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ASEN 5070
HW1

Problem 3: given $U = \mu/R$, solve for 2-body accel due to gravity
where $R = \sqrt{x^2+y^2+z^2}$

$$\ddot{\vec{r}} = \nabla U = \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}$$

$$\frac{\partial U}{\partial x} = \frac{\partial(\mu/R)}{\partial x} = \frac{\partial\left(\mu(x^2+y^2+z^2)^{-\frac{1}{2}}\right)}{\partial x}$$

$$= \frac{-\mu(x^2+y^2+z^2)^{-3/2}}{2} \cdot \frac{\partial}{\partial x}(x^2+y^2+z^2)$$

$$= \frac{-\mu(x^2+y^2+z^2)^{-3/2}}{2} \cdot 2x = \frac{-\mu x}{R^3}$$

similarly, $\frac{\partial U}{\partial y} = \frac{-\mu y}{R^3}$, $\frac{\partial U}{\partial z} = \frac{-\mu z}{R^3}$

$$\ddot{\vec{r}} = \nabla U = -\frac{\mu}{R^3}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\boxed{\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r}} \quad \text{where } r = \|\vec{r}\| = R$$