ASEN 5050 Mid-Term Exam
Due: 10:00 am Monday, 12/14/2015
Paper Copy Preferred (ECNT319) or D2L Dropbox

Read everything on this page before you take the exam!

This is a take-home exam. You may use whatever books and notes you have at your disposal. However, you may not communicate with other people (students or otherwise) about the exam. You may want to use a computer to help check the calculations you have made, but you will be graded based on the work you show in your answer. You may attach any code you have written in support of the exam, but it will in general not be used to grade your exam.

In general, a correct answer will not give you credit for a question unless you show your work. An incorrect answer may still give you partial credit if you show a correct process.

Write all your answers on separate sheets of paper and attach them to your exam! Circle your final answers asked for in the problem.

Use Appendix D-3 and D-4 of the book for all constants not given in the problem.

You must complete this exam within a time period of **48 hours**, but you may review the questions first to see if you have any questions for me. After **48 hours**, stop working. I expect you will finish in ~3-4 hours plus 1-2 hours to check your answers.

Sign the following statement, reflecting the Honor Code (electronic signatures are acceptable):

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work. I have used no more than 24 hours to complete this exam.

[signature]	John Clouse	12-13-15
Signature	Printed Name	Date

12-11-15 2:00 PM	12-13-15 11:00 AM
Date/Time Exam Started	Date/Time Exam Finished

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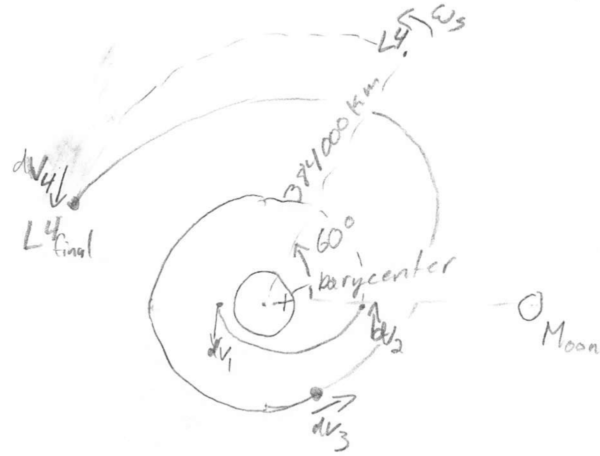
1. $h = 300$ Km
$i = 28°$
$r_{\text{ℂ}} = 384{,}000$ Km



Assuming circular orbits

$$\omega_s = \sqrt{\frac{\mu_E + \mu_M}{\left(384000 - \frac{384000\,\mu_M}{\mu_E + \mu_M}\right)^3}} = 2.71888 \times 10^{-6}$$

Need to match final trueanoms.
breaking it up into 3 transfers: - Hohmann to $h = 1000$ km
- circ. phasing orbit @ $h = 1000$ Km
- Hohmann to L4

$a_{xfer1} = R_\oplus + (300 + 1000)/2 = 7028$ km

$t_{xfer1} = 2\pi\sqrt{\frac{a_{xfer1}^3}{\mu_\oplus}}/2 = 2931$ s

$a_p = 7378$ km

$n_p = 9.9233 \times 10^{-4}$ rad/s

$a_{xfer2} = (R_\oplus + h_p + L_4)/2 = 195689$ Km

$t_{xfer2} = 430755$ s

total phasing time in 1000 Km orbit!

$$\pi + \pi + n_p t_p + \pi - revs \cdot 2\pi = \frac{\pi}{3} + \omega_s t_{xfer1} + \omega_s t_p + \omega_s t_{xfer2}$$

$$t_p = \frac{\frac{\pi}{3} + \omega_s (t_{xfer1} + t_{xfer2}) - 3\pi + revs \cdot 2\pi}{n_p - \omega_s}$$

for revs = 2, $\underline{t_p = 2240.876 \text{ sec}}$

$$dv_1 = \sqrt{\frac{2\mu_\oplus}{R_\oplus + 300} - \frac{\mu_\oplus}{7028}} - \sqrt{\frac{\mu_\oplus}{R_\oplus + 300}} = 0.1900 \text{ Km/s, vel-vec direction}$$

$$dv_2 = \sqrt{\frac{\mu_\oplus}{R_\oplus + 1000}} - \sqrt{\frac{2\mu_\oplus}{R_\oplus + 1000} - \frac{\mu_\oplus}{7029}} = 0.1854 \text{ Km/s, vel-vec direction}$$

$$dv_3 = \sqrt{\frac{2\mu_\oplus}{R_\oplus + 1000} - \frac{\mu_\oplus}{a_{xfer2}}} - \sqrt{\frac{\mu_\oplus}{R_\oplus + 1000}} = 2.9461 \text{ Km/s, vel-vec direction}$$

$$dv_4 = r_{L4} \cdot \omega_s - \sqrt{\frac{2\mu_\oplus}{r_{L4}} - \frac{\mu_\oplus}{a_{xfer2}}} = 0.8462 \text{ Km/s, vel-vec direction}$$

The maximum inclination change needed is $28° + 5.145396° = 33.145396°$, depending on Ω for the SC & Moon. It would have to take place on the shared line of nodes, which is the line connecting the SC, Earth, & Moon at the beginning (since SC/Moon are directly opposed). In my maneuver plan, there are 3 places to do it: dv_1, dv_2, or in the parking orbit after dv_2.

$$dV_{inc,max} = 2 \cdot \sqrt{\frac{\mu_\oplus}{R_\oplus + 300}} \sin\left(\frac{33.145°}{2}\right) = \boxed{4.4073 \text{ Km/s}}$$

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2.

$\uparrow 2\,m/s$

$40^\circ N,\ 105^\circ W,\ 6378.1363 + 1.6551$ km

$= 6379.7914$ km $= r$

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$$\vec{V}_{ECI} = \frac{\vec{r}_{ECI}}{\|\vec{r}\|_{ECI}} \cdot \underbrace{2\,m/s}_{=0.002\,Km/s} + \vec{\omega}_{earth} \times \vec{r}_{ECI}, \quad \omega_{earth} = 7.292115\times10^{-5}\ rad/s$$

$\theta_{LST} = \theta_{GMST} + \lambda \Rightarrow JD = 2457364.77200231$

$$T_{UT1} = \frac{JD - 2451545}{36525} = 0.15933667358$$

$$\theta_{GMST} = 67310.54841 + (876600 \cdot 3600 + 8640184.812866)T_{UT1} + 0.093104 T_{UT1}^2 - 6.2\times10^{-6} T_{UT3}^3$$

$= 5.042723099 \times 10^8 s$

$= 174.62440798^\circ$

$\theta_{LST} = 69.62440798^\circ$

$\vec{r}_{ECI} = R_3(\theta_{LST}) R_2(-(90-\phi_{gd})) \vec{r}_{SEZ}, \quad \vec{r}_{SEZ} = [0,0,r]^T$

$$\phi_{gd} = \operatorname{atan}\left(\frac{\tan \phi_{gc}}{1-e_{\oplus}^2}\right) = 40.1896106^\circ$$

$$\vec{r}_{ECI} = \begin{bmatrix} 1696.856707 \\ 4568.666352 \\ 4117.001747 \end{bmatrix} Km \Rightarrow \vec{V} = \begin{bmatrix} -0.332620496 \\ 0.125168988 \\ 0.001290638 \end{bmatrix} Km/s$$

cart2OE Results:

$a = 3193.123255$ km
$e = 0.997978$
$i = 40.189611^\circ$
$\omega = 339.624408^\circ$
$\Omega = 270.000653^\circ$
$\nu = 179.999347^\circ$
$r_a = 6379.791569$ km
$r_p = 6.454941$ km

$$\boxed{h_{max} = r_a - r = 2.0463609 \times 10^{-1} m}$$

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Problem 3: $m_{ast.} = 10^{15}\,kg$

a) $a_{ast} = 3AU$

$m_{sun} = 1.1891 \times 10^{30}\,kg$

$$\Rightarrow \left(\frac{m_{ast}}{m_{sun}}\right)^{2/5} \cdot a = \boxed{340.866263\text{ km SOI}}$$

b) $\mu_{ast} = G M_{ast} = 6.6740 \times 10^{-5}\,km^3/s^2$, $G = 6.672 \times 10^{-20}\,\frac{km^3}{kg\,s^2} = \frac{\mu_\oplus}{m_\oplus}$

$$P = 2\pi\sqrt{\frac{a_{sat}^3}{\mu_{ast}}} \boxed{\begin{aligned}&= 1.9460 \times 10^5\,s\\&= 54\text{ hrs}\end{aligned}}$$

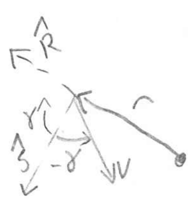
c) $V_{circ} = \sqrt{\frac{\mu}{40}} = 0.0013\,km/s = 1.3\,m/s$

$$v_{new}^2 = \frac{2\mu}{r} - \frac{\mu}{a_{xfer}} \Rightarrow \frac{1}{a_{xfer}} = \frac{2}{r} - \frac{v^2}{\mu} \Rightarrow a_{xfer} = \frac{1}{\frac{2}{r} - \frac{v^2}{\mu}} = 20.5228 \qquad \frac{a}{r_p} - 1 = e = 0.9491$$

$$M_f = 180^\circ + n \cdot 6.3600 = 289.7304^\circ \Rightarrow \nu = 190.9056^\circ$$

$$\boxed{r_f = \frac{a(1-e^2)}{1+e\cos\nu} = 29.9306\text{ km}}$$

d) $\gamma = \operatorname{atan2}(e_{xfer}\sin\nu,\ 1 + e_{xfer}\cos\nu) = -69.2332^\circ$



$$V_{circfinal} = \sqrt{\frac{\mu}{r_f}} = 0.00149\,km/s$$

$$V_{xferfinal} = \sqrt{\frac{2\mu}{r_f} - \frac{\mu}{a_{xfer}}} = 0.00110\,km/s$$

$$d\vec{V} = \begin{bmatrix} 0 \\ V_{circfinal} \\ 0 \end{bmatrix} - \begin{bmatrix} \sin\gamma \\ \cos\gamma \\ 0 \end{bmatrix} V_{xferfinal} = \boxed{\begin{bmatrix} 1.03 \\ 1.10 \\ 0 \end{bmatrix} m/s} \qquad \boxed{\|dV\| = 1.51\,m/s}$$

e) The inclination is 90°, so RAAN not affected ($\dot{\Omega} \propto \cos i$)
The orbit is circular, so ω's change is not important if using the argument of latitude, although ω will change secularly.
Mean anomaly will experience secular change. Operators should be concerned about this if they use Kepler's Problem to propagate the orbit.

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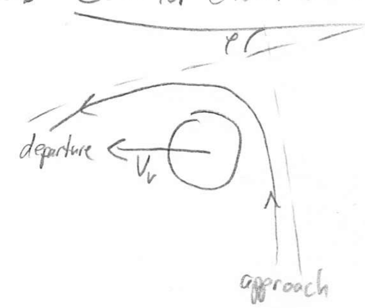
Problem 4. $SOI_{venus} = 6.1467e5 = r_\infty$

$$\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r_v} = \boxed{-796.7452\ J/kg}$$

$$\vec{V}_\infty = \vec{V}_{sc}^{Approach} - \vec{V}_{venus} = \boxed{\begin{bmatrix} 3.1988 \\ -7.0762 \\ 0 \end{bmatrix} km/s} \Rightarrow \boxed{\|\vec{V}_\infty\| = 7.7656\ km/s}$$

$$\mathcal{E}_{venus} = \frac{V_\infty^2}{2} - \frac{\mu_{venus}}{r_\infty} = \boxed{29.6244\ J/kg}$$

b) Counter Clockwise



Clockwise

$$V_\infty^{dep} = R_3(42.0°) \cdot V_\infty^{app} = \begin{bmatrix} -2.3577 \\ -7.3990 \\ 0 \end{bmatrix} km/s$$

$$\boxed{V_{sc}^{Dep} = \begin{bmatrix} -33.6800 \\ 8.3315 \\ 0 \end{bmatrix} km/s \quad \mathcal{E} = -627.7812\ J/kg}$$

$\boxed{\text{Increases heliocentric energy}}$

$$V_\infty^{dep} = R_3(-42°) \cdot V_\infty^{app} = \begin{bmatrix} 7.1121 \\ -3.1182 \\ 0 \end{bmatrix} km/s$$

$$\boxed{V_{sc}^{Dep} = \begin{bmatrix} -24.2102 \\ 12.6123 \\ 0 \end{bmatrix} \quad \mathcal{E} = -857.0560}$$

$\boxed{\text{Decreases Heliocentric energy}}$

$$r_p = \frac{\mu_v}{V_\infty^2} \cdot \left(\frac{1}{\cos\left(\frac{\pi - \varphi}{2}\right)} - 1 \right) = \boxed{75013.02\ km}$$

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Problem 5, $h = 185$

$\Delta V = 7\ \text{Km/s}$

a) $V_\infty = 7\ \text{Km/s} - (V_{esc} - V_{circ}) = 7 - \left(\sqrt{\frac{2\mu_\oplus}{185 + R_\oplus}} - \sqrt{\frac{\mu_\oplus}{185 + R_\oplus}}\right) = 3.7719\ \text{Km/s}$

$V_\oplus = 29.78469$

<u>for farthest point:</u>

$r = r_p = 1\ \text{AU}$

$$v^2 = \frac{2\mu_\odot}{r_\oplus} - \frac{\mu_\odot}{a_{xfer}} \Rightarrow \frac{1}{a_{xfer}} = \frac{2}{r_\oplus} - \frac{v^2}{\mu_\odot}$$

$$\Rightarrow a_{xfer} = \frac{1}{\frac{2}{r_\oplus} - \frac{v^2}{\mu_\odot}}$$

$V_p = V_\infty + V_\oplus = 33.5566\ \text{Km/s}$

$a_{xfer} = 204{,}730{,}865.759\ \text{Km} = 1.3686\ \text{AU}$

$r_a = 2a_{xfer} - r_p = \boxed{259{,}876{,}746.615\ \text{Km} = 1.7372\ \text{AU}}$

b) $r = r_a = 1\ \text{Au}$

$V_a = V_\oplus - V_\infty = 26.0128\ \text{Km/s}$

$a_{xfer} = 120{,}112{,}324.470\ \text{Km}$

$r_p = 2a_{xfer} - r_a = \boxed{92{,}226{,}778.239\ \text{Km} = 0.6165\ \text{AU}}$

c) 

$i = \cancel{i_0}\ 7^\circ + \text{asin}\ \frac{V_\infty}{V_\oplus} = \boxed{7.275^\circ}$

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Problem 6

$r_p = 7000$ km

$r_a = 8000$ Km

$i = 100°$

$P = 6000$ s

$R_{planet} = 6000$ Km

$r_{planet} = 2$ AU

$$a = \frac{r_p + r_a}{2} = 7500 \text{ Km}$$

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \Rightarrow \mu = 2\pi\frac{a^3}{P^2} = \boxed{7.363108 \times 10^4 \text{ Km}^3/s^2}$$

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.06667$$

$$P_{planet} = 2\pi\sqrt{\frac{r_{planet}^3}{\mu_\odot}} = 8.926 \times 10^7 s$$

$$\dot{\Omega}_{ss} = \frac{360°}{P_{planet}} = 7.03919 \times 10^{-8} \text{ rad/s}$$

$$= -\sqrt{\frac{\mu}{a^3}}\frac{3R_{planet}^2 J_2}{2a^2(1-e^2)^2}\cos(i)$$

$$\Rightarrow J_2 = -\dot{\Omega}\sqrt{\frac{a^3}{\mu_{planet}}} \cdot \frac{2a^2(1-e^2)^2}{3R_{planet}^2\cos(i)}$$

$$\boxed{= 0.0010017827}$$