9-9.2-39

AI24BTECH11012 - Pushkar Gudla

Question: The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

1) $\frac{3}{8}$ sq units
2) $\frac{5}{8}$ sq units
3) $\frac{7}{8}$ sq units
4) $\frac{9}{8}$ sq units

Solution:

Variable	Description
$g(x)$	Equation of Conic
L	Equation of line
$\mathbf{h}$	A point on line L
$\mathbf{m}$	Direction vector of line L
$\mathbf{x_1}$ and $\mathbf{x_2}$	Points of intersection of L and $g(x)$

TABLE 4: Variables and given data

$$g(x) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{4.1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{4.2}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{4.3}$$

$$f = 0 \tag{4.4}$$

$$L : \mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{4.5}$$

$$\mathbf{h} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{4.6}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \tag{4.7}$$

$$\mathbf{x_i} = \mathbf{h} + k_i\mathbf{m} \tag{4.8}$$

$$k_1 = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-m^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) + \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{4.9}$$

$$k_2 = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-m^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) - \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{4.10}$$

$$\mathbf{x_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{4.11}$$

$$\mathbf{x_2} = \begin{pmatrix} -1 \\ \frac{1}{4} \end{pmatrix} \tag{4.12}$$

The area bounded by the curve $y = \frac{x^2}{4}$ and line $y = \frac{x}{4} + \frac{1}{2}$ is given by:

$$\int_{-1}^{2} \left(\frac{x}{4} + \frac{1}{2} - \frac{x^2}{4} \right) dx = \frac{9}{8} \tag{4.13}$$

Hence, the area bounded by the curve and the line is $\frac{9}{8}$ sq units.