John Clouse, Homework 7

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a)

```
close all

A = [-10 0 -10 0; 0 -0.7 9 0; 0 -1 -0.7 0;1 0 0 0];
B = [20 2.8; 0 -3.13; 0 0; 0 0];
C = [0,0,1,0;0,0,0,1];
C_fake = eye(4); %to capture the state to calculate control input

system = ss(A,B,C,0);

cont_mat = ctrb(system);

rank(cont_mat)
%It is reachable!

[V,d] = eig(A);

lambda = diag(d)
% There is a pole at zero, so the open loop system is unstable.
% The negaitve-real part is assymptotically stable if there are no unstable
% poles.
% The complex-conjugate pair will cause oscillations that assymptotically
% reach stability if there are no unstable poles.
```

```
ans =

     4


lambda =

   0.000000000000000 + 0.000000000000000i
 -10.000000000000000 + 0.000000000000000i
  -0.700000000000000 + 3.000000000000000i
  -0.700000000000000 - 3.000000000000000i
```

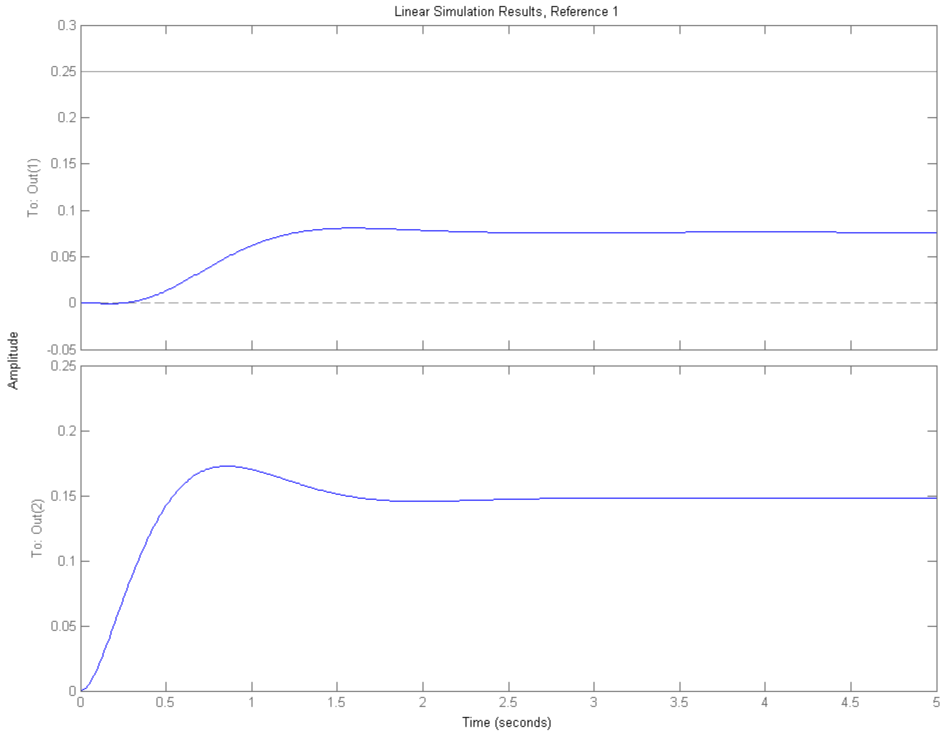
b)

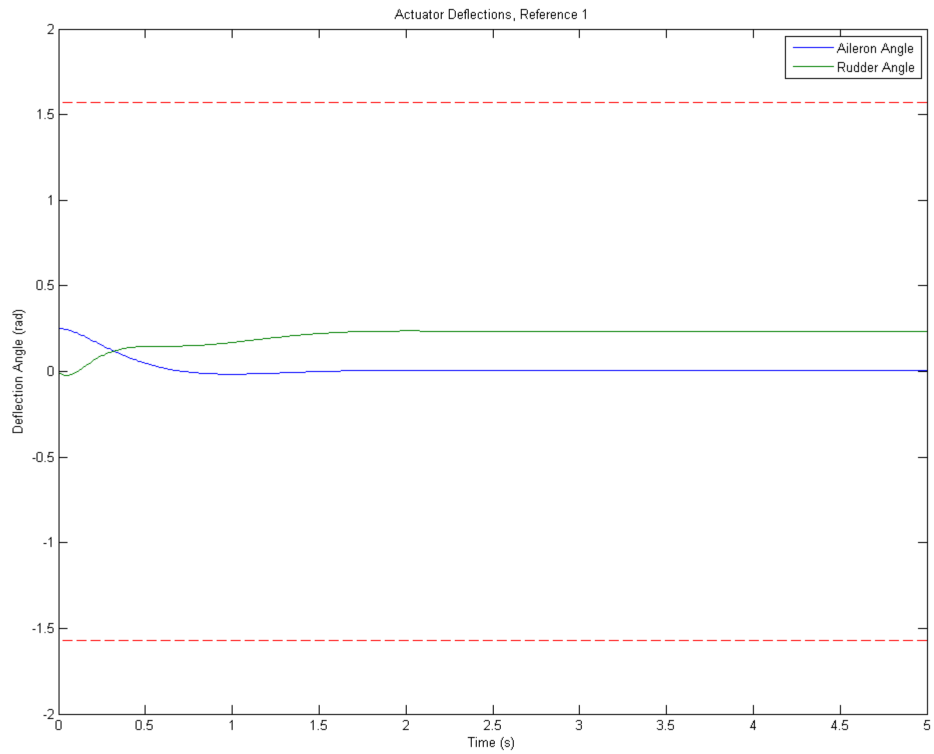
The design parameters

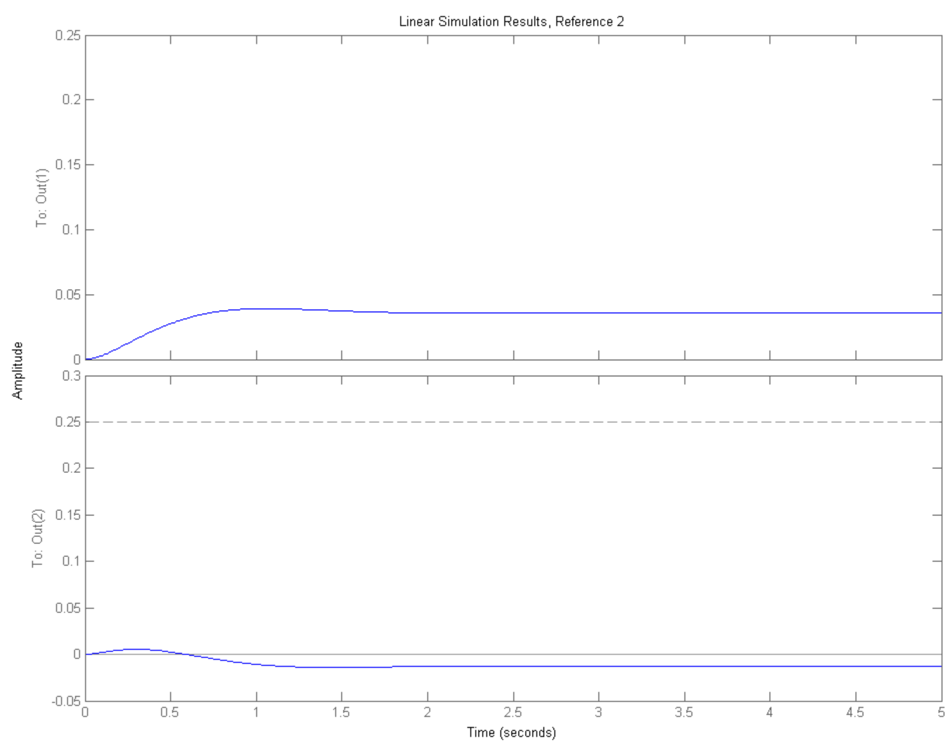
```
PO_desired = 10/100;
PS_desired = 5/100;
PO = 9/100;
PS = 4/100; %Settle percentage
Ts = 1.5;

% Get the desired dominant poles
damp_times_wn = -log(PS)/Ts
damping_ratio = -log(PO)/sqrt(pi*pi+(log(PO))^2);
wn = damp_times_wn/damping_ratio;
wd = wn*sqrt(1-damping_ratio^2)

P = [-5 -10 complex(-damp_times_wn, wd) complex(-damp_times_wn, -wd)];
K = place(A,B,P);
F = eye(2);

A_CL = A-B*K;
B_CL = B*F;

CL_system = ss(A_CL, B_CL, C,0);
CL_system_Fake = ss(A_CL, B_CL, C_fake,0);
t = 0:0.01:5;
r1 = [.25;0];
r2 = [0;0.25];

% No amount of tuning can eliminate the steady-state error for this system
% since F != inv[C*inv[A-BK]*B]. The feedforward gain is chosen to produce
% zero steady-state error in this manner.

% Reference 1
figure('Position',[0 0 hw_pub.figWidth hw_pub.figHeight])
lsim(CL_system,repmat(r1,1,length(t)),t)
title('Linear Simulation Results, Reference 1')
y1 = lsim(CL_system_Fake,repmat(r1,1,length(t)),t);
plot_CL_ctrl_outputs(r1,F,K,y1,t);
title('Actuator Deflections, Reference 1')
print_design_params(y1,t,PO_desired,PS_desired,Ts,3)

% Reference 2
figure('Position',[0 0 hw_pub.figWidth hw_pub.figHeight])
lsim(CL_system,repmat(r2,1,length(t)),t)
title('Linear Simulation Results, Reference 2')
y2 = lsim(CL_system_Fake,repmat(r2,1,length(t)),t);
plot_CL_ctrl_outputs(r2,F,K,y2,t);
title('Actuator Deflections, Reference 2')
print_design_params(y2,t,PO_desired,PS_desired,Ts,4)
```

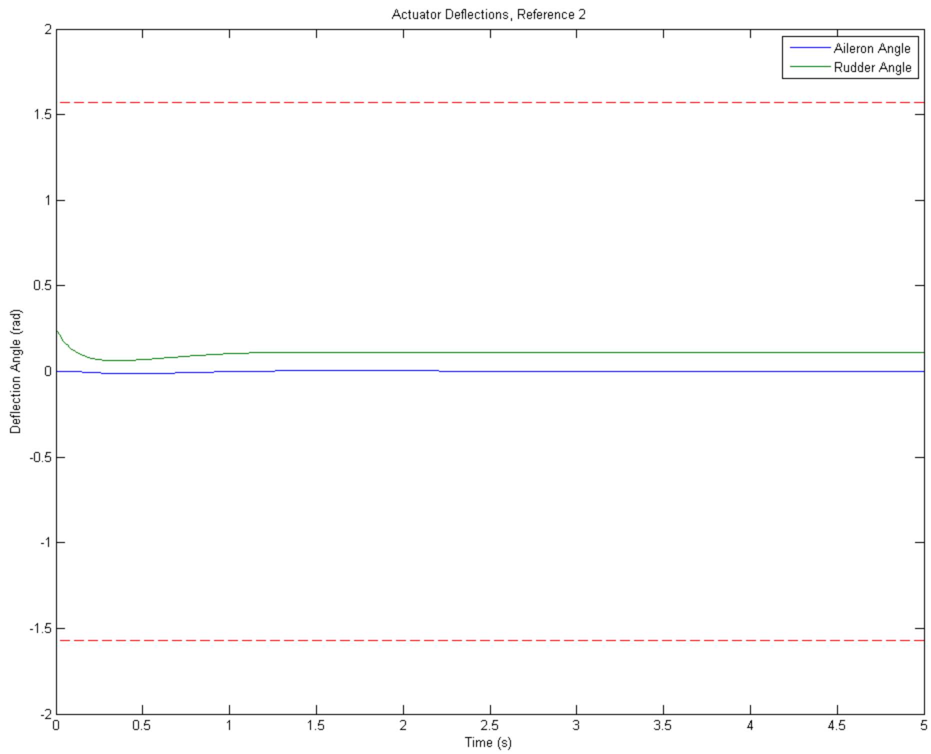
```
damp_times_wn =
```

```
   2.145917216578801

wd =

   2.7997300846800 33

Max overshoot within 10%: True (0.081)
Settled within 1.5 sec: True

Max overshoot within 10%: True (0.005)
Settled within 1.5 sec: False
```




Actuator Deflections, Reference 1
Aileron Angle
Rudder Angle
Deflection Angle (rad)
Time (s)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Linear Simulation Results, Reference 2
Amplitude
To: Out(1)
To: Out(2)
Time (seconds)
0.25
0.2
0.15
0.1
0.05
0
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

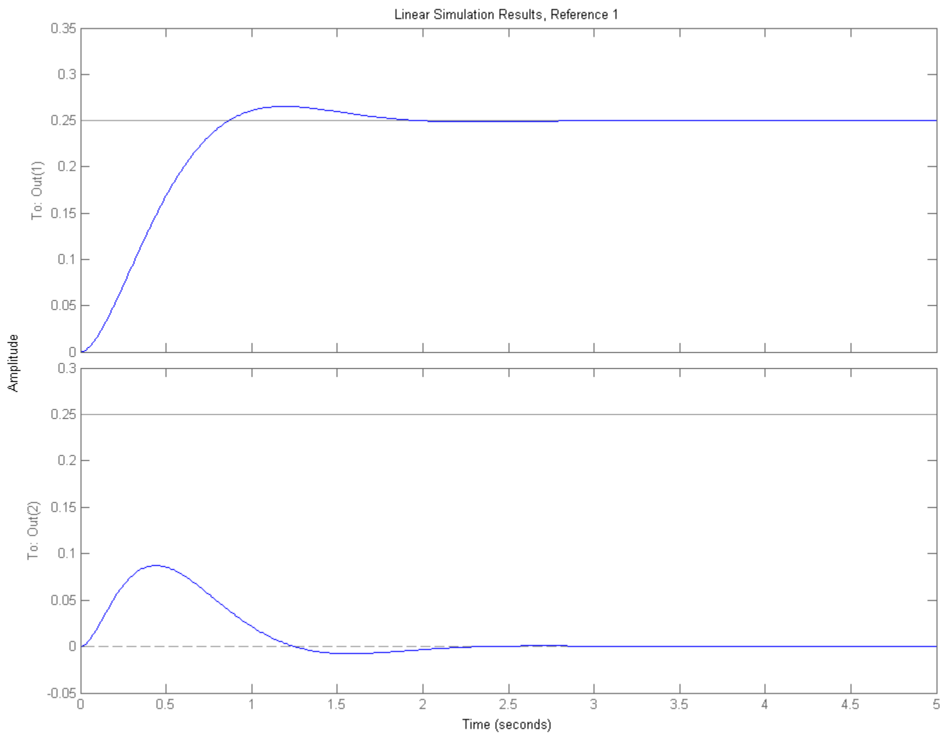


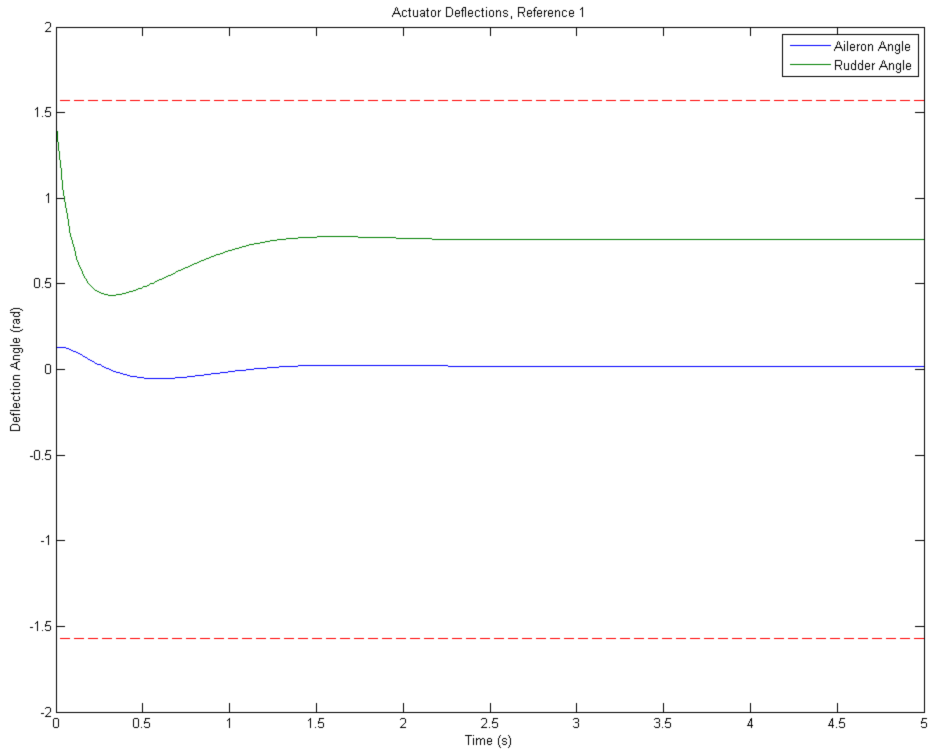
Part c)

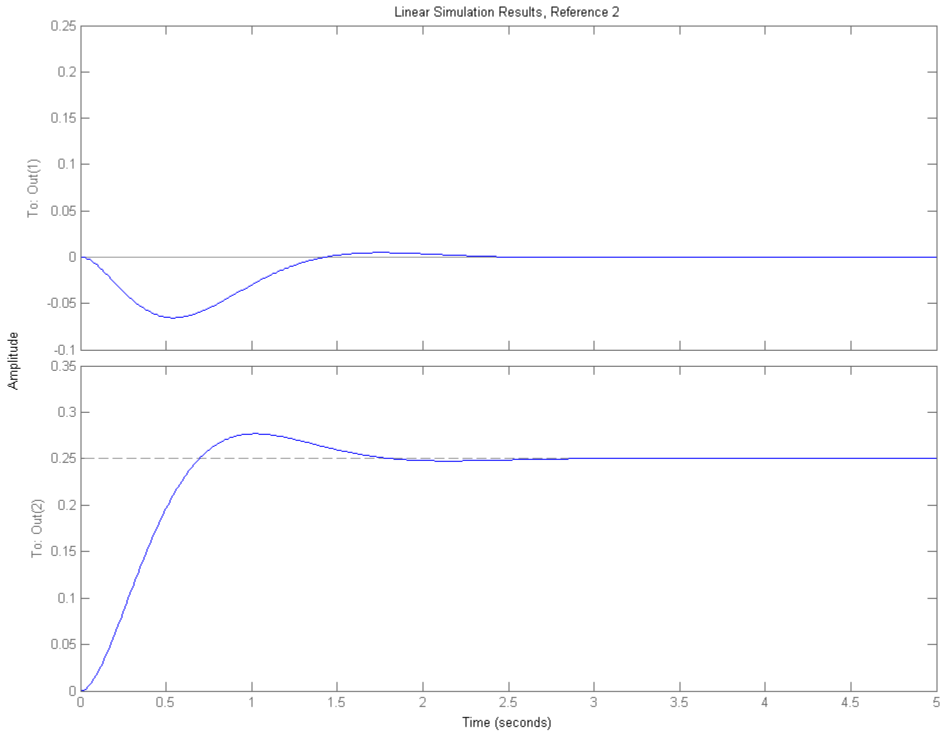
```
F = inv(C*inv(-A+B*K)*B);
B_CL = B*F;
CL_system_FF = ss(A_CL, B_CL, C,0);
CL_system_FF_Fake = ss(A_CL, B_CL, C_fake,0);

% To tune K, complex conj. poles were picked s.t. the natural frequency of
% damping ratio of a second-order system would adhere to the design
% parameters. The design parameters were given some margin to more easily
% place the remaining poles. The remaining poles were picked to be negative
% real numbers beyond (damping ratio)*(natural frequency). Their exact
% values were adjusted until the desired design was achieved.

% Reference 1
figure('Position',[0 0 hw_pub.figWidth hw_pub.figHeight])
lsim(CL_system_FF,repmat(r1,1,length(t)),t);
title('Linear Simulation Results, Reference 1')
y1 = lsim(CL_system_FF_Fake,repmat(r1,1,length(t)),t);
plot_CL_ctrl_outputs(r1,F,K,y1,t); % The control outputs
title('Actuator Deflections, Reference 1')
print_design_params(y1,t,PO_desired,PS_desired,Ts,3) % The g

% Reference 2
figure('Position',[0 0 hw_pub.figWidth hw_pub.figHeight])
```

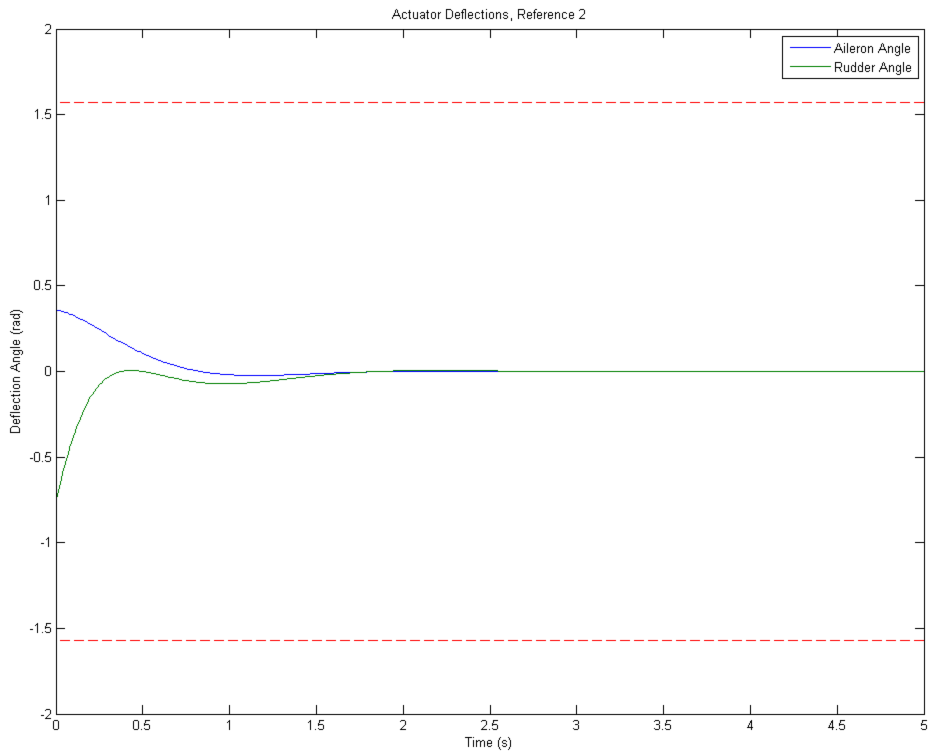
```
lsim(CL_system_FF,repmat(r2,1,length(t)),t)
title('Linear Simulation Results, Reference 2')
y2 = lsim(CL_system_FF_Fake,repmat(r2,1,length(t)),t);
plot_CL_ctrl_outputs(r2,F,K,y2,t);
title('Actuator Deflections, Reference 2')
print_design_params(y2,t,PO_desired,PS_desired,Ts,4)
```

```
Max overshoot within 10%: True (0.265)

Settled within 1.5 sec: True

Max overshoot within 10%: True (0.277)
Settled within 1.5 sec: True
```




Actuator Deflections, Reference 1
Aileron Angle
Rudder Angle
Deflection Angle (rad)
Time (s)


Linear Simulation Results, Reference 2
To: Out(1)
To: Out(2)
Amplitude
Time (seconds)

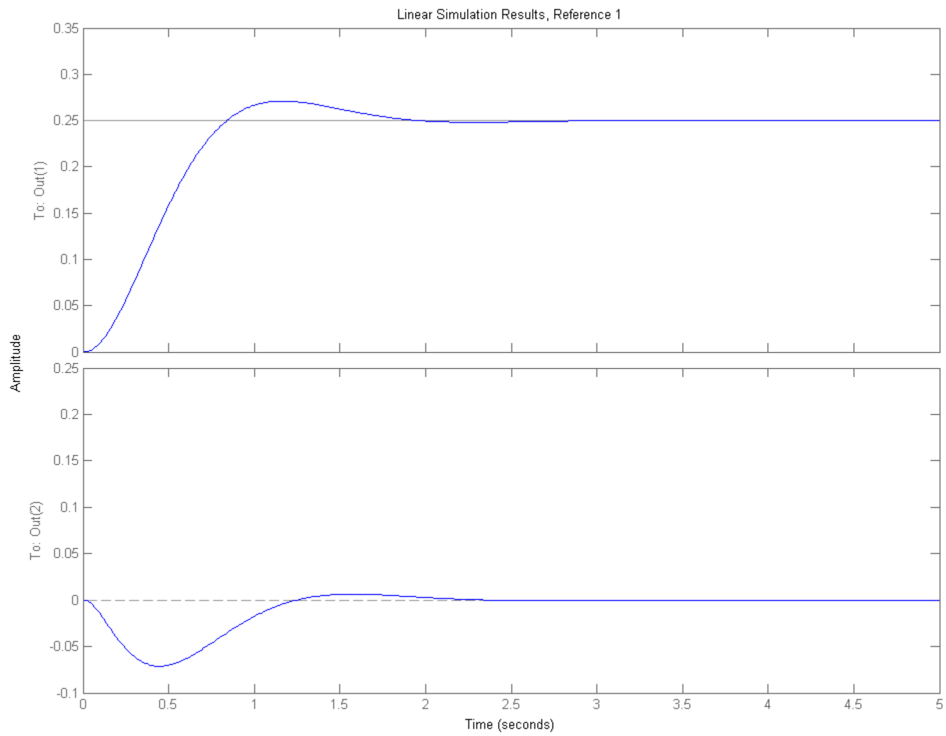


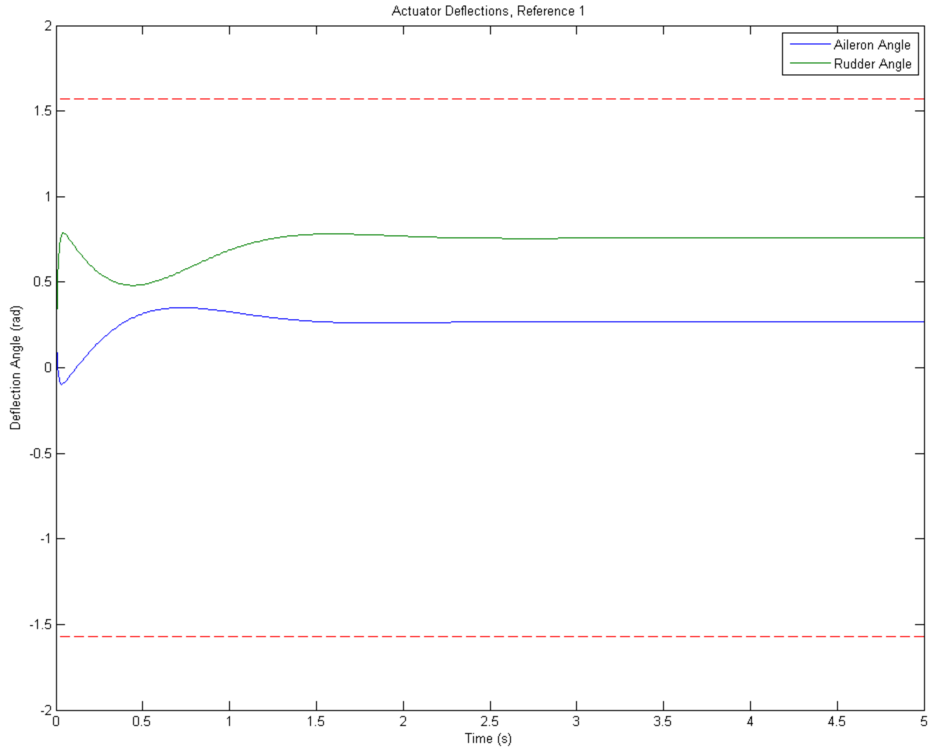
Part d)

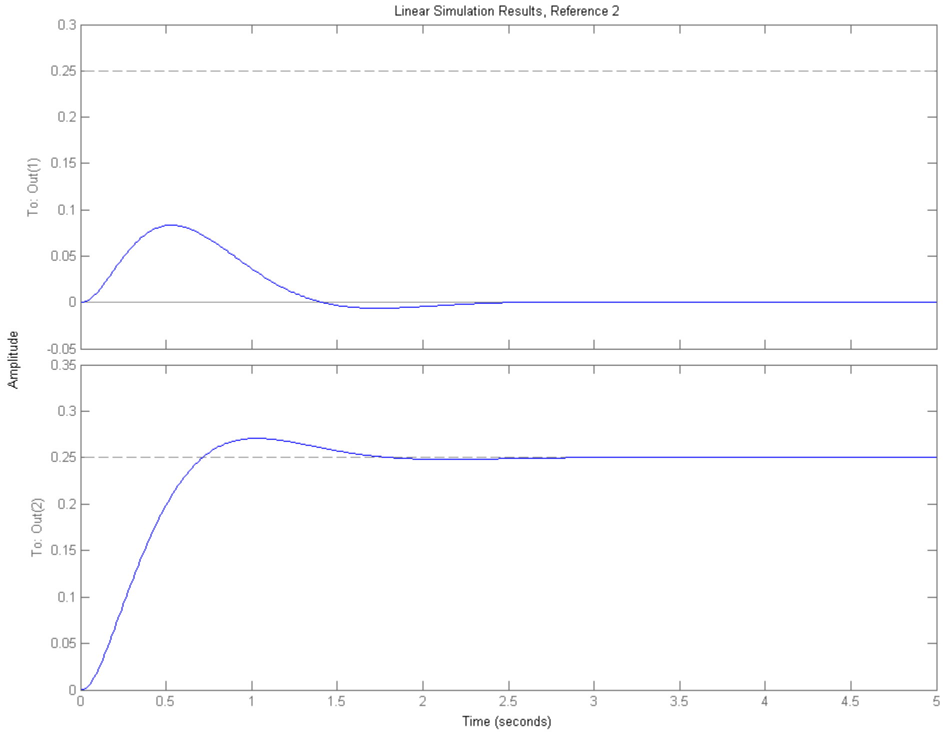
```
F = eye(2);
A_OL_Aug = [A,zeros(4,2);-C, zeros(2,2)];
B_OL_Aug = [B;zeros(2,2)];
P_Aug = [-100,-100.1,P];
K_Aug = place(A_OL_Aug,B_OL_Aug,P_Aug);
K = K_Aug(1:2,1:4); % gain for the nominal states
KI = K_Aug(1:2,5:6); % Integral gain
A_CL_Aug = [A-B*K, -B*KI; -C, zeros(2,2)];
B_CL_Aug = [zeros(4,2);eye(2)];

CL_system_Integral = ss(A_CL_Aug, B_CL_Aug,[C zeros(2)],0);
CL_system_Integral_Fake = ss(A_CL_Aug, B_CL_Aug,eye(6),0);

% Tuning K was done by placing the poles just like part c, with the
% additional poles being very far to the negative side of the Real axis.
% This controller will be better at rejecting unexpected disturbances or
% model errors than c), since it actively controls the error to zero.
% However, this controller requires an additional state for each desired
% output to be driven to a reference.

% Reference 1
figure('Position',[0 0 hw_pub.figWidth hw_pub.figHeight])
lsim(CL_system_Integral,repmat(r1,1,length(t)),t)
```

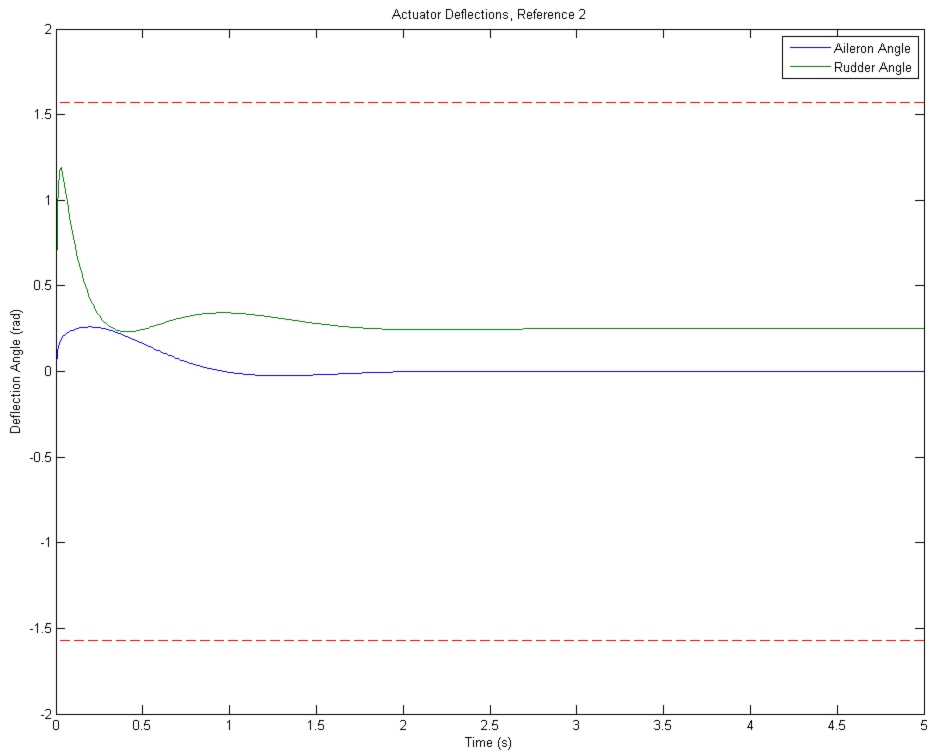
```
title('Linear Simulation Results, Reference 1')
y1 = lsim(CL_system_Integral_Fake,repmat(r1,1,length(t)),t);
plot_CL_ctrl_outputs(r1,F,K_Aug,y1,t);
title('Actuator Deflections, Reference 1')
print_design_params(y1,t,PO_desired,PS_desired,Ts,3)

% Reference 2
figure('Position',[0 0 hw_pub.figWidth hw_pub.figHeight])
lsim(CL_system_Integral,repmat(r2,1,length(t)),t)
title('Linear Simulation Results, Reference 2')
y2 = lsim(CL_system_Integral_Fake,repmat(r2,1,length(t)),t);
plot_CL_ctrl_outputs(r2,F,K_Aug,y2,t);
title('Actuator Deflections, Reference 2')
print_design_params(y2,t,PO_desired,PS_desired,Ts,4)
```

```
Max overshoot within 10%: True (0.271)

Settled within 1.5 sec: True

Max overshoot within 10%: True (0.271)
Settled within 1.5 sec: True
```




Actuator Deflections, Reference 1
Aileron Angle
Rudder Angle
Deflection Angle (rad)
Time (s)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Linear Simulation Results, Reference 2
Amplitude
To: Out(1)
To: Out(2)
Time (seconds)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0.35
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



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Single-Axis Control of a Solar Sail Through a Gimbal

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Contents

I. Introduction

Single-axis control of a solar-sail-driven interplanetary spacecraft (sailcraft) is proposed. The attitude control system will be responsible for ensuring that the steering angle between the force and velocity vectors is within the tolerance necessary for an interplanetary voyage. This steering angle is dependent on the mission parameters and the orbital position of the spacecraft. It, and the sun vector, will be treated as external commands to the system. The spacecraft will perform all its thrusting in the orbit plane.

The primary actuation mechanism will be a gimbaled control boom between the sail subsystem and the spacecraft bus, which contains the majority of the spacecraft mass. With the center of mass between the thrust point and the sun, expected disturbances will cause oscillation about some angle between the sun and the axis normal to the sail, α, for a locked gimbal. Changing the gimbal angle, δ, will dampen this oscilation with the right conrol law. Roll and pitch angles wil be held to zero for this analysis. Star trackers will determin attitude.

The state-space model is expected to have four states: the sun angle (α), the rate of the sun angle ($\dot{\alpha}$), the gimbal angle (δ), and the gimbal angle rate ($\dot{\delta}$). Depending on the vane implementation, there may be up to two more states for vane angles.

The sail and boom will be modeled as rigid bodies, justified by the slow actuation of the gimbal throughout the flight. The sail will be modeled as a thin plate, rather than a billowed sail. Solar pressure torques (about the non-steered axis) will be controlled against. Disturbance torques from thruster firings may also be modeled.

The state-space model will be obtained in a similar manner to that presented by Wie. The equations of motion for a gimbaled thrust vector are obtained for the yaw axis.

System performance will be judged by the response to errors, both with a step-error and a flight-like error where the steering angle constantly-but-slowly changes. Mitigation of disturbance torques will also be examined.

II. State Space Representation

The equations of motion were linearized about the state $\alpha = \dot{\alpha} = \delta = \dot{\delta} = 0$. This state is in equilibrium, due to the the force resulting from the solar radiation pressure acting through the sailcraft's center of mass. Any disturbance to α would cause oscillation about $\alpha = 0$. The linearized equations are shown below:

$$\begin{bmatrix} \dot{\alpha} \\ \ddot{\alpha} \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{d}{J_s} F_n & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m_p l}{J_p m + m_s m_p l^2} F_t & 0 & -\frac{m_p l}{J_p m + m_s m_p l^2} F_n & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{J_s} \\ 0 \\ \frac{1}{J_p + \frac{m_s m_p}{m} l^2} \end{bmatrix} T_{gimbal} \tag{1}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u \tag{2}$$

$$F_n = PA(1 + \rho_s + \frac{2}{3}\rho_d) \tag{3}$$

$$F_t = PA(1 - \rho_s) \tag{4}$$

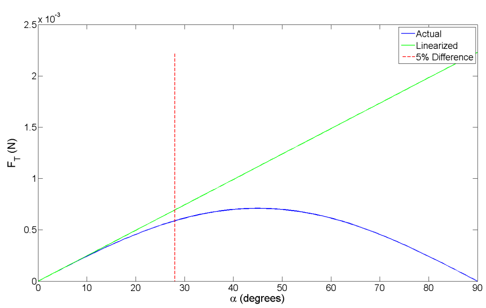
Table 1. Sailcraft characteristics.

Characteristic	Value
m_s	40 kg
m_p	116 kg
m	156 kg
J_s	6000 kg·m^2
J_p	20 kg·m^2
P	4.563e-6 kg/m^2
A_{sail}	1800 m^2
l	2 m
d	1.487 m
ρ_s	0.8272
ρ_d	-0.5949

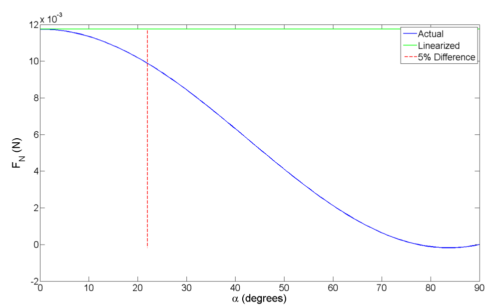
Using the sailcraft characteristics in Table 1, the eigenvalues are found to be:

$$\lambda_i = \{\pm 1.1200 \times 10^{-2} i, \pm 5.9395 \times 10^{-4} i\}.$$

The complex eigenvalues with no real parts indicate that the uncontrolled, linearized system is marginally stable. It will oscillate undamped when perturbed by a small amount, but a large disturbance could excite the modes and make the output y unbounded. However, as α and δ each approach $\pm 90°$, the assumption becomes invalid. Indeed, in Figure 1, one can see that a five-percent error between the non-linear and linearized sail force occurs at approximately $\pm 20°$.



(a) Transverse force



(b) Normal force

Figure 1. Linear solutions of the sail forces vs. sun angle.

The controller will be constrained to keep the sun angle less than or equal to 20° to keep the system model within the realm of linearity. It will have to dampen the oscillations induced induced by disturbances so that the sail can provide a force in the desired direction.

III. Homework 7

The system is found to be controllable since the four-row controllability matrix is full rank.

For full-state feedback control, pole placement will have to be performed such that the real parts are negative. The gimbal angle cannot exceed $\pm 90°$, which is a physical constraint. To maintain the linearity-about-zero assumption, the sun angle α should not exceed $\pm 20°$. The controller will track the specified sun angle with zero steady-state error. Due to the slow nature of the changing reference, a nearly-critical damped response will be sufficient.

The system is found to be observable since the observability matrix is full rank.

The uncontrolled step response of the system is shown in Figure 2. The response matches the eigenvalues: two sine waves superimposed on eachother, each corresponding to one of the complex conjugate pairs. Because the eigenvalues have no real part, the motion is undamped.

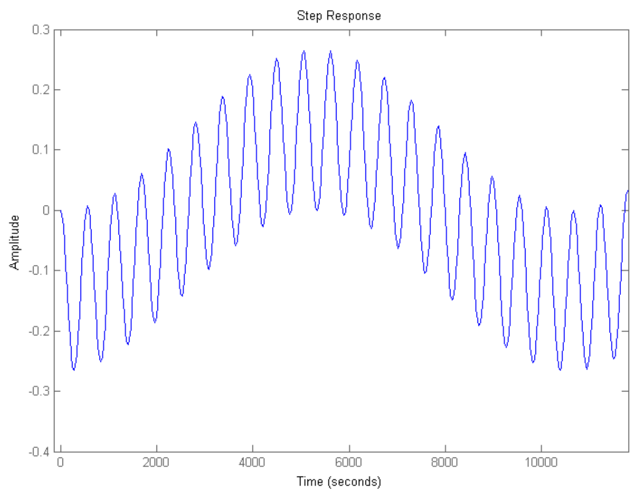


Figure 2. Linear solutions of the sail forces vs. sun angle.