HW 7

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a)

```
clear
A = [-10 0 -10 0;
    0 -0.7 9 0;
    0 -1 -0.7 0;
    1 0 0 0];
B = [20 3; 0 0 ; 0 0; 0 0];

% Calculate the eigenvalues:
[V, e_vals] = eig(A);
fprintf('Eigenvalues:\n')
disp(diag(e_vals))

% Not Hurwitz. But the eigenvalues are all distinct, so we can calculate
% B_tilde and figure out if it's reachable.

B_tilde = inv(V)*B
% Not all rows are reachable, since there is a pole of zero. The
% unreachable modes are the complex conjugate pair. However, the
% unreachable modes have real values on the LHP. Therefore, A is
% stabilizable.
```

```
Eigenvalues:
  0.000000000000000 + 0.000000000000000i
-10.000000000000000 + 0.000000000000000i
 -0.700000000000000 + 3.000000000000000i
 -0.700000000000000 - 3.000000000000000i


B_tilde =

   2.000000000000000   0.300000000000000
  20.099751242241780   3.014962686336267
                   0                   0
                   0                   0
```

b)

```
clear
A = [-5 1 0; 0 -5 0; 0 0 -10];
B = [0;1;1];

% Calculate the eigenvalues:
e_vals = eig(A);
fprintf('Eigenvalues:\n')
disp((e_vals))

% A is Hurwitz. Compute the Grammian to see if its rank matches rank(A)
sys = ss(A,B,zeros(0,length(A)),0);
G = gram(sys,'c')
fprintf('rank(A): ')
disp(rank(A))
fprintf('rank(G): ')
disp(rank(G))
% The system is reachable. For the controllability effort, get the min
% eigen value of G:
effort = min(eig(G));
fprintf('Reachability effort is %f.\n', 1/effort)
```

```
Eigenvalues:
    -5
    -5
   -10


G =

   0.002000000000000   0.010000000000000   0.004444444444444
   0.010000000000000   0.100000000000000   0.066666666666667
   0.004444444444444   0.066666666666667   0.050000000000000

rank(A):      3

rank(G):      3

Reachability effort is 11709.833341.
```

c)

```
A = [-3 1 0 0;0 -3 0 0;0 0 -2 1; 1 0 0 -2];
B = [0;0.001;0;1];

% Calculate the eigenvalues:
[V, e_vals] = eig(A);
fprintf('Eigenvalues:\n')
disp(diag(e_vals))

% A is Hurwitz. Compute the Grammian to see if its rank matches rank(A)
sys = ss(A,B,zeros(0,length(A)),0);
```

```
G = gram(sys,'c')
fprintf('rank(A): ')
disp(rank(A))
fprintf('rank(G): ')
disp(rank(G))
% The system is reachable. For the controllability effort, get the min
% eigen value of G:
effort = min(eig(G));
fprintf('Reachability effort is %f.\n', 1/effort)
```

```
Eigenvalues:
    -2
    -2
    -3
    -3


G =

  Columns 1 through 3

   0.000000009259259   0.000000027777778   0.000016000814815
   0.000000027777778   0.000000166666667   0.000040001111111
   0.000016000814815   0.000040001111111   0.031254500287037
   0.000040002962963   0.000200005555556   0.062509000574074

  Column 4

   0.000040002962963
   0.000200005555556
   0.062509000574074
   0.250020001481482

rank(A):      4

rank(G):      4

Reachability effort is 413364250915.476810.
```

d)

```
A = [5 -1 -3; 0 5.5 0; 0 0 -6];
B = [1;0;5];

% Calculate the eigenvalues:
[V, e_vals] = eig(A);
fprintf('Eigenvalues:\n')
disp(diag(e_vals))

% Not Hurwitz. But the eigenvalues are all distinct, so we can calculate
% B_tilde and figure out if it's reachable.
B_tilde = inv(V)*B
```

```
% The system is not reachable, the second mode does not have a non-zero
% value for its control input. It's not stabilizable, because that
% eigenvalue has a positive real part.
```

```
Eigenvalues:
   5.000000000000000
   5.500000000000000
  -6.000000000000000


B_tilde =

  -0.363636363636363
                   0
   5.182615568632444
```

Conclusion

For the reachable systems, the system in b) took much less effort to control than that of part c). You can tell that the Grammian in part c) was close to being singular in the first two columns, compared to part b)'s Grammian.

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Single-Axis Control of a Solar Sail Through a Gimbal

Johnathan Clouse*
University of Colorado, Boulder, CO, 80309-0429, USA

*Graduate Students, Aerospace Engineering Sciences, 1111 Engineering Drive, Boulder, CO, 80309-0429

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I. Introduction

Single-axis control of a solar-sail-driven interplanetary spacecraft (sailcraft) is proposed. The attitude control system will be responsible for ensuring that the steering angle between the force and velocity vectors is within the tolerance necessary for an interplanetary voyage. This steering angle is dependent on the mission parameters and the orbital position of the spacecraft. It, and the sun vector, will be treated as external commands to the system. The spacecraft will perform all its thrusting in the orbit plane.

The primary actuation mechanism will be a gimbaled control boom between the sail subsystem and the spacecraft bus, which contains the majority of the spacecraft mass. With the center of mass between the thrust point and the sun, expected disturbances will cause oscillation about some angle between the sun and the axis normal to the sail, α, for a locked gimbal. Changing the gimbal angle, δ, will dampen this oscilation with the right conrol law. Roll and pitch angles wil be held to zero for this analysis. Star trackers will determin attitude.

The state-space model is expected to have four states: the sun angle (α), the rate of the sun angle ($\dot{\alpha}$), the gimbal angle (δ), and the gimbal angle rate ($\dot{\delta}$). Depending on the vane implementation, there may be up to two more states for vane angles.

The sail and boom will be modeled as rigid bodies, justified by the slow actuation of the gimbal throughout the flight. The sail will be modeled as a thin plate, rather than a billowed sail. Solar pressure torques (about the non-steered axis) will be controlled against. Disturbance torques from thruster firings may also be modeled.

The state-space model will be obtained in a similar manner to that presented by Wie. The equations of motion for a gimbaled thrust vector are obtained for the yaw axis.

System performance will be judged by the response to errors, both with a step-error and a flight-like error where the steering angle constantly-but-slowly changes. Mitigation of disturbance torques will also be examined.

II. State Space Representation

The equations of motion were linearized about the state $\alpha = \dot{\alpha} = \delta = \dot{\delta} = 0$. This state is in equilibrium, due to the the force resulting from the solar radiation pressure acting through the sailcraft's center of mass. Any disturbance to α would cause oscillation about $\alpha = 0$. The linearized equations are shown below:

$$\begin{bmatrix} \dot{\alpha} \\ \ddot{\alpha} \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{d}{J_s} F_n & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m_p l}{J_p m + m_s m_p l^2} F_t & 0 & -\frac{m_p l}{J_p m + m_s m_p l^2} F_n & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{J_s} \\ 0 \\ \frac{1}{J_p + \frac{m_s m_p}{m} l^2} \end{bmatrix} T_{gimbal} \tag{1}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u \tag{2}$$

$$F_n = PA(1 + \rho_s + \frac{2}{3}\rho_d) \tag{3}$$

$$F_t = PA(1 - \rho_s) \tag{4}$$

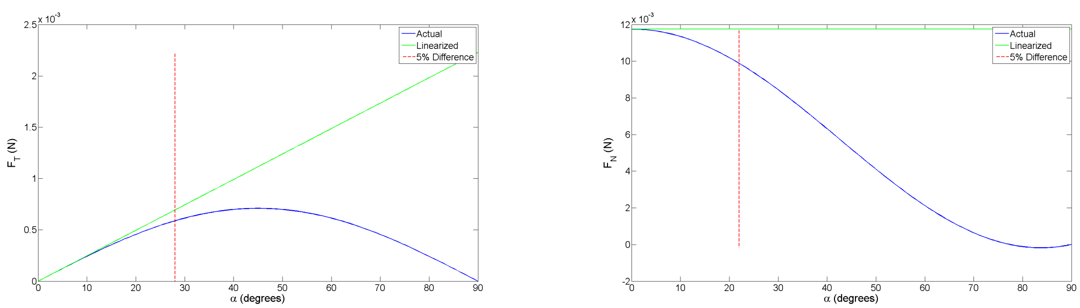
Table 1. Sailcraft characteristics.

Characteristic	Value
m_s	40 kg
m_p	116 kg
m	156 kg
J_s	6000 kg·m^2
J_p	20 kg·m^2
P	4.563e-6 kg/m^2
A_{sail}	1800 m^2
l	2 m
d	1.487 m
ρ_s	0.8272
ρ_d	-0.5949

Using the sailcraft characteristics in Table 1, the eigenvalues are found to be:

$$\lambda_i = \{\pm 1.1200 \times 10^{-2} i, \pm 5.9395 \times 10^{-4} i\}.$$

The complex eigenvalues with no real parts indicate that the uncontrolled, linearized system is marginally stable. It will oscillate undamped when perturbed by a small amount, but a large disturbance could excite the modes and make the output y unbounded. However, as α and δ each approach $\pm 90°$, the assumption becomes invalid. Indeed, in Figure 1, one can see that a five-percent error between the non-linear and linearized sail force occurs at approximately $\pm 20°$.



(a) Transverse force

(b) Normal force

Figure 1. Linear solutions of the sail forces vs. sun angle.

The controller will be constrained to keep the sun angle less than or equal to 20° to keep the system model within the realm of linearity. It will have to dampen the oscillations induced induced by disturbances so that the sail can provide a force in the desired direction.