

# 9-9.2-39

AI24BTECH11012 - Pushkar Gudla

**Question:** The area of the region bounded by the curve  $x^2 = 4y$  and the straight line  $x = 4y - 2$  is

- 1)  $\frac{3}{8}$  sq units
- 2)  $\frac{1}{8}$  sq units
- 3)  $\frac{1}{4}$  sq units
- 4)  $\frac{3}{4}$  sq units

**Solution:**

Variable	Description
$g(x)$	Equation of Conic
$L$	Equation of line
$\mathbf{h}$	A point on line $L$
$\mathbf{m}$	Direction vector of line $L$
$\mathbf{x}_1$ and $\mathbf{x}_2$	Points of intersection of $L$ and $g(x)$

TABLE 4: Variables and given data

$$g(x) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (4.1)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (4.2)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (4.3)$$

$$f = 0 \quad (4.4)$$

$$L : \mathbf{x} = \mathbf{h} + k\mathbf{m} \quad (4.5)$$

$$\mathbf{h} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (4.6)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \quad (4.7)$$

$$\mathbf{x}_i = \mathbf{h} + k_i \mathbf{m} \quad (4.8)$$

$$k_1 = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left( -m^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) + \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (4.9)$$

$$k_2 = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left( -m^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) - \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (4.10)$$

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (4.11)$$

$$\mathbf{x}_2 = \begin{pmatrix} -1 \\ \frac{1}{4} \end{pmatrix} \quad (4.12)$$

The area bounded by the curve  $y = \frac{x^2}{4}$  and line  $y = \frac{x}{4} + \frac{1}{2}$  is given by:

$$\int_{-1}^2 \left( \frac{x}{4} + \frac{1}{2} - \frac{x^2}{4} \right) dx = \frac{7}{8} \quad (4.13)$$

Hence, the area bounded by the curve and the line is  $\frac{7}{8}$  sq units.

