30-01-2023 shift-2(16-20)

AI24BTECH11012- Pushkar Gudla

1) If a plane passes through the points (-1, k, 0), (2, k, -1), (1, 1, 2) and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is
   a) $\frac{17}{5}$
   b) $\frac{5}{17}$
   c) $\frac{6}{13}$
   d) $\frac{13}{6}$
2) Let a, b, $c > 1$, a^3, b^3, and c^3 be in A.P., and $\log_a b$, $\log_c a$, and $\log_b c$ be in G.P. If the sum of the first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a+8b-c}{10}$, is -444, then abc is equal to
   a) 343
   b) 216
   c) $\frac{343}{8}$
   d) $\frac{125}{8}$
3) Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots, a_{100}$ is 25. Then S is
   a) ϕ
   b) $\{99\}$
   c) $\mathbb{N}$
   d) $\{9\}$
4) $\lim_{n\to\infty} \frac{3}{n}\left(4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \ldots + \left(3 - \frac{1}{n}\right)^2\right)$ is equal to
   a) 12
   b) $\frac{19}{3}$
   c) 0
   d) 19
5) For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations
   $x - y + z = 5$
   $2x + 2y + \alpha z = 8$
   $3x - y + 4z = \beta$
   has infinitely many solutions. Then α and β are the roots of
   a) $2x^2 - 10x + 16 = 0$
   b) $2x^2 + 18x + 56 = 0$
   c) $2x^2 - 18x + 56 = 0$
   d) $2x^2 + 14x + 24 = 0$
6) The 50th root of a number x is 12 and the 50th root of another number y is 18. Then the remainder obtained on dividing $x + y$ by 25 is ____________.
7) Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is ____________.
8) Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ____________.

9) The 8th common term of the series
$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots$
$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots$
is ______________.

10) Let a line L pass through the point $P(2, 3, 1)$ and be parallel to the line $x+3y-2z-2 = 0 = x-y+2z$. If the distance of L from the point $(5, 3, 8)$ is α, then $3\alpha^2$ is equal to ______________.

11) $\int \sqrt{\sec 2x - 1}\,dx = \alpha \log \left|\cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta} x\right)}\right| + C$, then $\beta - \alpha$ is equal to ______________.

12) If the value of the real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$, then β is equal to ______________.

13) The number of seven-digit odd numbers that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is ______________.

14) A bag contains six balls of different colors. Two balls are drawn in succession with replacement. The probability that both balls are of the same color is p. Next, four balls are drawn in succession with replacement, and the probability that exactly three balls are of the same color is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to ______________.

15) Let A be the area of the region $\{(x, y) : y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$. Then $540A$ is equal to ______________.