## AI24BTECH11012 - Pushkar Gudla

**Question:** The area of the region bounded by the curve  $x^2 = 4y$  and the straight line x = 4y - 2 is

- 3/8 sq units
  5/8 sq units
  7/8 sq units
  9/8 sq units

## **Solution:**

Variable	Description
g(x)	Equation of Conic
L	Equation of line
h	A point on line L
m	Direction vector of line L
$\mathbf{x_1}$ and $\mathbf{x_2}$	Points of intersection of $L$ and $g(x)$

TABLE 4: Variables and given data

$$g(x) = \mathbf{x}^{\mathsf{T}} \mathbf{V} \mathbf{x} + 2\mathbf{u}^{\mathsf{T}} \mathbf{x} + f = 0 \tag{4.1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{4.2}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{4.3}$$

$$f = 0 \tag{4.4}$$

$$L: \mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{4.5}$$

$$\mathbf{h} = \begin{pmatrix} -2\\0 \end{pmatrix} \tag{4.6}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \tag{4.7}$$

$$\mathbf{x_i} = \mathbf{h} + k_i \mathbf{m} \tag{4.8}$$

$$k_1 = \frac{1}{\mathbf{m}^{\mathsf{T}} \mathbf{V} \mathbf{m}} \left( -m^{\mathsf{T}} \left( \mathbf{V} \mathbf{h} + \mathbf{u} \right) + \sqrt{\left[ \mathbf{m}^{\mathsf{T}} \left( \mathbf{V} \mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h}) \left( \mathbf{m}^{\mathsf{T}} \mathbf{V} \mathbf{m} \right)} \right)$$
(4.9)

$$k_2 = \frac{1}{\mathbf{m}^{\top} \mathbf{V} \mathbf{m}} \left( -m^{\top} \left( \mathbf{V} \mathbf{h} + \mathbf{u} \right) - \sqrt{\left[ \mathbf{m}^{\top} \left( \mathbf{V} \mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h}) \left( \mathbf{m}^{\top} \mathbf{V} \mathbf{m} \right)} \right)$$
(4.10)

$$\mathbf{x_1} = \begin{pmatrix} 2\\1 \end{pmatrix} \tag{4.11}$$

$$\mathbf{x_2} = \begin{pmatrix} -1\\ \frac{1}{4} \end{pmatrix} \tag{4.12}$$

The area bounded by the curve  $y = \frac{x^2}{4}$  and line  $y = \frac{x}{4} + \frac{1}{2}$  is given by:

$$\int_{-1}^{2} \left( \frac{x}{4} + \frac{1}{2} - \frac{x^2}{4} \right) dx = \frac{7}{8}$$
 (4.13)

Hence, the area bounded by the curve and the line is  $\frac{7}{8}$  sq units.

