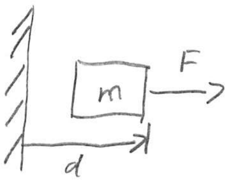
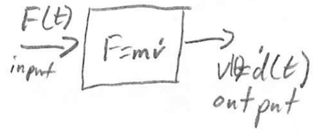
Clouse
ASEN 5014
HW1

Problem 1: Find the SS representation of the below system



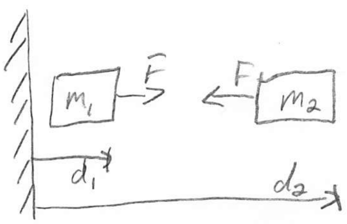


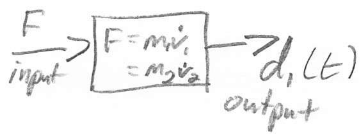
$$\int_{t_0}^{t} \dot{v}(\tau)\,d\tau = \frac{1}{m}\int_{t_0}^{t} F(\tau)\,d\tau = V(t)-v(t_0)$$

$$\Rightarrow v(t) = v(t_0) + \frac{1}{m}F(t)$$ ← for known $u(t)=F(t)$ over $[t_0,t_1]$, only one initial cond., so one state var.

$$\boxed{\begin{matrix} \vec{x}=[v], & \dot{\vec{x}}=[0]\vec{x}+\frac{1}{m}\cdot\vec{u} \\ \vec{u}=[F] & y=[1]\cdot\vec{x}+[0]\cdot\vec{u} \end{matrix}}$$

Problem 2: Find the SS representation of the below system





$$\int_{t_0}^{t} \ddot{d}_1(\tau)\,d\tau = \dot{d}_1(t)-\dot{d}_1(t_0) = v_1(t)-v_1(t_0) = \frac{1}{m_1}\int_{t_0}^{t_1} F(\tau)\,d\tau$$

$$\int_{t_0}^{t} \dot{d}_1(\tau)\,d\tau = d_1(t)-d_1(t_0) = \int_{t_0}^{t} v_1(\tau)\,d\tau = \int_{t_0}^{t}\left(v_1(t_0)+\frac{1}{m_1}\int_{t_0}^{t} F(\tau)\,d\tau\right)d\tau$$

$$\Rightarrow d_1(t) = d_1(t_0)+v_1(t_0)+\frac{1}{m_1}\int_{t_0}^{t}\int_{t_0}^{t} F(\tau)\,d\tau$$ ← 2 IC's, so 2 state vars

$$\boxed{\begin{matrix} u=F(t) & \dot{x}=\begin{matrix}v\\\dot{v}\end{matrix}=\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}d\\v\end{pmatrix}+\begin{pmatrix}0\\\frac{1}{m}\end{pmatrix}u \\ x=\begin{pmatrix}d\\v\end{pmatrix} & \\ & y=[1\ 0]\,x+0\cdot u \end{matrix}}$$

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Problem 3: Brogan Problem 3.1 in Controllable Canonical Form.

a) $\dfrac{Y(s)}{U(s)} = \dfrac{1}{s+\alpha} \Rightarrow Y(s)(s+\alpha) = U(s)$

$\xrightarrow{\mathcal{L}^{-1} \text{ w/ 0 IC's}}$ $\dot{y} + \alpha y = u$ (ODE)

$\boxed{x(t) = y(t), \quad \dot{x}(t) = \dot{y} = -\alpha y + u = \underline{-\alpha x + u}}$

b) $\dfrac{Y(s)}{U(s)} = \dfrac{s+\beta}{s+\alpha} \Rightarrow Y(s)(s+\alpha) = U(s)(s+\beta)$

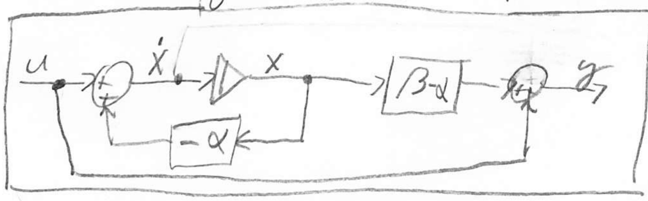
$\dfrac{G(s)}{U(s)} = \dfrac{1}{s+\alpha} \Rightarrow G(s)(s+\alpha) = U(s)$

$\xrightarrow{\mathcal{L}^{-1} \text{ w/ 0 IC's}}$ $\dot{g} + \alpha g = u$

$\boxed{\begin{aligned} &\text{let } x = g \\ &\dot{x} = -\alpha x + u \end{aligned}}$

$Y(s) = G(s)B(s) = G(s)(s+\beta)$

$\xrightarrow{\mathcal{L}^{-1} \text{ w/ 0 IC's}}$ $\boxed{y(t) = \dot{g} + \beta g = \dot{x} + \beta x = (-\alpha + \beta)x + u}$



Problem 3 (cont.)

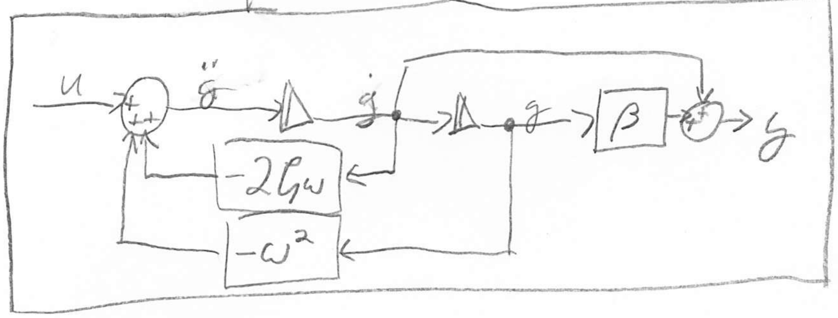
c) $\dfrac{Y(s)}{U(s)} = \dfrac{s+\beta}{s^2+2\zeta\omega s+\omega^2} \Rightarrow \dfrac{G(s)}{U(s)} = \dfrac{1}{s^2+2\zeta\omega s+\omega^2}$

$\xrightarrow{\mathcal{L}^{-1}\text{ w/ 0 IC's}} \ddot{g} + 2\zeta\omega\dot{g} + \omega^2 g = u$

$$\boxed{\text{let } \vec{x} = \begin{pmatrix} g \\ \dot{g} \end{pmatrix} \Rightarrow \dot{\vec{x}} = \begin{pmatrix} \dot{g} \\ \ddot{g} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega^2 & -2\zeta\omega \end{pmatrix}\vec{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u}$$

$Y(s) = G(s)B(s) = G(s)(s+\beta)$

$\xrightarrow{\mathcal{L}^{-1}\text{ w/ 0 IC's}} \boxed{y = \dot{g} + \beta g = [\beta \;\; 1]\vec{x} + 0u}$



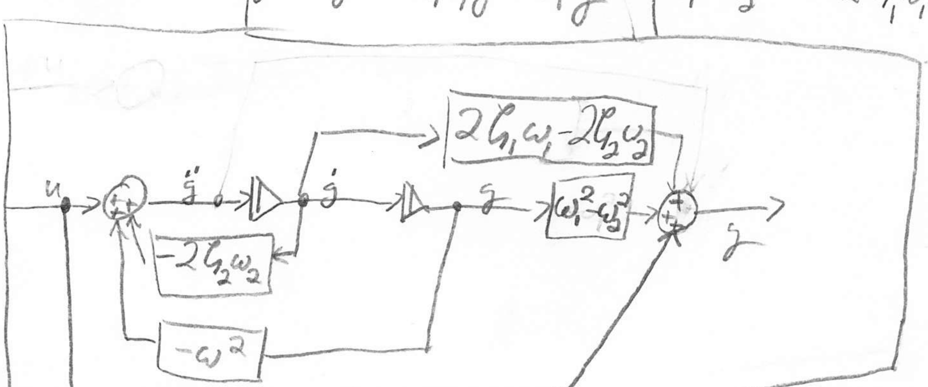
d) $\dfrac{Y(s)}{U(s)} = \dfrac{s^2+2\zeta_1\omega_1 s+\omega_1^2}{s^2+2\zeta_2\omega_2 s+\omega_2^2} \Rightarrow \dfrac{G(s)}{U(s)} = \dfrac{1}{s^2+2\zeta_2\omega_2 s+\omega_2^2}$

$\xrightarrow{\mathcal{L}^{-1}\text{ w/ 0 IC's}} \ddot{g} + 2\zeta_2\omega_2\dot{g} + \omega_2^2 g = u$

$$\boxed{\text{let } \vec{x} = \begin{pmatrix} g \\ \dot{g} \end{pmatrix},\quad \dot{\vec{x}} = \begin{pmatrix} 0 & 1 \\ -2\zeta_2\omega_2 & \omega_2^2 \end{pmatrix}\vec{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u}$$

$Y(s) = G(s)B(s) = G(s)(s^2+2\zeta_1\omega_1 s+\omega_1^2)$

$\xrightarrow{\mathcal{L}^{-1}\text{ w/ 0 IC's}} \boxed{y = \ddot{g} + 2\zeta_1\omega_1\dot{g} + \omega_1^2 g = (\omega_1^2-\omega_2^2 \quad 2\zeta_1\omega_1 - 2\zeta_2\omega_2)\vec{x} + 1 \cdot u}$



Problem 4: Find SS representation for:

$$\dot{y}_1 + 3(y_1 + y_2) = u_1 \qquad \Rightarrow \qquad \dot{y}_1 = u_1 - 3y_1 - 3y_2$$

$$\ddot{y}_2 + 4\dot{y}_2 + 3y_2 = u_2 \qquad\qquad \ddot{y}_2 = u_2 - 4\dot{y}_2 - 3y_2$$

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

let $\vec{x} = \begin{pmatrix} y_1 \\ y_2 \\ \dot{y}_2 \end{pmatrix}$; $\dot{\vec{x}} = \begin{bmatrix} -3 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix} \vec{x} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \vec{u}$

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \vec{u}$$

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Challenge 1: SS realization for $\ddot{y} + a\dot{y} + by = v + c\dot{v}$, where y is the output and v is the control

let $u = v$

$Y(s)(s^2+as+b) = V(s)(cs+1) = U(s)(cs+1)$

$\Rightarrow \frac{B(s)}{A(s)} U(s) = \frac{cs+1}{(s^2+as+b)} U(s)$

$G(s)A(s) = U(s)$

$\Rightarrow \ddot{g} + a\dot{g} + bg = u$

$Y(s) = G(s)B(s) \Rightarrow \boxed{c\dot{g} + g = y}$

$$\boxed{\vec{x} = \begin{pmatrix} g \\ \dot{g} \end{pmatrix} \quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix}\begin{pmatrix} g \\ \dot{g} \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad u = v}$$

Challenge 2: $G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s-1}$

$$K(s) = \frac{U(s)}{V(s)} = \frac{s-1}{s+1} \Rightarrow C(s) = K(s)G(s) = \frac{s-1}{(s-1)(s+1)} = \frac{B(s)}{A(s)}$$

$$\frac{H(s)}{U(s)} = \frac{1}{s^2-1} \Rightarrow H(s)(s^2-1) = U(s)$$

$$\xrightarrow{\mathcal{L}^{-1}\ w/\ 0\ IC's} h'' - h = u$$ ← complementary sol'n is undamped osc. find particular sol'n

try $h_p = Ae^t \rightarrow \frac{d}{dt}(e^t + Ae^t) - Ae^t = e^t + e^t + Ae^t - Ae^t = u$

$$\Rightarrow u = 2e^t$$

try $h_p = e^{ut} \rightarrow \frac{d}{dt}(ue^{ut}) - e^{ut} = \dot{u}e^{ut} + u^2e^{ut} - e^{ut} = u$

try $h_p = Au \rightarrow A\ddot{u} - Au = u \Rightarrow A\ddot{u} - (A+1)u$

~~find soln. for u: $r^2 - \frac{(A+1)}{A} = 0 \Rightarrow r = \pm\sqrt{1+\frac{1}{A}}$~~

~~$u = c_1 e^{\sqrt{1+\frac{1}{A}}t} + c_2 e^{-\sqrt{1+\frac{1}{A}}t}$~~

try $h_p = -u \rightarrow -\ddot{u} + u = u \Rightarrow \ddot{u} = 0$