Assignment-1

AI24BTECH11012- Pushkar Gudla

SECTION-A

F. MATCH THE FOLLOWING

1) Match the following:

a) $\int_0^{\frac{\pi}{2}} (\sin x)^{\cos x} \left(\cos x \cot x - \log (\sin x)^{\sin x} \right) dx$ a) 1

b) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ b) 0

c) Cosine of the angle of intersection of curves   c) $6 \ln(2)$
$y = 3^{x-1} \log x$ and $y = x^x - 1$ is   d) $\frac{4}{3}$

d) Let $\frac{dy}{dx} = \frac{6}{x+y}$ where $y(0) = 0$ then value of y
when $x + y = 6$ is

2) Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4x4 matrix given in the ORS.

Column I

a) $\int_{-1}^{1} \frac{dx}{1+x^2}$

b) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

c) $\int_2^3 \frac{dx}{1-x^2}$

d) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$

Column II

a) $\frac{1}{2} \log \left(\frac{2}{3} \right)$

b) $2 \log \left(\frac{2}{3} \right)$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{2}$

3)

List-I

a) The number of polynimials $f(x)$ with non-negative integer coeffecients of *degree* ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x) dx = 1$, is

b) The number of points in the interval $\left[-\sqrt{13}, \sqrt{13} \right]$ at which $f(x) = \sin x^2 + \cos x^2$ attains its maximum value is

c) $\int_{-2}^2 \frac{3x^2}{(1+e^x)} dx$ equals

d) $\dfrac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos 2x \log \left(\frac{1+x}{1-x} \right) dx \right)}{\left(\int_0^{\frac{1}{2}} \cos 2x \log \left(\frac{1+x}{1-x} \right) dx \right)}$

List-II

a) 8

b) 2

c) 4

d) 0

P Q R S

a) 3241

b) 3214

c) 2341

d) 2314

SECTION-B JEE MAIN/And the y-axis is:

a) The area (*insq.units*) of the region $\{(x, y) : y^2 \geq 2xand x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is:

[JEE M 2016]

   i) $\pi - \frac{4\sqrt{2}}{3}$

   ii) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

   iii) $\pi - \frac{4}{3}$

   iv) $\pi - \frac{8}{3}$

b) The area (*insq.units*) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y and y \leq 1 + \sqrt{x}\}$ is:

[JEE M 2017]

   i) $\frac{5}{2}$

   ii) $\frac{59}{12}$

   iii) $\frac{3}{2}$

   iv) $\frac{7}{3}$

c) The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to:

[JEE M 2017]

   i) -1

   ii) -2

   iii) 42

   iv) 4

d) Let $g(x) = \cos x^2, f(x) = \sqrt{x}, and \alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (*insq.units*) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta and y = 0$, is:

[JEE M 2018]

   i) $\frac{1}{2}\left(\sqrt{3} + 1\right)$

   ii) $\frac{1}{2}\left(\sqrt{3} - \sqrt{2}\right)$

   iii) $\frac{1}{2}\left(\sqrt{2} - 1\right)$

   iv) $\frac{1}{2}\left(\sqrt{3} - 1\right)$

e) The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx$ is:

[JEE M 2018]

   i) $\frac{\pi}{2}$

   ii) 4π

   iii) $\frac{\pi}{4}$

   iv) $\frac{\pi}{8}$

f) The value of $\int_0^{\pi} |\cos x|^3 dx$ is:

[JEE M 2019-9 Jan(M)]

   i) 0

   ii) $\frac{4}{3}$

   iii) $\frac{2}{3}$

   iv) $\frac{-2}{3}$

g) The area (*insq.units*) bounded by the parabola $y = x^2 - 1$, the tangent at the point $(2, 3)$ to it

   i) $\frac{8}{3}$

   ii) $\frac{32}{3}$

   iii) $\frac{56}{3}$

   iv) $\frac{14}{3}$

h) The value of $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x+\cos x}dx$ is:

[JEE M 2019-9 April(M)]

   i) $\frac{\pi-2}{8}$

   ii) $\frac{\pi-1}{4}$

   iii) $\frac{\pi-2}{4}$

   iv) $\frac{\pi-1}{2}$

i) The area (*insq.units*) of the region $A = \{(x, y) : x^2 \leq y \leq x + 2\}$ is:

[JEE M 2019-9 April(M)]

   i) $\frac{10}{3}$

   ii) $\frac{9}{2}$

   iii) $\frac{31}{6}$

   iv) $\frac{13}{6}$