Autonomous Mobile Robotics

Bayesian Filter

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Contents

Basic of Probability

Let X be a random variable and x be a specific value that is a outcome of sample space of X. For example, coin flip, where sample space consists of two values: head and tail

- $p(X = x)$ probability that the random variable X has value x

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- $p(y)$ does not depend on x. Thus, p(y) is same for any value of x, which yields $p(y)^{-1}$ as a normalizer, i.e., $p(x|y) = \eta p(y|x)p(x)$

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- $p(x|y,z) = \frac{p(y|x,z)p(x|z)}{p(y|z)}$ as long as $p(y|z) > 0$

- $$p(x|y,z) = \frac{p(y|x,z)p(x|z)}{p(y|z)}, \quad p(y|z) = \Sigma_x p(y|z,x)p(x|z)dx$$

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- p(x,y|z) = p(x|z,y)p(y|z,x) = p(x|z)p(y|z)
  Conditional independent does not imply absolute independence

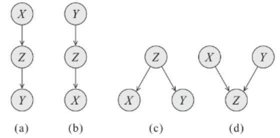

Figure 3.5 The four possible two-edge trails from X to Y via Z: (a) An indirect causal effect; (b) An indirect evidential effect; (c) A common cause; (d) A common effect.

https://mitpress.mit.edu/9780262013192/

- $E[X] = \Sigma_x x p(x)$ discrete

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- $Cov[X] = E[X - E[X]]^2 = E[X^2] - E[X]^2$

Example 01

Let's say, robot has to find a path between current pose and target pose. There are three path planning algorithms: A^*, RRT^*, and D^*, each of them are having 0.4, 0.4, and 0.2 possibility to find the optimal path between current and goal location. However, since the kinematics are incorporated when planning path, possibility of reaching to the target pose has following percentages: 40%, 80%, and 30%, respectively.

- Determine what would be probability that robot could not reach the target pose?
- The robot could not reach to the target. What is the probability that this happened on the path that A* is proposed?

Example 01

- Let random variable U be a selecting a path, where $A \in u_1, u_2, u_3$. Events: $u_1, u_2, \text{and}, u_3$, denoted A^* path, RRT^* path, and D^* path, respectively. Further, random variable V be a percentage of not reaching to the target pose. Thus, we can estimate p(V) as follows:

$$\begin{aligned} p(V) &= \Sigma_{u_i \in U}\, p(V|U = u_i)p(u_i) \\ &= 0.4 * 0.6 + 0.4 * 0.2 + 0.2 * 0.7 = 0.46 \end{aligned} \tag{1}$$

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- $$\begin{aligned} p(u_1|V) = \frac{p(B|u_1)p(u_1)}{\Sigma_{u_i \in U}\, p(V|U = u_i)p(u_i)} \\ = (0.4 \cdot 0.6)/0.46 \end{aligned} \tag{2}$$

Example 02

https://arxiv.org/pdf/1711.02144.pdf

Example 02

You have trained a neural network for detecting space is free or occupied. Probability of correctly detecting space is free 0.3, and occupied 0.7, respectively. To conform detected information, second image processing-based technique is used. In that method, it can classify correctly 60% if the network detected object space as free. Similarly, it can classify correctly 20% if the network detected space is occupied.

- What is the probability that neural network detected space is free if image processing technique also classified as free?

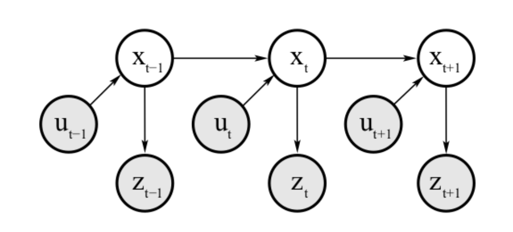
Example 02

- X - detecting a space, Z - recognize detected space by neural network as same as by image processing technique

$$\begin{aligned} p(X = x_1 | Z = z_1) &= \\ \frac{p(Z = z_1 | X = x_1) p(X = x_1)}{p(Z = z_1 | X = x_1) p(X = x_1) + p(Z = z_1 | X = \bar{x}_1) p(X = \bar{x}_1)} \\ &= \frac{0.3 \cdot 0.6}{0.3 \cdot 0.6 + 0.7 \cdot 0.8} \end{aligned} \quad (3)$$

, where $x_1 = free, \bar{x}_1 = obs, z_1 = free, \bar{z}_1 = obs$

Probabilistic Generative Laws



- $p(x_t|x_{0:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t|x_{t-1}, u_t)$, assuming system state is complete, i.e., x_t only depends on previous state information. $p(x_t|x_{t-1}, u_t)$ state transition probability
- $p(z_t|x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t|x_t)$, which we define as measurement probability

Markov Assumption

For any event x_k, which depends on previous events $x_{k-1}, \ldots, x_0$

Definition

$$p(x_k | x_{0:k-1}) = p(x_k | x_{k-1})$$

Markov assumption:

- world is static
- independence noise
- a perfect model

Estimation from Measurements

$$
\begin{aligned}
p(x_k|z_{1:k}) &= \frac{p(z_k|x_k, z_1, ..., z_{k-1})p(x_k|z_1, ..., z_{k-1})}{p(z_k|z_{1:k-1})} \\
&= \frac{p(z_k|x_k, z_{1:k-1})p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})} = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})}
\end{aligned} \tag{4}
$$

- $p(x_k|z_{1:k})$ state probability distribution at time step k, updated with measurement data

$$
\begin{aligned}
p(x_k|z_{1:k}) &= \frac{p(z_k|x_k, z_1, ..., z_{k-1})p(x_k|z_1, ..., z_{k-1})}{p(z_k|z_{1:k-1})} \\
&= \frac{p(z_k|x_k, z_{1:k-1})p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})} = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})}
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- $p(x_k|z_{1:k})$ state probability distribution at time step k, updated with measurement data
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p(x_k|z_{1:k}) &= \frac{p(z_k|x_k, z_1, ..., z_{k-1})p(x_k|z_1, ..., z_{k-1})}{p(z_k|z_{1:k-1})} \\
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- $p(x_k|z_{1:k})$ state probability distribution at time step k, updated with measurement data
- $p(z_k|x_k, z_{1:k-1})$ measurement probability distribution
- $p(x_k|z_{k-1})$ predicted state probability distribution

Estimation from Measurements

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p(x_k|z_{1:k}) &= \frac{p(z_k|x_k, z_1, ..., z_{k-1})p(x_k|z_1, ..., z_{k-1})}{p(z_k|z_{1:k-1})} \\
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- $p(z_k|x_k, z_{1:k-1})$ measurement probability distribution
- $p(x_k|z_{k-1})$ predicted state probability distribution
- $p(z_k|z_{1:k-1})$ measurement probability distribution

Estimation from Measurements

- Prediction step, i.e., updates the state based on previous measurements, proof: $p(x_k|x_{0:k-1}, z_{1:k-1}))$

$$p(x_k|z_{1:k-1}) = \Sigma_{x_{k-1}\in X}\, p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1}) \tag{5}$$

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- Correction step

$$p(x_k|z_{1:k}) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})} \tag{6}$$

, where

$$p(z_k|z_{1:k-1}) = \Sigma_{x_k\in X}\, p(z_k|x_k)p(x_k|z_{1:k-1})$$

Example 03

Consider the example 02,

- What is the probability that the detected space is free from the neural network if the image processing technique detects again as a free?
- What is the probability that the detected space is free from the neural network if the image processing technique detects the following sequence $z_{1:3} = \mathit{free}, \mathit{free}, \mathit{obs}$?

Example 03

Consider the example 02,

- What is the probability that the detected object is free from the neural network if the image processing technique detects again as free? Consider, in time step k = 1, image processing technique estimated as free $z_1 = \mathit{free}$, neural network estimated as free $x_1 = \mathit{free}$, then we can estimate the following:

$$
\begin{aligned}
p(x_1|z_1) &= \frac{0.3 \cdot 0.6}{0.3 \cdot 0.6 + 0.7 \cdot 0.8} = \frac{0.18}{0.74} = 0.24324 \\
p(\bar{x}_1|z_1) &= \frac{0.7 \cdot 0.8}{0.3 \cdot 0.6 + 0.7 \cdot 0.8} = \frac{0.42}{0.74} = 0.5675
\end{aligned}
\tag{7}
$$

Example 03

Consider the example 02,

- In the next time step k = 2, let $p(x_2|z_1)$ be the predicted probability

$$
\begin{aligned}
p(x_k|z_{1:k-1}) &= \Sigma_{x_{k-1}\in X}\; p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1}) \\
p(x_2|z_1) &= \Sigma_{x_1\in X}\; p(x_2|x_1)p(x_1|z_1) \\
&= p(x_2|x_1)p(x_1|z_1) + p(x_2|\bar{x}_1)p(\bar{x}_1|z_1) \\
&= 1\cdot 0.24324 + 0\cdot 0.5675 = 0.24324
\end{aligned}
\tag{8}
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- In the next time step k = 2, let $p(x_2|z_1)$ be the predicted probability

$$\begin{aligned} p(x_k|z_{1:k-1}) &= \Sigma_{x_{k-1}\in X}\ p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1}) \\ p(x_2|z_1) &= \Sigma_{x_1\in X}\ p(x_2|x_1)p(x_1|z_1) \\ &= p(x_2|x_1)p(x_1|z_1) + p(x_2|\bar{x}_1)p(\bar{x}_1|z_1) \\ &= 1 \cdot 0.24324 + 0 \cdot 0.5675 = 0.24324 \end{aligned} \tag{8}$$

- Now we can incorporate the measurement

$$\begin{aligned} p(x_k|z_{1:k}) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})} &= \frac{p(z_2|x_2)p(x_2|z_1)}{p(z_2|z_1)} \\ &= \frac{0.6 \cdot 0.24324}{p(z_2|z_1)} \end{aligned} \tag{9}$$

Example 03

Consider the example 02,

- In the next time step k = 2, summation of all possible state combinations, i.e., $p(z_2|z_1)$ can be estimated as follows:

$$\begin{aligned} p(z_k|z_{1:k-1}) &= \Sigma_{z_k \in X}\, p(z_k|x_k)p(x_k|z_{1:k-1}) \\ p(z_2|z_1) &= \Sigma_{x_2 \in X}\, p(z_2|x_2)p(x_2|z_1) \\ &= p(z_2|x_2)p(x_2|z_1) + p(z_2|\bar{x}_2)p(\bar{x}_2|z_1) \\ &= 0.6 \cdot 0.24324 + 0.8 \cdot 0.5675 = 0.599 \end{aligned} \tag{10}$$

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- In the next time step k = 2, summation of all possible state combinations, i.e., $p(z_2|z_1)$ can be estimated as follows:

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- Finally, we can estimate the $p(x_2|z_{1:2})$:

$$p(x_2|z_{1:2}) = \frac{0.6 \cdot 0.24324}{0.599} = 0.2436 \tag{11}$$

Estimation from Measurements and Controls

$$p(x_k|z_{1:k}, u_{0:k-1}) = \frac{p(z_k|x_k, z_1, ..., z_{k-1}, u_{0:k-1})p(x_k|z_1, ..., z_{k-1}, u_{0:k-1})}{p(z_k|z_{1:k-1}, u_{0:k-1})} = \frac{p(z_k|x_k)p(x_k|z_{1:k-1}, u_{0:k-1})}{p(z_k|z_{1:k-1}, u_{0:k-1})} \tag{12}$$

- $p(x_k|z_{1:k}, u_{0:k-1})$ state probability distribution at time step k, updated with measurement data and control inputs
- $p(z_k|x_k, z_{1:k-1}, u_{0:k-1})$ measurement probability distribution (if previous actions and measurements up to time step k-1 are known)
- $p(x_k|z_{k-1}, u_{0:k-1})$ predicted state probability distribution
- $p(z_k|z_{1:k-1}, u_{0:k-1})$ measurement probability distribution

Estimation from Measurements and Controls

- Prediction step

$$p(x_k|z_{1:k-1}, u_{1:k-1}) = \Sigma_{x_{k-1} \in X}\ p(x_k|x_{k-1}, u_{k-1}) p(x_{k-1}|z_{1:k-1}, u_{0:k-1}) \quad (13)$$

- Prediction step

$$p(x_k|z_{1:k-1}, u_{1:k-1}) = \Sigma_{x_{k-1} \in X}\ p(x_k|x_{k-1}, u_{k-1})p(x_{k-1}|z_{1:k-1}, u_{0:k-1}) \tag{13}$$

- Correction step

$$p(x_k|z_{1:k}, u_{0:k-1}) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1}, u_{0:k-1})}{p(z_k|z_{1:k-1}, u_{0:k-1})} \tag{14}$$

, where

$$p(z_k|z_{1:k-1}, u_{0:k-1}) = \Sigma_{x_k \in X}\ p(z_k|x_k)p(x_k|z_{1:k-1}, u_{0:k-1})$$

- Belief

$$
\begin{aligned}
bel(x_k) = p(x_k|z_{1:k}, u_{0:k-1}) = \eta p(z_k|x_k, u_k)bel_p(x_k) \\
bel_p(x_k) = p(x_k|z_{1:k-1}, u_{0:k-1}) \\
= \Sigma_{x_{k-1}\in X}\, p(x_k|x_{k-1}, u_{k-1})bel(x_{k-1})
\end{aligned}
\tag{15}
$$

- Belief

$$\begin{aligned} bel(x_k) = p(x_k|z_{1:k}, u_{0:k-1}) = \eta p(z_k|x_k, u_k) bel_p(x_k) \\ bel_p(x_k) = p(x_k|z_{1:k-1}, u_{0:k-1}) \\ = \Sigma_{x_{k-1}\in X}\, p(x_k|x_{k-1}, u_{k-1}) bel(x_{k-1}) \end{aligned} \tag{15}$$

- The normalization factor

$$\eta = \frac{1}{\alpha} = \frac{1}{p(z_k|z_{1:k-1}, u_{0:k-1})} \tag{16}$$

function BAYESIAN_FILTER($bel(x_{k-1}),\ u_{k-1},\ z_k$)
  $\alpha \leftarrow 0$
  for all x_k **do**
    $bel_p(x_k) \leftarrow \int p(x_k|x_{k-1}, u_{k-1}) bel(x_{k-1})\, dx_{k-1}$
    $bel'(x_k) \leftarrow p(z_k|x_k) bel_p(x_k)$
    $\alpha \leftarrow \alpha + bel'(x_k)$
  end for
  for all x_k **do**
    $bel(x_k) \leftarrow \frac{1}{\alpha} bel'(x_k)$
  end for
  return $bel(x_k)$
end function

Example 04

Consider the example 02 and example 03, loss of generality let consider the following notation: $X_k \in \{free, obs\}$ from neural network as $X_k \in \{x_k, \bar{x}_k\}$, $Z_k \in \{free, obs\}$ from image processing technique as $Z_k \in \{z_k, \bar{z}_k\}$, and $U_k \in \{move, \text{not move}\}$ from the motor control as $U_k \in \{u_k, \bar{u}_k\}$, where vehicle moves only if there are no obstacles.
The initial belief that road is without obstacles

$$bel(X_0 = free) = 0.5$$

We have following information about the sensor measurements:

$$p(z_k = free|x_k = free) = 0.8, p(z_k = obs|x_k = free) = 0.2,$$
$$p(z_k = obs|x_k = obs) = 0.9, andp(z_k = free|x_k = obs) = 0.1$$

Example 04

Based on current state and control input, next state is performed that is distributed as follows:

$$
\begin{aligned}
p(x_k = free | x_{k-1} = free, u_{k-1} = move) &= 1.0,\\
p(x_k = obs | x_{k-1} = free, u_{k-1} = move) &= 0.0,\\
p(x_k = free | x_{k-1} = obs, u_{k-1} = move) &= 0.8,\\
p(x_k = obs | x_{k-1} = obs, u_{k-1} = move) &= 0.2
\end{aligned}
$$

When control input are zero,

$$
\begin{aligned}
p(x_k = free | x_{k-1} = free, u_{k-1} = \text{not move}) &= 1.0,\\
p(x_k = obs | x_{k-1} = free, u_{k-1} = \text{not move}) &= 0.0,\\
p(x_k = free | x_{k-1} = obs, u_{k-1} = \text{not move}) &= 0.0,\\
p(x_k = obs | x_{k-1} = obs, u_{k-1} = \text{not move}) &= 1.0
\end{aligned}
$$

Example 04

For the following control inputs and sensor decision sequence, determine the belief $bel_p(x_k)$ (prediction) based on control inputs and the belief based on measurements $bel(x_k)$ (correction)

k	u_{k-1}	z_k
1	not move	obs
2	move	free
3	move	free

Example 04

- In the time step k=1,

$$
\begin{aligned}
bel_p(x_1) &= \Sigma_{x_0 \in X} p(x_1|x_0, \bar{u}_0) bel(x_0) \\
&= p(x_1|\bar{x}_0, \bar{u}_0) bel(\bar{x}_0) + p(x_1|x_0, \bar{u}_0) bel(x_0) \\
&= 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5 \\
bel_p(\bar{x}_1) &= \Sigma_{x_0 \in X} p(\bar{x}_1|x_0, \bar{u}_0) bel(x_0) \\
&= p(\bar{x}_1|\bar{x}_0, \bar{u}_0) bel(\bar{x}_0) + p(\bar{x}_1|x_0, \bar{u}_0) bel(x_0) \\
&= 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5
\end{aligned}
\tag{17}
$$

Example 04

- Based on the measurement the belief can be corrected;

$$\begin{aligned} bel(x_1) &= \eta p(\bar{z}_1|x_1)bel_p(x_1) = \eta 0.2 \cdot 0.5 = \eta 0.1 \\ bel(\bar{x}_1) &= \eta p(\bar{z}_1|\bar{x}_1)bel_p(\bar{x}_1) = \eta 0.9 \cdot 0.5 = \eta 0.45 \\ \eta &= \frac{1}{0.1 + 0.45} = 1.81 \end{aligned} \tag{18}$$

Example 04

- Based on the measurement the belief can be corrected;

$$\begin{aligned} bel(x_1) &= \eta p(\bar{z}_1|x_1)bel_p(x_1) = \eta 0.2 \cdot 0.5 = \eta 0.1 \\ bel(\bar{x}_1) &= \eta p(\bar{z}_1|\bar{x}_1)bel_p(\bar{x}_1) = \eta 0.9 \cdot 0.5 = \eta 0.45 \\ \eta &= \frac{1}{0.1 + 0.45} = 1.81 \end{aligned} \tag{18}$$

- Same procedure can be repeated for k =2 and k =3

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