km
% site3.r = site3.r*1e-3; %km
% ri = ri*1e-3;%km
% vi = vi*1e-3;%km/s

state = [ri; vi; mu; J2; drag.Cd; site(1).r; site(2).r; site(3).r];

% Set up propagator options
propagator_opts.mu = mu;

propagator_opts.drag = drag;
propagator_opts.drag.use = 1;
propagator_opts.drag.model_params.rho0 = 3.614e-13; %kg/m3
propagator_opts.drag.model_params.r0 = 700000+6378136.3;
propagator_opts.drag.model_params.H = 88667;
propagator_opts.drag.model_params.theta_dot = theta_dot;

propagator_opts.J2.use = 1;
propagator_opts.J2.params.J2 = J2;
```

```
propagator_opts.J2.params.mu = mu;
propagator_opts.J2.params.Re = Re;

propagator_opts.OD.use = 1;
propagator_opts.OD.state_len = 18;
propagator_opts.OD.A_mat_handle = @stat_od_proj_A;
propagator_opts.OD.A_params.Re = Re;
propagator_opts.OD.A_params.area = drag.A;
propagator_opts.OD.A_params.rho = propagator_opts.drag.model_params.rho0;
propagator_opts.OD.A_params.theta_dot = theta_dot;
propagator_opts.OD.A_params.m = drag.m;
propagator_opts.OD.A_params.H = propagator_opts.drag.model_params.H;
important_block = [9 9]; %rows, cols
propagator_opts.OD.A_params.important_block = important_block;
propagator_opts.OD.A_params.state_len = propagator_opts.OD.state_len;
STM_i = eye(propagator_opts.OD.state_len);
state = [state; reshape(STM_i(1:important_block(1),1:important_block(2)),...
    important_block(1)*important_block(2),1)];
```

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```
function state_dot = two_body_state_dot(t, state, opts)
%two_body_state_dot   Return state_dot given state. Used for numerical
%integration
% The first 6 elements are assumed to be r and v. If a state transition
% matrix (STM) is to be calculated as well, opts.OD needs to be set up.
% Currently assumes r/v are the only state elements that have non-zero
% derivatives.
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

state_dot = zeros(length(state),1);
state_dot(1:3) = state(4:6);
mu = 3.986e5; % km3/s2
if isfield(opts, 'mu')
    mu = opts.mu;
end

r_vec = (state(1:3));
r = norm(r_vec);
state_dot(4:6) = -mu * r_vec/(r*r*r);

if isfield(opts, 'J2') && isfield(opts.J2, 'use')
    if opts.J2.use == 1
        if isfield(opts.J2, 'params')
            state_dot(4:6) = state_dot(4:6) + J2_accel(state(1:3), opts.J2.params)
        else
            state_dot(4:6) = state_dot(4:6) + J2_accel( state(1:3) );
        end
    end
end
if isfield(opts, 'drag') && isfield(opts.drag, 'use')
    if opts.drag.use == 1
        state_dot(4:6) = state_dot(4:6) + drag_accel( state, opts.drag );
    end
end

if isfield(opts, 'OD')
    if opts.OD.use == 1
        opts.OD.state_len;
        % The OD.state_len is the length of the estimation state. The rest
        % is the STM, numerically propagated with the A-Matrix
        A = opts.OD.A_mat_handle(state(1:opts.OD.state_len),opts.OD.A_params);

        % Block matrix multiplication
%         long_dim = 9;
%         STM = zeros(long_dim);
        STM = reshape(state(opts.OD.state_len+1:end),...
            opts.OD.A_params.important_block(1),...
            opts.OD.A_params.important_block(2));
        STM_dot = ...
            A(1:opts.OD.A_params.important_block(1),1:opts.OD.A_params.important_b
            *STM;
        % Pack up the important stuff
```

```
        state_dot(opts.OD.state_len+1:end) = reshape(STM_dot,...
            opts.OD.A_params.important_block(1)*opts.OD.A_params.important_block(2
    end
end
```

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