Trajectory tracking based on NMPC for MAVs

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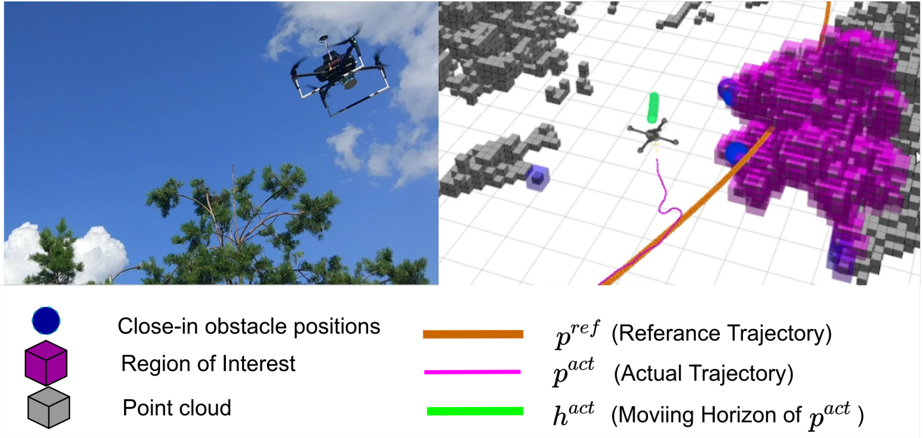
What we need for autonomous navigation?



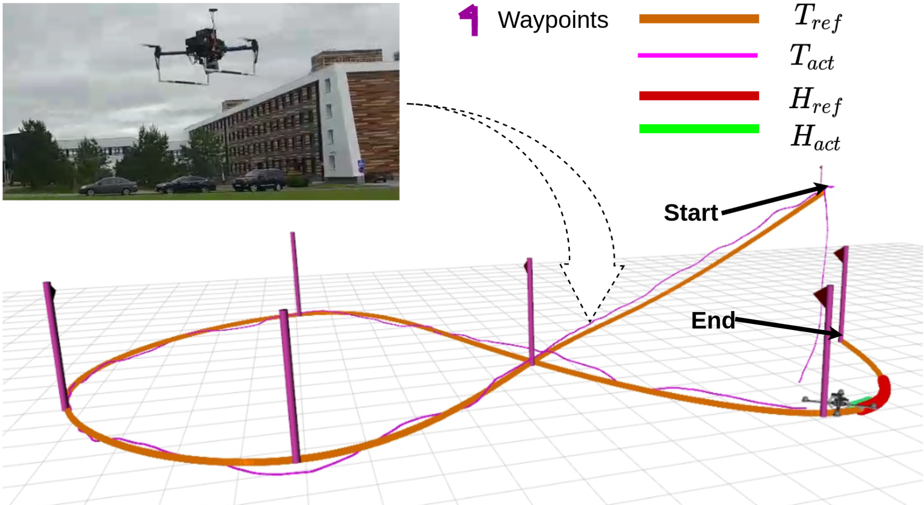
Motion Model Selection

$$\dot{\mathbf{x}}_k = \mathbf{f}(\mathbf{x_k}, \mathbf{u_k}) \qquad\qquad \mathbf{x_{k+1}} = \mathbf{f_d}(\mathbf{x_k}, \mathbf{u_k})$$

$$\begin{bmatrix} \dot{p}_k^x \\ \dot{p}_k^y \\ \dot{p}_k^z \\ \dot{\alpha}_k^z \end{bmatrix} = \begin{bmatrix} v_k^x cos(\alpha_k^z) - v_k^y sin(\alpha_k^z) \\ v_k^x sin(\alpha_k^z) + v_k^y cos(\alpha_k^z) \\ v_k^z \\ \omega_k^z \end{bmatrix} \xrightarrow[\text{Euler Discretization}]{\delta = 0.05s} \begin{bmatrix} p_k^x \\ p_k^y \\ p_k^z \\ \alpha_k^z \end{bmatrix} + \delta \begin{bmatrix} v_k^x cos(\alpha_k^z) - v_k^y sin(\alpha_k^z) \\ v_k^x sin(\alpha_k^z) + v_k^y cos(\alpha_k^z) \\ v_k^z \\ \omega_k^z \end{bmatrix}$$

1. $\mathbf{x}_k = [p_k^x, p_k^y, p_k^z, \alpha_k^z]^T \in \mathbb{R}^{n_x}$ and $\mathbf{u}_k = [v_k^x, v_k^y, v_k^z, \omega_k^z]^T \in \mathbb{R}^{n_u}$
2. p_k^i and $v_k^i, i \in \{x, y, z\}$ center position(m) and velocity (m/s) at time t = k in the world coordinate frame
3. yaw angle (rad) α_k^z and yaw rate (rad/s) ω_k^z

Reference Trajectory Generation



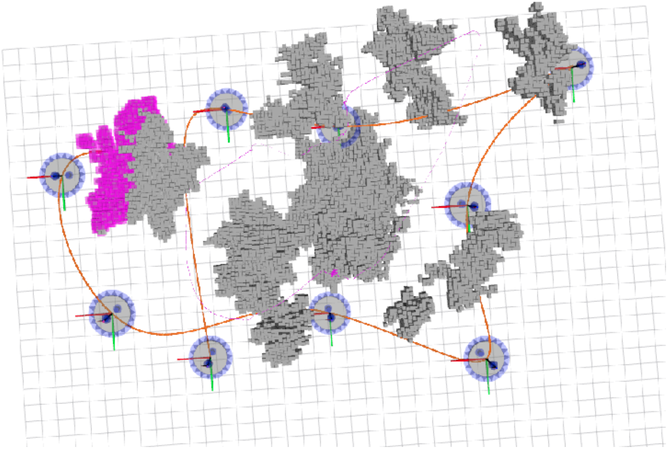
Reference Trajectory Generation

1. The d^{th}, i.e., d=3 order uniform B-Spline
2. knot sequence, i.e., $p^{knot} = \{t_0, t_1, ..., t_{n_k}\}$ and control points, i.e., $p^{ref} = \{p_0, p_1, ..., p_{n_p}\}$ where $t_* \in \mathbb{R}$, $p_* \in \mathbb{R}^d$ and $n_k = n_p + d + 1$; $*$ denotes the indexing of p^{ref} and p^{knot}
3. p_i^{ref} represents position i.e., $p_i^{ref} =< x_i, y_i, z_i >$ in $\mathbb{R}^3$ where $i = 0, ..., n_p$
4. For a given time index, t corresponding position, $c^{ref}(t)$

$$c^{ref}(t) = DeBoorCox(t, p^{ref}), \quad c^{ref}(t) \in \mathbf{R}^3 \tag{1}$$

5. To estimate reference velocity and acceleration can be estimated by taking first and second derivative of p^{ref} and $c^{ref^{(*)}}(t) = DeBoorCox(t, p^{ref^{(*)}})$

Reference Trajectory Generation



Tracking Problem Synthesis

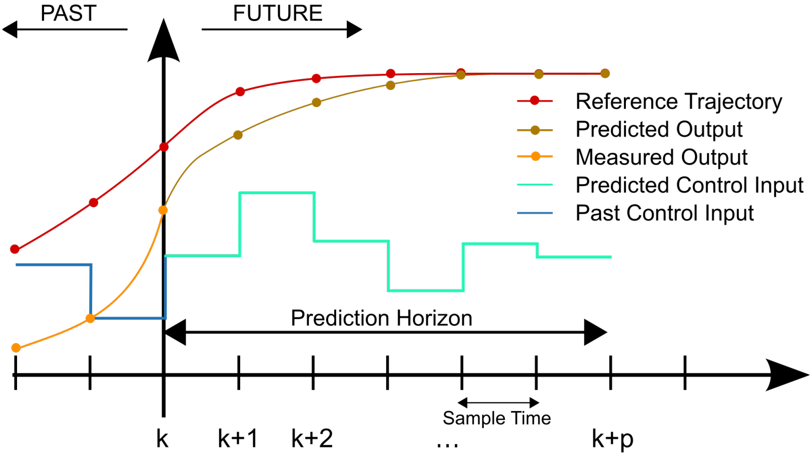


Figure: Model Predictive Control @ [1]

How to define the trajectory tracking problem?

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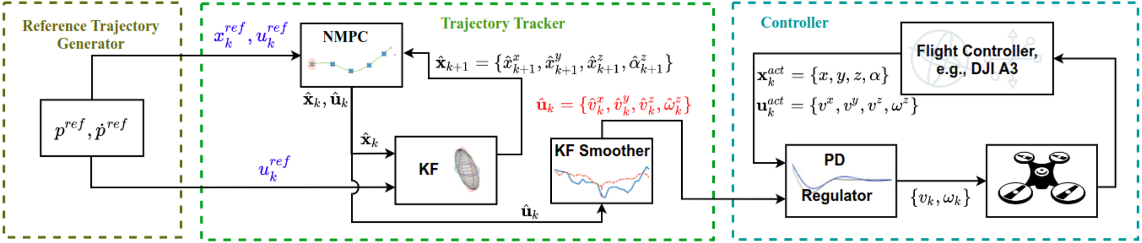
$$
\begin{aligned}
J_{N_e}(\mathbf{x}_k, \mathbf{u}_k) &= \sum_{h=0}^{N_e} \left\| \mathbf{x}_{k+h} - \mathbf{x}_{k+h}^{ref} \right\|_Q^2 + \left\| \mathbf{u}_{k+h} - \mathbf{u}_{k+h}^{ref} \right\|_R^2 \\
\min_{w} \quad & J_{N_e}(\mathbf{x}_k, \mathbf{u}_k) \\
\text{s.t.} \quad & g_1(w) = 0, \quad g_2(w) \leq 0 \\
g_1(w) &= \begin{bmatrix} \bar{\mathbf{x}}_k - \mathbf{x}_k \\ \vdots \\ f_d(\mathbf{x}_{k+h}, \mathbf{u}_{k+h}) - \mathbf{x}_{k+h+1} \\ \vdots \\ f_d(\mathbf{x}_{k+N_e-1}, \mathbf{u}_{k+N_e-1}) - \mathbf{x}_{k+N_e} \end{bmatrix}
\end{aligned}
\tag{2}
$$

Local Planner



PD Regulator:

$$
\begin{gathered}
v_k = k_p(\hat{\mathbf{p}}_k - \mathbf{p}_k^{act}) - k_d \mathbf{v_k}^{act}, \quad \omega_k = k_a(\hat{\alpha}^z_k - \alpha^{act}_k) - k_v \omega^{act}_k \\
\hat{\mathbf{x}}_k = < \underbrace{\hat{x}^x_k, \hat{x}^y_k, \hat{x}^z_k}_{\hat{\mathbf{p}}_k}, \hat{\alpha}^z_k > \quad \hat{\mathbf{u}}_k = < \underbrace{\hat{v}^x_k, \hat{v}^y_k, \hat{v}^z_k}_{\hat{\mathbf{v}}_k}, \hat{\omega}^z_k >
\end{gathered} \tag{3}
$$

D. Simon, "Model predictive control in flight control design: Stability and reference tracking," Ph.D. dissertation, Linköping University Electronic Press, 2014.