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1. A spacecraft in Earth orbit has a periapsis altitude of 600 km, and an apoapsis altitude of 800 km. The orbit is inclined at 28.5 degrees to the equator.

- **What is the orbit eccentricity?**
- **What is the orbit semi-major axis?**
- **What is the spacecraft velocity at periapsis?**
- **What is the apsidal rotation rate?**
- **What is the nodal regression rate?**
- **Where is the spacecraft true anomaly 10 minutes after passing periapsis?**
- **What velocity change is required to circularize the orbit to 700 km?**
- **In the preceding question where is the velocity change applied?**

- Eccentricity: 0.014128
- Semi-major axis: 7078.0 km
- Vp: 7.61 km/s
- Apsidal rotation rate: 9.91 deg/day
- Nodal regression rate: -6.85 deg/day
- True anomaly at 10 minutes: 37.42 deg
- Circularization dV: 0.106 km/s, 90.40 deg from the direction of motion toward the orbit interior.
- Circularization occurs at true anom: 90.81 deg

2. A spacecraft is in equatorial Earth orbit at 270 km altitude needs to be transferred to a geostationary position. Define the velocity changes required to perform this orbit adjustment using a minimum velocity change.

- dV 1: 2.43 km/s in velocity vector direction
- dV 2: 1.47 km/s in velocity vector direction

3. The Voyager 2 grand tour of the outer planets started with a launch on 20 August 1977 from ETR and ended with the encounter of Neptune on 24 August 1989. Gravity assist maneuvers at Jupiter, Saturn, and Uranus took advantage of the planetary alignment that allows this mission occurs once every 176 years. The mission time from Earth to Neptune was 12 years. What velocity changes would be required for a Hohmann transfer between Earth and Neptune without the gravity-assist maneuvers? What is the time required for the spacecraft transit between Earth and Neptune. Assume circular coplanar orbits.

- dV: 11.67 km/s
- Transfer Time: 30.55 years

4. A 862 kg communications satellite has attained a periapsis altitude of 41,756 km in the equatorial plane with an eccentricity of 0.0061. What is the minimum velocity change to place the satellite in a geosynchronous orbit? The final altitude must be between 35,776 to 35,796 km.

- dV 1: -0.106 km/s (anti-velocity-vector direction)

- dV 2: -0.100 km/s (anti-velocity-vector direction)

5. **A spacecraft is leaving Earth on a departure hyperbola with a hyperbolic excess velocity (Vhe) of 4.256 km/sec. What velocity was required to escape the Earth's gravitational pull?**
6. What is the radius of the sphere of influence at Mars where a spacecraft would transition between the Sun's gravitational pull and the planet's gravitational pull?

- V_esc: 11.18 km/s

6. What is the radius of the sphere of influence at Mars where a spacecraft would transition between the Sun's gravitational pull and the planet's gravitational pull?

- Sphere of Influence: 576189.393044 km (Using Laplace eqn)

7. **Consider an elliptical equatorial Earth orbit with a semimajor axis of 12,500 km and an eccentricity of 0.472. What is the time from periapsis passage to a position with a true anomaly of 205 degrees?**

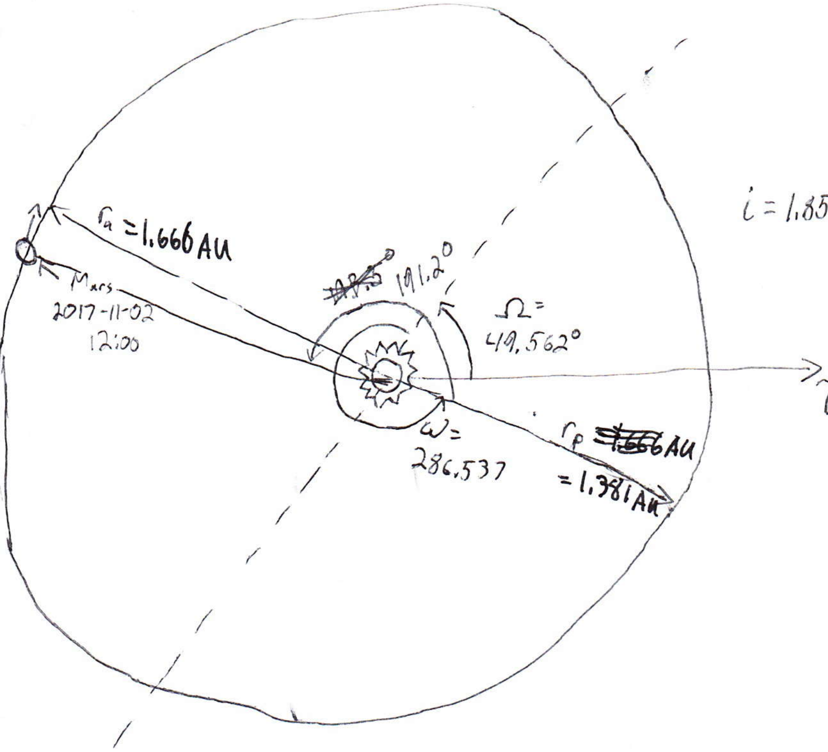
- Time of periapsis passage @ 205 deg: 9204.00 s

8. A spacecraft is in a circular orbit at an altitude of 400 km at 28.5 degrees inclination. What velocity change is required to put the spacecraft into an equatorial orbit?

- dV applied at node: 3.78 km/s
- If descending at node, dV is toward the North Pole.
- If ascending at node, dV is toward the South Pole.

9. Create a sketch (hand drawn OK) in the heliocentric reference frame showing the position of Mars on it's orbit on November 2, 2017. Locate the perapsis and apoapsis points in relation to the First Line of Aries. Indicate all other orbit properties required to define the planet location on the sketch.

Problem 9



$i = 1.85°$ to ecliptic

J2000 @ 2000-01-01_T12:00

$M_{J2000} = 19.3564°$

Elapsed Julian time
$= 6515$ days

$P = 686.971$ days

2017-11-02-T12:00

$M_f = 3433.5°$ (total travelled angle)

$M = 193.5°$

$t_p = 369.2$ days

$E = 192.32°$

$f =$ ~~168~~
191.2°

$$e = \frac{r_a - r_p}{r_a + r_p} = \underline{0.0935}$$

$r_a, r_p, M_{J200}, \Omega, \omega, i$ from Wikipedia

Extra Credit

10. It's known that the launch vehicle for the spacecraft released the satellite at an altitude of 700 km with a velocity of 7.8 km/sec. 12 minutes after separation spacecraft telemetry indicated that the main engine on the spacecraft fired, imparting an instantaneous 750 meter/sec velocity change tangent to the orbit. 30 minutes later the main engine fired again, imparting an instantaneous 500 meter/sec velocity change tangent to the orbit. What are the final orbit properties? All thrusting is done tangent to the orbit.

- Final semimajor axis: 12082.486372 km
- Final eccentricity: 0.339653
- Final perigee altitude: 1600.638305 km
- Final apogee altitude: 9808.334440 km
- Final period: 13217.389617 s

Mission Design HW

```
peri_alt = 600; %km
apo_alt = 800; %km
i = 28.5*pi/180; %rad
Re = 6378; %km
mu = 3.986e5; %km3/s2
J2 = 0.00108263; %J2 of Earth

a = (peri_alt + 2*Re + apo_alt)/2; %sma
c = a - peri_alt - Re; % from center to focus
e = c/a; %eccentricity
vp = sqrt(2*mu/(peri_alt+Re) - mu/a); %km/s, velocity at perigee
% Apsidal rotation
n = sqrt(mu/(a*a*a));
dw_dt = 3*n*J2*Re*Re*(4-5*sin(i)*sin(i))/(4*a*a*(1-e*e)*(1-e*e));
% Nodal regression
dW_dt = -3*n*J2*Re*Re*cos(i)/(2*a*a*(1-e*e)*(1-e*e));

E_10_min = t_to_E(600, e, n);
f_10_min = acos((cos(E_10_min)-e)/(1-e*cos(E_10_min)));

% circularization
% One instantaneous burn
circ_alt = 700;
v_circ = sqrt(mu/(circ_alt+Re));
v_at_circ_alt = sqrt(2*mu/(circ_alt+Re) - mu/a);
delta_v_circ = v_at_circ_alt - v_circ;
% this is zero! meaning dV can't be in the velocity vec direction

f = acos((peri_alt + Re)*(1+e)/(circ_alt+Re)/e - 1/e); % true anom @ burn

fpa_burn = atan2(e*sin(f),1+e*cos(f));
delta_v_circ = 2*v_circ*tan(fpa_burn/2);
burn_direction = pi - (pi-fpa_burn)/2;


fprintf('Problem 1\n')
fprintf('Eccentricity: %f\n', e)
fprintf('Semi-major axis: %f km\n', a)
fprintf('Vp: %f km/s\n', vp)
fprintf('Apsidal rotation rate: %f deg/day\n', dw_dt*180/pi*86400)
fprintf('Nodal regression rate: %f deg/day\n', dW_dt*180/pi*86400)
fprintf('True anomaly at 10 minutes: %f deg\n', f_10_min * 180/pi)
fprintf('Circularization dV: %f km/s\n', delta_v_circ)
fprintf('Circularization dV direction: %f deg\n', burn_direction*180/pi)
fprintf('Circularization occurs at true anom: %f deg\n', f*180/pi)
fprintf('\n')
%TODO  real dv?

% Problem 2
clearvars -except Re mu J2
```

```
alt = 270; %km
geo_alt = 35786; %km, Brown p. 102
% first dV, made at the Hohmann xfer's perigee
v_orig = sqrt(mu/(alt+Re));
a_hohmann = (alt + 2*Re + geo_alt)/2;
v_hoh_peri = sqrt(2*mu/(alt+Re)-mu/a_hohmann);
dv1 = v_hoh_peri - v_orig; %km/s, in direction of motion (being positive)

% Second dV, made at the Hohmann xfer's apogee
v_hoh_apo = sqrt(2*mu/(geo_alt+Re)-mu/a_hohmann);
v_final = sqrt(mu/(geo_alt + Re));
dv2 = v_final - v_hoh_apo; %km/s, in direction of motion.

fprintf('Problem 2\n')
fprintf('dV 1: %f km/s in velocity vector direction\n', dv1)
fprintf('dV 2: %f km/s in velocity vector direction\n', dv2)
fprintf('\n')

% Problem 3
clearvars -except Re mu J2

AU = 149.597870e6; %km, Brown p. 110
Earth_R = 1*AU;
Neptune_R = 30.011 * AU; %km, Brown p. 111
Earth_V = 29.77; %km/s
mu_sun = 132712440018; %km3/s2

a_hoh = (Earth_R + Neptune_R)/2;
v_hoh_peri = sqrt(2*mu_sun/(Earth_R)-mu_sun/a_hoh);
dV = v_hoh_peri-Earth_V;
t_xfer = 2*pi/sqrt(mu_sun/(a_hoh*a_hoh*a_hoh)) /2;

fprintf('Problem 3\n')
fprintf('dV: %f km/s \n', dV)
fprintf('Transfer Time: %f years\n', t_xfer/3600/24/365)
fprintf('\n')

% Problem 4
clearvars -except Re mu J2
alt_p = 41756; %km
e = 0.0061;
geo_alt = 35786; %km, Brown p. 102
% we are shrinking the orbit, so dv's will be in direction opp. to motion
a_hoh = (alt_p+2*Re+geo_alt)/2;
a_orig = (alt_p + Re)/(1-e);
v_p_orig = sqrt(2*mu/(alt_p + Re) - mu/a_orig);
v_hoh_apo = sqrt(2*mu/(alt_p + Re) - mu/a_hoh);
dv1 = v_hoh_apo-v_p_orig;

v_hoh_peri = sqrt(2*mu/(geo_alt + Re) - mu/a_hoh);
v_final = sqrt(mu/(geo_alt + Re));
dv2 = v_final - v_hoh_peri;
% TODOtry the other way?
```

```
fprintf('Problem 4\n')
fprintf('dV 1: %f km/s (anti-velocity-vector direction)\n', dv1)
fprintf('dV 2: %f km/s (anti-velocity-vector direction)\n', dv2)
fprintf('\n')

ra = a_orig*(1+e);
a_hoh = (ra+Re+geo_alt)/2;
v_a_orig = sqrt(2*mu/ra - mu/a_orig);
v_hoh_apo = sqrt(2*mu/(ra) - mu/a_hoh);
dv1 = v_hoh_apo-v_a_orig

v_hoh_peri = sqrt(2*mu/(geo_alt + Re) - mu/a_hoh);


% Problem 5
clearvars -except Re mu J2
v_esc = sqrt(2*mu/Re); %km/s, from the surface of the earth.
fprintf('Problem 5\n')
fprintf('V_esc: %f km/s\n', v_esc)
fprintf('\n')

% Problem 6
clearvars -except Re mu J2
% (using Laplace)
M_Mars = 639e21; %kg
M_Sun = 1.989e30; %kg
R_Mars = 227939100;
Rs = R_Mars*(M_Mars/M_Sun)^(2/5);
fprintf('Problem 6\n')
fprintf('Sphere of Influence: %f km (Using Laplace eqn)\n', Rs)
fprintf('\n')

% Problem 7
clearvars -except Re mu J2
a = 12500; %km
e = 0.472;
n = sqrt(mu/a/a/a);
f = 205*pi/180; %rad
E = acos((e+cos(f))/(1+e*cos(f)));
if f > pi
    E = 2*pi - E;
end
t = (E-e*sin(E))/n;
fprintf('Problem 7\n')
fprintf('Time of periapsis passage @ 205 deg: %f s\n', t)
fprintf('\n')

% Problem 8
clearvars -except Re mu J2
alt = 400; %km
i = 28.5*pi/180; %rad
v_i = sqrt(mu/(alt+Re));
dV = 2*v_i*sin(i/2);
fprintf('Problem 8\n')
```

```
fprintf('dV applied at node: %f km/s\n', dV)
fprintf('If descending at node, dV is toward the North Pole.\n')
fprintf('If ascending at node, dV is toward the South Pole.\n')
fprintf('\n')

% Problem 10
clearvars -except Re mu J2

% Orbit 1 conditions
rp = 700 + Re; % km
vp = 7.8; % km/s
a1 = mu*rp/(2*mu-vp*vp*rp);
e1 = 1-rp/a1;
n1 = sqrt(mu/a1/a1/a1);

% burn 1 conditions, new orbit
t_12 = 12*60;
E_12 = t_to_E(t_12, e1, n1);
f_12 = E_to_f(E_12, e1);
fpa_burn1 = atan2(e1*sin(f_12),1+e1*cos(f_12));
r_burn1 = a1*(1-e1*e1)/(1+e1*cos(f_12));
v_burn1_pre = sqrt(2*mu/r_burn1 - mu/a1);
v_burn1_post = v_burn1_pre + 0.75;
a2 = mu*r_burn1/(2*mu-v_burn1_post*v_burn1_post*r_burn1);
H2 = r_burn1*v_burn1_post*cos(fpa_burn1);
e2 = sqrt(1-H2*H2/mu/a2);
n2 = sqrt(mu/a2/a2/a2);
f2_burn1_post = fpa_to_f(fpa_burn1, e2);
E2_burn1_post = f_to_E(f2_burn1_post,e2);
tpp_burn1_post = (E2_burn1_post-e2*sin(E2_burn1_post))/n2;

% burn 2 conditions, new orbit
tpp_burn2 = 30*60+tpp_burn1_post;
f2_burn2 = E_to_f(t_to_E(tpp_burn2, e2, n2), e2);
fpa_burn2 = atan2(e2*sin(f2_burn2),1+e2*cos(f2_burn2));
r_burn2 = a2*(1-e2*e2)/(1+e2*cos(f2_burn2));
v_burn2_pre = sqrt(2*mu/r_burn2 - mu/a2);
v_burn2_post = v_burn2_pre + .5;
a3 = mu*r_burn2/(2*mu-v_burn2_post*v_burn2_post*r_burn2);
H3 = r_burn2*v_burn2_post*cos(fpa_burn2);
e3 = sqrt(1-H3*H3/mu/a3);
n3 = sqrt(mu/a3/a3/a3);
P3 = 2*pi/n3;
ra3 = a3*(1+e3);
rp3 = a3*(1-e3);
fprintf('Problem 10\n')
fprintf('Final semimajor axis: %f km\n', a3)
fprintf('Final eccentricity: %f \n', e3)
fprintf('Final perigee altitude: %f km\n', rp3-Re)
fprintf('Final apogee altitude: %f km\n', ra3-Re)
fprintf('Final period: %f s\n', P3)
```

```
Problem 1
Eccentricity: 0.014128
```

```
Semi-major axis: 7078.000000 km
Vp: 7.611138 km/s
Apsidal rotation rate: 9.906275 deg/day
Nodal regression rate: -6.084576 deg/day
True anomaly at 10 minutes: 37.423888 deg
Circularization dV: 0.106029 km/s
Circularization dV direction: 90.404759 deg
Circularization occurs at true anom: 90.809518 deg

Problem 2
dV 1: 2.434361 km/s in velocity vector direction
dV 2: 1.469961 km/s in velocity vector direction

Problem 3
dV: 11.667205 km/s
Transfer Time: 30.549508 years

Problem 4
dV 1: -0.105518 km/s (anti-velocity-vector direction)
dV 2: -0.100013 km/s (anti-velocity-vector direction)


dv1 =

   -0.0964

Problem 5
V_esc: 11.179989 km/s

Problem 6
Sphere of Influence: 576189.393044 km (Using Laplace eqn)

Problem 7
Time of periapsis passage @ 205 deg: 9204.005153 s

Problem 8
dV applied at node: 3.775318 km/s
If descending at node, dV is toward the North Pole.
If ascending at node, dV is toward the South Pole.

Problem 10
Final semimajor axis: 12082.486372 km
Final eccentricity: 0.339653
Final perigee altitude: 1600.638305 km
Final apogee altitude: 9808.334440 km
Final period: 13217.389617 s
```

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```
function f = E_to_f(E,e)
f = acos((cos(E)-e)/(1-e*cos(E)));
```

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```
function E = f_to_E(f,e)
E = acos((e+cos(f))/(1+e*cos(f)));
```

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```
function f = fpa_to_f(fpa, e)

% t in seconds
% n in rad/s

tol = 1e-6;
f0 = 0;
f1 = 0;
diff = tol + 1;
while abs(diff) > tol
    f0 = f1; % from the last round
    F = tan(fpa) + e*tan(fpa)*cos(f0) - sin(f0);
    F_prime = -e*tan(fpa)*sin(f0) - cos(f0);

    f1 = f0 - F/F_prime;
    diff = f1 - f0;
end

f = f1;
```

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```
function E = t_to_E(t, e, n)

% t in seconds
% n in rad/s

tol = 1e-6;
E0 = 0;
E1 = 0;
diff = tol + 1;
while abs(diff) > tol
    E0 = E1; % from the last round
    F = (E0-e*sin(E0))/n - t;
    F_prime = (1-e*cos(E0))/n;

    E1 = E0 - F/F_prime;
    diff = E1 - E0;
end

E = E1;
% if t > pi/n
%     E = 2*pi-E;
% end
```

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