Two-Axis Spacecraft Control

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Underactuation of spacecraft attitude can arise for several reasons. Whether by design or by fault, many spacecraft must maintain a pointing requirement to complete their missions. Perfect PD control about two axes is considered to drive a 1-2-1 Euler angle sequence to zero, one axis at a time. Investigated scenarios include elimination of attitude error with and without body rates, as well as inertia tensors that aren't aligned with the body frame. ¡Summarize the paper results¿

I. Introduction

UNDERACTUATED spacecraft attitude scenarios arise when control authority is not available about every body axis. Such scenarios can happen due to system design, as low cost and hardware simplicity are desirable in spacecraft systems. Propellant or energy saving presents another case, depending on the system design and the mission. Finally, faulted actuators could require underactuated control to make mission success possible.

Underactuated control is a well-studied topic. Coverstone showed how a variable structure controller can detumble a spacecraft with two axes of control.[1] Krishnan et al. considers stabilization of a vehicle with two momentum wheels.[2] Much of the literature focuses on solutions that are not easily implemented during mission operations. A simpler control design can lead to less-expensive and quicker solutions for craft already in flight. This paper investigates PD control about two body axes. Attitude errors are reduced by sequentially driving Euler angles in a 1-2-1 sequence to zero.

II. Problem Statement

This simulation considers the spacecraft's body frame aligned with the principle inertia frame. Control authority is about the b_1 and b_2 axes. An Euler Angle set in the 1-2-1 sequence is being used to drive the attitude error to zero, one axis at a time. Using such a control scheme, the body angular rates are assumed to be about one axis at a time (pure spin about one axis only). Thus, the general rotational equations of motion[3]

$$[I]\,\dot{\boldsymbol{\omega}} = -\,[\tilde{\boldsymbol{\omega}}]\,[I]\,\boldsymbol{\omega} + \mathbf{u}$$

can be simplifed to:

$$\dot{\theta}_i = \frac{u_i}{I_{ii}}$$

for the controlled axes. The uncontrolled axis has no control authority, so $\dot{\theta} = 0$ for b_3 .

A controller is chosen such that the equations of motion about the controlled axes resemble a damped harmonic oscillator (a PD controller). Thus, the controller torque is defined as:

$$u_i = -K_d\dot{\theta}_i + -K_p\theta_i$$

III. Controller Design

The control gains are calculated from the desired parameters $T_s = 70$ seconds and $\zeta = 0.98$. For each axis, the relationship of these parameters to the gains Kd and Kp are defined as[45]:

$$\zeta = \frac{K_d}{2\sqrt{K_p I}}$$

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$$T_s = \frac{-\ln(tolerance\sqrt{1-\zeta^2})}{\zeta\omega_n}$$

where

$$\omega_n = \sqrt{\frac{K_p}{I}}$$

Using a body inertia tensor of

$$I = \begin{bmatrix} 14 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 19 \end{bmatrix}$$

the controller gains are found to be

$$K_{p1} = 0.0909 \quad K_{d1} = 2.2106 \quad K_{p2} = 0.1038 \quad K_{d2} = 2.5264$$

IV. Simulation Design

The simulator propagates Modified Rodrigues Parameters (MRPs) using 4th-order Runge-Kutta integration. Use of MRPs allows for a compact set, while allowing a switch to the shadow set as the propagated parameters approach a singularity.[3] To test the simulation, the rigid body dynamics were propagated for 10 minutes with no control or disturbance torques. The results can be seen in Figures 1 and 2, which used initial conditions of $\theta_{121}(t_0) = (45°, 60°, -75°)^T$, $\boldsymbol{\omega}(t_0) = (1°/\text{s}, -2°/\text{s}, 0.5°/\text{s})^T$, and the previously defined inertia tensor.

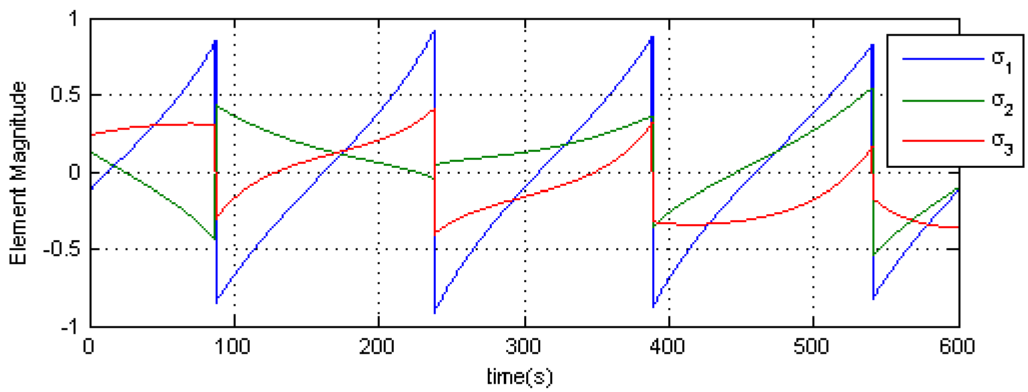

Figure 1: Propagated MRP attitude.

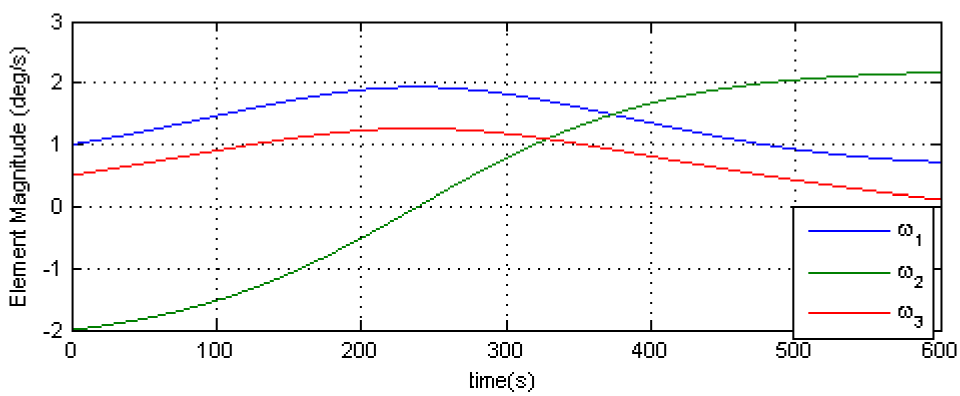

Figure 2: Propagated body rates.

Since there are no external torques present, angular momentum is conserved. Kinetic energy in this simulation is also guaranteed to be conserved, due to the assumption of a completely rigid body. Figures 3 and 4 demonstrate the change in angular momentum and kinetic energy in the simulation, caused by numerical integration error, to be very small.

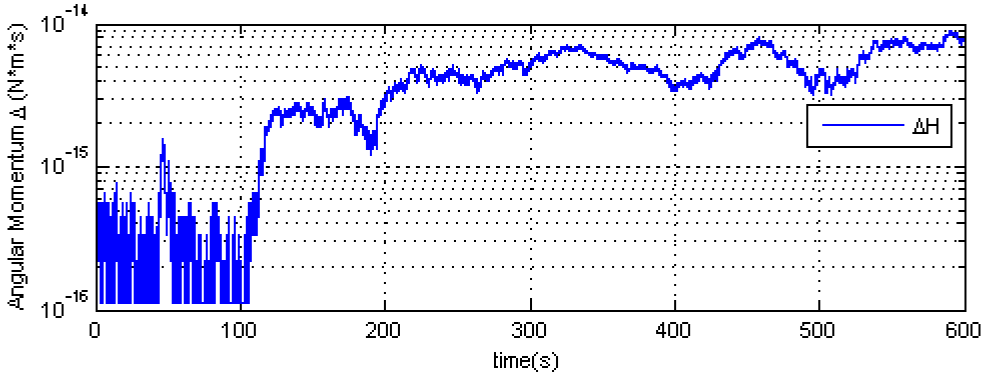

Figure 3: Simulation change in angular momentum, no external torques.

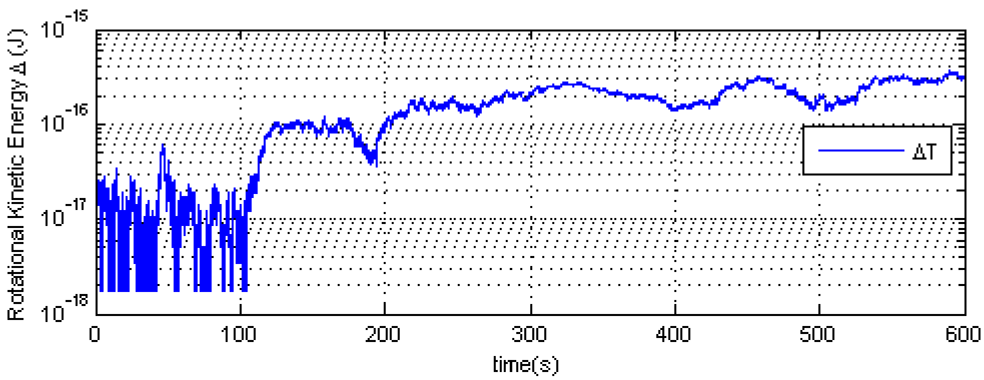

Figure 4: Simulation change in kinetic energy, no external torques.

V. Numerical Simulations

Discuss any numerical simulations that you are going to use to illustrate your analytical results. Be sure to fully explain how the simulations were achieved. Imagine yourself reading this paper, and having to duplicate this work using only this reference. Provide all required initial conditions, state what dynamical systems were integrated, etc. Also, be sure to reference and discuss any result figures that you show here.

VI. Conclusion

The conclusion you briefly summarize the paper. Discuss what problem was solved, how it was solved, any limitations of the method. Contrary to the introduction, you do briefly discuss the results here. It is also ok to briefly address future research in this area.

Acknowledgment

Acknowledge any sponsors, or other person who are not authors, who helped create this paper.

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