HW 7

Table of Contents

a)

```
clear
A = [-10 0 -10 0;
    0 -0.7 9 0;
    0 -1 -0.7 0;
    1 0 0 0];
B = [20 3; 0 0 ; 0 0; 0 0];

% Calculate the eigenvalues:
[V, e_vals] = eig(A);
fprintf('Eigenvalues:\n')
disp(diag(e_vals))

% Not Hurwitz. But the eigenvalues are all distinct, so we can calculate
% B_tilde and figure out if it's reachable.

B_tilde = inv(V)*B
% Not all rows are reachable, since there is a pole of zero. The
% unreachable modes are the complex conjugate pair. However, the
% unreachable modes have real values on the LHP. Therefore, A is
% stabilizable.
```

```
Eigenvalues:
  0.000000000000000 + 0.000000000000000i
-10.000000000000000 + 0.000000000000000i
 -0.700000000000000 + 3.000000000000000i
 -0.700000000000000 - 3.000000000000000i


B_tilde =

   2.000000000000000   0.300000000000000
  20.099751242241780   3.014962686336267
                   0                   0
                   0                   0
```

b)

```
clear
A = [-5 1 0; 0 -5 0; 0 0 -10];
B = [0;1;1];

% Calculate the eigenvalues:
e_vals = eig(A);
fprintf('Eigenvalues:\n')
disp((e_vals))

% A is Hurwitz. Compute the Grammian to see if its rank matches rank(A)
sys = ss(A,B,zeros(0,length(A)),0);
G = gram(sys,'c')
fprintf('rank(A): ')
disp(rank(A))
fprintf('rank(G): ')
disp(rank(G))
% The system is reachable. For the controllability effort, get the min
% eigen value of G:
effort = min(eig(G));
fprintf('Reachability effort is %f.\n', 1/effort)
```

```
Eigenvalues:
    -5
    -5
   -10


G =

   0.002000000000000   0.010000000000000   0.004444444444444
   0.010000000000000   0.100000000000000   0.066666666666667
   0.004444444444444   0.066666666666667   0.050000000000000

rank(A):      3

rank(G):      3

Reachability effort is 11709.833341.
```

c)

```
A = [-3 1 0 0;0 -3 0 0;0 0 -2 1; 1 0 0 -2];
B = [0;0.001;0;1];

% Calculate the eigenvalues:
[V, e_vals] = eig(A);
fprintf('Eigenvalues:\n')
disp(diag(e_vals))

% A is Hurwitz. Compute the Grammian to see if its rank matches rank(A)
sys = ss(A,B,zeros(0,length(A)),0);
```

```
G = gram(sys,'c')
fprintf('rank(A): ')
disp(rank(A))
fprintf('rank(G): ')
disp(rank(G))
% The system is reachable. For the controllability effort, get the min
% eigen value of G:
effort = min(eig(G));
fprintf('Reachability effort is %f.\n', 1/effort)
```

```
Eigenvalues:
    -2
    -2
    -3
    -3


G =

  Columns 1 through 3

   0.000000009259259   0.000000027777778   0.000016000814815
   0.000000027777778   0.000000166666667   0.000040001111111
   0.000016000814815   0.000040001111111   0.031254500287037
   0.000040002962963   0.000200005555556   0.062509000574074

  Column 4

   0.000040002962963
   0.000200005555556
   0.062509000574074
   0.250020001481482

rank(A):      4

rank(G):      4

Reachability effort is 413364250915.476810.
```

d)

```
A = [5 -1 -3; 0 5.5 0; 0 0 -6];
B = [1;0;5];

% Calculate the eigenvalues:
[V, e_vals] = eig(A);
fprintf('Eigenvalues:\n')
disp(diag(e_vals))

% Not Hurwitz. But the eigenvalues are all distinct, so we can calculate
% B_tilde and figure out if it's reachable.
B_tilde = inv(V)*B
```

```
% The system is not reachable, the second mode does not have a non-zero
% value for its control input. It's not stabilizable, because that
% eigenvalue has a positive real part.
```

```
Eigenvalues:
   5.000000000000000
   5.500000000000000
  -6.000000000000000


B_tilde =

  -0.363636363636363
                   0
   5.182615568632444
```

Conclusion

For the reachable systems, the system in b) took much less effort to control than that of part c). You can tell that the Grammian in part c) was close to being singular in the first two columns, compared to part b)'s Grammian.

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