Low Thrust Interplanetary Mission Trajectory Optimization using Differential Evolution

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Interplanetary Transfers with Electrically Propelled Spacecraft

- Only gradual velocity changes are possible.
- These spacecraft do not travel along conics.
- Thrust vector is a control variable which is to be determined to optimize flight duration and/or propellant consumption.
- Thrust vector magnitude and direction are the controls for spacecraft dynamics.

Electric propulsion (EP) power-plant types considered

- Nuclear electric propulsion (NEP)
  - Constant power availability
- Solar electric propulsion (SEP)
  - Inverse square law model (first approximation to available power)
  - Williams and Coverstone-Carroll model (from experimental data[Coverstone-Carroll, 1997])

Problem description

- Spacecraft in initial heliocentric orbit.
- To be transferred to a final orbit under heliocentric gravitational dynamics.
- Optimize flight duration and/or propellant mass.
- Coasting is to be allowed and should arise naturally out of the solution.
- Indirect approach to optimal control has been chosen to solve this problem.

Equations of motion

$$\dot{x} = v_x \tag{1}$$

$$\dot{y} = v_y \tag{2}$$

$$\dot{v}_x = -\frac{\mu_s x}{r^3} + a_x \tag{3}$$

$$\dot{v}_y = -\frac{\mu_s y}{r^3} + a_y \tag{4}$$

$$\dot{m} = -\frac{m\sqrt{a_x^2 + a_y^2}}{g_0 I_{sp}} \tag{5}$$

The above equations govern both time and fuel optimal trajectories.

Cost functions

$$J_{time} = \Phi_f + \int_{t_0}^{t_f} dt \qquad \text{subject to} \qquad m\sqrt{a_x^2 + a_y^2} = T_{max} \tag{6}$$

$$J_{fuel} = \Phi_f + \int_{t_0}^{t_f} \frac{m\sqrt{a_x^2 + a_y^2}}{g_0 I_{sp}} dt \qquad \text{subject to} \qquad m\sqrt{a_x^2 + a_y^2} \leq T_{max} \tag{7}$$

Φ_f represents the error in achieving the final desired orbit.

Two Point Boundary Value Problem (TPBVP)

Variables

- States - $[x\ y\ v_x\ v_y\ m]$
- Costates - $[\lambda_x\ \lambda_y\ \lambda_{v_x}\ \lambda_{v_y}\ \lambda_m]$
- Controls - $[a_x\ a_y]$

TPBVP formulation

- System dynamics, cost functionals and costates form the Hamiltonians [Kirk (2012)].
- Pontryagin's minimum principle gives the optimal control law.
- Initial and final states are partly known.
- Problem is reduced to the determination of initial costates such that the final state is achieved with maximum accuracy.

Optimal Control Law

- Pontryagin's minimum principle to be applied.
- Results in constrained minimization problem.
- Lagrange multipliers or the Karush-Kuhn-Tucker (KKT) conditions have to be utilized to obtain the control law.

$$l = \frac{\sqrt{\lambda_{v_x}^2 + \lambda_{v_y}^2}}{m} - \frac{1-\lambda_m}{g_0 I_{sp}} \qquad k = -\frac{T_{max}/m}{\sqrt{\lambda_{v_x}^2 + \lambda_{v_y}^2}} \tag{8}$$

$$\text{Fuel optimal - } \begin{cases} \text{If } l \geq 0, & a_x = k\lambda_{v_x} \quad a_y = k\lambda_{v_y} \\ \text{If } l < 0, & a_x = 0 \quad a_y = 0 \end{cases} \tag{9}$$

$$\text{Time optimal - } a_x = k\lambda_{v_x} \quad a_y = k\lambda_{v_y} \tag{10}$$

Differential Evolution (DE)[Storn and Price, 1997]

- DE is an evolutionary algorithm.
- Search based global optimization method.
- Utilizes crossover, mutation and selection.

DE parameters

- Crossover ratio (CR) - Typical value = 0.80
- Mutation factor (F) - Typical value = 0.80
- Population size (NP) - Typical value = 5 to 10 times problem dimensionality

- The DE algorithm used requires the selection of 3 distinct members out of the population.
- The Durstenfeld version of the Fischer-Yates shuffle is utilized.

Model validation

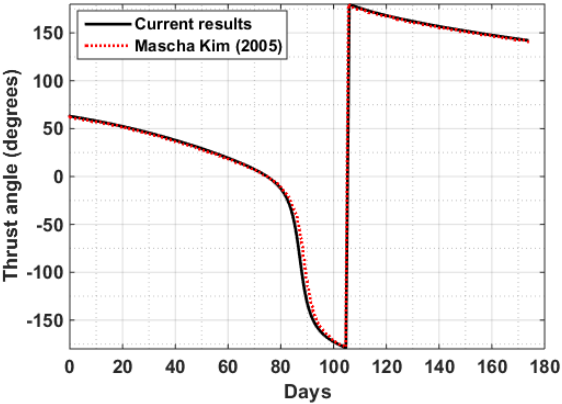

Figure: Comparison of obtained results with literature.

DE robustness with input bounds and seed

Table: Sample problem results with varying bounds, seeds and threads.

	Final Cost<1e-7	NP=250	CR=0.1	F=0.8	**All run on a 2 core machine**	Random seeds
25 dimensional Rastrigin function			1 thread	2 threads	4 threads	
Lower bounds	Upper bounds	Generations	Time(ms)	Time(ms)	Time(ms)	Max Speedup
-1	1	1000	857.626	546.38	379.27	2.261254515
-10	10	1750	1460.03	934.662	621.439	2.349434136
-100	100	1950	1613.14	1021.72	682.483	2.363633966
-1000	1000	2125	1764.25	1088.09	751.529	2.347547467
-10000	10000	2350	1954.39	1202.36	816.597	2.393334778
	Solution	(0,0, . . . ,0)				

- The above table shows the robustness of DE for the standard test problem taken. (25 dimensional Rastrigin function)
- The efficiency of multi-threading has also been simultaneously demonstrated.
- This provides confidence to apply DE to solve the TPBVP formed by indirect optimal control.

DE performance

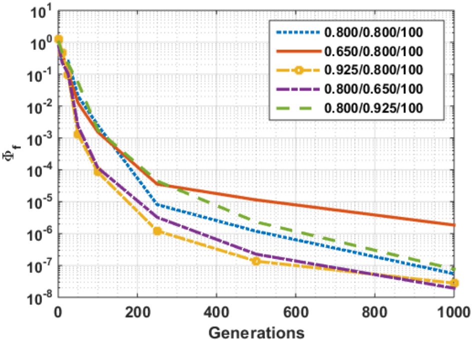

Figure: DE performance for various CR/F/NP values. (200 day Earth-Mars fuel optimal results)

It is observed that CR/F ratios greater than 1 are suitable for rapid convergence.

Time optimal results

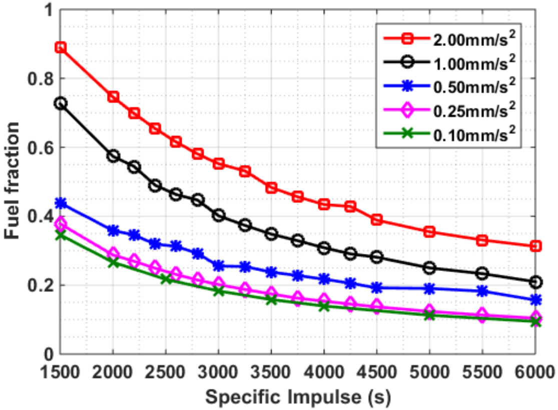

Figure: Fuel fraction required for Earth-Mars transfer with varying initial acceleration levels.

Fuel optimal results

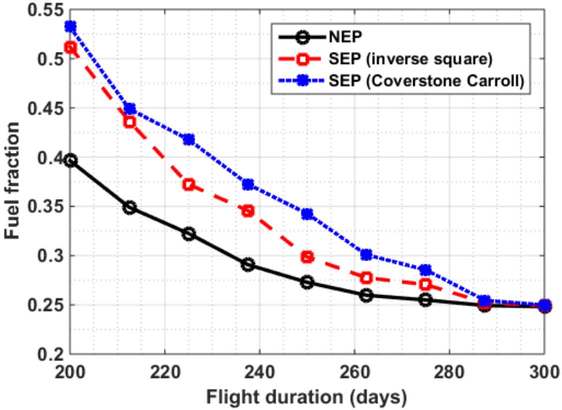

Figure: Fuel fraction required for Earth-Mars transfer with varying flight duration and electrical power models.

Sample trajectory - Ellipse to Circle - Heliocentric

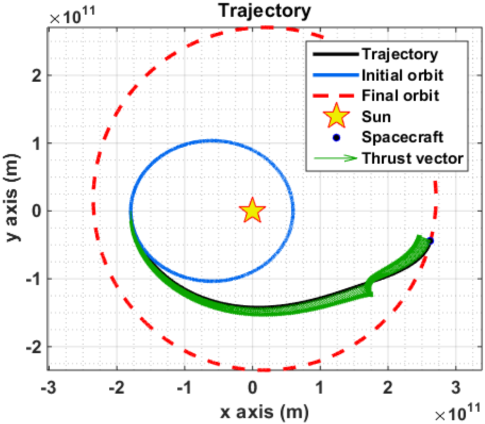

Figure: Sample trajectory for an ellipse to circle time optimal transfer which graphically depicts thrust vector steering.

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