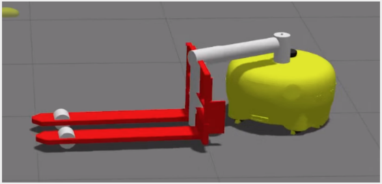
Autonomous Mobile Robotics

Motion Planning and Control

Geesara Kulathunga



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Control of Mobile Robots

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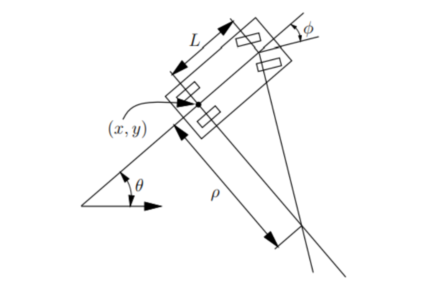


- If the speed v and steering angle ϕ are directly specified by the control inputs u_s and u_ϕ, respectively, transition equation for simple car is

$$\begin{aligned} \dot{x} &= u_s \cdot cos(\theta) \\ \dot{y} &= u_s \cdot sin(\theta) \\ \dot{\theta} &= \frac{u_s}{L} \cdot tan(u_\phi) \end{aligned} \tag{1}$$

- What steering angles are possible? $[-\pi/2, \pi/2]$. It was assumed that the car moves in the direction that the rear wheels are pointing. When $\phi = \pi/2$, the front wheel perpendicular to the rear wheels, then car has to rotate in place. In other words, $\dot{x} = \dot{y} = 0$ because because the center of the rear axle does not translate. This behaviour is usually not possible because the front wheels would collide with the front axle when turned to $\phi = \pi/2$. Thus,

$$|\phi| \leq \phi_{max}$$

- A simple car is moving slowly to safely neglect dynamics, let's assume $|u_s| \leq 1$, where $u_s \in \{-1, 0, 1\}$. However, $0 < u_s < 1$, in this case, car can not drive in reverse

Several interesting variations are possible

- **Tricycle:** $U = [-1, -1] \times [-\pi/2, \pi/2]$, Assuming front-wheel drive, the vehicle can rotate in place if $u_\theta = \pi/2$. This kind of motion can be obtained using unicycle model
- **Simple car:** $U = [-1, -1] \times [-\phi_{max}, \phi_{max}]$, by requiring that $|u_\phi| < \phi_{max} < \pi/2$, a car with minimum turning radius $\rho_{min} = \frac{L}{tan\phi_{max}}$ is obtained
- **Reeds-Sheepp Car:** Further restrict the speed of the car, i.e., $u_s = \{-1, 0, 1\}$ or a car with three gears: reverse, park, and forward.
- **Dubins Car:** After removing reverse speed $u_s = -1$ from a Reeds-Sheepp car, $u_s = \{0, 1\}$ as the only possible speeds

- If you are given start and target poses (s and e), how can you estimate the shortest-path problem with respect to Reeds-Sheepp Car or Dubins Car model?

A bounded-curvature shortest-path problem

- If you are given start and target poses (s and e), how can you estimate the shortest-path problem with respect to Reeds-Sheepp Car or Dubins Car model?
- **Objective**: to minimize the length of the curve as the car travels between s and e

A bounded-curvature shortest-path problem

- If you are given start and target poses (s and e), how can you estimate the shortest-path problem with respect to Reeds-Sheepp Car or Dubins Car model?
- **Objective**: to minimize the length of the curve as the car travels between s and e
- However, ρ_{min}, curvature has to be bounded

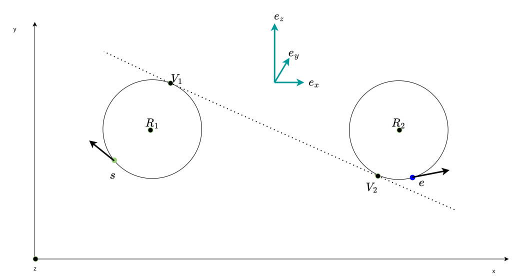
- C: circular arc with minimum turning radius
- $C_{>\pi}$: circular arc with minimum turning radius , angle $> \pi$
- S : straight-line segments

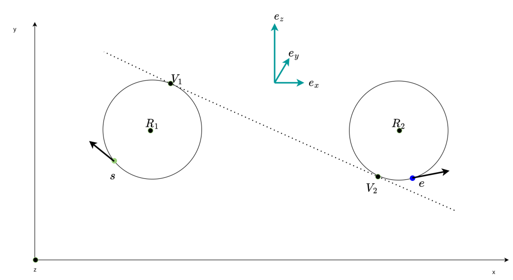
Dubins Path

Shortest paths are either CSC or $CC_{>\pi}C$

- C: R right turn, L left turn
- Dubins curves = {LRL, RLR, LSL, LSR, RSL, RSR}
- After specifying the duration of each primitive, Dubins curves = $\{L_\alpha R_\beta L_\gamma, R_\alpha L_\beta R_\gamma, L_\alpha S_d L_\gamma, L_\alpha SdR_\gamma, R_\alpha S_d L_\gamma, R_\alpha S_d R_\gamma\}$, where $\alpha \in [0, 2\pi), \beta \in (\pi, 2\pi)$, and $d \geq 0$.
- if $\beta < \pi$, there must be another path that is optimal

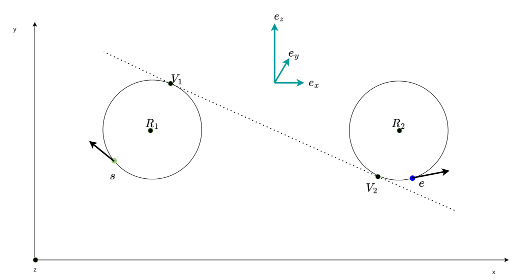
Dubins Path Planning (RSL)





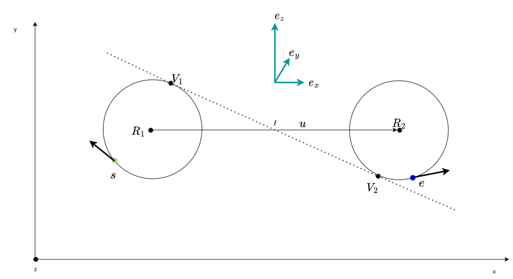
- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$

Dubins Path Planning (RSL)



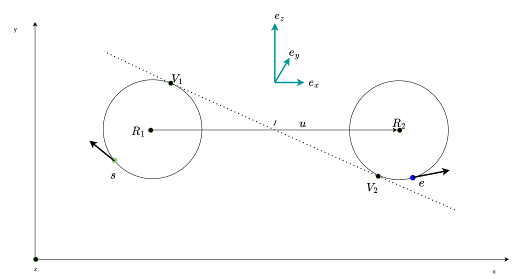
- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$
- $\mathbf{R}_2 = \mathbf{e} - r \cdot (\mathbf{e}_e \times \mathbf{e}_z)$

Dubins Path Planning (RSL)



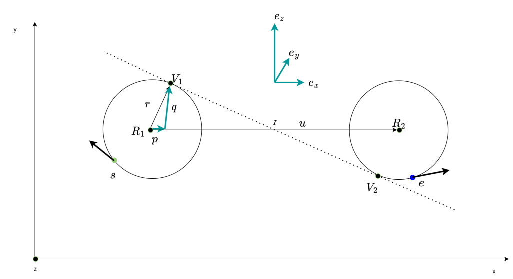
- $\mathbf{u} = \mathbf{R}_2 - \mathbf{R}_1$

Dubins Path Planning (RSL)



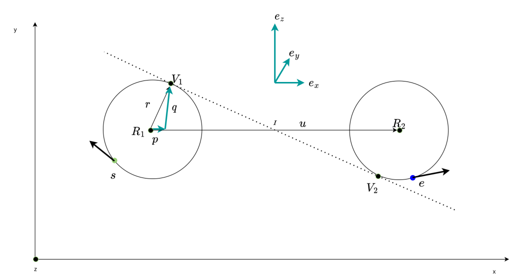
- $\mathbf{u} = \mathbf{R}_2 - \mathbf{R}_1$
- $\mathbf{I} = \mathbf{R}_1 + \frac{\mathbf{u}}{2}$

Dubins Path Planning (RSL)



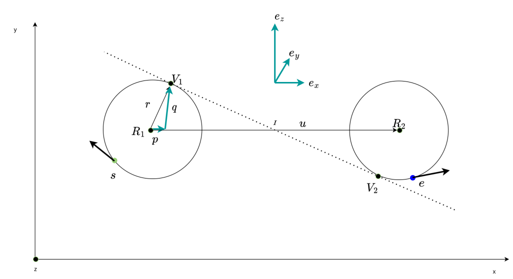
- $\alpha = arccos(\frac{2 \cdot r}{|\mathbf{u}|})$

Dubins Path Planning (RSL)

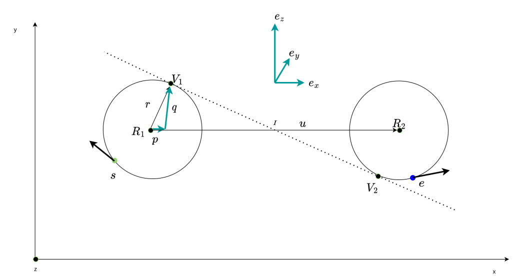


- $\alpha = arccos(\frac{2 \cdot r}{|\mathbf{u}|})$
- $\mathbf{p} = r \cdot cos(\alpha) \cdot \mathbf{e}_u$

Dubins Path Planning (RSL)

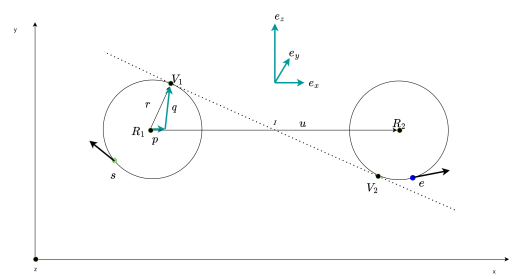


- $\alpha = arccos(\frac{2 \cdot r}{|\mathbf{u}|})$
- $\mathbf{p} = r \cdot cos(\alpha) \cdot \mathbf{e}_u$
- $\mathbf{q} = r \cdot sin(\alpha) \cdot (\mathbf{e}_z \times \mathbf{e}_u)$



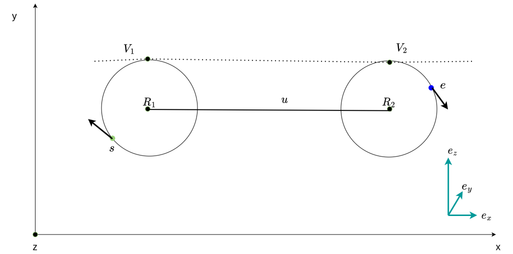
- $\alpha = arccos(\frac{2 \cdot r}{|\mathbf{u}|})$
- $\mathbf{p} = r \cdot cos(\alpha) \cdot \mathbf{e}_u$
- $\mathbf{q} = r \cdot sin(\alpha) \cdot (\mathbf{e}_z \times \mathbf{e}_u)$
- $\mathbf{V}_1 = \mathbf{R}_1 + \mathbf{p} + \mathbf{q}$

Dubins Path Planning (RSL)



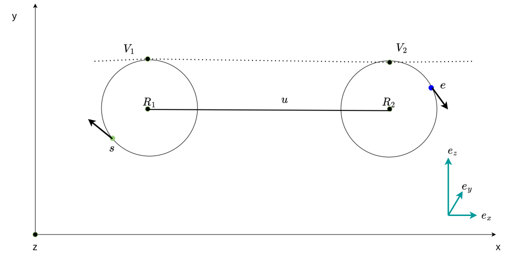
- $\alpha = arccos(\frac{2 \cdot r}{|\mathbf{u}|})$
- $\mathbf{p} = r \cdot cos(\alpha) \cdot \mathbf{e}_u$
- $\mathbf{q} = r \cdot sin(\alpha) \cdot (\mathbf{e}_z \times \mathbf{e}_u)$
- $\mathbf{V}_1 = \mathbf{R}_1 + \mathbf{p} + \mathbf{q}$
- $\mathbf{V}_2 = \mathbf{V}_1 + 2 \cdot (\mathbf{I} - \mathbf{V}_1)$

Dubins Path Planning (RSR)



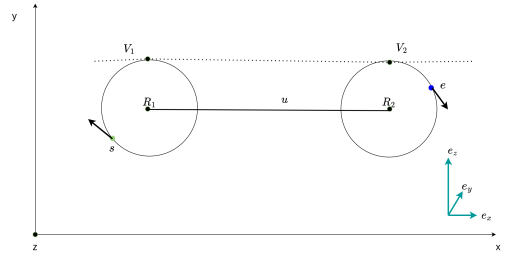
- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$

Dubins Path Planning (RSR)



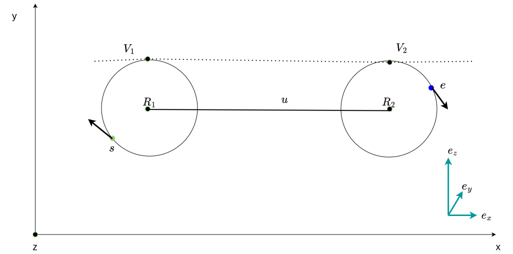
- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$
- $\mathbf{R}_2 = \mathbf{e} + r \cdot (\mathbf{e}_e \times \mathbf{e}_z)$

Dubins Path Planning (RSR)

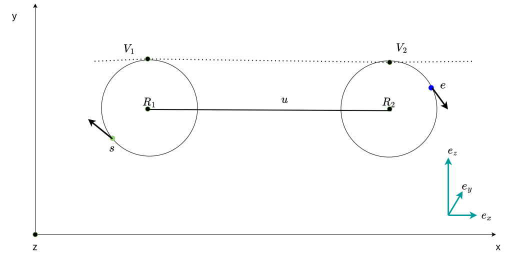


- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$
- $\mathbf{R}_2 = \mathbf{e} + r \cdot (\mathbf{e}_e \times \mathbf{e}_z)$
- $\mathbf{u} = \mathbf{R}_2 - \mathbf{R}_1$

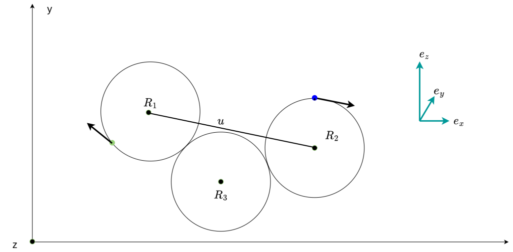
Dubins Path Planning (RSR)



- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$
- $\mathbf{R}_2 = \mathbf{e} + r \cdot (\mathbf{e}_e \times \mathbf{e}_z)$
- $\mathbf{u} = \mathbf{R}_2 - \mathbf{R}_1$
- $\mathbf{V}_1 = \mathbf{R}_1 + r \cdot (\mathbf{e}_z \times \mathbf{e}_u)$

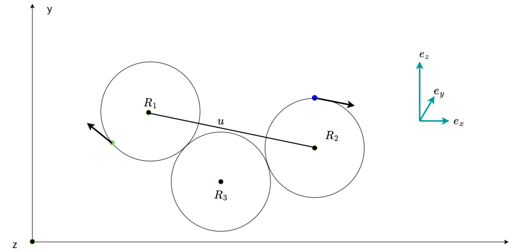


- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$
- $\mathbf{R}_2 = \mathbf{e} + r \cdot (\mathbf{e}_e \times \mathbf{e}_z)$
- $\mathbf{u} = \mathbf{R}_2 - \mathbf{R}_1$
- $\mathbf{V}_1 = \mathbf{R}_1 + r \cdot (\mathbf{e}_z \times \mathbf{e}_u)$
- $\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{u}$



- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$

Dubins Path Planning (RLR)



- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$
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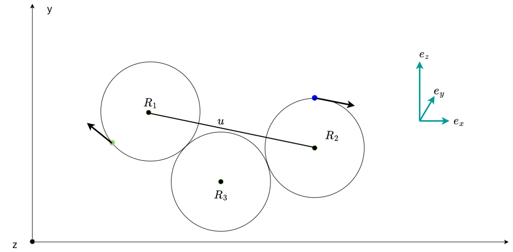
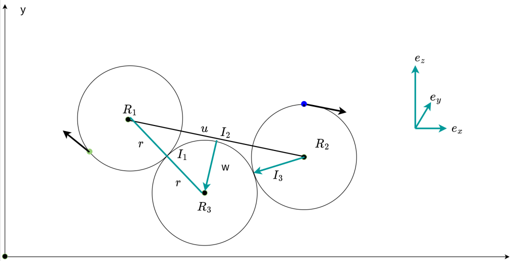
Dubins Path Planning (RLR)

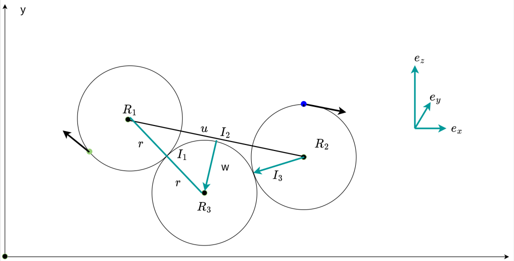


- $\mathbf{R}_1 = \mathbf{s} + r \cdot (\mathbf{e}_s \times \mathbf{e}_z)$
- $\mathbf{R}_2 = \mathbf{e} + r \cdot (\mathbf{e}_e \times \mathbf{e}_z)$
- $\mathbf{u} = \mathbf{R}_2 - \mathbf{R}_1$

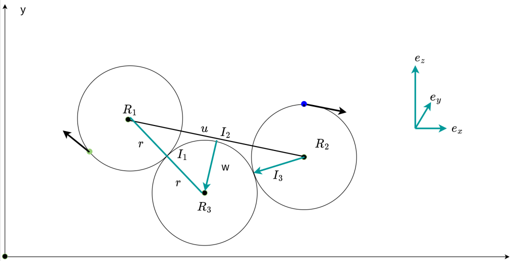
Dubins Path Planning (RLR)



- $\mathbf{I}_2 = \mathbf{R}_1 + \frac{\mathbf{u}}{2}$

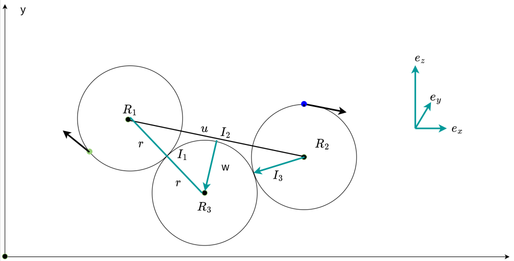


- $\mathbf{I}_2 = \mathbf{R}_1 + \frac{\mathbf{u}}{2}$
- $\mathbf{w} = \sqrt{4 \cdot r^2 - \frac{|u|^2}{4}} \cdot (\mathbf{e}_u \times \mathbf{e}_z)$

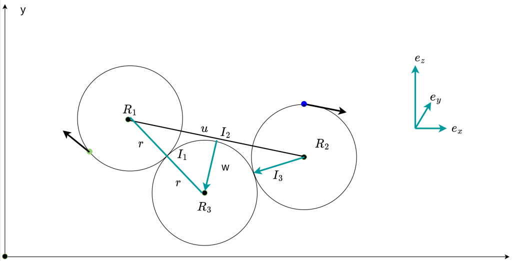


- $\mathbf{I}_2 = \mathbf{R}_1 + \frac{\mathbf{u}}{2}$
- $\mathbf{w} = \sqrt{4 \cdot r^2 - \frac{|u|^2}{4}} \cdot (\mathbf{e}_u \times \mathbf{e}_z)$
- $\mathbf{R}_3 = \mathbf{I}_2 + \mathbf{w}$

Dubins Path Planning (RLR)

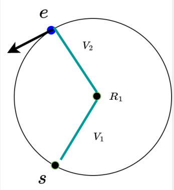


- $\mathbf{I}_2 = \mathbf{R}_1 + \frac{\mathbf{u}}{2}$
- $\mathbf{w} = \sqrt{4 \cdot r^2 - \frac{|u|^2}{4}} \cdot (\mathbf{e}_u \times \mathbf{e}_z)$
- $\mathbf{R}_3 = \mathbf{I}_2 + \mathbf{w}$
- $\mathbf{I}_3 = \mathbf{R}_2 + \frac{1}{2}(\mathbf{R}_3 - \mathbf{R}_2)$



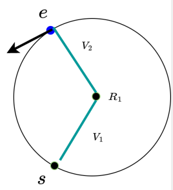
- $\mathbf{I}_2 = \mathbf{R}_1 + \frac{\mathbf{u}}{2}$
- $\mathbf{w} = \sqrt{4 \cdot r^2 - \frac{|u|^2}{4}} \cdot (\mathbf{e}_u \times \mathbf{e}_z)$
- $\mathbf{R}_3 = \mathbf{I}_2 + \mathbf{w}$
- $\mathbf{I}_3 = \mathbf{R}_2 + \frac{1}{2}(\mathbf{R}_3 - \mathbf{R}_2)$
- $\mathbf{I}_1 = \mathbf{R}_1 + \frac{1}{2}(\mathbf{R}_3 - \mathbf{R}_1)$

Computing Arc Length



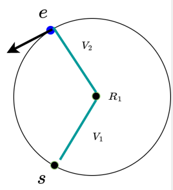
- arc length l = $r\theta$, where the angle between e and s is θ, and radius of circle is r

COMPUTING ARC LENGTH



- arc length l = $r\theta$, where the angle between *e* and *s* is θ, and radius of circle is r
- Let $\mathbf{v}_1 = \mathbf{e} - \mathbf{R}_1$, and $\mathbf{v}_2 = \mathbf{s} - \mathbf{R}_1$ be two vectors that connect start and target position with the center of the circle

Computing Arc Length



- arc length l = $r\theta$, where the angle between e and s is θ, and radius of circle is r
- Let $\mathbf{v}_1 = \mathbf{e} - \mathbf{R}_1$, and $\mathbf{v}_2 = \mathbf{s} - \mathbf{R}_1$ be two vectors that connect start and target position with the center of the circle
- Then depending on the direction of $\theta = atan2(\mathbf{v}_1) - atan2(\mathbf{v}_2)$, i.e, what direction that $\mathbf{v}_1$ rotates to end up at $\mathbf{v}_2$ direction of rotation can be defined: positive rotation is left turn and negative rotation is the right turn

Algorithm 1: Calculate arc length

Input: $\mathbf{v}_1, \mathbf{v}_2, r, d \in left, right$

Result: $|\theta * r|$

$\theta = atan2(\mathbf{v}_2) - atan2(\mathbf{v}_1)$

if $\theta < 0 \quad and \quad d = left$ **then**

  $\theta = \theta + 2\pi$

else

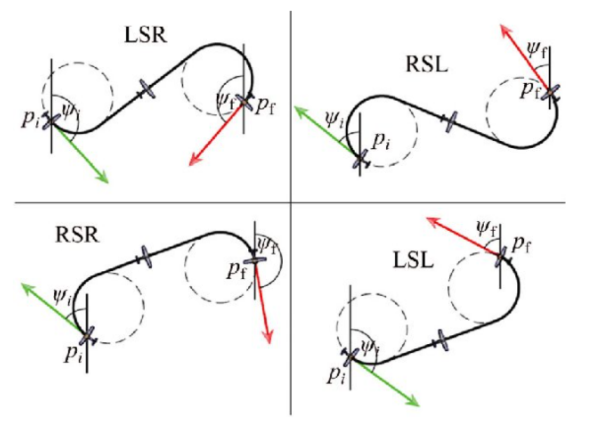
  if $\theta > 0 \quad and \quad d = right$ **then**

    $\theta = \theta - 2\pi$

  end

end

Dubins Path Planning



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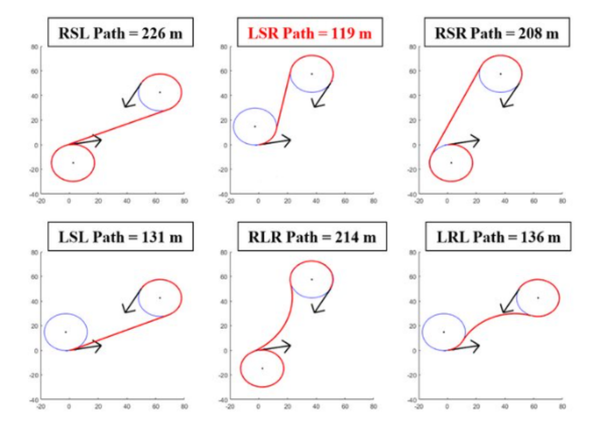

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