# Low Thrust Interplanetary Mission Trajectory Optimization using Differential Evolution

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# Interplanetary Transfers with Electrically Propelled Spacecraft

- Electric propulsion offers a good option for interplanetary transfers.
- Spacecraft in initial heliocentric orbit.
- To rendezvous with a final heliocentric orbit under heliocentric gravitational dynamics.
- Thrust vector magnitude and direction are the control variables that are to be determined to optimize:
  - Flight duration
  - Propellant consumption
- Only gradual velocity changes are possible due to low thrust.
- Coasting is to be allowed and should arise naturally out of the solution.
- Indirect approach to optimal control has been followed to solve this problem.

# Electric propulsion (EP) power-plant types considered

- Nuclear electric propulsion (NEP)
    - Constant power availability
- Solar electric propulsion (SEP)
    - Inverse square law model (first approximation to available power)
    - Williams and Coverstone-Carroll model (from experimental data)

# Equations of motion

$$\dot{x} = v_x \tag{1}$$

$$\dot{y} = v_y \tag{2}$$

$$\dot{v}_x = -\frac{\mu_s x}{r^3} + a_x \tag{3}$$

$$\dot{v}_y = -\frac{\mu_s y}{r^3} + a_y \tag{4}$$

$$\dot{m} = -\frac{m\sqrt{a_x^2 + a_y^2}}{g_0 I_{sp}} \tag{5}$$

The above equations govern both time and fuel optimal trajectories.

# Cost functions

$$J_{time} = \Phi_f + \int_{t_0}^{t_f} dt \qquad \text{subject to} \qquad m\sqrt{a_x^2 + a_y^2} = T_{max} \tag{6}$$

$$J_{fuel} = \Phi_f + \int_{t_0}^{t_f} \frac{m\sqrt{a_x^2 + a_y^2}}{g_0 I_{sp}} dt \qquad \text{subject to} \qquad m\sqrt{a_x^2 + a_y^2} \leq T_{max} \tag{7}$$

$\Phi_f$ represents the error in achieving the final desired orbit.

# Two Point Boundary Value Problem (TPBVP) Formulation

## TPBVP formulation

- Costates are introduced and the Hamiltonian is formed
- System dynamics, cost functionals and costates form the Hamiltonians.
- Pontryagin's minimum principle gives the costate dynamics and the optimal control law.
- Initial costates are unknown.
- Problem is reduced to the determination of initial costates such that the final state is achieved with maximum accuracy.

## Variables

- States - $[x\ y\ v_x\ v_y\ m]$, Costates - $[\lambda_x\ \lambda_y\ \lambda_{v_x}\ \lambda_{v_y}\ \lambda_m]$
- Controls - $[a_x\ a_y]$

# Optimal Control Law with Constraints

- Results in constrained minimization problem.
- Lagrange multipliers or the Karush-Kuhn-Tucker (KKT) conditions are utilized to obtain the control law.

$$l = \frac{\sqrt{\lambda_{v_x}^2 + \lambda_{v_y}^2}}{m} - \frac{1-\lambda_m}{g_0 I_{sp}} \qquad k = -\frac{T_{max}/m}{\sqrt{\lambda_{v_x}^2 + \lambda_{v_y}^2}} \tag{8}$$

$$\text{Fuel optimal - } \begin{cases} \text{If } l \geq 0, & a_x = k\lambda_{v_x} \quad a_y = k\lambda_{v_y} \\ \text{If } l < 0, & a_x = 0 \quad a_y = 0 \end{cases} \tag{9}$$

$$\text{Time optimal - } a_x = k\lambda_{v_x} \quad a_y = k\lambda_{v_y} \tag{10}$$

# Differential Evolution (DE)

- DE is used to determine the initial unknown costates.
- DE is an evolutionary algorithm.
- Search based global optimization method.
- Utilizes three operation: crossover, mutation and selection.

## DE parameters that influence the convergence

- Crossover ratio (CR)
- Mutation factor (F)
- Population size (NP)

- The DE algorithm requires the selection of 3 distinct members from the population to generate a trial vector.
- The Durstenfeld version of the Fischer-Yates shuffle is utilized.

# DE robustness and performance

Table: Test problem for optimization.

| 25 dimensional Rastrigin function | | | 1 thread | 2 threads | 4 threads | |
|---|---|---|---|---|---|---|
| Lower bounds | Upper bounds | Generations | Time(ms) | Time(ms) | Time(ms) | Max Speedup |
| -1 | 1 | 1000 | 857.626 | 546.38 | 379.27 | 2.26 |
| -10 | 10 | 1750 | 1460.03 | 934.662 | 621.439 | 2.35 |
| -100 | 100 | 1950 | 1613.14 | 1021.72 | 682.483 | 2.36 |
| -1000 | 1000 | 2125 | 1764.25 | 1088.09 | 751.529 | 2.35 |
| -10000 | 10000 | 2350 | 1954.39 | 1202.36 | 816.597 | 2.39 |
| | Solution | (0,0, . . . ,0) | | | | |

- Final cost $< 10^{-7}$, NP=250, CR=0.1, F=0.8, run on a 2 core machine with random seeds.
- The above table shows the robustness of DE for the standard test problem taken. (25 dimensional Rastrigin function)
- The efficiency of multi-threading has also been simultaneously demonstrated.
- This provides confidence to apply DE to solve the TPBVP formed by indirect optimal control.

# Model validation

1AU to 1.5AU time optimal transfer, 6000s $I_{sp}$ at $1mm/s^2$ initial acceleration level.

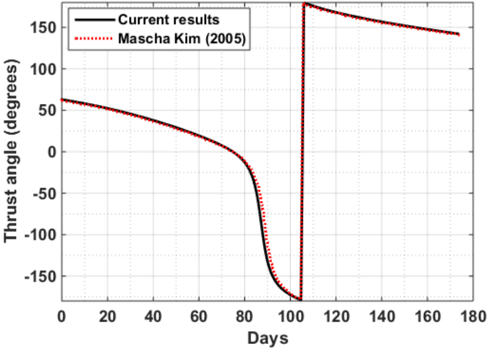
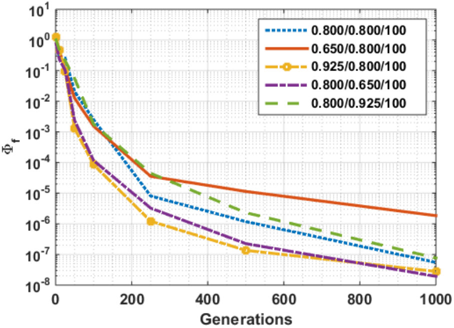



Figure: Comparison of obtained results with literature.

# DE performance for Earth-Mars fuel optimal transfers

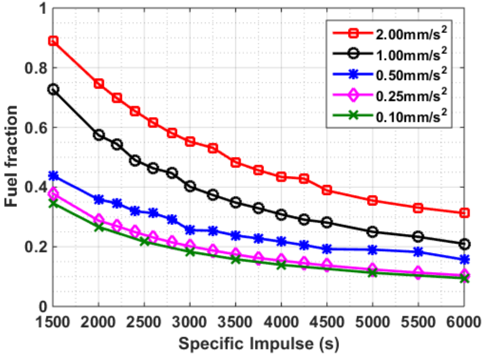
- Different CR/F/NP combinations are values. (200 days, 2000s $I_{sp}$), $1mm/s^2$




It is observed that CR/F ratios greater than 1 are suitable for rapid convergence. CR=0.9 and F=0.8 has been chosen for further analysis.

# Time optimal transfers

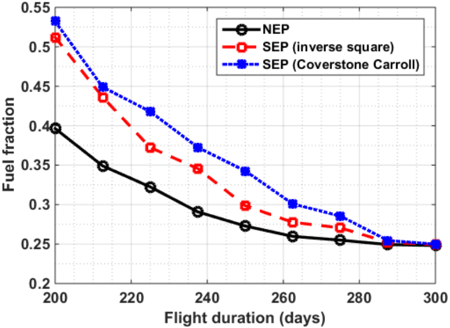
- Earth-Mars transfer with varying initial acceleration levels.




Low acceleration levels results in spiral transfers which lead to similar fuel fractions.

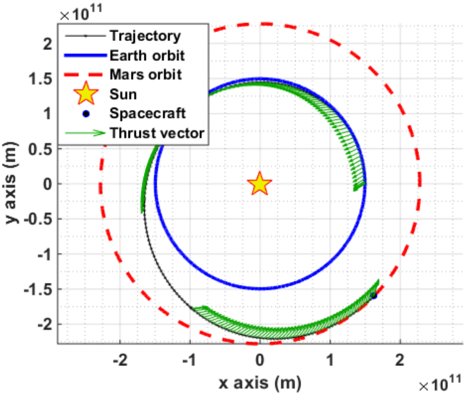
# Earth-Mars fuel optimal transfers

- NEP and SEP power models have been used.
- For small flight durations, NEP consumes much lower fuel than SEP.
- For large flight durations, all power models converge to the same fuel fraction.



# Sample trajectory - Earth to Mars - Heliocentric

- 400 day fuel optimal transfer, 2000s $I_{sp}$, initial mass $1000kg$, thrust level $236mN$. (NEXT class thruster)



# Thank You