ASEN 5050 Mid-Term Exam
Due: Monday, 10/26/2015
Paper Copy Preferred (ECNT319) or D2L Dropbox

Read everything on this page before you take the exam!

This is a take-home exam. You may use whatever books and notes you have at your disposal. However, you may not communicate with other people (students or otherwise) about the exam. You may want to use a computer to help check the calculations you have made, but you will be graded based on the work you show in your answer. You may attach any code you have written in support of the exam, but it will in general not be used to grade your exam.

In general, a correct answer will not give you credit for a question unless you show your work. An incorrect answer may still give you partial credit if you show a correct process.

Use Appendix D of the book for all constants not given in the problem.

You must complete this exam within a time period of **24 hours**, but you may review the questions first to see if you have any questions for me. After **24 hours**, stop working. I expect you will finish in ~3-4 hours.

Sign the following statement, reflecting the Honor Code (electronic signatures are acceptable):

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work. I have used no more than 24 hours to complete this exam.

[signature]	John Clause	10-23-15
Signature	Printed Name	Date

10-23-15 11:00 AM MDT	10-23-15 6:30PM MDT
Date/Time Exam Started	Date/Time Exam Finished

1. $h_p = 700$
$i = 112°$
$e = 0.22$
$\Omega = 0°$
$\omega = 310°$

$r_p = h_p + R_e = 7078.1363$ km

$a = r_p/(1-e) = 7078.1363/(1-0.22) = 5520.946$ Km

$\underline{h} = \sqrt{\mu/a^3} = 0.001539035$ rad/s

$M_2 - M_1 = n\underline{\underline{\Delta t}}$

$\nu_1 = 50°$
$\nu_2 = 50° + 180° = 230°$

$\tan(E) = \frac{\sin(\nu)\sqrt{1-e^2}}{e+\cos(\nu)} \Rightarrow \begin{matrix} E_1 = 0.713778 \text{ rad} \\ E_2 = 4.10752 \text{ rad} \end{matrix}$

$M = E - e\sin E \Rightarrow \begin{matrix} M_1 = 0.569745 \text{ rad} \\ M_2 = 4.38900 \text{ rad} \end{matrix}$

$\Delta t = \frac{M2 - M_1}{n} = 2481.65s = \boxed{0.689 \text{ hr}}$

2. $a = 8580$ Km
$e = 0.39$
$i = 150.9°$
$\Omega = 275°$
$\omega = 110.1°$
$\nu = 230°$

a) Impact if $r_p < R_e$

$\boxed{r_p = a(1-e) = 5233.8 < R_e}$

b) $r = \frac{p}{1+e\cos(\nu)} \Rightarrow 1 + e\cos(\nu) = \frac{p}{r} \Rightarrow \nu_{imp} = a\cos\left(\frac{\frac{p}{r} - 1}{e}\right)$

$p = a(1-e^2) = 7274.982$

$\nu_{imp} = a\cos\left(\frac{\frac{7274.982}{6378.1363} - 1}{0.39}\right) = \begin{cases} 68.86° \leftarrow \text{launch} \\ \underline{\underline{291.1°}} \leftarrow \text{Impact} \end{cases}$

$\Delta t = \frac{M_2 - M_1}{n}$ $\quad n = \sqrt{\frac{\mu}{a^3}} = 7.94398e{-4}$

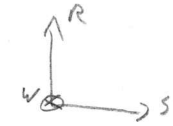
$\tan E = \frac{\sin(\nu)\sqrt{1-e^2}}{e+\cos(\nu)} \Rightarrow \begin{matrix} E_1 = 4.36828 \text{ rad} \\ E_{imp} = 5.43657 \text{ rad} \end{matrix}$

$M = E - e\sin E \Rightarrow \begin{matrix} M_1 = 4.73542 \text{ rad rad} \\ M_{imp} = 5.72424 \text{ rad rad} \end{matrix}$

$\boxed{\Delta t = 1244.8s = 0.346 \text{ hr}}$

c) $v_{imp} = \sqrt{\frac{2\mu}{r_{imp}} - \frac{\mu}{a}} = \boxed{8.8619 \text{ km/s}}$

$r_{imp} = 6378.1363$

3) 

$\vec{V}_{A_1} = \begin{pmatrix} 0 \\ 0 \\ 0.4 \end{pmatrix} m/s$ $\quad \vec{V}_{B_1} = \begin{pmatrix} 0 \\ 0.2 \\ 0 \end{pmatrix} m/s$, $\vec{r}_{A_1} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \vec{r}_{B_1}$

a) $\omega = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{\mu}{(6378.1363+380)^3}} = 0.0011363193 \text{ rad/s}$

$t_2 = 8 min = 480 sec$

$x_{B2} = \cancelto{0}{\frac{\dot{x}_0}{\omega}} \sin(\omega t) - (3\cancelto{0}{x_0} + \frac{2\dot{y}_0}{\omega})\cos(\omega t) + (4\cancelto{0}{x_0} + \frac{2\dot{y}_0}{\omega}) = \frac{-2\dot{y}_0}{\omega}\cos(\omega t) + \frac{2\dot{y}_0}{\omega}$

$= 51.079 m$ @ 8min

$y_{B2} = (6\cancelto{0}{x_0} + \frac{4\dot{y}_0}{\omega})\sin(\omega t) + \frac{2\cancelto{0}{\dot{x}_0}}{\omega}\cos(\omega t) - (6\omega \cancelto{0}{x_0} + 3\dot{y}_0)t + \cancelto{0}{y_0} - \frac{2\cancelto{0}{\dot{x}_0}}{\omega}$

$= \frac{4\dot{y}_0}{\omega}\sin(\omega t) - 3\dot{y}_0 t = \dot{y}_0\left(\frac{4}{\omega}\sin(\omega t) - 3t\right) = 77.239 m$ @ 8min

z_{B2}: z-motion is decoupled from y & x motion, since $\dot{z}_{B_1} = 0$, $z_B = 0$

$$\boxed{\vec{V}_{B@8min} = \begin{pmatrix} 51.079 \\ 77.239 \\ 0 \end{pmatrix} m}$$

$x_{A2} = 0$

$y_{A2} = 0$

$z_{A2} = \cancelto{0}{z_0}\cos(\omega t) + \frac{\dot{z}_0}{\omega}\sin(\omega t) = 182.62 m$

$$\Rightarrow \boxed{\vec{V}_{A@8min} = \begin{pmatrix} 0 \\ 0 \\ 182.62 \end{pmatrix} m}$$

b) $\vec{r}_{B@20} = \begin{pmatrix} 279.6 \\ -31.065 \\ 0 \end{pmatrix} m$ (using eqns from part A)

where t = 12 minutes

$$\vec{r}_{B@20} = \vec{r}_{A@20} = \begin{matrix} \frac{\dot{x}_0}{\omega}\sin(\omega t) - \frac{2\dot{y}_0}{\omega}\cos(\omega t) + \frac{2\dot{y}_0}{\omega} \\ \frac{4\dot{y}_0}{\omega}\sin(\omega t) + \frac{2\dot{x}_0}{\omega}\cos(\omega t) - 3\dot{y}_0 t - \frac{2\dot{x}_0}{\omega} \\ z_0\cos(\omega t) + \frac{\dot{z}_0}{\omega}\sin(\omega t) \end{matrix} = \begin{bmatrix} \frac{\sin(\omega t)}{\omega} & \frac{2}{\omega}(1-\cos\omega t) & 0 \\ \frac{2}{\omega}\cos\omega t - \frac{2}{\omega} & \frac{4}{\omega}\sin(\omega t) - 3t & 0 \\ 0 & 0 & \frac{\sin(\omega t)}{\omega} \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z_{A_2}\cos(\omega t) \end{pmatrix}$$

$\vec{V}_{A\_impt} = \begin{pmatrix} 0.2299 \\ 0.2369 \\ -0.1943 \end{pmatrix}$ m/s wrt shuttle, but there is already $\dot{z}_{A2} \neq 0$...

$\dot{z}_{A2} = 0.4 \cdot \cos(\omega t) = 0.3420 m/s$

$\therefore \vec{v}_{burn A} = \vec{v}_f - \vec{v}_i = \begin{pmatrix} 0.2299 \\ 0.2369 \\ -0.5363 \end{pmatrix}$ m/s wrt shuttle

$V_{burn A \, wrt B} = \vec{V}_{burn A} - \vec{V}_B$

$\vec{V}_{B@} = \begin{pmatrix} 2\dot{y}_0 \sin\omega t \\ 4\dot{y}_0\cos\omega t - 3\dot{y}_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.2075 \\ 0.0839 \\ 0 \end{pmatrix} \Rightarrow \boxed{V_{burn A \, wrt B} = \begin{pmatrix} 0.0223 \\ 0.1530 \\ -0.5363 \end{pmatrix} \frac{m}{s}}$

3. cont.)

c) $\vec{v}_B @20 = \begin{pmatrix} 0.3115 \\ 0.6411 \\ 0 \end{pmatrix} m/s$

$$\vec{V}_A @20 = \begin{pmatrix} \dot{x}_{A@8} \cos(\omega t) + 2\dot{y}_{A@8} \sin(\omega t) \\ 4\dot{y}_{A@8} \cos(\omega t) - 2\dot{x}_{A@8} \sin(\omega t) - 3\dot{y}_{A@8} \\ -z_{A@8} \sin(\omega t) + \dot{z}_{A@8} \cos(\omega t) \end{pmatrix} = \begin{pmatrix} 0.5630 \\ -0.3986 \\ -0.2843 \end{pmatrix} m/s.$$

$$V_{A\,dv\,burn\,wrt\,B} = (\vec{v}_f - \vec{v}_i) - \vec{v}_B = \vec{V}_{B@20} - \vec{V}_{A@20} - \vec{V}_{B@20}$$

$$\Rightarrow V_{A\,final\,wrt\,B} = \begin{pmatrix} -0.5630 \\ +0.3986 \\ +0.2843 \end{pmatrix} m/s$$

d) It is risky, Satellite A follows a purely sinusoidal motion in $\hat{w}$ after the initial impulse, if propulsion fails after that, it will collide with the shuttle. This can also happen with pure x velocity.

4)

for prograde, latitude = inclination
for retrograde, $i = 180 - \phi$

$$\boxed{i = 124^\circ}$$

$$\Delta\lambda = 195^\circ - 220^\circ = 25^\circ$$

Let $\dot{\theta}_{Earth} = \frac{360^\circ}{24} \cdot 0.99726968 = 4.15529e^3$

$$P = \frac{25^\circ}{\dot{\theta}_{Eart}} = 6016s = 1.67hr = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$\Rightarrow a = \sqrt[3]{\left(\frac{P}{2\pi}\right)^2 \cdot \mu} = 7149.655\ km$$

$$h = a - R_e = \boxed{771.5\ km}$$

5) a) $P = 2\pi\sqrt{\frac{a^3}{\mu}}$

$a = \frac{r_p + r_a}{2} = \underline{384{,}000\ km}$

$P = 2\pi\sqrt{\frac{384000}{398600.4415}} = 2.368e6\ sec = \boxed{27.41\ days}$

b) $\nu_@ r = 390{,}000 = a\cos\left(\frac{\frac{p}{390{,}000} - 1}{e}\right)$

$e = \left(\frac{r_a - r_p}{r_a + r_p}\right) = 0.05573$, $p = a(1-e^2) = 382{,}807\ km$

$\nu = \underline{\pm}\ 82.38°$, Total eclipse when $-82.38° \le \nu \le 82.38°$

$n = \sqrt{\mu/a^3} = 2.6532e{-6}$

$\tan E = \frac{\sin(\nu)\sqrt{1-e^2}}{e + \cos(\nu)} \Rightarrow E = 1.3828\ rad$

$M = E - e\sin E = 1.3280\ rad$

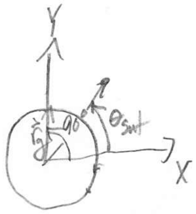
$\Delta t = \frac{2M}{n} = 1.001e6\ sec = \boxed{11.586\ days}$

c) The Moon is close enough for a total eclipse $\underline{42.27\%}$ of the time

6) $\theta_{t-3h} = 45^\circ$, $\theta_{t_0} = 45^\circ + 3 \cdot 15^\circ/_{hr} = 90^\circ$

$\|\vec{r}\| = 10,435.516$ km

$\sin\phi = \frac{z}{\|\vec{r}\|} \Rightarrow \boxed{31.81^\circ = \phi}$



$\tan\theta_{sat} = \frac{y}{x} \Rightarrow \theta_{sat} = -7.125^\circ$

$\boxed{\lambda = -97.125^\circ}$ or, 97.125° W

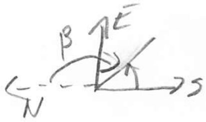
For Az/El, need to transform $r_{ECI} \to r_{SEZ}$ for Boulder

$\theta_{LST} = 90^\circ - 105.27^\circ = -15.27^\circ$

$\vec{r}_{SEZ} = R_3(\theta_{LST})\, R_2(90^\circ - \phi)\, \vec{r}_{ECI} = \begin{pmatrix} 1432 \\ 1256 \\ 10260 \end{pmatrix}$ Km

$\vec{\rho} = \vec{r}_{SEZ} - \vec{r}_{Boulder\,SEZ} = \vec{r}_{SEZ} - \begin{pmatrix} 0 \\ 0 \\ 6379.77 \end{pmatrix}$ Km $= \begin{pmatrix} 1432 \\ 1256 \\ 3880 \end{pmatrix}$

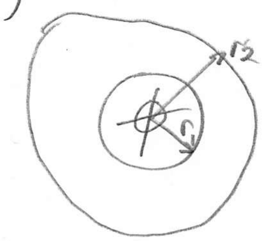
$el = \mathrm{asin}\left(\frac{\rho_z}{\|\vec{\rho}\|}\right) = \boxed{63.85^\circ = \text{elevation}}$



$\beta = az = 180 - \mathrm{atan}\left(\frac{\rho_E}{\rho_S}\right)$

$\boxed{az = 138.75^\circ}$

7)



$r_{2ideal} = 3r_1$

$r_{2act} = 2r_1$

$\Rightarrow a_{xferideal} = \frac{3r_1 + r_1}{2} = 2r_1$

$a_{xferactual} = \frac{2r_1 + r_1}{2} = 1.5r_1$

$\Rightarrow V_1 = \sqrt{\frac{\mu}{r_1}}$

$V_{2ideal} = \sqrt{\frac{\mu}{3r_1}}$

$V_{2act} = \sqrt{\frac{\mu}{2r_1}}$

$$V_{p\,ideal} = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{2r_1}} = \sqrt{\frac{4\mu - \mu}{2r_1}} = \sqrt{\frac{3\mu}{2r_1}}$$

$$V_{a\,ideal} = \sqrt{\frac{2\mu}{3r_1} - \frac{\mu}{2r_1}} = \sqrt{\frac{4\mu - 3\mu}{6r_1}} = \sqrt{\frac{\mu}{6r_1}}$$

$$\Delta V_{ideal} = \Delta V_{tot} = V_{pideal} - V_1 + V_{2ideal} - V_{aideal} = \sqrt{\frac{3\mu}{2r_1}} - \sqrt{\frac{\mu}{r_1}} + \sqrt{\frac{\mu}{3r_1}} - \sqrt{\frac{\mu}{6r_1}}$$

$$\Delta V_{tot} = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{3}{2}} - 1 + \sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}}\right)$$

$$V_{pact} = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{1.5r_1}} = \sqrt{\frac{3\mu - \mu}{1.5r_1}} = \sqrt{\frac{2\mu}{3/2\,r_1}} = \sqrt{\frac{4}{3}\frac{\mu}{r_1}}$$

$$V_{aact} = \sqrt{\frac{\cancel{2}\mu}{\cancel{2}r_1} - \frac{\mu}{3/2\,r_1}} = \sqrt{\frac{3/2\,\mu - \mu}{3/2\,r_1}} = \sqrt{\frac{\frac{1}{2}\mu}{3/2\,r_1}} = \sqrt{\frac{2\mu}{6r_1}} = \sqrt{\frac{\mu}{3r_1}}$$

$$\Delta V_{used} = \sqrt{\frac{\mu}{r_1}}\left(2\sqrt{\frac{4}{3}} - 1 + \sqrt{\frac{1}{2}} - \sqrt{\frac{1}{3}}\right) = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{1}{3}} - 1 + \sqrt{\frac{1}{2}}\right)$$

$$\Delta V_{remaining} = \Delta V_{tot} - \Delta V_{used} = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{3}{2}} - \sqrt{\frac{1}{6}} - \sqrt{\frac{1}{2}}\right)$$

For circ orbits:

$$\Delta V_{incl.} = 2v\sin\left(\frac{i}{2}\right) = 2\sqrt{\frac{\mu}{2r_1}}\sin\left(\frac{i}{2}\right) = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{3}{2}} - \sqrt{\frac{1}{6}} - \sqrt{\frac{1}{2}}\right)$$

$$\Rightarrow \frac{2}{\sqrt{2}}\sin\frac{i}{2} = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}\sqrt{3}} - \frac{1}{\sqrt{2}}$$

$$\Rightarrow 2\sin\frac{i}{2} = \sqrt{3} - \frac{1}{\sqrt{3}} - 1$$

$$\Rightarrow i = 2\cdot \operatorname{asin}\left(\frac{\sqrt{3} - \frac{1}{\sqrt{3}} - 1}{2}\right)$$

$$\boxed{i = 8.87^\circ}$$