

# 22-07-2021 Shift-2

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- 1) The number of solutions of  $\sin^7 x + \cos^7 x = 1$ , where  $x \in [0, 4\pi]$ , is equal to:
  - a) 11
  - b) 7
  - c) 5
  - d) 9
- 2) If the domain of the function  $f(x) = \frac{\cos^{-1} \sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}(\frac{2x-1}{2})}}$  is the interval  $(\alpha, \beta]$ , then  $\alpha + \beta$  is equal to:
  - a)  $\frac{3}{2}$
  - b) 2
  - c)  $\frac{1}{2}$
  - d) 1
- 3) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined as:  $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e \left( \frac{1+2xe^{-2x}}{(1-xe^{-x})^2} \right) & , x \neq 0 \\ \alpha & , x = 0 \end{cases}$   
 If  $f$  is continuous at  $x = 0$ , then  $\alpha$  is equal to:
  - a) 1
  - b) 3
  - c) 0
  - d) 2
- 4) Let a line  $L : 2x + y = k$ ,  $k > 0$ , be a tangent to the hyperbola  $x^2 - y^2 = 3$ . If  $L$  is also a tangent to the parabola  $y^2 = \alpha x$ , then  $\alpha$  is equal to:
  - a) 12
  - b) -12
  - c) 24
  - d) -24
- 5) Let  $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$ . Let  $E_2$  be another ellipse such that it touches the endpoints of the major axis of  $E_1$ , and the foci of  $E_2$  are the endpoints of the minor axis of  $E_1$ . If  $E_1$  and  $E_2$  have the same eccentricities, then its value is:
  - a)  $\frac{-1+\sqrt{5}}{2}$
  - b)  $\frac{-1+\sqrt{8}}{2}$
  - c)  $\frac{-1+\sqrt{3}}{2}$
  - d)  $\frac{-1+\sqrt{6}}{2}$
- 6) Let  $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ . The number of bijective functions  $f : A \rightarrow A$  such that  $f(1) + f(2) = 3 - f(3)$  is equal to
- 7) If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to \_\_\_\_\_.
- 8) Let  $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ . The number of  $3 \times 3$  matrices  $B$  with entries from the set  $\{1, 2, 3, 4, 5\}$  and satisfying  $AB = BA$  is equal to \_\_\_\_\_.

9) Consider the following frequency distribution:

**Class:**     0 – 6   6 – 12   12 – 18   18 – 24   24 – 30

**Frequency:**      $a$          $b$         12        9        5

If the mean is  $\frac{309}{22}$  and the median is 14, then the value  $(a - b)^2$  is equal to \_\_\_\_\_.

10) The sum of all the elements in the set  $\{n \in \{1, 2, \dots, 100\} \mid \text{H.C.F. of } n \text{ and } 2040 = 1\}$  is equal to \_\_\_\_\_.

11) The area (in square units) of the region bounded by the curves  $x^2 + 2y - 1 = 0$ ,  $y^2 + 4x - 4 = 0$ , and  $y^2 - 4x - 4 = 0$  in the upper half-plane is equal to \_\_\_\_\_.

12) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined as:

$$f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } |x| \leq 2 \\ 0 & \text{if } |x| > 2 \end{cases}.$$

Let  $g : \mathbb{R} \rightarrow \mathbb{R}$  be given by  $g(x) = f(x + 2) - f(x - 2)$ . If  $n$  and  $m$  denote the number of points in  $\mathbb{R}$  where  $g$  is not continuous and not differentiable, respectively, then  $n + m$  is equal to \_\_\_\_\_.

13) If the constant term in the binomial expansion of  $\left(2x^r + \frac{1}{x^2}\right)^{10}$  is 180, then  $r$  is equal to \_\_\_\_\_.

14) Let  $y = y(x)$  be the solution of the differential equation  $\left((x + 2)e^{\frac{y+1}{x+2}} + (y + 1)\right)dx = (x + 2)dy$ ,  $y(1) = 1$ . If the domain of  $y = y(x)$  is an open interval  $(\alpha, \beta)$ , then  $|\alpha + \beta|$  is equal to \_\_\_\_\_.

15) The number of elements in the set  $\{n \in \{1, 2, 3, \dots, 100\} \mid 11^n > 10^n + 9^n\}$  is \_\_\_\_\_.