ASEN 5090 Introduction to GNSS

FINAL EXAM

December 16, 2014

Format – There are two problems worth a total of 100 points. This is an open-book exam.

Calculations - Calculator use is allowed. Other types of computer aids are NOT permitted. Please assume that the speed of light, c = 3 x 10^8 m/s, for the calculations on this exam.

Time - You may use up to 2.5 hours for the exam.

Extra Paper - You may use your own paper if you need more space. Please put your name and the problem number on each extra sheet, and staple all sheets together in their proper order.

CAETE students - please scan your completed exam and upload it as a single pdf document to the D2L site.

Date: 12-18-14

Start Time: 7:00 PM

Stop Time: 9:30 PM

On my honor, as a University of Colorado Boulder student, I have followed all of the above instructions and have neither given nor received unauthorized assistance on this work.

Signature

John Clouse

Printed name

PROBLEM 1. (50 points) (a-f)

A hypothetical telecommunications company is housed in a building located at N 0, E 0. The company has an antenna on the roof of the building to receive GPS signals. It uses a receiver connected to this antenna to solve for a clock bias that allows the company to synchronize timing with other sites around the world. The height of the antenna above the Earth is about 30 m, but this height is not perfectly known. You may assume that the horizontal position of the antenna is given exactly by the latitude and longitude given above.

Also assume that the Earth is a sphere of radius RE = 6,378,000 m and that the GPS satellites are all in perfectly circular orbits with radius RS = 26,600,000 m.

At midnight local time the receiver tracks 5 satellites with azimuth and elevation given in the table below. The measured pseudorange to each satellite is also shown.
(The only errors included in the measurements are receiver clock bias and noise.)

PRN	EL (deg)	AZ (deg)	Pseudorange (m)	Expected Range (m)	dy (m)
1	90	0	20,222,010	20,221,970	-20
2	0	90	25,824,076	25,824,034.02	41.99
3	0	270	25,824,075	25,824,034.02	40.98
4	45	0	21,704,980	21,704,694.86	15,14
5	45	180	21,704,980	21,704,694.86	15.14

a) Compute the geometrical range to each satellite (ignore the time of flight correction) (Hint: For full credit, calculate it using actual geometry; however, if you are stuck on some of them, make a reasonable guess/assumption based on the ones that you do know.)

b) If this were a real-world example, what would you expect the five largest sources of error between the measured pseudorange and your computed geometrical range? Give an order of magnitude estimate for each, where possible.

c) Set up the equations for a least-squares solution to solve only for the receiver height and clock bias error using the measurements to all 5 satellites. (Hint: if you are not sure how to do this, start by setting up the full solution and then think about how to reduce it to just height and clock.) It is not necessary to compute the solution, just compute all the required matrixes. (Give numerical values for each matrix).

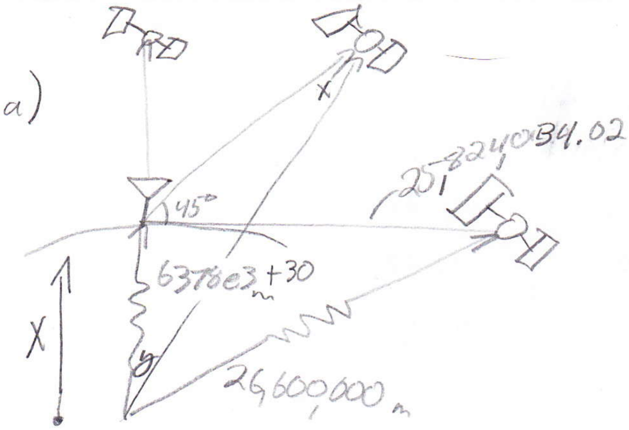
d) Compute the G Matrix $(A^TA)^{-1}$ and the solution for the height and clock bias using only the two measurements for PRNs 1 and 2. What are the VDOP (vertical or up) and TDOP (time dilution of precision) for this case? Would you expect this solution to be the same or different from the solution using all 5 satellites?

e) If you wanted to form a solution for height and clock bias using only two satellites, which two you would choose and why?

f) If the receiver's "known" horizontal position is actually off by a few meters, will this produce a clock error that is much larger, approximately equal to, or much smaller than the horizontal position error? Explain

John Clouse

1.a)



$$\frac{\sin(45° + 90°)}{26{,}600{,}000} = \frac{\sin x}{6378030} \Rightarrow x = 0.170369 \text{ rad}$$

$$= \frac{\sin y}{r} \qquad y = 0.615021$$

1.b) Troposphere ~ 1's m
Ionosphere ~ 1's m
Clock ~ 100's km
Multipath ~ <1 m
Relativity ~ 1's m

1.c) $\vec{x}_0 = \begin{pmatrix} 6378030 \\ 0 \\ 0 \end{pmatrix}$ m

$$A = \begin{array}{c} \begin{array}{cccc} x & y & z & s \end{array} \\ \begin{pmatrix} \frac{\rho}{\rho} = 1 & 0 & 0 & 1 \\ \frac{6378030 - 6378030}{25{,}824{,}034} & 1 - \frac{0}{\rho} & 0 & 1 \\ \frac{6378030 - 6378030}{25{,}824{,}034} & -1 & 0 & 1 \\ \frac{21704695 \sin 45 + R_e - R_e}{21704695} & 0 & \sin(45°) & 1 \\ \sin 45° & 0 & -\sin(45°) & 1 \end{pmatrix} \end{array}$$

cont →

1.c) cont.

$$A = \begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & +1 & 0 & 1 \\ -\sin 45^\circ & 0 & -\sin 45^\circ & 1 \\ -\sin 45^\circ & 0 & +\sin 45^\circ & 1 \end{pmatrix} \; ; \; d_y = \begin{pmatrix} -20 \\ 41.98 \\ 40.98 \\ 15.14 \\ 15.14 \end{pmatrix}$$

$$\underline{d_y = A \begin{bmatrix} \delta \vec{x} \\ b \end{bmatrix} + \vec{\varepsilon}}$$

1.d) removing horizontal stuff, non-singular otherwise

$$A = \begin{matrix} -1 & 1 \\ 0 & 1 \end{matrix} \Rightarrow G = (A^T A)^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$VDOP = \sqrt{G_{11}} = \underline{\sqrt{2}}$$

$$TDOP = \sqrt{G_{44}} = \underline{1}$$

1.e) PRN 1, it is most sensitive to vertical
then PRN 3 Because it has the smallest error (only clock + noise)

1.f) much smaller than. The impact of horizontal pos. by that much, to the clock, is small

PROBLEM 2. (50 Points) (a-j)

This question deals with a simplified version of the GPS L5 civil signal. Assume the following:

- Carrier Frequency, fcarrier = 1176.45 MHz
- Code chip rate, fcode = 10.23 MHz, Full code length = 102,300 chips (or bits)

Answer the following **10 questions** that refer to this signal structure. Each part is worth 5 points. Correct answers will get full credit. Partial credit will be awarded based on descriptions of method or thinking. ***Please assume that the speed of light c = 3x10⁸ m/s.***

a) What is the duration of 1 code chip in seconds? In meters?

$T_{chip} = \frac{1}{10.23e6} = 9.77517 \times 10^{-8} s$

$l_{chip} = 29.33m = cT_{chip}$

b) What is the duration of the entire simplified L5 code in seconds? In meters? Given that the delay from the GPS satellites to the Earth is approximately 70 ms how many full code lengths are there along the path from satellite to receiver?

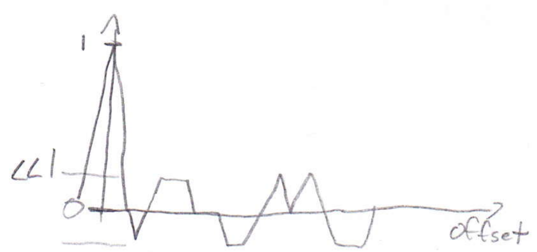
L_5 Duration $= T_{chip} \cdot 102,300 = \underline{0.01 s} = 10ms$

L_5 Duration(m) $= l_{chip} \cdot 102,300 = \underline{3e6\ m}$

7 full code lengths from sat → rec.

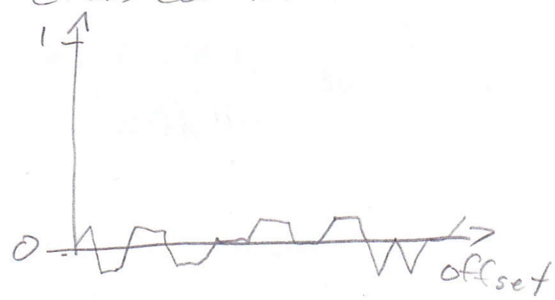
c) Sketch the autocorrelation function for the code, R(τ) assuming that it is very similar to that of a maximal length code. Assuming that the codes for different satellites have been selected well, sketch what the cross-correlation between two codes should look like. Label your axes in both sketches.

Autocorrelation

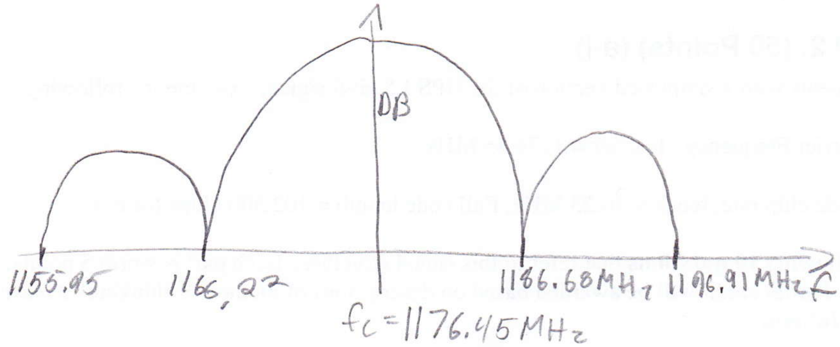


Can be zero, even # chips

cross correlation



d) Sketch the overall spectrum of this simplified L5 Code carefully identifying the center frequency, main lobe, and side lobe widths. How does it compare with the L1 C/A code?



L1 C/A: centered @ 1575.42, side lobes 1.023 MHz wide

e) Is the simplified L5 spectrum continuous? If not, sketch the small scale structure of the spectrum (zoom in on your previous sketch) and compute the spectral line spacing. What could be done in the signal design to make the signal more continuous? No, they have spectral lines @ $\frac{10230000}{102300} = 100 Hz$

lengthen the code or lessen the chip rate

f) Ignoring atmospheric losses, how much power must be transmitted by the satellite such that the signal power received by an antenna with Gain = 3 dB, located 20,000 km away is -155 dBW? Assume that the transmitting satellite antenna gain is 10 dB.

$$P_{dB} = P_{T_{dB}} + G_{T_{dB}} + G_{R_{dB}} - 20\log_{10} 20e6 - 11$$

$$\Rightarrow P_{T_{dB}} = -155 - 3 - 10 + 20\log_{10} 20e6 + 11$$

$$= -11\, dBW$$

g) If the receiver has a C/No value of 40 dB-Hz for a pseudorange (code) measurements on both L1 and L5, what is the ratio of the standard deviation of the DLL tracking error due to noise on L5 compared to L1 ($\sigma_{L5} / \sigma_{L1}$). Assume that the coherent integration time for both is 10 ms. Which provides a more precise pseudorange measurement?

h) How will the ionospheric delays for L5 compare to L1? Compute the L5 ionospheric delay that would be observed when the L1 ionospheric delay is 15 m.

$f_{L1} = 1575.42 MHz$

$f_{L5} = 1176.45 MHz$ L5 has greater delay

$I_\rho = \frac{40.3 TEC}{f^2}$, $TEC = I_{\rho f_1} \frac{f_1^2}{40.3}$

$I_{\rho f_{L5}} = \frac{15 \cdot 1575.42^2}{1176.45^2} = \boxed{26.9 m}$

i) How does the acquisition search for the simplified L5 code compare to L1 C/A? Comment on both the delay and Doppler ranges that must be considered.

j) Give 2 significant reasons that the L5 signals are being added to GPS.

precision due to longer code + faster chipping rate for civil users

2nd civil signal w/ protection from RFI