Matgeo Presentation

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Contents

Problem Statement

The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

Setup and Variable Definitions

Variable	**Description**
$g(x)$	Equation of Conic
L	Equation of line
$\mathbf{h}$	A point on line L
$\mathbf{m}$	Direction vector of line L
$\mathbf{x_1}$ and $\mathbf{x_2}$	Points of intersection of L and $g(x)$

Table: Variables and given data

Converting the equations to Matrix form

To simplify the calculation of points of intersection, the curve $x^2 = 4y$ is represented in standard conic matrix form:

$$g(x) = x^\top \mathbf{V} x + 2\mathbf{u}^\top x + f = 0 \tag{3.1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{3.2}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{3.3}$$

$$f = 0 \tag{3.4}$$

Parametric Form of the Line

$$L : \mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{3.5}$$

where k is a scalar constant

$$\mathbf{h} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{3.6}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \tag{3.7}$$

$$\mathbf{x_i} = \mathbf{h} + k_i\mathbf{m} \tag{3.8}$$

Finding Points of Intersection

On substituting L in $g(x)$ we get a quadratic equation in k. We find that the discriminant is not equal to zero and thus we get two values k_1 and k_2. The point obtained by substituting k_1 in L is $\mathbf{x_1}$ and similarly on substituting k_2 in L we get $\mathbf{x_2}$.

$$k_1 = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-m^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) + \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{3.9}$$

$$k_2 = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-m^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) - \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{3.10}$$

On solving for k_1 and k_2 we get $k_1 = 4$ and $k_2 = 1$. Thus

$$\mathbf{x_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{3.11}$$

$$\mathbf{x_2} = \begin{pmatrix} -1 \\ \frac{1}{4} \end{pmatrix} \tag{3.12}$$

Finding Area through Integration

The area between the curve and the line from $x = -1$ to $x = 2$ is calculated using the following integration:
Area $= \int_{-1}^{2} \left(\frac{x}{4} + \frac{1}{2} - \frac{x^2}{4} \right) dx$
Solving this integral, we get Area$=\frac{9}{8}$.
Hence, the area bound between the curve $x^2 = 4y$ and the line $x = 4y - 2$ is $\frac{9}{8}$.

C code to verify the Area

We can us the following code to verify that the area we found is correct:

https://github.com/GPushkar16/EE1030/blob/main/Matgeo/question-4/codes/area_between_curves.c

Generating Parabola and Plotting the Figure

We can use the following C code to generate points that lie on the parabola:

`https://github.com/GPushkar16/EE1030/blob/main/Matgeo/question-4/codes/generate_parabola.c`

We can then use the following Python code to generate the figure:

`https://github.com/GPushkar16/EE1030/blob/main/Matgeo/question-4/codes/plot.py`

Figure