Calculo integral

en los ejercicios 19 a 36, determinar si la integral impropia es divergente o convergente. evaluar la integral si es convergente.

19) $\int_1^{\infty} \frac{1}{x^3}\,dx$

integral indefinida $= \int \frac{1}{x^3}\,dx = -\frac{1}{2x^2} + C$

limite $= \int_1^{\infty} \frac{1}{x^3}\,dx = 0 - \left(-\frac{1}{2}\right)$

$= 0 - \left(\frac{1}{2}\right)$

$= \boxed{\frac{1}{2}}$

20) $\int_1^{\infty} \frac{3}{x^5}\,dx$

$\int \frac{3}{x^5}\,dx \qquad = \int \frac{3}{x^5}\,dx \qquad = 3x \int \frac{1}{x^5}\,dx \qquad = 3x\left(-\frac{1}{4x^4}\right.$

$= -\frac{3}{4x^4} \qquad = \frac{3}{4x^4}\Big|_1^a \qquad = \left(\frac{3}{4a^4}\right) + \frac{3}{4} \qquad = \boxed{\frac{3}{4}}$

21) $\int_1^{\infty} \frac{3}{\sqrt[3]{x}}\,dx$ = es Divergente

$= 3x\,\frac{3\sqrt[3]{x^2}}{2} \qquad = \frac{9\sqrt[3]{x^2}}{2} \qquad = \frac{9\sqrt[3]{x^2}}{2}\Big|_1^a$

$= \frac{9\sqrt[3]{a^2}}{2} - \frac{9\sqrt[3]{1^2}}{2} \qquad = \frac{9\sqrt[3]{a^2} - 9}{2} \qquad = \lim_{a\to\infty}\left(\frac{9\sqrt[3]{a^2} - 9}{2}\right)$

$= \lim_{a\to+\infty} (2) + \infty \qquad \boxed{= +\infty}$

22) $\int_1^{\infty} \frac{4}{\sqrt[4]{x}}\, dx$ = es Divergente

$\lim\limits_{a \to +\infty} \left(\int_1^a \frac{4}{\sqrt[4]{x}}\, dx \right) = \int \frac{4}{\sqrt[4]{x}}\, dx = 4 \times \int \frac{1}{\sqrt[4]{x}}\, dx$

$4 \times \int \frac{1}{x^{\frac{1}{4}}}\, dx = 4 \times \frac{4\sqrt[4]{x^3}}{3} = \frac{16\sqrt[4]{x^3}}{3}$

$= \frac{16\sqrt[4]{x^3}}{3} \Big|_1^a = \frac{16\sqrt[4]{a^3} - 16}{3} = \lim\limits_{a \to \infty} \left(\frac{16\sqrt[4]{a^3} - 16}{3} \right)$

$= +\infty$

23) $\int_{-\infty}^{0} x e^{-4x}\, dx$ = es Divergente

$= \frac{1}{16} \left(-4e^{-4x} x - e^{-4x} \right) + c$

$= +\infty$

24) $\int_0^{\infty} x e^{-\frac{x}{4}}\, dx$

$= -4e^{-\frac{x}{4}} - 16e^{-\frac{x}{4}} + C$

$= 0 - (16)$

$= 16$

25) $\int_0^{\infty} x^2 e^{-x} dx$

$= x^2 \cdot (-e^{-x}) - \int -e^{-x} \times 2x\, dx$ $\quad = x^2 \times (-e^{-x}) - 1 \times (-2) \times \int e^{-x} \times dx$

$= x^2 \times (-e^{-x}) + 2(x \cdot (-e^{-x}) + \int e^{-x} dx$ $\quad = -x^2 e^{-x} - 2xe^{-x} - 2e^{-x}$

$= (-x^2 e^{-x} - 2xe^{-x} - 2e^{-x}) \Big|_0^a$ $\quad = -a^2 e^{-a} - 2ae^{-a} - 2e^{-a} + 2$

$= \lim_{a \to \infty} (-a^2 e^{-a} - 2ae^{-a} - 2e^{-a} + 2)$

$= 2$

26) $\int_0^{\infty} (x-1) e^{-x} dx$

$\int (x-1) e^{-x} dx$ $\quad = \int (x-1) \times \frac{1}{e^x} dx$ $\quad = \int \frac{x-1}{e^x} dx$

$= \int \frac{x}{e^x} dx - \int \frac{1}{e^x} dx$ $\quad = -\frac{x+1}{e^x} + \frac{1}{e^x}$ $\quad = -\frac{x}{e^x} \Big|_0^a$

$= -\frac{a}{e^a} \cdot \left(-\frac{0}{e^0}\right)$ $\quad = -\lim_{a \to +\infty} \left(\frac{a}{e^a}\right)$

$= 0$

27) $\int_0^{\infty} e^{-x} \cos x\, dx$

$= \frac{e^{-x} \sin(x)}{2} - \frac{e^{-x} \cos(x)}{2} + c$

$= 0 - \left(-\frac{1}{2}\right)$

$= \frac{1}{2}$

28) $\int_0^{\infty} e^{-ax} \operatorname{sen} bx \, dx, \; a>0$

$= \frac{1}{a^2} \sin(b)(-ae^{-ax}x - e^{-ax}) + c$

$= 0 - \left(-\frac{1}{a^2} \sin(b)\right)$

$= 0 \cdot \left(-\frac{1}{a^2} \sin(b)\right)$

$= \frac{\sin(b)}{a^2}$

29) $\int_4^{\infty} \frac{1}{x(\ln x)^3} dx$

$= \int \frac{1}{x \cdot \ln(x)^3} dx \qquad = \int \frac{1}{t^3} dt \qquad = -\frac{1}{2t^2}$

$= -\frac{1}{2\ln(x)^2} \Big|_4^a \qquad = -\frac{1}{2\ln(a)^2} - \left(-\frac{1}{2\ln(4)^2}\right)$

$= \lim_{a \to \infty} \left(-\frac{1}{2\ln(a)^2}\right) + \lim_{a \to +\infty} \left(\frac{1}{8\ln(2)^2}\right)$

$= \lim_{a \to \infty} \left(\frac{1}{2\ln(a)^2}\right) + \frac{1}{8\ln(2)^2} \qquad = -0 + \frac{1}{8\ln(2)^2}$

$= \frac{1}{8\ln(2)^2}$

30) $\int_1^{\infty} \frac{\ln x}{x} dx$ → es Divergente

$= \int \frac{\ln(x)}{x} dx \quad = \int t\, dt \quad = \frac{t^2}{2} \quad = \frac{\ln(x)^2}{2}$

$= \frac{\ln(x)^2}{2}\Big|_1^a \quad = \frac{\ln(a)^2}{2} - \frac{\ln(1)^2}{2} \quad = \frac{\ln(a)^2}{2}$

$\lim_{a \to +\infty} \left(\frac{\ln(a)^2}{2}\right) \quad = +\infty \quad \lim_{a \to \infty} (2)$

$= +\infty$

31) $\int_{-\infty}^{\infty} \frac{4}{16+x^2} dx$

$= \lim_{a \to -\infty} \left(\int_a^0 \frac{4}{16+x^2} dx\right) + \int_0^{+\infty} \frac{4}{16+x^2} dx$

$= \int \frac{4}{16+x^2} dx \quad = 4 \times \int \frac{1}{16+x^2} dx \quad = 4 \times \frac{1}{4} \times \arctan\left(\frac{x}{4}\right)$

$= \arctan\left(\frac{x}{4}\right)\Big|_a^0 \quad = \arctan\left(\frac{0}{4}\right) - \arctan\left(\frac{a}{4}\right)$

$= \lim_{a \to -\infty} \left(-\arctan\left(\frac{a}{4}\right)\right) + \lim_{a \to +\infty} \left(\arctan\left(\frac{a}{4}\right)\right)$

$= \lim_{a \to -\infty} \left(\arctan\left(\frac{a}{4}\right)\right) \quad = -\arctan\left(\lim_{a \to -\infty}\left(\frac{a}{4}\right)\right)$

$= -\arctan(-\infty)$

$= -\frac{-\pi}{2} \quad = \frac{\pi}{2}$

$= \frac{\pi}{2} + \frac{\pi}{2} \quad = \pi$

$= \pi$

32) $\int_0^{\infty} \frac{x^3}{(x^2+1)^2}\,dx$ $=$ es Divergente

$= \lim_{a \to +\infty}\left(\int_0^{a} \frac{x^3}{(x^2+1)^2}\,dx\right)$ $= \int \frac{x^3}{(x^2+1)^2}\,dx$

$= \int \frac{t-1}{2t^2}\,dt$ $= \frac{1}{2} \times \int \frac{t-1}{t^2}\,dt$

$= \frac{1}{2} \times \int \frac{t}{t^2} - \frac{1}{t^2}\,dt$ $= \frac{1}{2} \times \left(\int \frac{1}{t}\,dt - \int \frac{1}{t^2}\,dt\right)$

$= \frac{1}{2} \times \ln(x^2+1) + \frac{1}{2x^2+2}$ $= \left(\frac{1}{2} \times \ln(x^2+1) + \frac{1}{2x^2+2}\right)\Big|_0^a$

$= \frac{1}{2} \times \ln(a^2+1) + \frac{1}{2a^2+2} - \frac{1}{2}$

$= \lim_{a \to \infty}\left(\frac{1}{2} \times \ln(a^2+1) + \frac{1}{2a^2+2} - \frac{1}{2}\right)$

$= +\infty$

33) $\int_0^{\infty} \frac{1}{e^x + e^{-x}}\,dx$

$= \int \frac{1}{e^x + e^{-x}}\,dx$ $= \int \frac{1}{\frac{e^{2x}+1}{e^x}}\,dx$

$= \int \frac{1}{t^2+1}\,dt$ $= \frac{1}{1} \times \arctan\left(\frac{t}{1}\right)$

$= \frac{1}{1} \times \arctan\left(\frac{e^x}{1}\right)$ $= \arctan(e^x)\Big|_0^a$

$= \arctan(e^a) - \frac{\pi}{4}$ $= \lim_{a \to \infty}\left(\arctan(e^a)\right) - \lim_{a \to +\infty}\left(\frac{\pi}{4}\right)$

$= \arctan(+\infty) - \frac{\pi}{4}$ $= \frac{\pi}{2} - \frac{\pi}{4}$

$= \frac{\pi}{4}$

34) $\int_0^{\infty} \frac{e^x}{1+e^x} dx$ = es Divergente

$= \int \frac{e^x}{1+e^x} dx$ $= \int \frac{1}{t} dt$ $\ln = (1+1)$

$= \ln(|1+e^x|)$ $= \ln(1+e^x)$ $= \ln(1+e^x)\Big|_0^a$

$= \ln(1+e^a) - \ln(1+e^0)$ $= \ln(1+e^a) - \ln(2)$

$= \lim\limits_{a \to \infty} \left(\ln(1+e^a) - \ln(2)\right)$ $= \lim\limits_{a \to \infty} \left(\ln(2)\right)$

$= +\infty \ \ln(2)$

35) $\int_0^{\infty} \cos \pi x \, dx$ = es Divergente

$= \int \cos(\pi x) \, dx$

$= \frac{1}{\pi} \sin(\pi x) + c$

$= \lim\limits_{a \to \infty} \left(\frac{1}{\pi} \sin(\pi x)\right)$

$= +\infty$

36) $\int_0^{\infty} \text{sen} \frac{x}{2} \, dx$ $=$ es Divergente

$= \int \sin\left(\frac{x}{2}\right) dx$

$= -2\cos\left(\frac{x}{2}\right) + C$

$= \lim_{a \to \infty} \left(-2\cos\left(\frac{x}{2}\right)\right)$

$= +\infty$