22-07-2021 Shift-2(16-30)

AI24BTECH11012- Pushkar Gudla

1) The number of solutions of $\sin^7 x + \cos^7 x = 1$, where $x \in [0, 4\pi]$, is equal to:
   a) 11
   b) 7
   c) 5
   d) 9
2) If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to:
   a) $\frac{3}{2}$
   b) 2
   c) $\frac{1}{2}$
   d) 1
3) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as: $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , x \neq 0 \\ \alpha & , x = 0 \end{cases}$
   If f is continuous at $x = 0$, then α is equal to:
   a) 1
   b) 3
   c) 0
   d) 2
4) Let a line $L : 2x + y = k$, $k > 0$, be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to:
   a) 12
   b) -12
   c) 24
   d) -24
5) Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b$. Let E_2 be another ellipse such that it touches the endpoints of the major axis of E_1, and the foci of E_2 are the endpoints of the minor axis of E_1. If E_1 and E_2 have the same eccentricities, then its value is:
   a) $\frac{-1+\sqrt{5}}{2}$
   b) $\frac{-1+\sqrt{8}}{2}$
   c) $\frac{-1+\sqrt{3}}{2}$
   d) $\frac{-1+\sqrt{6}}{2}$
6) Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. The number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to
7) If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _____.
8) Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. The number of 3×3 matrices B with entries from the set $\{1, 2, 3, 4, 5\}$ and satisfying $AB = BA$ is equal to _____.

9) Consider the following frequency distribution:

Class:	$0-6$	$6-12$	$12-18$	$18-24$	$24-30$
Frequency:	a	b	12	9	5

If the mean is $\frac{309}{22}$ and the median is 14, then the value $(a-b)^2$ is equal to _____.

10) The sum of all the elements in the set $\{n \in \{1, 2, \ldots, 100\} \mid \text{H.C.F. of } n \text{ and } 2040 = 1\}$ is equal to _____.

11) The area (in square units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$, and $y^2 - 4x - 4 = 0$ in the upper half-plane is equal to _____.

12) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as:

$$f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } |x| \leq 2 \\ 0 & \text{if } |x| > 2 \end{cases}.$$

Let $g : \mathbb{R} \to \mathbb{R}$ be given by $g(x) = f(x+2) - f(x-2)$. If n and m denote the number of points in $\mathbb{R}$ where g is not continuous and not differentiable, respectively, then $n + m$ is equal to _____.

13) If the constant term in the binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _____.

14) Let $y = y(x)$ be the solution of the differential equation $\left((x+2)\, e^{\frac{y+1}{x+2}} + (y+1)\right) dx = (x+2)\, dy$, $y(1) = 1$. If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha + \beta|$ is equal to _____.

15) The number of elements in the set $\{n \in \{1, 2, 3, \ldots, 100\} \mid 11^n > 10^n + 9^n\}$ is _____.