Autonomous Mobile Robotics

Environmental Mapping and Localization

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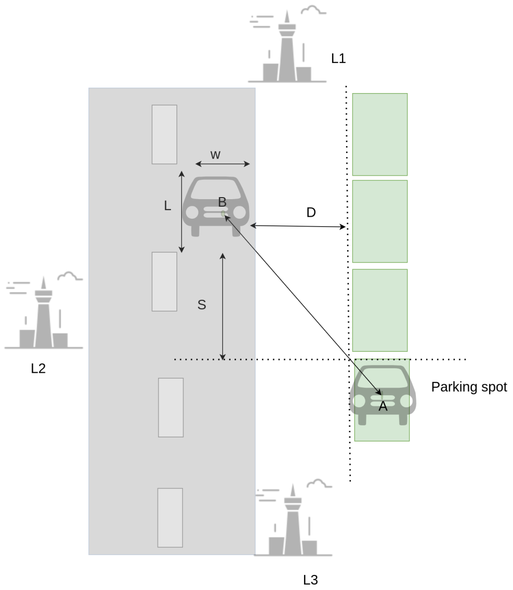
Simultaneous localization and mapping (SLAM)

https://d.facebook.com/169553396712821/photos/a.195338390800988/698095017191987/?type=3&
__kn__=EH-R

Simultaneous localization and mapping (SLAM) is the problem of acquiring a map of an unknown environment, while simultaneously localizing the robot relative to the map. Why can't we rely on sensors' incremental egomotion for robot position estimation (e.g., odometry, inertial navigation) [2]? What is the main problem with these sensors?

Problem Description



- There are three landmarks: L1 (5, 30), L2 (5, -30), and L3 (-5, 0), which can be seen by the sensor attached to car. Sensor readings are obtained in the following way

$$\underbrace{\begin{bmatrix} r_k^i \\ \theta_k^i \end{bmatrix}}_{z_k^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ atan2(m_{j,y} - y, m_{j,x} - x) - \theta \end{pmatrix}}_{h(x_{t,j,m})} + N(0, R) \qquad (1)$$

, where $m_{j,x}, m_{j,y}$ denotes the coordinates of jth landmark detection at time t. The white noise of each sensor reading, the optimal robot current location estimation, and the vehicle heading angle is given by $R = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}$,
$\mathbf{x}_{k,x}^- = x, \mathbf{x}_{k,y}^- = y$, and θ, respectively.

1. Let's say we have n landmarks and now we need to incorporate those locations into the state vector (x_k)?

$$x_k = (x, y, \theta, m_x^1, m_y^1, ..., m_x^n, m_y^n)^T \tag{2}$$

2. Design the system model (Φ_k)?
3. Can you derive the the general form of $\bar{x}_k^-$ and P_k^-?
4. When we perform the state prediction which part of the matrix (P_k^-) get updated?

$$
\underbrace{\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix}}_{\mu}
\underbrace{\begin{pmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{m_{n,x}} & \sigma_{m_{n,y}} \\
\sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\
\sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} & \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\
\sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} & \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} & \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \cdots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\
\sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} & \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \cdots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}}
\end{pmatrix}}_{\Sigma}
$$

The prediction state representation. Here, μ and Σ are equal to $\bar{x}_k^-$ and P_k^- respectively

$$
\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu}
\underbrace{\begin{pmatrix}
\Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\
\Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\
\vdots & \vdots & \ddots & \vdots \\
\Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n}
\end{pmatrix}}_{\Sigma}
$$

The prediction state representation (more compact form). Here, μ and Σ are equal to $\bar{x}_k^-$ and P_k^- respectively

Problem Formulation

$$\underbrace{\begin{pmatrix} x \\ m \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix}}_{\Sigma}$$

The prediction state representation (even more compact form). Here, μ and Σ are equal to $\bar{x}_k^-$ and P_k^- respectively

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