03-09-2020 shift-1

AI24BTECH11012- Pushkar Gudla

1) The value of $(2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 - \cdots$ up to 51st term) + $(1! - 2! + 3! - \cdots$ up to 51st term) is equal to:
   a) $1 - 51(51)!$
   b) $1 + (52)!$
   c) 1
   d) $1 + (51)!$
2) Let P be a point on the parabola $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then:
   a) $PN = 4$
   b) $MQ = \frac{1}{3}$
   c) $PN = 3$
   d) $MQ = \frac{1}{4}$
3) If $\Delta = \begin{vmatrix} x^2 & 2x & 3 \\ 2x & 3x & 4 \\ 3x & 5x & 8 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to:
   a) 1
   b) -1
   c) -3
   d) 9
4) The foot of the perpendicular drawn from the point $(4, 2, 3)$ to the line joining the points $(1, -2, 3)$ and $(1, 1, 0)$ lies on the plane:
   a) $x - y - 2z = 1$
   b) $x - 2y + z = 1$
   c) $2x + y - z = 1$
   d) $x + 2y - z = 1$
5) If $y^2 + \log_e(\cos^2 x) = y,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then:
   a) $|y'(0)| + |y''(0)| = 1$
   b) $y''(0) = 0$
   c) $|y'(0)| + |y''(0)| = 3$
   d) $|y''(0)| = 2$
6) $2\pi - \left(\sin^{-1}\left(\frac{4}{5} - \sin^{-1}\right)\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right)$ is equal to:
   a) $\frac{5\pi}{4}$
   b) $\frac{3\pi}{2}$
   c) $\frac{7\pi}{4}$
   d) $\frac{\pi}{2}$
7) A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$. Then this hyperbola does not pass through which of the following points:
   a) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$
   b) $\left(1, \frac{-1}{\sqrt{2}}\right)$

c) $\left(\frac{1}{\sqrt{2}}, 0\right)$

d) $\left(-\sqrt{\frac{3}{2}}, 1\right)$

8) For the frequency distribution:
Variate(x) : $x_1, x_2, \ldots, x_{15}$
Frequency(f) : $f_1, f_2, \ldots, f_{15}$
where $0 < x_1 < x_2 < \cdots < x_{15} \leq 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be:

a) 1
b) 4
c) 6
d) 2

9) A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared at least once is:

a) $\frac{1}{3}$
b) $\frac{1}{4}$
c) $\frac{1}{8}$
d) $\frac{1}{9}$

10) If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value of n is:

a) 128
b) 248
c) 256
d) 264

11) $\int_{-\pi}^{\pi} |\pi - |x||dx$ is:

a) π^2
b) $\frac{\pi^2}{2}$
c) $\sqrt{2}\pi^2$
d) $2\pi^2$

12) Consider the two sets:
$A = \{m \in \mathbb{R} : \text{both the roots of } x^2 - (m+1)x + m + 4 = 0 \text{ are real}\}$ and $B = [-3, 5)$
Which of the following is not true?

a) $A - B = (-\infty, -3) \cup (5, \infty)$
b) $A \cap B = \{-3\}$
c) $B - A = (-3, 5)$
d) $A \cup B = \mathbb{R}$

13) The proposition $p \rightarrow \sim (p \wedge \sim q)$ is equivalent to:

a) $(\sim p) \vee (\sim q)$
b) $(\sim p) \wedge q$
c) q
d) $(\sim p) \vee q$

14) The function $f(x) = (3x - 7)x^{2/3}$, $x \in \mathbb{R}$, is increasing for all x lying in:

a) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$
b) $\left(-\infty, \frac{14}{15}\right)$
c) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$
d) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

15) If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15

terms, then the common difference of this A.P. is:

a) $\frac{1}{6}$
b) $\frac{1}{5}$
c) $\frac{1}{4}$
d) $\frac{1}{7}$