ASEN 5050 Mid-Term Exam
Due: 10:00 am Monday, 12/14/2015
Paper Copy Preferred (ECNT319) or D2L Dropbox

Read everything on this page before you take the exam!

This is a take-home exam. You may use whatever books and notes you have at your disposal. However, you may not communicate with other people (students or otherwise) about the exam. You may want to use a computer to help check the calculations you have made, but you will be graded based on the work you show in your answer. You may attach any code you have written in support of the exam, but it will in general not be used to grade your exam.

In general, a correct answer will not give you credit for a question unless you show your work. An incorrect answer may still give you partial credit if you show a correct process.

Write all your answers on separate sheets of paper and attach them to your exam! Circle your final answers asked for in the problem.

Use Appendix D-3 and D-4 of the book for all constants not given in the problem.

You must complete this exam within a time period of **48 hours**, but you may review the questions first to see if you have any questions for me. After **48 hours**, stop working. I expect you will finish in ~3-4 hours plus 1-2 hours to check your answers.

Sign the following statement, reflecting the Honor Code (electronic signatures are acceptable):

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work. I have used no more than 24 hours to complete this exam.

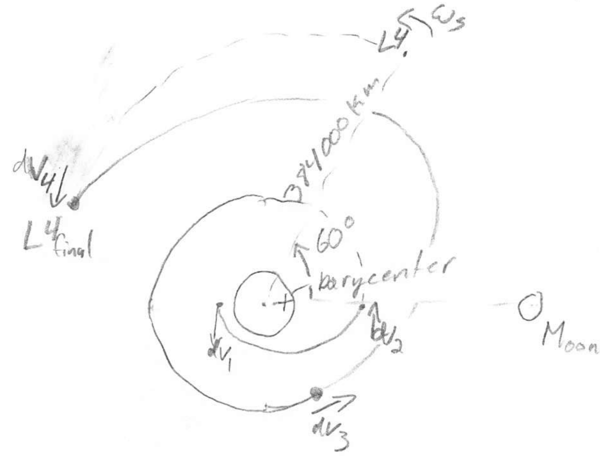
Signature	John Clouse Printed Name	12-13-15 Date

12-11-15 2:00 PM
Date/Time Exam Started

12-13-15 11:00 AM
Date/Time Exam Finished

Clouse
ASEN 5050
Final Exam

1. $h = 300$ Km
$i = 28^\circ$
$r_☾ = 384,000$ Km



Assuming circular orbits

$$\omega_s = \sqrt{\frac{\mu_E + \mu_m}{\left(384000 - \frac{384000 \mu_m}{\mu_E + \mu_m}\right)^3}} = 2.71888 \times 10^{-6}$$

Need to match final trueanoms.
breaking it up into 3 transfers: - Hohmann to $h = 1000$ km
- circ. phasing orbit @ $h = 1000$ Km
- Hohmann to L4

$a_{xfer1} = R_\oplus + (300 + 1000)/2 = 7028$ km

$t_{xfer1} = 2\pi \sqrt{\frac{a_{xfer1}^3}{\mu_\oplus}} / 2 = 2931$ s

$a_p = 7378$ km

$n_p = 9.9233 \times 10^{-4}$ rad/s

$a_{xfer2} = (R_\oplus + h_p + L_4)/2 = 195689$ Km

$t_{xfer2} = 430755$ s

total phasing time in 1000 Km orbit!

$$\pi + \pi + n_p t_p + \pi - revs \cdot 2\pi = \frac{\pi}{3} + \omega_s t_{xfer1} + \omega_s t_p + \omega_s t_{xfer2}$$

$$t_p = \frac{\frac{\pi}{3} + \omega_s (t_{xfer1} + t_{xfer2}) - 3\pi + revs \cdot 2\pi}{n_p - \omega_s}$$

for revs = 2, $\underline{t_p = 2240.876 \text{ sec}}$

$$dv_1 = \sqrt{\frac{2\mu_\oplus}{R_\oplus + 300} - \frac{\mu_\oplus}{7028}} - \sqrt{\frac{\mu_\oplus}{R_\oplus + 300}} = 0.1900 \text{ Km/s, vel-vec direction}$$

$$dv_2 = \sqrt{\frac{\mu_\oplus}{R_\oplus + 1000}} - \sqrt{\frac{2\mu_\oplus}{R_\oplus + 1000} - \frac{\mu_\oplus}{7028}} = 0.1854 \text{ Km/s, vel-vec direction}$$

$$dv_3 = \sqrt{\frac{2\mu_\oplus}{R_\oplus + 1000} - \frac{\mu_\oplus}{a_{xfer2}}} - \sqrt{\frac{\mu_\oplus}{R_\oplus + 1000}} = 2.9461 \text{ Km/s, vel-vec direction}$$

$$dv_4 = r_{L4} \cdot \omega_s - \sqrt{\frac{2\mu_\oplus}{r_{L4}} - \frac{\mu_\oplus}{a_{xfer2}}} = 0.8462 \text{ Km/s, vel-vec direction}$$

The maximum inclination change needed is $28^\circ + 5.145396^\circ = 33.145396^\circ$, depending on Ω for the SC & Moon. It would have to take place on the shared line of nodes, which is the line connecting the SC, Earth, & Moon at the beginning (since SC/Moon are directly opposed). In my maneuver plan, there are 3 places to do it: dv_1, dv_2, or in the parking orbit after dv_2.

$$dV_{inc,max} = 2 \cdot \sqrt{\frac{\mu_\oplus}{R_\oplus + 300}} \sin\left(\frac{33.145^\circ}{2}\right) = \boxed{4.4073 \text{ Km/s}}$$

Clouse
ASEN 5050
Final Exam

2.

$\uparrow 2\,m/s$

$40^\circ N,\ 105^\circ W,\ 6378.1363 + 1.6551$ Km

$= 6379.7914$ Km $= r$

13:31:41 MST 12-8-15

$$\vec{V}_{ECI} = \frac{\vec{r}_{ECI}}{\|\vec{r}\|_{ECI}} \cdot \underbrace{2\,m/s}_{=0.002\,Km/s} + \vec{\omega}_{earth} \times \vec{r}_{ECI}, \quad \omega_{earth} = 7.292115\times10^{-5}\ rad/s$$

$\theta_{LST} = \theta_{GMST} + \lambda \Rightarrow JD = 2457364.77200231$

$$T_{UT1} = \frac{JD - 2451545}{36525} = 0.15933667358$$

$$\theta_{GMST} = 67310.54841 + (876600 \cdot 3600 + 8640184.812866)T_{UT1} + 0.093104T_{UT1}^2 - 6.2\times10^{-6}T_{UT1}^3$$

$= 5.042723099\times10^8\,s$

$= 174.62440798^\circ$

$\theta_{LST} = 69.62440798^\circ$

$\vec{r}_{ECI} = R_3(\theta_{LST})R_2(-(90-\phi_{gd}))\vec{r}_{SEZ}, \quad \vec{r}_{SEZ} = [0,0,r]^T$

$$\phi_{gd} = \text{atan}\left(\frac{\tan\phi_{gc}}{1-e_\oplus^2}\right) = 40.1896106^\circ$$

$$\vec{r}_{ECI} = \begin{bmatrix} 1696.856707 \\ 4568.666352 \\ 4117.001747 \end{bmatrix} Km \Rightarrow \vec{V} = \begin{bmatrix} -0.332620496 \\ 0.125168988 \\ 0.001290638 \end{bmatrix} Km/s$$

cart2OE Results:

$a = 3193.123255$ km
$e = 0.997978$
$i = 40.189611^\circ$
$\omega = 339.624408^\circ$
$\Omega = 270.000653^\circ$
$\nu = 179.999347^\circ$
$r_a = 6379.791569$ km
$r_p = 6.454941$ km

$$\boxed{h_{max} = r_a - r = 2.046369\times10^{-1}\,m}$$

Problem 3: $M_{ast.} = 10^{15}\, kg$

a) $a_{ast} = 3AU$ $\Rightarrow \left(\frac{M_{ast}}{M_{sun}}\right)^{2/5} \cdot a = \boxed{340.866263\ km\ \ SOI}$

$M_{sun} = 1.1891 \times 10^{30}\, kg$

b) $\mu_{ast} = GM_{ast} = 6.6740 \times 10^{-5}\, km^3/s^2$, $G = 6.672 \times 10^{-20}\ \frac{km^3}{kg\, s^2} = \frac{\mu_\oplus}{m_\oplus}$

$P = 2\pi\sqrt{\frac{a_{sat}^3}{\mu_{ast}}} = \boxed{1.9460 \times 10^5 s = 54\ hrs}$

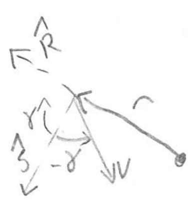
c) $V_{circ} = \sqrt{\frac{\mu}{40}} = 0.0013\ km/s = 1.3\ m/s$

$V_{new}^2 = \frac{2\mu}{r} - \frac{\mu}{a_{xfer}} \Rightarrow \frac{1}{a_{xfer}} = \frac{2}{r} - \frac{V^2}{\mu} \Rightarrow a_{xfer} = \frac{1}{\frac{2}{r} - \frac{V^2}{\mu}} = 20.5228$ $\qquad \frac{a}{r_p} - 1 = e = 0.01411$

$M_f = 180° + n \cdot 6.3600 = 288.7304° \Rightarrow \nu = 190.9056°$

$$\boxed{r_f = \frac{a(1-e^2)}{1+e\cos\nu} = 20.9306\ km}$$

d) $\gamma = atan2(e_{xfer}\sin\nu,\ 1 + e_{xfer}\cos\nu) = -69.2332°$



$V_{circfinal} = \sqrt{\frac{\mu}{r_f}} = 0.00149\ km/s$

$V_{xferfinal} = \sqrt{\frac{2\mu}{r_f} - \frac{\mu}{a_{xfer}}} = 0.00110\ km/s$

$$d\vec{V} = \begin{bmatrix} 0 \\ V_{circfinal} \\ 0 \end{bmatrix} - \begin{bmatrix} \sin\gamma \\ \cos\gamma \\ 0 \end{bmatrix} V_{xferfinal} = \boxed{\begin{bmatrix} 1.03 \\ 1.10 \\ 0 \end{bmatrix} m/s} \qquad \boxed{\|dV\| = 1.51\ m/s}$$

e) The inclination is 90°, so RAAN not affected ($\dot{\Omega} \propto \cos i$)
The orbit is circular, so ω's change is not important if using the argument of latitude, although ω will change secularly.
Mean anomaly will experience secular change. Operators should be concerned about this if they use Kepler's Problem to propagate the orbit.

Clouse
ASEN 5050
Final Exam

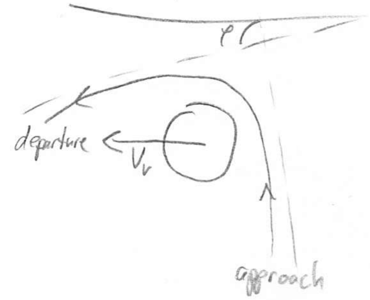
Problem 4. $SOI_{venus} = 6.1467e5 = r_\infty$

$$\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r_v} = \boxed{-796.745\ J/kg}$$

$$\vec{V}_\infty = \vec{V}_{sc}^{Approach} - \vec{V}_{venus} = \boxed{\begin{bmatrix} 3.1988 \\ -7.0762 \\ 0 \end{bmatrix} km/s} \Rightarrow \boxed{\|\vec{V}_\infty\| = 7.7656\ km/s}$$

$$\mathcal{E}_{venus} = \frac{V_\infty^2}{2} - \frac{\mu_{venus}}{r_\infty} = \boxed{29.6244\ J/kg}$$

b) Counter Clockwise



Clockwise

$$V_\infty^{dep} = R_3(42.0^\circ) \cdot V_\infty^{app} = \begin{bmatrix} -2.3577 \\ -7.3990 \\ 0 \end{bmatrix} km/s$$

$$\boxed{V_{sc}^{Dep} = \begin{bmatrix} -33.6800 \\ 8.3315 \\ 0 \end{bmatrix} km/s \quad \mathcal{E} = -627.7812\ J/kg}$$

$\boxed{\text{Increases heliocentric energy}}$

$$V_\infty^{dep} = R_3(-42^\circ) \cdot V_\infty^{app} = \begin{bmatrix} 7.1121 \\ -3.1182 \\ 0 \end{bmatrix} km/s$$

$$\boxed{V_{sc}^{Dep} = \begin{bmatrix} -24.2102 \\ 12.6123 \\ 0 \end{bmatrix} \quad \mathcal{E} = -857.0560}$$

$\boxed{\text{Decreases Heliocentric energy}}$

$$r_p = \frac{\mu_v}{V_\infty^2} \cdot \left(\frac{1}{\cos\left(\frac{\pi - \varphi}{2}\right)} - 1 \right) = \boxed{75013.02\ km}$$

Clouse
ASEN 5050
Final Exam

Problem 5, $h = 185$

$\Delta V = 7\ Km/s$

a) $V_\infty = 7\ Km/s - (V_{esc} - V_{circ}) = 7 - \left(\sqrt{\frac{2\mu_\oplus}{185 + R_\oplus}} - \sqrt{\frac{\mu_\oplus}{185 + R_\oplus}}\right) = 3.7719\ Km/s$

$V_\oplus = 29.78469$

for farthest point:

$r = r_p = 1AU$

$v^2 = \frac{2\mu_\odot}{r_\oplus} - \frac{\mu_\odot}{a_{xfer}} \Rightarrow \frac{1}{a_{xfer}} = \frac{2}{r_\oplus} - \frac{v^2}{\mu_\odot}$

$\Rightarrow a_{xfer} = \frac{1}{\frac{2}{r_\oplus} - \frac{v^2}{\mu_\odot}}$

$V_p = V_\infty + V_\oplus = 33.5566\ Km/s$

$a_{xfer} = 204{,}730{,}865.759\ Km = 1.3686\ AU$

$r_a = 2a_{xfer} - r_p = \boxed{259{,}876{,}746.615\ Km = 1.7372\ AU}$

b) $r = r_a = 1Au$

$V_a = V_\oplus - V_\infty = 26.0128\ Km/s$

$a_{xfer} = 120{,}912{,}324.470\ Km$

$r_p = 2a_{xfer} - r_a = \boxed{92{,}226{,}778.239\ Km = 0.6165\ AU}$

c) 

$i = \cancel{0°}\ 7° + \text{asin} \frac{V_\infty}{V_\oplus} = \boxed{7.275°}$

Clonse
ASEN 5050
Final Exam

Problem 6

$r_p = 7000$ km

$r_a = 8000$ Km

$i = 100°$

$P = 6000$ s

$R_{planet} = 6000$ Km

$r_{planet} = 2$ AU

$$a = \frac{r_p + r_a}{2} = 7500 \text{ Km}$$

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \Rightarrow \mu = 2\pi \frac{a^3}{P^2} = \boxed{7.363108 \times 10^4 \text{ Km}^3/\text{s}^2}$$

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.06667$$

$$P_{planet} = 2\pi\sqrt{\frac{r_{planet}^3}{\mu_\odot}} = 8.926 \times 10^7 \text{ s}$$

$$\dot{\Omega}_{ss} = \frac{360°}{P_{planet}} = 7.03919 \times 10^{-8} \text{ rad/s}$$

$$= -\sqrt{\frac{\mu}{a^3}} \frac{3R_{planet}^2 J_2}{2a^2(1-e^2)^2} \cos(i)$$

$$\Rightarrow J_2 = -\dot{\Omega}\sqrt{\frac{a^3}{\mu_{planet}}} \cdot \frac{2a^2(1-e^2)^2}{3R_{planet}^2 \cos(i)}$$

$$\boxed{= 0.0010017827}$$

Table of Contents

Problem 1

```
clear all
CelestialConstants

ws = sqrt((Earth.mu+Moon.mu)/(384000-(384000*Moon.m/(Earth.m+Moon.m)))^3)

% T_tot = 5/3*pi/ws

h_phasing=1000;
a_xfer1 = Earth.R+(300+h_phasing)/2;
t_xfer1 = 2*pi*sqrt(a_xfer1^3/Earth.mu)/2

n_p = sqrt(Earth.mu/(Earth.R+h_phasing)^3)

a_xferf = (Earth.R+h_phasing+384000)/2
t_xferf = 2*pi*sqrt(a_xferf^3/Earth.mu)/2
n_xfer = sqrt(Earth.mu/a_xferf^3)

% L4_init_phase = atan((Moon.m/(Earth.m+Moon.m)-1/2))
L4_phase_angle_during_init_transfer = t_xfer1*ws
L4_phase_angle_during_final_transfer = t_xferf*ws

revs = 2;
phase_time = (pi/3-pi+L4_phase_angle_during_init_transfer+...
    L4_phase_angle_during_final_transfer -3*pi + revs*2*pi)/(n_p-ws)

dv1 = sqrt(2*Earth.mu/(Earth.R+300)-Earth.mu/a_xfer1)...
    -sqrt(Earth.mu/(Earth.R+300))
dv2 = sqrt(Earth.mu/(Earth.R+1000))...
    -sqrt(2*Earth.mu/(Earth.R+1000)-Earth.mu/a_xfer1)
dv3 = sqrt(2*Earth.mu/(Earth.R+1000)-Earth.mu/a_xferf)...
    -sqrt(Earth.mu/(Earth.R+1000))
dv4 = 384000*ws - sqrt(2*Earth.mu/384000-Earth.mu/a_xferf)

dv_incl_max = 2*sqrt(Earth.mu/(Earth.R+300))*sind(33.145396/2)
% v_L4 = sqrt()
```

Problem 2

```
r = 6378.1363+1.655064; %km
```

```
JD = computeJD(2015,12,8,13-7,31,41);
T_UT1 = ((JD)-2451545)/36525;

GMST = 67310.54841 ...
    + (876600*3600+8640184.812866) * T_UT1 ...
    + 0.093104 * T_UT1 * T_UT1 ...
    - 6.2e-6 * T_UT1 * T_UT1 * T_UT1;
while GMST > 86400
GMST = GMST-86400
end
LST = GMST/240-105

phi_gd = atand(tand(40)/(1-Earth.oblate_ecc^2));
r_eci = Euler2DCM('23', [-(90-phi_gd), -LST]*pi/180)*[0;0;r]
v = r_eci/r*0.002+cross([0;0;Earth.spin_rate],r_eci)
% v = cross([0;0;Earth.spin_rate],r_ecf)

[a,e,i,w,RAAN,f] = cart2OE(r_eci,v,Earth.mu)
ra = a*(1+e)
rp = a*(1-e)
h_max = ra-r
```

Problem 3

```
clear
CelestialConstants
a = 3*149597870.7;
m_ast = 1e15;
SOI = (m_ast/1.9891e30)^(2/5)*a

G = Earth.mu/Earth.m;%6.674e-20;
mu_ast = G*m_ast
P = 2*pi*sqrt(40^3/mu_ast)
P/3600
v_circ = sqrt(mu_ast/40)
a_new = 1/(2/40-(v_circ-.001)^2/mu_ast)
e_xfer = 40/a_new-1
n_xfer = sqrt(mu_ast/a_new^3)
Mf = pi+n_xfer*6*3600
Mf*180/pi
f = E2f(M2E(Mf,e_xfer),e_xfer)
f*180/pi
rf = a_new*(1-e_xfer^2)/(1+e_xfer*cos(f))
fpa = atan2(e_xfer*sin(f),1+e_xfer*cos(f))
fpa*180/pi
v_circ_f = sqrt(mu_ast/rf)
v_xfer_f = sqrt(2*mu_ast/rf-mu_ast/a_new)
dV = [0;v_circ_f]-[sin(fpa);cos(fpa)]*v_xfer_f
norm(dV)
```

Problem 4

```
clear
```

```
CelestialConstants

r_venus = [48965315.1 96179438.8 0.0]'; %km
v_venus = [-31.322263 15.730492 0.0]'; %km

v_approach = [-28.123456 8.654321 0.0]'; %km

SOI = (Venus.m/Sun.m)^(2/5)*norm(r_venus)
energy = norm(v_approach)^2/2-Sun.mu/(norm(r_venus))
V_inf_app = v_approach-v_venus
norm(V_inf_app)
energy_venus = norm(V_inf_app)^2/2-Venus.mu/SOI

%ccw
V_inf_dep_ccw = Euler2DCM('3',[42*pi/180])*V_inf_app
V_dep_ccw = V_inf_dep_ccw+v_venus
energy = norm(V_dep_ccw)^2/2-Sun.mu/(norm(r_venus))

%cw
V_inf_dep_cw = Euler2DCM('3',[-42*pi/180])*V_inf_app
V_dep_cw = V_inf_dep_cw+v_venus
energy = norm(V_dep_cw)^2/2-Sun.mu/(norm(r_venus))

rp = Venus.mu/norm(V_inf_app)*(1/cosd((180-42)/2)-1)
```

Problem 5

```
clear
CelestialConstants
h = 185; %km
v_inf = 7-(sqrt(2*Earth.mu/(h+Earth.R)) - sqrt(Earth.mu/(h+Earth.R)))

v_earth = sqrt(Sun.mu/au2km)

vp = v_inf+v_earth
a_xfer1 = 1/(2/au2km-norm(vp)^2/Sun.mu)
a_xfer1/au2km
ra1 = 2*a_xfer1-au2km
ra1/au2km

%b
va = v_earth-v_inf
a_xfer2 = 1/(2/au2km-norm(va)^2/Sun.mu)
a_xfer2/au2km
rp2 = 2*a_xfer2-au2km
rp2/au2km

%c
i = asind(v_inf/v_earth)
```

Problem 6

```
clear
```

```
CelestialConstants
rp = 7e3;
ra = 8e3;
i = 100;
P = 60*100; %s
R = 6e3;
r_planet = 2*au2km;

a = (rp+ra)/2
mu = 2*pi*a^3/P^2

P_planet = 2*pi*sqrt(r_planet^3/Sun.mu)
RAAN_dot_ss = 2*pi/P_planet
e = (ra-rp)/(ra+rp)

J2 = -RAAN_dot_ss*sqrt(a^3/mu)*2*a^2*(1-e*e)^2/(3*R*R*cosd(i))
```

Published with MATLAB® R2013b

```
function [a,e,i,RAAN,w,f] = cart2OE( r, v ,mu)
%cart2OE return classical orbital elements from cartesian coords
% Only valid for e < 1
% units in radians
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

h = cross(r,v);
n = cross([0;0;1],h);
ecc_vec = ((norm(v)*norm(v)-mu/norm(r))*r - dot(r,v)*v)/mu;

e = norm(ecc_vec);
a = 0;
if e < 1.0
    specific_energy = norm(v)*norm(v)/2-mu/norm(r);
    a = -mu/2/specific_energy;
end

i = acos(h(3)/norm(h));
RAAN = acos(n(1)/norm(n));
if n(2) < 0
    RAAN = 2*pi-RAAN;
end
w = acos(dot(n,ecc_vec)/(norm(n)*norm(ecc_vec)));
if ecc_vec(3) < 0
    w = 2*pi-w;
end
f = acos(dot(ecc_vec,r)/(norm(ecc_vec)*norm(r)));
if dot(r,v) < 0
    f = 2*pi-f;
end
```

Published with MATLAB® R2013b

```
function DCM = Euler2DCM( seq_string, angle_vector )
%Euler2DCM Turn an Euler Angle set into a DCM
%   Angle vector in radians
fcnPrintQueue(mfilename('fullpath'))

DCM = eye(3);
%get the trig functions
num_rot = length(seq_string);
c = zeros(num_rot,1);
s = zeros(num_rot,1);

for idx = 1:num_rot
c(idx) = cos(angle_vector(idx));
s(idx) = sin(angle_vector(idx));
end


for idx = num_rot:-1:1
    if strcmp(seq_string(idx),'1')
        M = [1 0 0; 0 c(idx) s(idx); 0 -s(idx) c(idx)];
        DCM = DCM*M;
    elseif strcmp(seq_string(idx),'2')
        M = [c(idx) 0 -s(idx); 0 1 0; s(idx) 0 c(idx)];
        DCM = DCM*M;
    elseif strcmp(seq_string(idx),'3')
        M = [c(idx) s(idx) 0; -s(idx) c(idx) 0; 0 0 1];
        DCM = DCM*M;
    else
        fprintf('%s is not a valid axis\n', seq_string(idx))
    end
end


end
```

Published with MATLAB® R2013b

```
function f = E2f( E, e )
%E2f Eccentric anomaly (E) to true anomaly (f)
% Only valid for e < 1
% units in radians
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

% Vallado eqn 2-10
f = acos((cos(E) - e)/(1-e*cos(E)));
if E > pi
    f = 2*pi - f;
end
```

Published with MATLAB® R2013b

```
function E = M2E( M, e )
%M2E Mean anom (M) to eccentric anom (E)
fcnPrintQueue(mfilename('fullpath')) % Add this code to code app

tol = 1e-5;

if (M < 0 && M > -pi) || M > pi
    E_1 = M - e;
else
    E_1 = M + e;
end

E = E_1 + tol + 1;
while abs(E_1-E) > tol
    E = E_1;
    E_1 = E - (E - e*sin(E) - M)/(1 - e*cos(E));
end
```

Published with MATLAB® R2013b