Assignment-1

AI24BTECH11012- Pushkar Gudla

SECTION-A

F. MATCH THE FOLLOWING

1) Match the following:

<div align="right">(2006-6M)</div>

a) $\int_0^{\frac{\pi}{2}} (\sin x)^{\cos x} \left(\cos x \cot x - \log (\sin x)^{\sin x}\right) dx$ a) 1

b) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ b) 0

c) Cosine of the angle of intersection of curves   c) $6\ln(2)$
   $y = 3^{x-1} \log x$ and $y = x^x - 1$ is   d) $\frac{4}{3}$

d) Let $\frac{dy}{dx} = \frac{6}{x+y}$ where $y(0) = 0$ then value of y
   when $x + y = 6$ is

2) Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening
   the appropriate bubbles in the $4x4$ matrix given in the ORS.

<div align="right">(2007-6M)</div>

Column I

a) $\int_{-1}^{1} \frac{dx}{1+x^2}$

b) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

c) $\int_2^3 \frac{dx}{1-x^2}$

d) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$

Column II

a) $\frac{1}{2} \log\left(\frac{2}{3}\right)$

b) $2 \log\left(\frac{2}{3}\right)$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{2}$

3)

<div align="right">(JEE Adv. 2014)</div>

List-I

a) The number of polynimials $f(x)$ with non-
   negative integer coeffecients of degree ≤ 2,
   satisfying $f(0) = 0$ and $\int_0^1 f(x)\,dx = 1$, is

b) The number of points in the interval
   $\left(-\sqrt{13}, \sqrt{13}\right)$ at which $f(x) = \sin x^2 +$
   $\cos x^2$ attains its maximum value is

c) $\int_{-2}^2 \frac{3x^2}{(1+e^x)} dx$ equals

d) $\dfrac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_0^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}$

List-II

a) 8

b) 2

c) 4

d) 0

P Q R S

a) 3241

b) 3214

c) 2341

d) 2314

SECTION-B JEE MAIN/AIEEE

1) The area (*insq.units*) of the region $(x, y) : y^2 \geq 2x$ and $x^2 + y^2 \leq 4x, x \geq 0, y \geq 0$ is:

[JEE M 2016]

 a) $\pi - \frac{4\sqrt{2}}{3}$
 b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$
 c) $\pi - \frac{4}{3}$
 d) $\pi - \frac{8}{3}$

2) The area (*insq.units*) of the region $(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y$ and $y \leq 1 + \sqrt{x}$ is:

[JEE M 2017]

 a) $\frac{5}{2}$
 b) $\frac{59}{12}$
 c) $\frac{3}{2}$
 d) $\frac{7}{3}$

3) The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to:

[JEE M 2017]

 a) -1
 b) -2
 c) 42
 d) 4

4) Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (*insq.units*) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is:

[JEE M 2018]

 a) $\frac{1}{2}\left(\sqrt{3} + 1\right)$
 b) $\frac{1}{2}\left(\sqrt{3} - \sqrt{2}\right)$
 c) $\frac{1}{2}\left(\sqrt{2} - 1\right)$
 d) $\frac{1}{2}\left(\sqrt{3} - 1\right)$

5) The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is:

[JEE M 2018]

 a) $\frac{\pi}{2}$
 b) 4π
 c) $\frac{\pi}{4}$
 d) $\frac{\pi}{8}$

6) The value of $\int_0^{\pi} |\cos x|^3 \, dx$ is:

[JEE M 2019-9 Jan(M)]

 a) 0
 b) $\frac{4}{3}$
 c) $\frac{2}{3}$
 d) $\frac{-2}{3}$

7) The area (*insq.units*) bounded by the parabola $y = x^2 - 1$, the tangent at the point $(2, 3)$ to it and the y-axis is:

[JEE M 2019-9Jan(M)]

 a) $\frac{8}{3}$
 b) $\frac{32}{3}$
 c) $\frac{56}{3}$
 d) $\frac{14}{3}$

8) The value of $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x + \cos x} dx$ is:

  a) $\frac{\pi - 2}{8}$
  b) $\frac{\pi - 1}{4}$
  c) $\frac{\pi - 2}{4}$
  d) $\frac{\pi - 1}{2}$

9) The area (*insq.units*) of the region $A = \{(x, y) : x^2 \leq y \leq x + 2\}$ is:

  a) $\frac{10}{3}$
  b) $\frac{9}{2}$
  c) $\frac{31}{6}$
  d) $\frac{13}{6}$