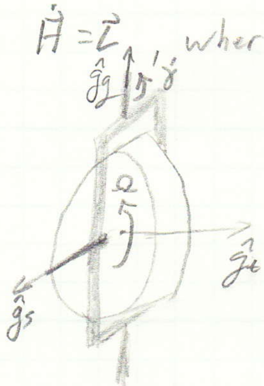
ASEN 5010 John Clause

Problem 6: Derive EOM for rigid S/c w/ N VSCMGs attached

$\dot{\vec{H}} = \vec{L}$, where $\vec{H} = \vec{H}_S + \vec{H}_G + \vec{H}_W$



$$\vec{H}_{S/c} = [I_S]\,\vec{\omega}_{B/N}$$

$$\vec{H}_G = [I_G]\,\vec{\omega}_{G/N} = [I_G](\vec{\omega}_{G/B} + \vec{\omega}_{B/N})$$

$$\vec{H}_G = [BG]\,{}^G[I_G][BG]^T\vec{\omega}_{B/N} + [I_G]\,\vec{\omega}_{G/B}\begin{pmatrix}0\\0\\\dot{\gamma}\end{pmatrix}, \text{ where } [BG] = [\hat{g}_s\ \hat{g}_T\ \hat{g}_g]$$

$$I_G = {}^G\begin{bmatrix} I_{G_s} & 0 & 0 \\ 0 & I_{G_T} & 0 \\ 0 & 0 & I_{G_g} \end{bmatrix}$$

$$\vec{H}_G = (I_{G_s}\hat{g}_s\hat{g}_s^T + I_{G_T}\hat{g}_t\hat{g}_t^T + I_{G_g}\hat{g}_g\hat{g}_g^T)\vec{\omega}_{B/N} + I_{G_g}\dot{\gamma}\hat{g}_g$$

$$\left.\begin{aligned} \omega_s &= \hat{g}_s^T\vec{\omega}_{B/N} \\ \omega_t &= \hat{g}_t^T\vec{\omega}_{B/N} \\ \omega_g &= \hat{g}_g^T\vec{\omega}_{B/N} \end{aligned}\right\} \text{Projection of } \vec{\omega}_{B/N} \text{ onto gimbal frame}$$

$$\vec{H}_G = I_{G_s}\omega_s\hat{g}_s + I_{G_t}\omega_t\hat{g}_t + I_{G_g}(\omega_g + \dot{\gamma})\hat{g}_g$$

$$\vec{H}_W = [I_W]\,\vec{\omega}_{W/N} = [I_W](\vec{\omega}_{W/G} + \vec{\omega}_{G/B} + \vec{\omega}_{B/N}) = [I_W](\Omega\hat{g}_s + \dot{\gamma}\hat{g}_g + \vec{\omega}_{B/N})$$

$$I_W = {}^G\begin{bmatrix} I_{W_s} & 0 & 0 \\ 0 & I_{W_t} & 0 \\ 0 & 0 & I_{W_t} \end{bmatrix}$$

$$\vec{H}_W = (I_{W_s}\hat{g}_s\hat{g}_s^T + I_{W_t}\hat{g}_t\hat{g}_t^T + I_{W_t}\hat{g}_g\hat{g}_g^T)\vec{\omega}_{B/N} + I_{W_s}\Omega\hat{g}_s + I_{W_t}\dot{\gamma}\hat{g}_g$$

$$= I_{W_s}(\omega_s + \Omega)\hat{g}_s + I_{W_t}\omega_t\hat{g}_t + I_{W_t}(\omega_g + \dot{\gamma})\hat{g}_g$$

$$\dot{\vec{H}}_S = [I_S]\dot{\vec{\omega}}_{B/N} + \vec{\omega}_{B/N}\times[I_S]\vec{\omega}_{B/N}$$

$\dot{\vec{H}}_W, \dot{\vec{H}}_G$: $\dot{\hat{g}}_s = \frac{{}^Bd}{dt}(\hat{g}_s) + \vec{\omega}\times\hat{g}_s$, ${}^G\vec{\omega} = \omega_s\hat{g}_s + \omega_t\hat{g}_t + \omega_g\hat{g}_g$

$${}^B\hat{g}_s(t) = \cos(\gamma(t)-\gamma_0)\,{}^B\hat{g}_s(t_0) + \sin(\gamma(t)-\gamma_0)\,{}^B\hat{g}_t(t_0);\quad \frac{{}^Bd}{dt}\hat{g}_s = \dot{\gamma}\hat{g}_t \text{ (instantaneous deriv)}$$

$$\dot{\hat{g}}_s = \dot{\gamma}\hat{g}_t - \omega_t\hat{g}_g + \omega_g\hat{g}_t = (\dot{\gamma}+\omega_g)\hat{g}_t - \omega_t\hat{g}_g$$

$$\dot{\hat{g}}_t = \frac{{}^Bd}{dt}(\hat{g}_t) + \vec{\omega}\times\hat{g}_t$$

$${}^B\hat{g}_t(t) = -\sin(\gamma(t)-\gamma_0)\hat{g}_s(t_0) + \cos(\gamma(t)-\gamma_0)\hat{g}_t(t_0);\quad \frac{{}^Bd}{dt}\hat{g}_t = -\dot{\gamma}\hat{g}_s$$

$$\dot{\hat{g}}_t = -\dot{\gamma}\hat{g}_s + \omega_s\hat{g}_g - \omega_g\hat{g}_s = -(\dot{\gamma}+\omega_g)\hat{g}_s + \omega_s\hat{g}_g$$

$$\dot{\hat{g}}_g = \frac{{}^Bd}{dt}(\hat{g}_g) + \vec{\omega}\times\hat{g}_g = 0 - \omega_s\hat{g}_t + \omega_t\hat{g}_s$$

cont →

Problem 6 cont:

$$\dot\omega_s = \dot{\hat g}_s^T \vec\omega_{B/N} + \hat g_s^T \dot{\vec\omega}_{B/N} = 0 + \underline{\dot\gamma \omega_t} + \cancel{\omega_g\omega_t - \omega_g\omega_t} + \hat g_s^T\dot{\vec\omega}_{B/N}$$

$$\dot\omega_t = -\dot\gamma\omega_s \cancel{-\omega_g\omega_s + \omega_g\omega_s} + \hat g_t^T \dot{\vec\omega}_{B/N}$$

$$\dot\omega_g = \cancel{-\omega_s\omega_t + \omega_t\omega_s} + \hat g_g^T\dot{\vec\omega}_{B/N}$$

$$\dot{\vec H}_W = I_{W_s}(\dot\gamma\omega_t + \hat g_s^T\dot{\vec\omega}_{B/N} + \dot\Omega)\hat g_s + I_{W_s}(\omega_s+\Omega)[(\dot\gamma+\omega_g)\hat g_t - \omega_t\hat g_g]$$
$$+ I_{W_t}(-\dot\gamma\omega_s + \hat g_t^T\dot{\vec\omega}_{B/N})\hat g_t + I_{W_t}\omega_t[-(\dot\gamma+\cancel{\omega_g})\hat g_s + \omega_s\hat g_g]$$
$$+ I_{W_t}(\hat g_g^T\dot{\vec\omega}_{B/N} + \ddot\gamma)\hat g_g + I_{W_t}(\omega_g+\dot\gamma)[\cancel{\omega_t\hat g_s} - \omega_s\hat g_t]$$

$$\dot{\vec H}_W = I_{W_s}(\dot\gamma\omega_t + \hat g_s^T\dot{\vec\omega}_{B/N} + \dot\Omega)\hat g_s$$
$$+[I_{W_s}(\omega_s+\Omega)(\dot\gamma+\omega_g) + I_{W_t}(\hat g_t^T\dot{\vec\omega}_{B/N} - \omega_g\omega_s - 2\dot\gamma\omega_s)]\hat g_t$$
$$+[-I_{W_s}\omega_t(\omega_s+\Omega) + I_{W_t}(\omega_t\omega_s + \hat g_g^T\dot{\vec\omega}_{B/N} + \ddot\gamma)]\hat g_g$$

$$\dot{\vec H}_G = I_{G_s}(\dot\gamma\omega_t + \hat g_s^T\dot{\vec\omega}_{B/N})\hat g_s + I_{G_s}\omega_s[(\dot\gamma+\omega_g)\hat g_t - \omega_t\hat g_g]$$
$$+I_{G_t}(-\dot\gamma\omega_s + \hat g_t^T\dot{\vec\omega}_{B/N})\hat g_t + I_{G_t}\omega_t[-(\dot\gamma+\omega_g)\hat g_s + \omega_s\hat g_g]$$
$$+I_{G_g}(\hat g_g^T\dot{\vec\omega}_{B/N}+\ddot\gamma)\hat g_g + I_{G_g}(\omega_g+\dot\gamma)[\omega_t\hat g_s - \omega_s\hat g_t]$$

$$=[(I_{G_s}-I_{G_t}+I_{G_g})\dot\gamma\omega_t + I_{G_s}\hat g_s^T\dot{\vec\omega}_{B/N} + (-I_{G_t}+I_{G_g})(\omega_g\omega_t)]\hat g_s$$
$$+[(I_{G_s}-I_{G_t}-I_{G_g})\dot\gamma\omega_s + I_{G_t}\hat g_t^T\dot{\vec\omega}_{B/N} + (I_{G_s}-I_{G_g})\omega_s\omega_g]\hat g_t$$
$$+[I_{G_g}(\hat g_g^T\dot{\vec\omega}_{B/N}+\ddot\gamma) + (I_{G_t}-I_{G_s})\omega_s\omega_t]\hat g_g$$

$$[J] = [I_G] + [I_W] = {}^{G}\begin{bmatrix} J_s & 0 & 0 \\ 0 & J_t & 0 \\ 0 & 0 & J_g \end{bmatrix} \quad ; [I] = [I_s] + [J]$$

add $(I_{W_t}-I_{W_g})\omega_g\omega_t$ for cross-prod. form

add $(I_{W_t}-I_{W_g})$

$$\dot{\vec H} = \dot{\vec H}_s + \dot{\vec H}_G + \dot{\vec H}_W = \hat g_s[I_{W_s}\hat g_s^T\dot{\vec\omega}_{B/N} + I_{G_s}\hat g_s^T\dot{\vec\omega}_{B/N} + (I_{G_g}-I_{G_t})\omega_g\omega_t + \dot\Omega I_{W_s} + \dot\gamma\omega_t(I_{W_s}+I_{G_s}-I_{G_t}+I_{G_g})]$$
$$+\hat g_t[I_{W_t}\hat g_t^T\dot{\vec\omega}_{B/N} + I_{G_t}\hat g_t^T\dot{\vec\omega}_{B/N} + \omega_s\omega_g((I_{W_s}-I_{W_g})+(I_{G_s}-I_{G_g})) + \dot\gamma\omega_s(I_{G_s}-I_{G_t}-I_{G_g}+I_{W_s}-2I_{W_t})$$
$$+I_{W_s}(\Omega\dot\gamma + \Omega\omega_g)]$$
$$+\hat g_g[I_{W_t}\hat g_g^T\dot{\vec\omega}_{B/N} + I_{G_g}\hat g_g^T\dot{\vec\omega}_{B/N} + (I_{W_t}-I_{W_s}+I_{G_t}-I_{G_s})\omega_s\omega_t + I_{G_g}\ddot\gamma - I_{W_s}\omega_t\Omega + I_{W_t}\ddot\gamma] + [I_s]\dot{\vec\omega}_{B/N}$$
$$+\vec\omega_{B/N}\times[I_s]\vec\omega_{B/N} = \vec L$$

$$\vec\omega_{B/N}\times[J]\vec\omega_{B/N} = (J_g-J_t)\omega_t\omega_g\hat g_s + (J_s-J_g)\omega_s\omega_g\hat g_t + (J_t-J_s)\omega_s\omega_t\hat g_g$$

cont →

Problem 6 cont.

$$\Rightarrow [I]\dot{\vec{\omega}}_{B/N} = -\vec{\omega}_{B/N} \times [I]\omega_{B/N} - \hat{g}_s[I_{ws}\dot{\Omega} + \dot{\gamma}\omega_t(J_s + J_g - J_t)]$$
$$-\hat{g}_t[\dot{\gamma}\omega_s(J_s - J_t - J_g) + I_{ws}\Omega(\dot{\gamma} + \omega_g)]$$
$$-\hat{g}_g[J_g\ddot{\gamma} - I_{ws}\omega_t\Omega] + \vec{L}$$

EOM for 1 VSCMG

Setup matrices for multiple VSCMGs (1 to N)

$$[G_s] = [\hat{g}_{s1}, \cdots, \hat{g}_{sN}]$$
$$[G_t] = [\hat{g}_{t1}, \cdots, \hat{g}_{tN}]$$
$$[G_g] = [\hat{g}_{g1}, \cdots, \hat{g}_{gN}]$$

$$\vec{\tau}_s = \begin{bmatrix} I_{ws_1}\dot{\Omega}_1 + \dot{\gamma}_1\omega_{t_1}(J_{s_1} + J_{g_1} - J_{t_1}) \\ \vdots \\ I_{ws_N}\dot{\Omega}_N + \dot{\gamma}_N\omega_{t_N}(J_{s_N} + J_{g_N} - J_{t_N}) \end{bmatrix}$$

$$\vec{\tau}_t = \begin{bmatrix} \dot{\gamma}_1\omega_{s_1}(J_{s_1} - J_{t_1} - J_{g_1}) + I_{ws_1}\dot{\Omega}_1(\dot{\gamma}_1 + \omega_{g_1}) \\ \vdots \\ \dot{\gamma}_N\omega_{s_N}(J_{s_N} - J_{t_N} - J_{g_N}) + I_{ws_N}\dot{\Omega}_N(\dot{\gamma}_N + \omega_{g_N}) \end{bmatrix}$$

$$\vec{\tau}_g = \begin{bmatrix} J_{g_1}\ddot{\gamma}_1 - I_{ws_1}\omega_{t_1}\Omega_1 \\ \vdots \\ J_{g_N}\ddot{\gamma}_N - I_{ws_N}\omega_{t_N}\Omega_N \end{bmatrix}$$

$$\vec{H} = \vec{H}_s + \sum_{i=1}^{N}(\vec{H}_{G_i} + \vec{H}_{w_i}), \quad [I] = [I_s] + \sum_{i=1}^{M}[J_i]$$

EOM for N VSCMGs:

$$[I]\dot{\vec{\omega}}_{B/N} = -\vec{\omega}_{B/N} \times [I]\vec{\omega}_{B/N} - [G_s]\vec{\tau}_s - [G_t]\vec{\tau}_t - [G_g]\vec{\tau}_g + \vec{L}$$