IMD HW 1 Problem 2

Table of Contents

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Initialization

```
clearvars -except T v_xfer_i hw_pub function_list
close all
lineWidth = 2;

mu_sun = 1.327e11; %km3/s2
mu_earth = 3.986e5; %km3/s2
mu_mars = 4.305e4; %km3/s2

km_per_AU = 1.4959787e8; %km
a_earth = 1*km_per_AU; %km
a_mars = 1.52367934*km_per_AU; %km
% r_earth = 6378.1363; %km
% r_mars = 3397.2; %km

r_earth = [-578441.002878924;
    -149596751.684464;
    0]; % km
v_earth = [29.7830732658560;
    -0.115161262358529;
    0]; % km/s

r_mars_f = [-578441.618274359;
    227938449.869731;
    0];

v_mars_f = [-24.1281802482527;
    -0.0612303173808154;
    0];

r_sat = [0;-km_per_AU;0];
v_sat = [v_xfer_i;0;0];

% Compute the initial location of Mars.
% Since we assume Mars is in circ. orbit and coplanar, just rotate the
% vector back.
% Find mean motion of Mars
% Rotate Mars final r,v by -n*T
```

```
n_mars = sqrt(mu_sun/a_mars^3);
r_mars_i = Euler2DCM('3',n_mars*T*3600*24)*r_mars_f;
v_mars_i = Euler2DCM('3',n_mars*T*3600*24)*v_mars_f;
```

Propagate with just sun accel

```
prop_opts.mu = mu_sun;
times = linspace(0, T*3600*24, 3000);

ode_opts = odeset('RelTol', 1e-12, 'AbsTol', 1e-16);
ode_opts = odeset('RelTol', 1e-3, 'AbsTol', 1e-6);
[T_out, X_out] = ode45(@two_body_state_dot, times, [r_sat;v_sat], ode_opts, prop_o
X_x_store = X_out(:,1);
X_y_store = X_out(:,2);
X_vx_store = X_out(:,4);
X_vy_store = X_out(:,5);
```

Propagate with Earth and Mars perturbations.

Need to track the states of earth and mars for each integration step

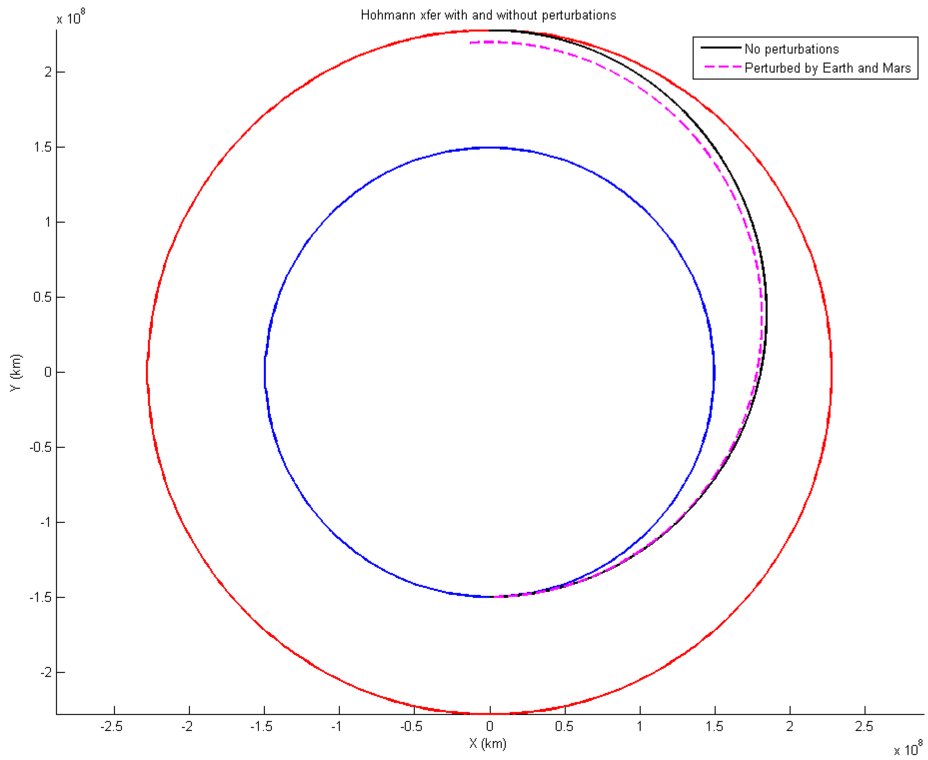
```
state = [r_earth; v_earth; r_mars_i; v_mars_i; r_sat; v_sat;];

% Anonymous function to calculate heliocentric acceleration
sun_accel = @(r) -mu_sun*r/(norm(r))^3;

% Anonymous function to calculate combined accel on satellite from earth,
% mars, and sun
total_accel = @(re, rm, rs) -mu_earth*(rs-re)/(norm(rs-re))^3 + ...
    -mu_mars*(rs-rm)/(norm(rs-rm))^3 + ...
    sun_accel(rs);

% This is the state derivative.
state_dot = @(t, state) [state(4:6); sun_accel(state(1:3));...
    state(10:12); sun_accel(state(7:9));...
    state(16:18); total_accel(state(1:3),state(7:9),state(13:15))];

[T_out, X_out] = ode45(state_dot, times, state, ode_opts);
pert_X_x_store = X_out(:,13);
pert_X_y_store = X_out(:,14);
pert_X_vx_store = X_out(:,16);
pert_X_vy_store = X_out(:,17);

earth_x_store = X_out(:,1);
earth_y_store = X_out(:,2);
mars_x_store = X_out(:,7);
mars_y_store = X_out(:,8);
```

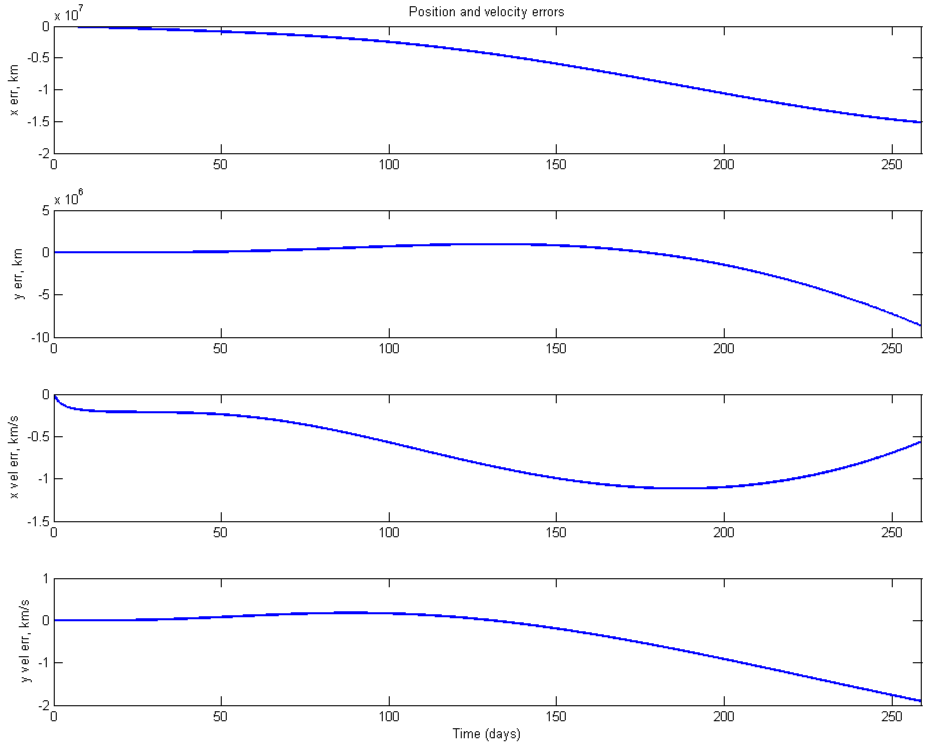
Plots

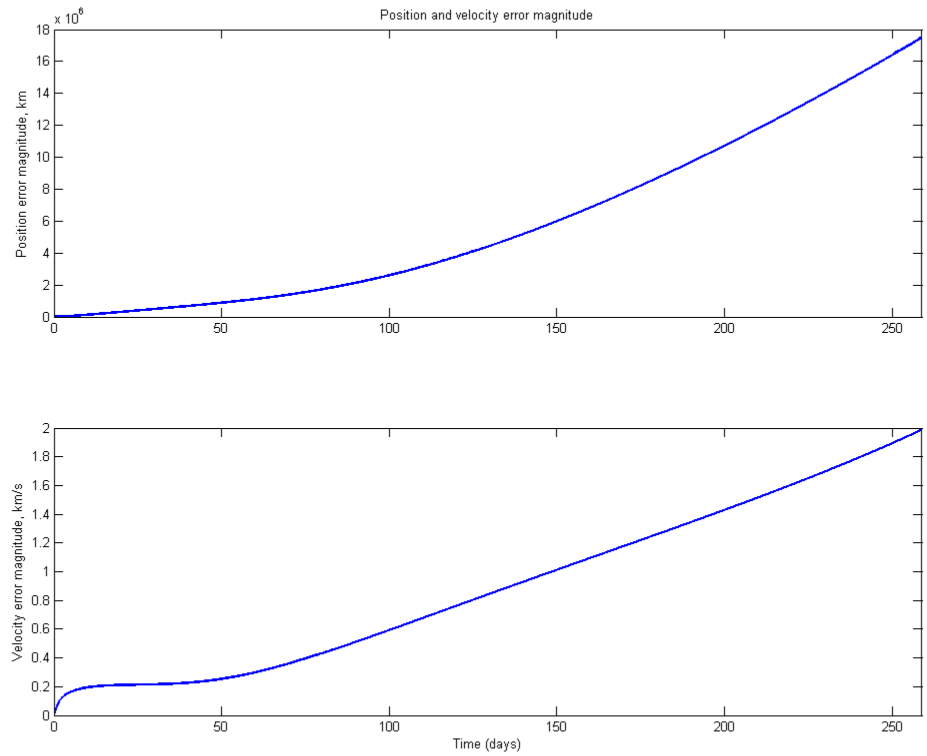
```
polar_plot = figure('Position', hw_pub.figPosn);
```

```
hold on
angles = 0:1:360;
plot(norm(r_earth)*cosd(angles),norm(r_earth)*sind(angles), ...
    'LineWidth', hw_pub.lineWidth)
plot(norm(r_mars_f)*cosd(angles),norm(r_mars_f)*sind(angles),'r', ...
    'LineWidth', hw_pub.lineWidth)
no_pert_plot = plot(X_x_store, X_y_store,'k', ...
    'LineWidth', hw_pub.lineWidth);
axis equal
pert_plot = plot(pert_X_x_store, pert_X_y_store,'m--', ...
    'LineWidth', hw_pub.lineWidth);
% plot(earth_x_store, earth_y_store,'m--') % Earth motion propagated
% plot(mars_x_store, mars_y_store,'b--') % Mars motion propagated
xlabel('X (km)');
ylabel('Y (km)');
title('Hohmann xfer with and without perturbations')
legend([no_pert_plot, pert_plot],{'No perturbations', ...
    'Perturbed by Earth and Mars'});

figure('Position', hw_pub.figPosn)
plot_times = times/3600/24;
lims = [0;plot_times(end)];
subplot(4,1,1)
plot(plot_times, pert_X_x_store-X_x_store, 'LineWidth', hw_pub.lineWidth)
title('Position and velocity errors')
ylabel('x err, km')
xlim(lims)
subplot(4,1,2)
plot(plot_times, pert_X_y_store-X_y_store, 'LineWidth', hw_pub.lineWidth)
ylabel('y err, km')
xlim(lims)
subplot(4,1,3)
plot(plot_times, pert_X_vx_store-X_vx_store, 'LineWidth', hw_pub.lineWidth)
ylabel('x vel err, km/s')
xlim(lims)
subplot(4,1,4)
plot(plot_times, pert_X_vy_store-X_vy_store, 'LineWidth', hw_pub.lineWidth)
ylabel('y vel err, km/s')
xlim(lims)
xlabel('Time (days)')

% norm of r_non_perturbed - r_perturbed
diff_r = sqrt((X_x_store-pert_X_x_store).*(X_x_store-pert_X_x_store) ...
    + (X_y_store-pert_X_y_store).*(X_y_store-pert_X_y_store));
% norm of v_non_perturbed - v_perturbed
diff_v = sqrt((X_vx_store-pert_X_vx_store).*(X_vx_store-pert_X_vx_store) ...
    + (X_vy_store-pert_X_vy_store).*(X_vy_store-pert_X_vy_store));

figure('Position', hw_pub.figPosn)
subplot(2,1,1)
plot(plot_times, diff_r, 'LineWidth', hw_pub.lineWidth)
title('Position and velocity error magnitude')
ylabel('Position error magnitude, km')
xlim(lims)
```

```
subplot(2,1,2)
plot(plot_times, diff_v, 'LineWidth', hw_pub.lineWidth)
ylabel('Velocity error magnitude, km/s')
xlim(lims)
xlabel('Time (days)')
```




Position and velocity errors
x 10^7
x err, km
x 10^6
y err, km
x vel err, km/s
y vel err, km/s
Time (days)


Position and velocity error magnitude
x 10^6
Position error magnitude, km
Velocity error magnitude, km/s
Time (days)

Conclusion

The perturbations that were modeled are slowing down the satellite with respect to the sun. At the beginning, Earth gravity is providing an acceleration against the satellite velocity vector. Toward the end, Mars is lagging in phase with the satellite and slowing it down in the velocity vector direction. These accelerations result in a lowered apoapsis.

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