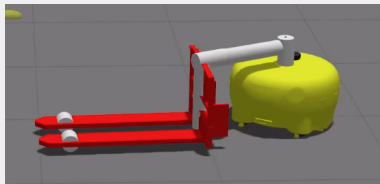


# AUTONOMOUS MOBILE ROBOTICS

## MOTION PLANNING AND CONTROL

GEESARA KULATHUNGA

JANUARY 24, 2023



# **CONTROL OF MOBILE ROBOTS**

- **Kinematics of wheeled mobile robots:** internal, external, direct, and inverse
  - ▶ Differential drive kinematics
  - ▶ Bicycle drive kinematics
  - ▶ Rear-wheel bicycle drive kinematics
  - ▶ Car(Ackermann) drive kinematics
- **Wheeled Mobile System Control: pose and orientation**
  - ▶ Control to reference pose
  - ▶ Control to reference pose via an intermediate point
  - ▶ Control to reference pose via an intermediate direction
  - ▶ Control by a straight line and a circular arc
  - ▶ Reference path control
- **Dubins path planning**

# KINEMATICS OF WHEELED MOBILE ROBOTS

- The process of moving an autonomous system from one place to another is called **Locomotion**



[www.proantic.com/en/display.php](http://www.proantic.com/en/display.php)

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- For mobile robotics **kinematic model is sufficient**



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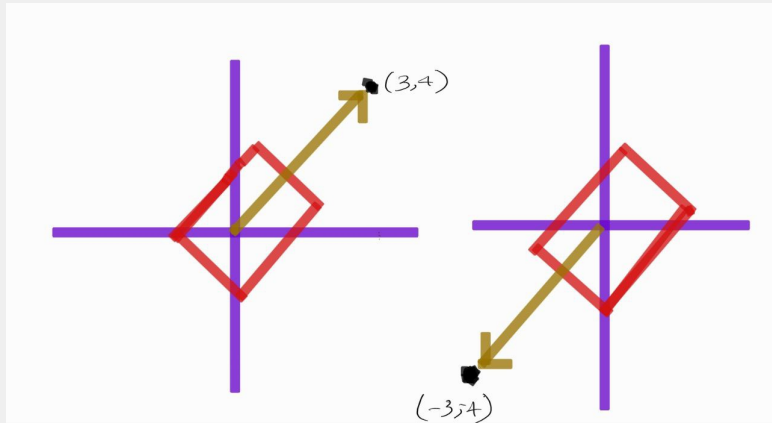


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- **Inverse kinematics**: robot inputs as a function of the desired robot pose

# THE DIFFERENCE BETWEEN ATAN AND ATAN2

Can you estimate the orientation of the robot?



# THE DIFFERENCE BETWEEN ATAN AND ATAN2

Quadrant	Angle		sin	cos	tan
I	$0$	$< \alpha < \pi/2$	+	+	+
II	$\pi/2$	$< \alpha < \pi$	+	-	-
III	$\pi$	$< \alpha < 3\pi/2$	-	-	+
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■  $|A \cdot B| = |A||B|\cos(\theta)$  and  $|A \times B| = |A||B|\sin(\theta)$

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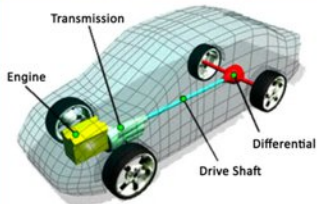
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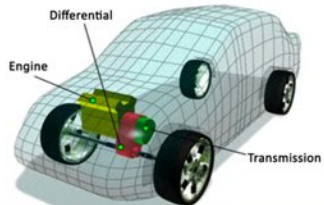
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- $\text{atan2}$ :  $-\pi < \text{atan2}(y,x) \leq \pi$  and  $\text{atan}$ :  $-\pi/2 < \text{atan}(y/x) < \pi/2$

# DIFFERENTIAL DRIVE KINEMATICS

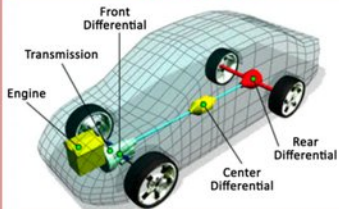
## Rear-Wheel Drive



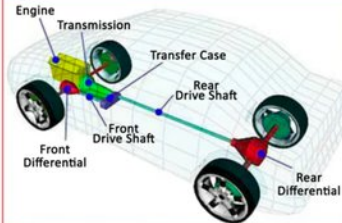
## Front-Wheel Drive



## All-Wheel Drive



## Four-Wheel Drive

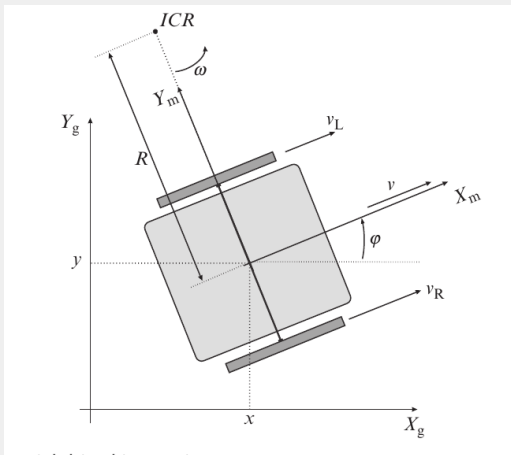


<https://cartreatments.com/types-of-differentials-how-they-work/>



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  - ▶ Wheel radius  $r$ , distance between wheels  $L$ , and term  $R(t)$  depicts the vehicle's instantaneous radius (ICR). **Angular velocity** is the **same** for **both left and right wheels around the ICR**.



# DIFFERENTIAL DRIVE KINEMATICS

## ■ Tangential velocity

$$\mathbf{v}(t) = \omega(t)R(t) = \frac{\mathbf{v}_R(t) + \mathbf{v}_L(t)}{2} \quad (1)$$

, where  $\omega = \mathbf{v}_L(t)/(R(t) - L/2) = \mathbf{v}_R(t)/(R(t) + L/2)$ . Hence,  $\omega$  and  $R(t)$  can be determined as follows:

$$\begin{aligned} \omega(t) &= \frac{\mathbf{v}_R(t) - \mathbf{v}_L(t)}{L} \\ R(t) &= \frac{L}{2} \frac{\mathbf{v}_R(t) + \mathbf{v}_L(t)}{\mathbf{v}_R(t) - \mathbf{v}_L(t)} \end{aligned} \quad (2)$$

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## ■ Wheels tangential velocities

$$\mathbf{v}_L = r\omega_L(t), \quad \mathbf{v}_R = r\omega_R(t) \quad (3)$$

# DIFFERENTIAL DRIVE KINEMATICS

## ■ Internal robot kinematics

$$\begin{bmatrix} \dot{x}_m(t) \\ \dot{y}_m(t) \\ \dot{\phi}(t) \end{bmatrix} = \begin{bmatrix} v_{x_m}(t) \\ v_{y_m}(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} \omega_L(t) \\ \omega_R(t) \end{bmatrix} \quad (4)$$

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## ■ Discrete time dynamics using Euler integration

$$\begin{aligned} x(k+1) &= x(k) + v(k)T_s \cos(\Phi(k)) \\ y(k+1) &= y(k) + v(k)T_s \sin(\Phi(k)) \\ \Phi(k+1) &= \Phi(k) + \omega(k)T_s \end{aligned} \quad (6)$$

, where discrete time instance  $t = kT_s$ ,  $k=0,1,2,\dots$ , for  $T_s$

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- Forward robot kinematics (given a set of wheel speeds, determine robot velocity)

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, where discrete time instance  $t = kT_s$ ,  $k=0,1,2,\dots$ , for  $T_s$  sampling time

- We can also try trapezoidal numerical integration for better approximation

$$\begin{aligned}x(k+1) &= x(k) + v(k)T_s \cos(\Phi(k) + \omega(k)T_s/2) \\y(k+1) &= y(k) + v(k)T_s \sin(\Phi(k) + \omega(k)T_s/2) \\ \Phi(k+1) &= \Phi(k) + \omega(k)T_s\end{aligned}\tag{8}$$

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- Inverse robot kinematics (given desired robot velocity, determine corresponding wheel velocities)
  - ▶ The **most challenging case compared to direct or forward kinematics**
  - ▶ Given the target pose **how many possible ways to get there?**
  - ▶ What if the **robot** goes can perform only **two types of motions: forward and rotations**

$$\begin{aligned} \mathbf{v}_R = \mathbf{v}_L = \mathbf{v}_R, \omega(t) = 0, \mathbf{v}(t) = \mathbf{v}_R // \text{forward} \\ \mathbf{v}_R = -\mathbf{v}_L = \mathbf{v}_R, \omega(t) = 2\mathbf{v}_R/L, \mathbf{v}(t) = 0 // \text{rotation} \end{aligned} \quad (9)$$

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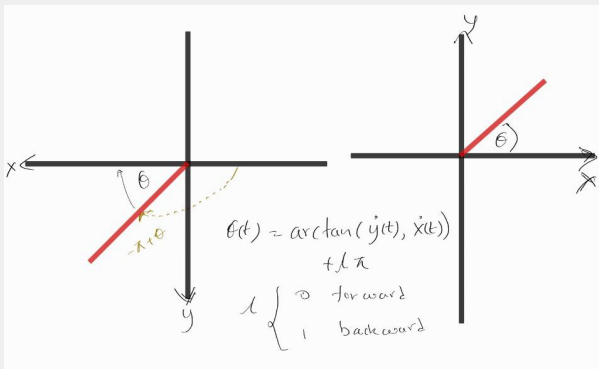
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  - If there is a disturbance in the trajectory and know the desired pose at time  $t$ , i.e.,  $x(t), y(t)$

$$\begin{aligned} \mathbf{v}(t) &= \pm \sqrt{\dot{x}^2(t) + \dot{y}^2(t)} // + \text{forward and - reverse} \\ \Phi(t) &= \arctan2(\dot{y}(t), \dot{x}(t)) + l\pi, \quad l \in \{0, 1\} \\ & // 0 \text{ forward and 1 reverse} \\ \omega(t) &= \frac{\dot{x}(t)\ddot{y}(t) - \dot{y}(t)\ddot{x}(t)}{\dot{x}^2(t) + \dot{y}^2(t)} = v(t)k(t) \end{aligned} \tag{10}$$

, where  $k(t)$  is the **path curvature** and  $\omega(t) = \dot{\Phi}(t)$

# DIFFERENTIAL DRIVE KINEMATICS



# MOTION CONTROL OF BICYCLE MOBILE ROBOTS

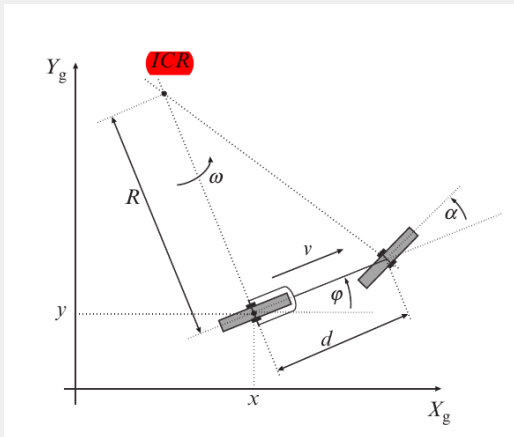


<https://helpfulcolin.com/bike-riding-robots-are-helped-by-gyroscopes-cameras/>

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According to Fig. 20,

- Steering angle  $\alpha$ , steering wheel angular velocity  $\omega_S$ , ICR point is defined by intersection of both wheel axes, and distance between wheels  $d$

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$$R(t) = d \tan\left(\frac{\pi}{2} - \alpha(t)\right) = \frac{d}{\tan(\alpha(t))} \quad (11)$$

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- Angular velocity  $\omega$  around ICR

$$\omega(t) = \dot{\Phi} = \frac{\mathbf{v}_S(t)}{\sqrt{d^2 + R^2}} = \frac{v_S(t)}{d} \sin(\alpha(t)) \quad (12)$$

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- Steering wheel velocity

$$\mathbf{v}_S(t) = \omega_S(t)r \quad (13)$$

# BICYCLE MOBILE (FRONT WHEEL DRIVE)

## ■ Internal robot kinematics

$$\begin{aligned}\dot{x}_m(t) &= \mathbf{v}_S(t)\cos(\alpha(t)) \\ \dot{y}_m(t) &= 0 \\ \dot{\phi}(t) &= \frac{\mathbf{v}_S(t)}{d}\sin(\alpha(t))\end{aligned}\tag{14}$$

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## ■ External robot kinematics

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$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\Phi}(t) \end{bmatrix} = \begin{bmatrix} \cos(\Phi(t)) & 0 \\ \sin(\Phi(t)) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{v}(t) \\ \omega(t) \end{bmatrix}\tag{16}$$

, where  $\mathbf{v}(t) = \mathbf{v}_S(t)\cos(\alpha(t))$  and  $\omega(t) = \frac{\mathbf{v}_S(t)}{d}\sin(\alpha(t))$

# MOTION CONTROL OF REAR-WHEEL BICYCLE MOBILE ROBOTS

## ■ Internal robot kinematics

$$\begin{aligned}\dot{x}_m(t) &= \mathbf{v}_S(t)\cos(\alpha(t)) = \mathbf{v}_r(t) \\ \dot{y}_m(t) &= 0 \\ \dot{\Phi}(t) &= \frac{\mathbf{v}_r(t)}{d}\tan(\alpha(t))\end{aligned}\tag{17}$$



# MOTION CONTROL OF REAR-WHEEL BICYCLE MOBILE ROBOTS

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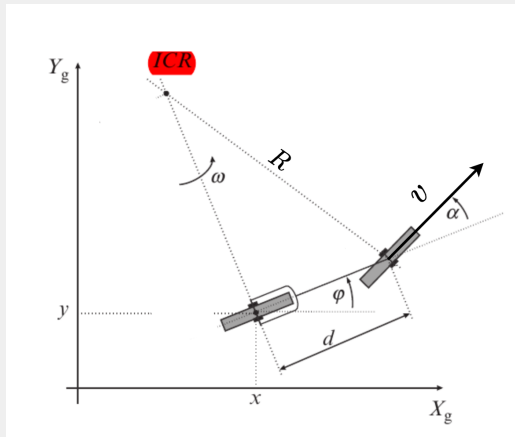
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, where  $\omega(t) = \frac{\mathbf{v}_r}{d}\tan(\alpha(t))$

# MOTION CONTROL OF BICYCLE MOBILE ROBOTS



## ■ External robot kinematics

$$\begin{aligned}\dot{x}(t) &= v \cdot \cos(\Phi(t) + \alpha(t)) \\ \dot{y}(t) &= v \cdot \sin(\Phi(t) + \alpha(t)) \\ \dot{\Phi}(t) &= v/R = v/(d/\sin(\alpha)) = v \cdot \sin(\alpha)/d \\ \dot{\alpha} &= \text{input (rate of change of steering angle)}\end{aligned}\tag{19}$$

# MOTION CONTROL OF REAR-WHEEL BICYCLE MOBILE ROBOTS

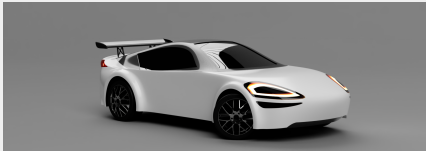


- Bicycle model imposes curvature constraint, where the curvature is defined by

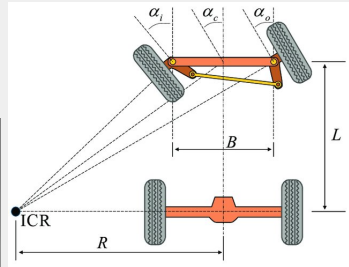
$$k = \frac{\dot{x}(t)\ddot{y}(t) - \dot{y}(t)\ddot{x}(t)}{(\dot{x}^2(t) + \dot{y}^2(t))^{3/2}}$$

- Curvature constraint is non-holonomic  $v^2 \leq \frac{a_{lat}}{k}$ , where  $a_{lat} \leq a_{lat_{max}}$

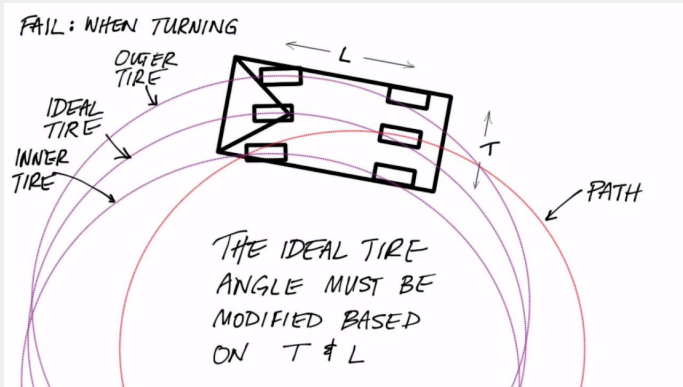
# MOTION CONTROL OF CAR(ACKERMANN) DRIVE MOBILE ROBOTS



<https://github.com/winstxnhdw/AutoCarROS2>, <https://doi.org/10.3390/s19214816>



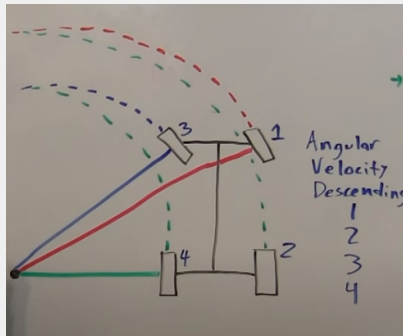
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<https://www.youtube.com/watch?v=i6uBwudwA5o>

# MOTION CONTROL OF CAR(ACKERMANN) DRIVE MOBILE ROBOTS

- Uses **steering principle**, i.e., the inner wheel, which is closer to its ICR, should steer for a bigger angle than the outer wheel. Consequently, the inner wheel travels at a slower speed than the outer wheel



**Figure:** Angular velocity speed descending order

# MOTION CONTROL OF CAR(ACKERMANN) DRIVE MOBILE ROBOTS

- **Ackermann geometry** is to **avoid** the need for tires to **slip sideways** when following the path around a curve which requires that the ICR point lies on a straight line defined by the rear wheels' axis



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- **Ackermann geometry** can be seen as **two bicycles welded together** side by side

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- **Ackermann geometry** can be seen as **two bicycles welded together** side by side
- For the differential drive it needs individual drives at each wheel which makes the system more complex
- **Ackerman steering** adjusts the **relative angles of the steerable wheels** so they both run **around a curve**.  
**Differentials** allow the **two driven wheels to run at different speeds** around **a curve**, which is quite a different requirement

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