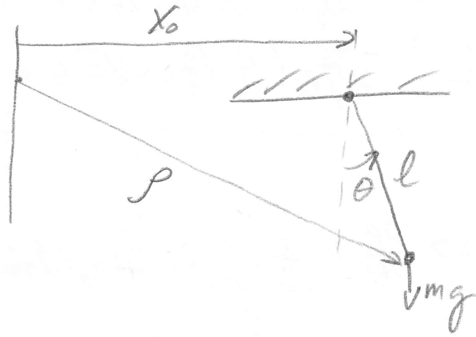

6.5 $X = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \qquad C = \begin{bmatrix} x_0 \\ g \end{bmatrix}$

$$\rho = \sqrt{(x_0 + l\sin\theta)^2 + (l\cos\theta)^2}$$
$$= x_0 + l\theta$$

$$\dot{X} = \begin{bmatrix} \dot{\theta} \\ -\frac{g}{l}\theta \end{bmatrix} \Rightarrow AX + Bc = x$$
$$= \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\theta}{l} \end{bmatrix} c$$

$$\theta = A\cos\left(\sqrt{\tfrac{g}{l}}t + \phi\right) \Rightarrow \theta(t_0 = 0) = A\cos\phi$$
$$= A\left(\cos\left(\sqrt{\tfrac{g}{l}}t\right)\cos\phi - \sin\left(\sqrt{\tfrac{g}{l}}t\right)\sin\phi\right)$$

$$\dot{\theta} = -A\sqrt{\tfrac{g}{l}}\sin\left(\sqrt{\tfrac{g}{l}}t + \phi\right) \Rightarrow \dot{\theta}(t_0 = 0) = -A\sqrt{\tfrac{g}{l}}\sin\phi$$
$$= -A\sqrt{\tfrac{g}{l}}\left(\sin\left(\sqrt{\tfrac{g}{l}}t\right)\cos\phi + \cos\left(\sqrt{\tfrac{g}{l}}t\right)\sin\phi\right)$$

$$\Phi(t, t_0) = \frac{\partial X}{\partial X(t_0)} \Rightarrow \frac{\partial\theta}{\partial\theta_0} = \cos\left(\sqrt{\tfrac{g}{l}}t\right)$$
$$\Rightarrow \frac{\partial\theta}{\partial\dot{\theta}_0} = \sqrt{\tfrac{l}{g}}\sin\left(\sqrt{\tfrac{g}{l}}t\right)$$
$$\Rightarrow \frac{\partial\dot{\theta}}{\partial\theta_0} = -\sqrt{\tfrac{g}{l}}\sin\left(\sqrt{\tfrac{g}{l}}t\right)$$
$$\Rightarrow \frac{\partial\dot{\theta}}{\partial\dot{\theta}_0} = \cos\left(\sqrt{\tfrac{g}{l}}t\right)$$

$$\Rightarrow \boxed{\Phi(t, t_0) = \begin{bmatrix} \cos\left(\sqrt{\tfrac{g}{l}}t\right) & \sqrt{\tfrac{l}{g}}\sin\left(\sqrt{\tfrac{g}{l}}t\right) \\ -\sqrt{\tfrac{g}{l}}\sin\left(\sqrt{\tfrac{g}{l}}t\right) & \cos\left(\sqrt{\tfrac{g}{l}}t\right) \end{bmatrix}}$$

$$\Theta(t, t_0) = \frac{\partial X}{\partial C(t_0)} \Rightarrow \frac{\partial\theta}{\partial x_0} = 0$$

$$\frac{\partial\theta}{\partial g} = A \cdot -\sin\left(\sqrt{\tfrac{g}{l}}t + \phi\right) \cdot \tfrac{1}{2}t \cdot \tfrac{1}{\sqrt{gl}}$$
$$= -\frac{t}{2\sqrt{gl}} \cdot A \cdot \left(\sin\left(\sqrt{\tfrac{g}{l}}t\right)\cos\phi + \cos\left(\sqrt{\tfrac{g}{l}}t\right)\sin\phi\right)$$
$$= -\frac{t}{2\sqrt{gl}} \cdot \left(\theta_0\sin\left(\sqrt{\tfrac{g}{l}}t\right) - \dot{\theta}_0 \cdot \sqrt{\tfrac{l}{g}}\cos\left(\sqrt{\tfrac{g}{l}}t\right)\right)$$

$$\frac{\partial\dot{\theta}}{\partial x_0} = 0$$

$$\frac{\partial\dot{\theta}}{\partial g} = -A\sqrt{\tfrac{g}{l}}\cos\left(\sqrt{\tfrac{g}{l}}t + \phi\right) \cdot \tfrac{1}{2}t\tfrac{1}{\sqrt{gl}}$$
$$= -\frac{t}{2\sqrt{gl}}A\sqrt{\tfrac{g}{l}}\left(\cos\left(\sqrt{\tfrac{g}{l}}t\right)\cos\phi - \sin\left(\sqrt{\tfrac{g}{l}}t\right)\sin\phi\right)$$
$$= -\frac{t}{2\sqrt{gl}}\left(\theta_0\sqrt{\tfrac{g}{l}}\cos\left(\sqrt{\tfrac{g}{l}}t\right) + \dot{\theta}_0\sin\left(\sqrt{\tfrac{g}{l}}t\right)\right)$$

cont →

$$\boxed{\Theta(t,t_0) = \begin{bmatrix} 0 & -\frac{t}{2\sqrt{gl}}\left(\theta_0 \sin\left(\sqrt{\frac{g}{l}}t\right) - \dot{\theta}_0\sqrt{\frac{l}{g}}\cos\left(\sqrt{\frac{g}{l}}t\right)\right) \\ 0 & -\frac{t}{2\sqrt{gl}}\left(\theta_0\sqrt{\frac{g}{l}}\cos\left(\sqrt{\frac{g}{l}}t\right) + \dot{\theta}_0 \sin\left(\sqrt{\frac{g}{l}}t\right)\right) \end{bmatrix}}$$

$$\tilde{H}_x = \frac{\partial \rho}{\partial X} \Rightarrow \frac{\partial \rho}{\partial \theta} = \frac{1}{2}\cdot\frac{1}{\rho}\cdot\left(2(x_0 + l\sin\theta)\cos\theta + 2l\cos\theta\cdot -\sin\theta\right)$$

$$= \frac{x_0\cos\theta}{\rho} \qquad \Rightarrow \underline{\tilde{H}_{x_i} = \begin{bmatrix} \frac{x_0\cos\theta}{\rho} & 0 \end{bmatrix}}$$

$$\tilde{H}_c = \frac{\partial \rho}{\partial C} \Rightarrow \frac{\partial \rho}{\partial x_0} = \frac{1}{2}\cdot\frac{1}{\rho}\cdot\left(2\cdot(x_0 + l\sin\theta)\right) = \frac{x_0 + l\sin\theta}{\rho}$$

$$\Rightarrow \underline{\tilde{H}_{c_i} = \begin{bmatrix} \frac{x_0 + l\sin\theta}{\rho} & 0 \end{bmatrix}}$$