Ejercicios de 19 a el 36

19 $\int_1^{\infty} \frac{1}{x^3}\,dx = \lim_{b \to \infty} \int_a^b \frac{1}{x^3}\,dx$

$= \lim_{b \to \infty} \left(-\frac{1}{2b^2}\right) - \left(-\frac{1}{2(1)^2}\right) = \lim_{b \to \infty} \left(-\frac{1}{2b^2}\right) + \frac{1}{2} = \frac{1}{2}$

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20) $\int_1^{\infty} \frac{3}{x^5}\, dx$

$\lim_{b \to \infty} \int_a^b \left(\frac{3}{x^5}\right) dx = \lim_{b \to \infty} \left(-\frac{3}{4x^4}\right) \Big]_1^b$

$\lim_{b \to \infty} \left(\frac{-3}{4b^4}\right) - \left(-\frac{3}{4(1)^2}\right) = \lim_{b \to \infty} \left(\frac{-3}{4b}\right) + \frac{3}{4} = \frac{3}{4}$

21) $\int_1^{\infty} \frac{3}{\sqrt[3]{x}}\, dx = \lim_{b \to \infty} \int_a^b \frac{3}{\sqrt[3]{x}}\, dx$

$\lim_{b \to \infty} \left(\frac{9\sqrt[3]{b^2}}{2}\right) - \left(\frac{9\sqrt[3]{1^2}}{2}\right) = \lim_{b \to \infty} \left(\frac{9\sqrt[3]{b^2}}{2}\right) - \left(\frac{9}{2}\right) = \frac{9}{2}$

22 $\int_1^{\infty} \frac{4}{\sqrt[4]{x}}\, dx$

$\lim_{b \to \infty} \left(\int_1^b \frac{4}{\sqrt[4]{x}}\, dx\right) = \int \frac{4}{\sqrt[4]{x}}\, dx = 4x \int \frac{1}{\sqrt[4]{x}}\, dx$

$= 4 \int \frac{1}{4^{1/4}}\, dx = 4x\, \frac{4\sqrt[4]{x^3}}{3} = \frac{16\sqrt[4]{x^3}}{3}$

$= \frac{16\sqrt[4]{x^3}}{3} \Big|_1^b = \frac{16\sqrt[4]{b^3} - 16}{3} = \lim_{b = \infty} \left(\frac{16\sqrt[4]{b^3} - 16}{3}\right)$

$= \infty$

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23) $\int_0^{\infty} xe^{-4x}\,dx$

$= \frac{1}{16}(-4e^{-4x}x - e^{-4x}) + e$

$= +\infty$

24) $\int_0^{\infty} xe^{\frac{-x}{4}}\,dx$

$= -4e^{-\frac{x}{4}} - 16e^{\frac{x}{4}} + e$

$= 0 - (16)$

$= -16$

25 $\int_0^{\infty} x^{2} e^{-x}\,dx$

$= x^2 \cdot (-e^{-x}) - \int -e^{-x} \cdot 2x\,dx \;= x^2 \cdot (e^{-x}) - 1x(-2)x \int e^{-x} x\,dx$

$= x^2 \cdot (-e^{-x}) + 2(x \cdot (-e^{-x})) + \int e^{-x}\,dx = -x^2 \cdot e^{-x} - 2xe^{-x} - 2e^{-x}$

$= (-x^2 e^{-x} - 2xe^{-x} - 2e^{-x})\Big|_0^b = -b^2 e^{-b} - 2bc^{-b} - 2e^{-b} + 2$

$= \lim_{b \to \infty} (-b^2 \cdot e^{-b} - 2bc^{-b} - 2e^{-b} + 2)$

$= 2$

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26) $\int_0^\infty (x-1)e^{-x}\,dx$

$\int (x-1)e^{x}\,dx = \int (x-1) \cdot \frac{1}{e^x}\,dx = \int \frac{x-1}{e^x}\,dx$

$= \int \frac{x}{e^x}\,dx - \int \frac{1}{e^x}\,dx = \frac{-x+1}{e^x} + \frac{1}{e^x} = \frac{-x}{e^x}\Big|_0^b$

$= -\frac{b}{e^b} - \left(-\frac{0}{e^0}\right) = \lim_{b\to\infty}\left(\frac{b}{e^b}\right) = 0$

27) $\int_0^\infty e^{-x}\cos x\,dx$

$= \frac{e^{-x}\sin(x)}{2} - \frac{e^{-x}\cos(x)}{2} + C$

$= 0 - (-1/2)$

$= 1/2$

28) $\int_0^\infty e^{-ax}\operatorname{sen} bx\,dx,\quad a>0$

$= \frac{1}{a^2}\sin(b)\left(-b e^{-ax}x - e^{-ax}\right) + C$

$= 0 - \left(-\frac{1}{a^2}\sin(b)\right)$

$= \frac{\sin(b)}{a^2}$

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29) $\int_4^{\infty} \frac{1}{x(\ln x)^3} dx$

$= \int_4^{\infty} \frac{1}{x \cdot \ln(x)^3} dx = \int \frac{1}{x^3} dx = -\frac{1}{2x^2}$

$= \int -\frac{1}{2\ln(x)^2} \Big|_4^a = -\frac{1}{2\ln(a)^2} - \left(-\frac{1}{2\ln(4)^2}\right)$

$= \lim_{a \to \infty} \left(\frac{1}{2\ln(a)^2}\right) + \frac{1}{8\ln(2)^2} = 0 + \frac{1}{8\ln(2)^2}$

$= \frac{1}{8\ln(2)^2}$

30) $\int_1^{\infty} \frac{\ln x}{x} dx$

$= \int t\, dt = \frac{t^2}{2} = \frac{\ln(x)^2}{2} = \frac{\ln(x)^2}{2}\Big|_1^a$

$= \frac{\ln(a)^2}{2} - \frac{\ln(1)^2}{2} = \frac{\ln(a)^2}{2}$

$\lim_{a \to \infty} \left(\frac{\ln(a)^2}{2}\right) = +\infty \quad \lim_{a \to \infty} (2)$

$= \infty$

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35) $\int_0^{\infty} \cos \pi x \, dx$

$= \int \cos (\pi x) dx$

$= \frac{1}{\pi} \sin (\pi x) + C$

$= \lim_{a \to \infty} \left(\frac{1}{\pi} \sin (\pi x) \right)$

$= +\infty$

36) $\int_0^{\infty} \sin \frac{x}{2} \, dx$

$= \int \sin \left(\frac{x}{2}\right) dx$

$= -2 \cos \left(\frac{x}{2}\right) + C$

$= \lim_{a \to \infty} \left(-2 \cos \left(\frac{x}{2}\right) \right)$

$= +\infty$