Low Thrust Interplanetary Mission Trajectory Optimization using Differential Evolution

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Interplanetary Transfers with Electrically Propelled Spacecraft

- Electric propulsion offers a good option for interplanetary transfers.
- Spacecraft in initial heliocentric orbit.
- To rendezvous with a final heliocentric orbit under heliocentric gravitational dynamics.
- Thrust vector magnitude and direction are the control variables that are to be determined to optimize:
  - Flight duration
  - Propellant consumption
- Only gradual velocity changes are possible due to low thrust.
- Coasting is to be allowed and should arise naturally out of the solution.
- Indirect approach to optimal control has been followed to solve this problem.

Electric propulsion (EP) power-plant types considered

- Nuclear electric propulsion (NEP)
  - Constant power availability
- Solar electric propulsion (SEP)
  - Inverse square law model (first approximation to available power)
  - Williams and Coverstone-Carroll model (from experimental data)

Equations of motion

$$\dot{x} = v_x \tag{1}$$

$$\dot{y} = v_y \tag{2}$$

$$\dot{v}_x = -\frac{\mu_s x}{r^3} + a_x \tag{3}$$

$$\dot{v}_y = -\frac{\mu_s y}{r^3} + a_y \tag{4}$$

$$\dot{m} = -\frac{m\sqrt{a_x^2 + a_y^2}}{g_0 I_{sp}} \tag{5}$$

The above equations govern both time and fuel optimal trajectories.

Cost functions

$$J_{time} = \Phi_f + \int_{t_0}^{t_f} dt \qquad \text{subject to} \qquad m\sqrt{a_x^2 + a_y^2} = T_{max} \tag{6}$$

$$J_{fuel} = \Phi_f + \int_{t_0}^{t_f} \frac{m\sqrt{a_x^2 + a_y^2}}{g_0 I_{sp}} dt \qquad \text{subject to} \qquad m\sqrt{a_x^2 + a_y^2} \leq T_{max} \tag{7}$$

Φ_f represents the error in achieving the final desired orbit.

Two Point Boundary Value Problem (TPBVP) Formulation

TPBVP formulation

- Costates are introduced and the Hamiltonian is formed
- System dynamics, cost functionals and costates form the Hamiltonians.
- Pontryagin's minimum principle gives the costate dynamics and the optimal control law.
- Initial costates are unknown.
- Problem is reduced to the determination of initial costates such that the final state is achieved with maximum accuracy.

Variables

- States - $[x \ y \ v_x \ v_y \ m]$, Costates - $[\lambda_x \ \lambda_y \ \lambda_{v_x} \ \lambda_{v_y} \ \lambda_m]$
- Controls - $[a_x \ a_y]$

Optimal Control Law with Constraints

- Results in constrained minimization problem.
- Lagrange multipliers or the Karush-Kuhn-Tucker (KKT) conditions are utilized to obtain the control law.

$$l = \frac{\sqrt{\lambda_{v_x}^2 + \lambda_{v_y}^2}}{m} - \frac{1-\lambda_m}{g_0 I_{sp}} \qquad k = -\frac{T_{max}/m}{\sqrt{\lambda_{v_x}^2 + \lambda_{v_y}^2}} \tag{8}$$

$$\text{Fuel optimal - } \begin{cases} \text{If } l \geq 0, & a_x = k\lambda_{v_x} \quad a_y = k\lambda_{v_y} \\ \text{If } l < 0, & a_x = 0 \quad\quad a_y = 0 \end{cases} \tag{9}$$

$$\text{Time optimal - } a_x = k\lambda_{v_x} \quad a_y = k\lambda_{v_y} \tag{10}$$

Differential Evolution (DE)

- DE is used to determine the initial unknown costates.
- DE is an evolutionary algorithm.
- Search based global optimization method.
- Utilizes three operation: crossover, mutation and selection.

DE parameters that influence the convergence

- Crossover ratio (CR)
- Mutation factor (F)
- Population size (NP)

- The DE algorithm requires the selection of 3 distinct members from the population to generate a trial vector.
- The Durstenfeld version of the Fischer-Yates shuffle is utilized.

DE robustness and performance

Table: Test problem for optimization.

25 dimensional Rastrigin function			1 thread	2 threads	4 threads	
Lower bounds	Upper bounds	Generations	Time(ms)	Time(ms)	Time(ms)	Max Speedup
-1	1	1000	857.626	546.38	379.27	2.26
-10	10	1750	1460.03	934.662	621.439	2.35
-100	100	1950	1613.14	1021.72	682.483	2.36
-1000	1000	2125	1764.25	1088.09	751.529	2.35
-10000	10000	2350	1954.39	1202.36	816.597	2.39
	Solution	(0,0, . . . ,0)				

- Final cost$< 10^{-7}$, NP=250, CR=0.1, F=0.8, run on a 2 core machine with random seeds.
- The above table shows the robustness of DE for the standard test problem taken. (25 dimensional Rastrigin function)
- The efficiency of multi-threading has also been simultaneously demonstrated.
- This provides confidence to apply DE to solve the TPBVP formed by indirect optimal control.

Model validation

1AU to 1.5AU time optimal transfer, 6000s I_{sp} at $1mm/s^2$ initial acceleration level.

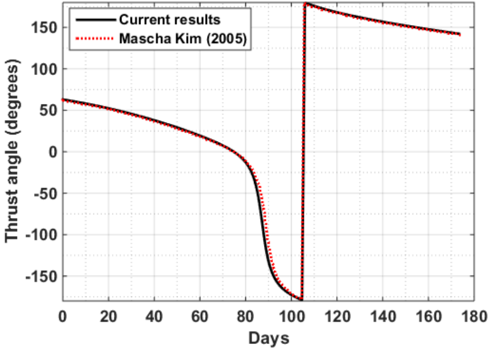
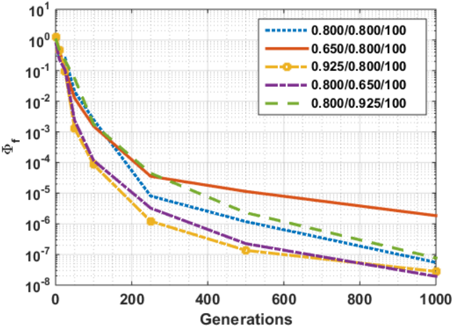


Figure: Comparison of obtained results with literature.

DE performance for Earth-Mars fuel optimal transfers

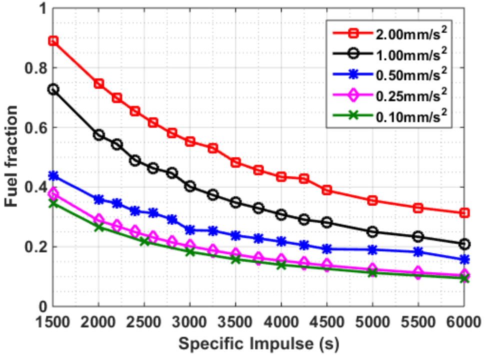
- Different CR/F/NP combinations are values. (200 days, 2000s I_{sp}), $1mm/s^2$



It is observed that CR/F ratios greater than 1 are suitable for rapid convergence. CR=0.9 and F=0.8 has been chosen for further analysis.

Time optimal transfers

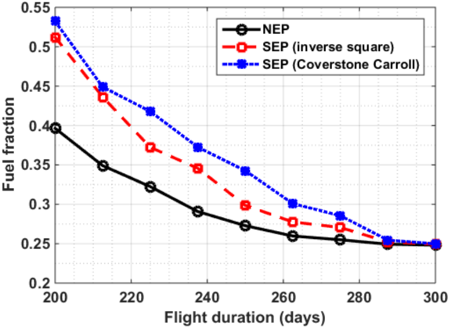
- Earth-Mars transfer with varying initial acceleration levels.



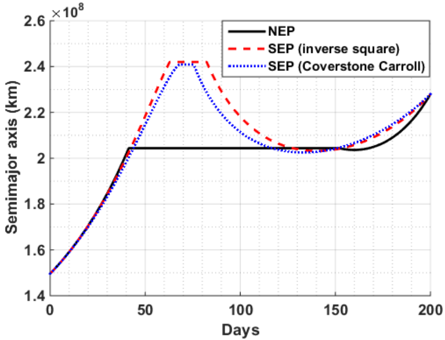
Very low acceleration levels result in spiral transfers. These transfers take similar fuel fractions.

Earth-Mars fuel optimal transfers

- NEP and SEP power models have been used.
- For small flight durations, NEP consumes much lower fuel than SEP.
- For large flight durations, all power models converge to the same fuel fraction.



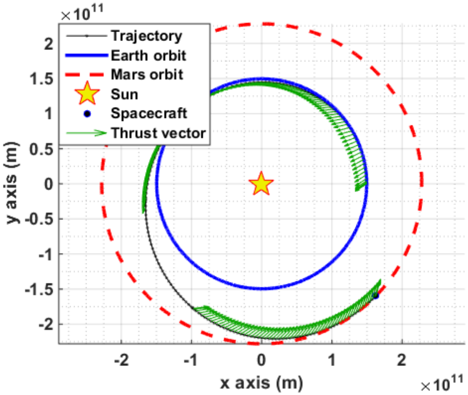
Earth-Mars fuel optimal transfers - Power models



SEP models for a 200 day transfer show that solar power models tend to waste energy to satisfy the flight time constraint. This is due to reduced thrust at larger radii from the Sun.

Sample trajectory - Earth to Mars - Heliocentric

- 400 day fuel optimal transfer, 2000s I_{sp}, initial mass $1000kg$, thrust level $236mN$. (NEXT class thruster)



Thank You