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# HW 10: Sequential Processor for the Term Project

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## Initialize

```
clearvars -except function_list pub_opt P_joseph_store x_est_batch P0_est_batch P_
global function_list;
function_list = {};
% close all

stat_od_proj_init
ObsData = load('ObsData.txt');

consts.Re = Re;
consts.area = drag.A;
consts.rho = compute_density(ri);
consts.theta_dot = theta_dot;
consts.m = drag.m;
consts.state_len = 18;

P0 = eye(consts.state_len)*1e6;
P0(7,7) = 1e20;
P0(10:12,10:12) = eye(3)*1e-10;

x0_ap = zeros(consts.state_len,1);

sig_range = 0.01; % m
sig_rangerate = 0.001; %m/s
W = [1/(sig_range*sig_range) 0; 0 1/(sig_rangerate*sig_rangerate)];
R = [(sig_range*sig_range) 0; 0 (sig_rangerate*sig_rangerate)];
```

## Sequential Processor

```
dt = 0.1;
times = 0:dt:18340;
ode_opts = odeset('RelTol', 1e-12, 'AbsTol', 1e-20);

% for iter = 1:3
[T,X] = ode45(@two_body_state_dot, times, state, ode_opts, propagator_opts);

% Store off every 20 seconds of data
```

```
X_store = X(mod(times,20) == 0,:);
T_store = T(mod(times,20) == 0);

[num_obs, ~] = size(ObsData);
chol_P0 = chol(P0, 'lower');
P0_inv = chol_P0'\inv(chol_P0);
info_mat = P0_inv;
norm_mat = P0_inv*x0_ap;

% Obs. deviation
y1 = zeros(num_obs,1);
y2 = zeros(num_obs,1);
for ii = 1:num_obs
    site_num = 0;
    for jj = 1:3
        if ObsData(ii, 2) == site(jj).id
            site_num = jj;
            break
        end
    end
    t_obs = ObsData(ii,1);
    ostate = X(T(:,1)==t_obs,1:6);

    r_comp = compute_range_ECFsite(ostate(1:3),...
        site(site_num).r,theta_dot*t_obs);
    rr_comp = compute_range_rate_ECFsite(ostate(1:6),...
        site(site_num).r,theta_dot*t_obs, theta_dot);

    y1(ii) = (ObsData(ii,3)-r_comp);
    y2(ii) = (ObsData(ii,4)-rr_comp);

end

% CKF init
x_est = x0_ap;
P = P0;
obs_time_last = ObsData(ii,1);

use_joseph = 1;
if use_joseph
    P_joseph_store = zeros(num_obs,1);
else
    P_trace_store = zeros(num_obs,1);
end
STM_accum = eye(consts.state_len);
pfr_store = zeros(2,num_obs);
% Run CKF
for ii = 1:num_obs
    obs_time = ObsData(ii,1);
    obs_site = ObsData(ii,2);

    % STM from last obs to this one.
    % Not very efficient, since I'm running the integrator again.
    if ii == 1
```

```
STM_obs2obs = eye(consts.state_len);
else
    times_temp = obs_time_last:dt:obs_time;
    last_state = X_store(T_store == ObsData(ii-1,1),:);
    STM_obs2obs = eye(consts.state_len);
    % Make the STM reflect an epoch time == the last msmnt time
    last_state(consts.state_len+1:end) = ...
        reshape(STM_obs2obs(1:important_block(1),1:important_block(2)),...
            important_block(1)*important_block(2),1);

    [T_temp,X_temp] = ...
        ode45(@two_body_state_dot, times_temp, last_state, ...
            ode_opts, propagator_opts);
    STM_obs2obs(1:important_block(1),1:important_block(2)) = ...
        reshape(X_temp(end,consts.state_len+1:end), ...
            important_block(1), important_block(2));
end
obs_time_last = obs_time;

% Time update
STM_accum = STM_obs2obs*STM_accum;
x_ap = STM_obs2obs*x_est;
P_ap = STM_obs2obs*P*STM_obs2obs';

% H~
consts.t = obs_time;
for xx = 1:3
    if site(xx).id == obs_site
        consts.site = xx;
        break
    end
end
state_at_obs = X_store(T_store == obs_time,1:consts.state_len);
H_tilda = stat_od_proj_H_tilda(state_at_obs, consts);

% Kalman gain
K = P_ap*H_tilda'/(H_tilda*P_ap*H_tilda'+R);

% Measurement Update
y = [y1(ii);y2(ii)];
x_est = x_ap + K*(y - H_tilda*x_ap);
I = eye(consts.state_len);
if use_joseph
    P = (I-K*H_tilda)*P_ap*(I-K*H_tilda)' + K*R*K';
    P_joseph_store(ii) = trace(P(1:6,1:6));
else
    P = (I-K*H_tilda)*P_ap;
    P_trace_store(ii) = trace(P(1:6,1:6));
end

if ii == 1
    x_est_CKF = x_est;
end
pfr = y-H_tilda*x_est;
```

```
pfr_store(:,ii) = pfr;

end

RMS_accum = 0;
for ii = 1:num_obs
    RMS_accum = RMS_accum + pfr_store(:,ii)'/R*pfr_store(:,ii);
end
RMS = sqrt(RMS_accum/num_obs)

rangerate_RMS = sqrt(sum(pfr_store(2,:).*pfr_store(2,:))/num_obs)
range_RMS = sqrt(sum(pfr_store(1,:).*pfr_store(1,:))/num_obs)

figure
subplot(2,1,1)
plot(1:num_obs, pfr_store(1,:))
hold on
plot(1:num_obs,3*sig_range*ones(1,num_obs),'r--')
plot(1:num_obs,-3*sig_range*ones(1,num_obs),'r--')
title(sprintf('Range RMS = %.4e m',range_RMS))
ylabel('m')
subplot(2,1,2)
plot(1:num_obs, pfr_store(2,:))
hold on
plot(1:num_obs,3*sig_rangerate*ones(1,num_obs),'r--')
plot(1:num_obs,-3*sig_rangerate*ones(1,num_obs),'r--')
title(sprintf('Range-Rate RMS = %.4e m/s',range_RMS))
ylabel('m/s'),xlabel('Observation')
```

## Covariance Matrix Traces

The Joseph formulation follows the same basic shape of the regular Kalman P formulation, but is generally slightly larger. The Joseph formulation will better consider measurements because of this.

```
figure
semilogy(ObsData(:,1)/3600,abs(P_trace_store))
hold on
semilogy(ObsData(:,1)/3600,abs(P_joseph_store),'r')
legend('Kalman P trace', 'Joseph P trace')
title('Traces of Covariance Matrix')
xlabel('Time (hr)'), ylabel('S/C Position/Velocity Trace')

if use_joseph
    CKF_joseph_init_state = x_est_CKF+state(1:consts.state_len);
    CKF_joseph_final_state = x_est+X_store(end,1:consts.state_len)';
else
    CKF_init_state = x_est_CKF+state(1:consts.state_len);
    CKF_final_state = x_est+X_store(end,1:consts.state_len)';
    conv_P_trace_store = P_trace_store;
end

% Ellipsoid
```

```
[U,Pprime] = eigs(P(1:3,1:3));
Semi = [sqrt(9*Pprime(1,1)) sqrt(9*Pprime(2,2)) sqrt(9*Pprime(3,3))];
figure
subplot(1,2,1)
plotEllipsoid(U,Semi);
% title('3-sigma Position Error Probability Ellipsoid - Batch')
xlabel('x (m)'),ylabel('y (m)'),zlabel('z (m)')
subplot(1,2,2)
plotEllipsoid(U,Semi);
% title('3-sigma Position Error Probability Ellipsoid - Batch')
xlabel('x (m)'),ylabel('y (m)'),zlabel('z (m)')
```

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