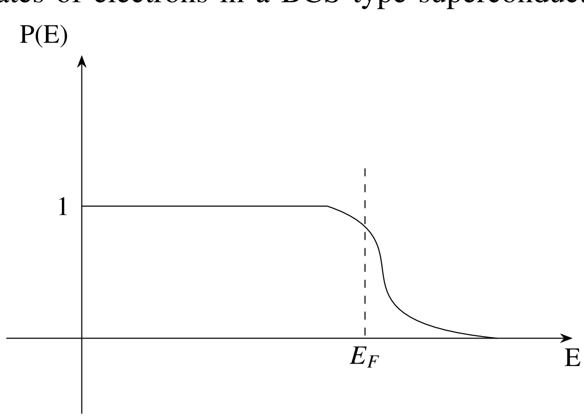
PH-2022 40-52

AI24BTECH11012- Pushkar Gudla

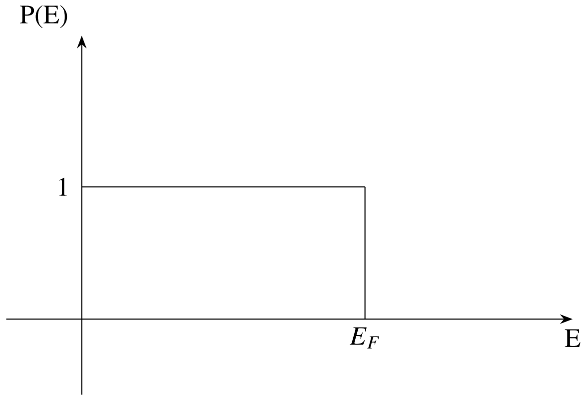
1) In cylindrical coordinates (s, φ, z), which of the following is a Hermitian operator?
   a) $\frac{1}{i}\frac{\partial}{\partial s}$
   b) $\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{s}\right)$
   c) $\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{2s}\right)$
   d) $\left(\frac{\partial}{\partial s} + \frac{1}{s}\right)$
2) A particle of mass 1 kg is released from a height of 1 m above the ground. When it reaches the ground, what is the value of Hamilton's action for this motion in J s? (g is the acceleration due to gravity; take gravitational potential to be zero on the ground)
   a) $-\frac{2}{3}\sqrt{2g}$
   b) $\frac{5}{3}\sqrt{2g}$
   c) $3\sqrt{2g}$
   d) $-\frac{1}{3}\sqrt{2g}$
3) If $(\dot{x}y + \alpha xy)$ is a constant of motion of a two-dimensional isotropic harmonic oscillator with Lagrangian

$$L = \frac{m(\dot{x}^2 + \dot{y}^2)}{2} - \frac{k(x^2 + y^2)}{2}$$

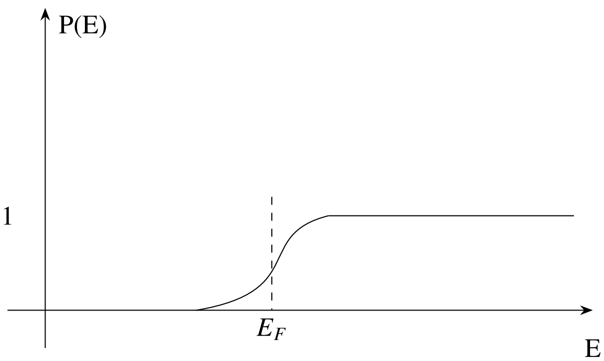
   then α is
   a) $+\frac{k}{m}$
   b) $-\frac{k}{m}$
   c) $-\frac{2k}{m}$
   d) 0
4) In a two-dimensional square lattice, frequency ω of phonons in the long wavelength limit changes linearly with the wave vector k. Then the density of states of phonons is proportional to
   a) ω
   b) ω^2
   c) $\sqrt{\omega}$
   d) $\frac{1}{\sqrt{\omega}}$
5) At $T = 0$K, which of the following diagram represents the occupation probability P(E) of energy states of electrons in a BCS type superconductor?



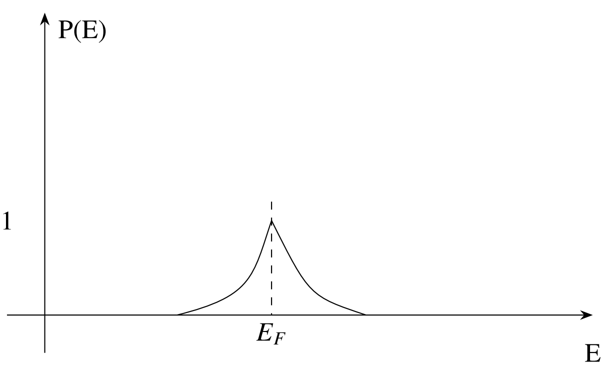
   a)



b)



c)



d)

6) For a one-dimensional harmonic oscillator, the creation operator ($a^\dagger$) acting on the nth state $|\psi_n\rangle$, where $n = 0, 1, 2, \ldots$, gives $a^\dagger|\psi_n\rangle = \sqrt{n+1}|\psi_{n+1}\rangle$. The matrix representation of the position operator

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$$

for the first three rows and columns is:

a) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{3} \end{pmatrix}$

b) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

c) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}$

d) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 1 & 0 & \sqrt{3} \\ 0 & 0 & 0 \\ \sqrt{3} & 0 & 1 \end{pmatrix}$

7) A piston of mass m is fitted to an airtight horizontal cylindrical jar. The cylinder and piston have identical unit area of cross-section. The gas inside the jar has volume V and is held at pressure $P = P_{\text{atmosphere}}$. The piston is pushed inside the jar very slowly over a small distance. On releasing, the piston performs an undamped simple harmonic motion of low frequency. Assuming that the gas is ideal and no heat is exchanged with the atmosphere, the frequency of the small oscillations is proportional to
   a) $\sqrt{\frac{P}{\gamma m V}}$
   b) $\sqrt{\frac{P\gamma}{Vm}}$
   c) $\sqrt{\frac{P}{mV^{\gamma-1}}}$
   d) $\sqrt{\frac{\gamma P}{mV^{\gamma-1}}}$
8) A paramagnetic salt of mass m is held at temperature T in a magnetic field H. If S is the entropy of the salt and M is its magnetization, then $dG = -S\,dT - M dH$, where G is the Gibbs free energy. If the magnetic field is changed adiabatically by $\Delta H \to 0$ and the corresponding infinitesimal changes in entropy and temperature are ΔS and ΔT, then which of the following statements are correct
   a) $\Delta S = -\frac{1}{T}\left(\frac{\partial G}{\partial T}\right)_H \Delta T$
   b) $\Delta S = 0$
   c) $\Delta T = \left(\frac{\partial M}{\partial T}\right)_H \Delta H$
   d) $\Delta T = 0$
9) A particle of mass m is moving inside a hollow spherical shell of radius a so that the potential is

$$V(r) = \begin{cases} 0 & \text{for } r < a \\ \infty & \text{for } r \geq a \end{cases}$$

   The ground state energy and wavefunction of the particle are E_0 and $R(r)$, respectively. Then which of the following options are correct?
   a) $E_0 = \frac{\hbar^2\pi^2}{2ma^2}$
   b) $-\frac{\hbar^2}{2mr^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = E_0 R \quad (r < a)$
   c) $-\frac{\hbar^2}{2mr^2}\frac{d^2R}{dr^2} = E_0 R \quad (r < a)$
   d) $R(r) = \frac{1}{r}\sin\left(\frac{\pi r}{a}\right) \quad (r < a)$
10) A particle of unit mass moves in a potential $V(r) = -V_0 e^{-r^2}$. If the angular momentum of the particle is $L = 0.5\sqrt{V_0}$, then which of the following statements are true?
   a) There are two equilibrium points along the radial coordinate
   b) There is one stable equilibrium point at r_1 and one unstable equilibrium point at $r_2 > r_1$
   c) There are two stable equilibrium points along the radial coordinate
   d) There is only one equilibrium point along the radial coordinate
11) In a diatomic molecule of mass M, electronic, rotational and vibrational energy scales are of magnitude E_e, E_R, and E_V, respectively. The spring constant for the vibrational energy is determined by E_e. If the electron mass is m then
   a) $E_R \sim \frac{m}{M}E_e$
   b) $E_R \sim \sqrt{\frac{m}{M}}E_e$
   c) $E_V \sim \sqrt{\frac{m}{M}}E_e$
   d) $E_V \sim \left(\frac{m}{M}\right)^{1/4} E_e$
12) Electronic specific heat of a solid at temperature T is $C = \gamma T$, where γ is a constant related to the thermal effective mass (m_{eff}) of the electrons. Then which of the following statements are correct?
   a) $\gamma \propto m_{\text{eff}}$

b) m_{eff} is greater than free electron mass for all solids
c) Temperature dependence of C depends on the dimensionality of the solid
d) The linear temperature dependence of C is observed at $T \ll$ Debye temperature

13) In a Hall effect experiment on an intrinsic semiconductor, which of the following statements are correct?
a) Hall voltage is always zero
b) Hall voltage is negative if the effective mass of holes is larger than those of electrons
c) Hall coefficient can be used to estimate the carrier concentration in the semiconductor
d) Hall voltage depends on the mobility of the carriers