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Problem 10: S&J 3.19: KDEs for $\beta_{1,2,3}$

$$\beta_1 = \frac{C_{23}}{4\beta_0} - \frac{C_{32}}{4\beta_0}$$

$$\dot{\beta}_1 = \frac{\dot{C}_{23} - \dot{C}_{32}}{4\beta_0} + \frac{C_{23} - C_{32}}{-4\beta_0^2}\cdot\dot{\beta}_0$$

$$[\dot{C}] = -[\tilde{\omega}][C]$$

$$= \begin{bmatrix} -C_{31}\omega_2 + C_{21}\omega_3 & -C_{32}\omega_2 + C_{22}\omega_3 & -C_{33}\omega_2 + C_{23}\omega_3 \\ -C_{11}\omega_3 + C_{31}\omega_1 & -C_{12}\omega_3 + C_{32}\omega_1 & -C_{13}\omega_3 + C_{33}\omega_1 \\ -C_{21}\omega_1 + C_{11}\omega_2 & -C_{22}\omega_1 + C_{12}\omega_2 & -C_{23}\omega_1 + C_{13}\omega_2 \end{bmatrix}$$

$$= \frac{-C_{13}\omega_3 + C_{33}\omega_1 + C_{22}\omega_1 - C_{12}\omega_2}{4\beta_0} - \beta_1\cdot\frac{\dot{\beta}_0}{\beta_0}$$

$$= \frac{(+C_{22} + C_{33})\omega_1 - C_{13}\omega_3 - C_{12}\omega_2}{4\beta_0} - \beta_1\cdot\frac{\frac{1}{2}(-\beta_1\omega_1 - \beta_2\omega_2 - \beta_3\omega_3)}{\beta_0}$$

$$= \frac{(+C_{22} + C_{33})\omega_1 - C_{13}\omega_3 - C_{12}\omega_2 + 2\beta_1^2\omega_1 + 2\beta_1\beta_2\omega_2 + 2\beta_1\beta_3\omega_3}{4\beta_0}$$

Substitute w/ eqn 3.93

$$= \frac{(\cancel{2}\beta_0^2 - \cancel{2}\beta_1^2)\omega_1 - \cancel{2\beta_1\beta_3\omega_3} + 2\beta_0\beta_2\omega_3 - \cancel{2\beta_1\beta_2\omega_2} - 2\beta_0\beta_3\omega_2 + 2\beta_1^2\omega_1 + \cancel{2\beta_1\beta_2\omega_2} + \cancel{2\beta_1\beta_3\omega_3}}{2\cancel{4}\beta_0}$$

$$\boxed{\dot{\beta}_1 = \frac{1}{2}(\beta_0\omega_1 + \beta_2\omega_3 - \beta_3\omega_2)}$$

$$\dot{\beta}_2 = \frac{\dot{C}_{31} - \dot{C}_{13}}{4\beta_0} - \beta_2\cdot\frac{\dot{\beta}_0}{\beta_0} = \frac{-C_{21}\omega_1 + C_{11}\omega_2 + C_{33}\omega_2 - C_{23}\omega_3}{4\beta_0} - \beta_2\cdot\frac{\dot{\beta}_0}{\beta_0}$$

$$= \frac{-\cancel{2\beta_1\beta_2\omega_1} + 2\beta_0\beta_3\omega_1 + (2\beta_0^2 - \cancel{2\beta_2^2})\omega_2 - \cancel{2\beta_2\beta_3\omega_3} - 2\beta_0\beta_1\omega_3 + \cancel{2\beta_2\beta_1\omega_1} + \cancel{2\beta_2^2\omega_2} + \cancel{2\beta_2\beta_3\omega_3}}{4\beta_0}$$

$$\boxed{\dot{\beta}_2 = \frac{1}{2}(\beta_0\omega_2 - \beta_1\omega_3 + \beta_3\omega_1)}$$

$$\dot{\beta}_3 = \frac{\dot{C}_{12} - \dot{C}_{21}}{4\beta_0} - \beta_3\cdot\frac{\dot{\beta}_0}{\beta_0} = \frac{-C_{32}\omega_2 + C_{22}\omega_3 + C_{11}\omega_3 - C_{31}\omega_1}{4\beta_0} - \beta_3\cdot\frac{\dot{\beta}_0}{\beta_0}$$

$$= \frac{-\cancel{2\beta_3\beta_2\omega_2} + 2\beta_0\beta_1\omega_2 + (2\beta_0^2 - \cancel{2\beta_3^2})\omega_3 - \cancel{2\beta_3\beta_1\omega_1} - 2\beta_0\beta_2\omega_1 + \cancel{2\beta_3\beta_1\omega_1} + \cancel{2\beta_3\beta_2\omega_2} + \cancel{2\beta_3^2\omega_3}}{4\beta_0}$$

$$\boxed{\dot{\beta}_3 = \frac{1}{2}(\beta_0\omega_3 + \beta_1\omega_2 - \beta_2\omega_1)}$$

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Problem 11: S&J 3.20

$$\vec{q} = \begin{pmatrix} \frac{\beta_1}{\beta_0} & \frac{\beta_2}{\beta_0} & \frac{\beta_3}{\beta_0} \end{pmatrix}^T$$

$$\beta_0 = \frac{1}{\sqrt{1+\vec{q}^T\vec{q}}} = \frac{1}{\sqrt{1+\frac{\beta_1^2}{\beta_0^2}+\frac{\beta_2^2}{\beta_0^2}+\frac{\beta_3^2}{\beta_0^2}}} = \frac{1}{\sqrt{1+\frac{1-\beta_0^2}{\beta_0^2}}} = \frac{1}{\sqrt{\cancel{1}+\frac{1}{\beta_0^2}-\cancel{1}}}$$

$$= \frac{1}{1/\beta_0} = \underline{\underline{\beta_0}}$$

$$\beta_i = \frac{a_i}{\sqrt{1+\vec{q}^T\vec{q}}} = \frac{\beta_i/\beta_0}{\sqrt{1+\vec{q}^T\vec{q}}} = \frac{\beta_i/\cancel{\beta_0}}{1/\cancel{\beta_0}} = \underline{\underline{\beta_i}}$$

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Problem 12: S+J 3.24

$$\vec{\sigma} = \begin{pmatrix} \frac{\beta_1}{1+\beta_0} & \frac{\beta_2}{1+\beta_0} & \frac{\beta_3}{1+\beta_0} \end{pmatrix}$$

$$\beta_0 = \frac{1-\sigma^2}{1+\sigma^2} = \frac{1-\frac{\beta_1^2+\beta_2^2+\beta_3^2}{(1+\beta_0)^2}}{1+\frac{\beta_1^2+\beta_2^2+\beta_3^2}{(1+\beta_0)^2}} = \frac{1-\frac{1-\beta_0^2}{(1+\beta_0)^2}}{1+\frac{1-\beta_0^2}{(1+\beta_0)^2}} = \frac{1-\frac{(1-\beta_0)\cancel{(1+\beta_0)}}{(1+\beta_0)^{\cancel{2}}}}{1+\frac{(1-\beta_0)\cancel{(1+\beta_0)}}{(1+\beta_0)^{\cancel{2}}}}$$

$$1-\beta_0 = (1+\beta_0)(-1)+2$$

$$\beta_0 = \frac{1-\frac{\cancel{(1+\beta_0)}(-1)}{\cancel{1+\beta_0}}-\frac{2}{1+\beta_0}}{1+\frac{\cancel{(1+\beta_0)}(-1)}{\cancel{1+\beta_0}}+\frac{2}{1+\beta_0}} = \frac{1+1-\frac{2}{1+\beta_0}}{1-1+\frac{2}{1+\beta_0}} = \frac{\cancel{2}1-\frac{\cancel{2}1}{1+\beta_0}}{\frac{\cancel{2}1}{1+\beta_0}}$$

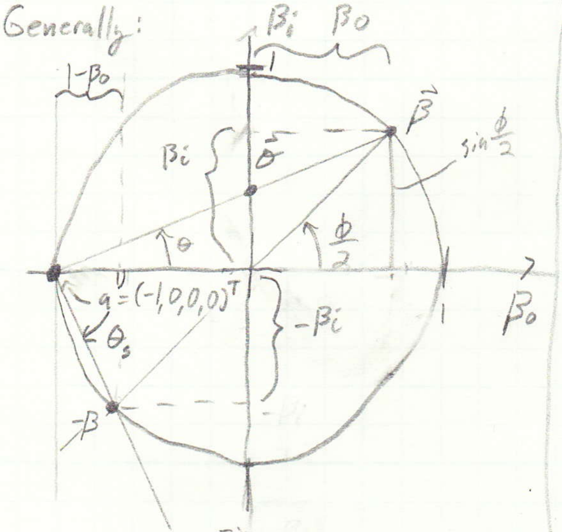
$$= 1+\beta_0-1 = \underline{\beta_0}$$

$$\beta_i = \frac{2\sigma_i}{1+\sigma^2} = \frac{\cancel{2}\cdot\frac{\beta_i}{\cancel{1+\beta_0}}}{\frac{\cancel{2}}{\cancel{1+\beta_0}}} = \underline{\beta_i}$$

from β_0 derivation

Problem 13: S&J 3.26

Generally:



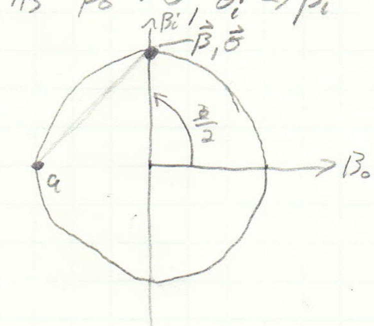
$$\tan\theta = \frac{\sigma_i}{1} = \frac{\beta_i}{1+\beta_0}$$

$$\Rightarrow \sigma_i = \frac{\beta_i}{1+\beta_0}$$

$$\tan\theta_s = \frac{\sigma_i^s}{1} = \frac{-\beta_i}{1-\beta_0}$$

$$\sigma_i = \frac{\beta_i}{1+\beta_i}$$

As $\beta_0 \to 0$, $\sigma_i \to \beta_i$



As $\beta_0 \to 1$, $\beta_i \to 0$ and therefor $\sigma_i \to 0$