Assignment-1

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SECTION-A

F. Match the Following

1) Match the following:

(2006-6M)

(A) $\int_0^{\frac{\pi}{2}} (\sin x)^{\cos x} \left(\cos x \cot x - \log(\sin x)^{\sin x}\right) dx$ (p) 1
(B) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ (q) 0
(C) Cosine of the angle of intersection of curves $y = 3^{x-1} \log x$ and $y = x^x - 1$ is (r) 6ln2
(D) Let $\frac{dy}{dx} = \frac{6}{x+y}$ where $y(0) = 0$ then value of y when $x + y = 6$ is (s) $\frac{4}{3}$

2) Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4*x*4 matrix given in the ORS.

(2007-6M)

Column I	**Column II**
(A) $\int_{-1}^{1} \frac{dx}{1+x^2}$	(p) $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$	(q) $2\log\left(\frac{2}{3}\right)$
(C) $\int_2^3 \frac{dx}{1-x^2}$	(r) $\frac{\pi}{3}$
(D) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$	(s) $\frac{\pi}{2}$

3) (JEE Adv. 2014)

List-I	**List-II**
P. The number of polynimials $f(x)$ with non-negative integer coeffecients of *degree* ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)\,dx = 1$, is	a) 8
Q. The number of points in the interval $\left[-\sqrt{13}, \sqrt{13}\right]$ at which $f(x) = \sin x^2 + \cos x^2$attains its maximum value is	b) 2
R. $\int_{-2}^{2} \frac{3x^2}{(1+e^x)} dx$ equals	c) 4
S. $\frac{\left(\int_{\frac{-1}{2}}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_0^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}$	d) 0

P Q R S	**P Q R S**
(a) 3 2 4 1	(b) 2 3 4 1
(c) 3 2 1 4	(a) 2 3 1 4

SECTION-B JEE MAIN/AIEEE

1) The area (*insq.units*) of the region $\{(x, y) : y^2 \geq 2x and x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is:
[JEE M 2016]
a) $\pi - \frac{4\sqrt{2}}{3}$
b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$
c) $\pi - \frac{4}{3}$
d) $\pi - \frac{8}{3}$

2) The area (*insq.units*) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y and y \leq 1 + \sqrt{x}\}$ is:
[JEE M 2017]
a) $\frac{5}{2}$
b) $\frac{59}{12}$
c) $\frac{3}{2}$
d) $\frac{7}{3}$

3) The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to:
[JEE M 2017]
a) -1
b) -2
c) 42
d) 4

4) Let $g(x) = \cos x^2, f(x) = \sqrt{x}, and \alpha, \beta(\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (*insq.units*) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta and y = 0$, is:
[JEE M 2018]
a) $\frac{1}{2}\left(\sqrt{3} + 1\right)$
b) $\frac{1}{2}\left(\sqrt{3} - \sqrt{2}\right)$
c) $\frac{1}{2}\left(\sqrt{2} - 1\right)$
d) $\frac{1}{2}\left(\sqrt{3} - 1\right)$

5) The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is:
[JEE M 2018]
a) $\frac{\pi}{2}$
b) 4π
c) $\frac{\pi}{4}$
d) $\frac{\pi}{8}$

6) The value of $\int_0^{\pi} |\cos x|^3 \, dx$ is:
[JEE M 2019-9 Jan(M)]
a) 0
b) $\frac{4}{3}$
c) $\frac{2}{3}$
d) $\frac{-2}{3}$

7) The area (*insq.units*) bounded by the parabola $y = x^2 - 1$, the tangent at the point $(2, 3)$ to it and the y-axis is:
[JEE M 2019-9Jan(M)]
a) $\frac{8}{3}$
b) $\frac{32}{3}$
c) $\frac{56}{3}$
d) $\frac{14}{3}$

8) The value of $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x + \cos x} dx$ is:
[JEE M 2019-9 April(M)]
a) $\frac{\pi-2}{8}$
b) $\frac{\pi-1}{4}$
c) $\frac{\pi-2}{4}$
d) $\frac{\pi-1}{2}$

9) The area (*insq.units*) of the region $A = \{(x, y) : x^2 \leq y \leq x + 2\}$ is:
[JEE M 2019-9 April(M)]
a) $\frac{10}{3}$
b) $\frac{9}{2}$
c) $\frac{31}{6}$
d) $\frac{13}{6}$