

30-01-2023 shift-2(16-30)

AI24BTECH11012- Pushkar Gudla

- 1) If a plane passes through the points $(-1, k, 0)$, $(2, k, -1)$, $(1, 1, 2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is
 - a) $\frac{17}{5}$
 - b) $\frac{5}{17}$
 - c) $\frac{6}{13}$
 - d) $\frac{13}{6}$
- 2) Let $a, b, c > 1$, a^3, b^3 , and c^3 be in A.P., and $\log_a b, \log_c a$, and $\log_b c$ be in G.P. If the sum of the first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a+8b-c}{10}$, is -444 , then abc is equal to
 - a) 343
 - b) 216
 - c) $\frac{343}{8}$
 - d) $\frac{125}{8}$
- 3) Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \dots, a_{100}$ is 25. Then S is
 - a) ϕ
 - b) $\{99\}$
 - c) \mathbb{N}
 - d) $\{9\}$
- 4) $\lim_{n \rightarrow \infty} \frac{3}{n} \left(4 + \left(2 + \frac{1}{n} \right)^2 + \left(2 + \frac{2}{n} \right)^2 + \dots + \left(3 - \frac{1}{n} \right)^2 \right)$ is equal to
 - a) 12
 - b) $\frac{19}{3}$
 - c) 0
 - d) 19
- 5) For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$$\begin{aligned} x - y + z &= 5 \\ 2x + 2y + \alpha z &= 8 \\ 3x - y + 4z &= \beta \end{aligned}$$
 has infinitely many solutions. Then α and β are the roots of
 - a) $2x^2 - 10x + 16 = 0$
 - b) $2x^2 + 18x + 56 = 0$
 - c) $2x^2 - 18x + 56 = 0$
 - d) $2x^2 + 14x + 24 = 0$
- 6) The 50th root of a number x is 12 and the 50th root of another number y is 18. Then the remainder obtained on dividing $x + y$ by 25 is _____.
- 7) Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \rightarrow A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is _____.
- 8) Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ . If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to _____.

- 9) The 8th common term of the series
 $S_1 = 3 + 7 + 11 + 15 + 19 + \dots$
 $S_2 = 1 + 6 + 11 + 16 + 21 + \dots$
 is _____.
- 10) Let a line L pass through the point $P(2, 3, 1)$ and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$. If the distance of L from the point $(5, 3, 8)$ is α , then $3\alpha^2$ is equal to _____.
- 11) $\int \sqrt{\sec 2x - 1} dx = \alpha \log \left| \cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta} x \right)} \right| + C$, then $\beta - \alpha$ is equal to _____.
- 12) If the value of the real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$, then β is equal to _____.
- 13) The number of seven-digit odd numbers that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _____.
- 14) A bag contains six balls of different colors. Two balls are drawn in succession with replacement. The probability that both balls are of the same color is p . Next, four balls are drawn in succession with replacement, and the probability that exactly three balls are of the same color is q . If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to _____.
- 15) Let A be the area of the region $\{(x, y) : y \geq x^2, y \geq (1 - x)^2, y \leq 2x(1 - x)\}$. Then $540A$ is equal to _____.