# 25-07-2021 Shift-1

AI24BTECH11012- Pushkar Gudla

1) If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is:
   a) $\frac{33}{2^{32}}$
   b) $\frac{33}{2^{29}}$
   c) $\frac{33}{2^{28}}$
   d) $\frac{33}{2^{27}}$
2) If the numbers appeared on the two throws of a fair six-faced die are $\alpha$ and $\beta$, then the probability that $x^2 + \alpha x + \beta > 0$ for all $x \in \mathbb{R}$ is:
   a) $\frac{17}{36}$
   b) $\frac{4}{9}$
   c) $\frac{1}{2}$
   d) $\frac{19}{36}$
3) The number of solutions of $|\cos x| = \sin x$ such that $-4\pi \leq x \leq 4\pi$ is:
   a) 4
   b) 6
   c) 8
   d) 12
4) A tower $PQ$ stands on a horizontal ground with base $Q$ on the ground. The point $R$ divides the tower in two parts such that $QR = 15\,m$. If from a point $A$ on the ground the angle of elevation of $R$ is $60°$ and the part $PR$ of the tower subtends an angle of $15°$ at $A$, then the height of the tower is:
   a) $5(2\sqrt{3} + 3)$m
   b) $5(\sqrt{3} + 3)$m
   c) $10(\sqrt{3} + 1)$m
   d) $10(2\sqrt{3} + 1)$m
5) Which of the following statements is a tautology?
   a) $((\sim p) \vee q) \implies p$
   b) $p \implies ((\sim p) \vee q)$
   c) $((\sim p) \vee q) \implies q$
   d) $q \implies ((\sim p) \vee q)$
6) Let $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ and $B = A - I$. If $\omega = \frac{\sqrt{3}i-1}{2}$, then the number of elements in the set $\{n \in \{1, 2, ..., 100\} : A^n + (\omega B)^n = A + B\}$ is equal to ____________.
7) The letters of the word "MANKIND" are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word "MANKIND" is ____________.
8) If the maximum value of the term independent of $t$ in the expansion of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is $K$, then $8K$ is equal to ____________.
9) Let $a, b$ be two non-zero real numbers. If $p$ and $r$ are the roots of the equation $x^2 - 8ax + 2a = 0$ and $q$ and $s$ are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to ____________

10) Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n b_n$ is equal to ______________.

11) Let $f(x) = \begin{cases} |4x^2 - 8x + 5| & \text{, if } 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5] & \text{, if } 8x^2 - 6x + 1 < 0 \end{cases}$
where $[\alpha]$ denotes the greatest integer less than or equal to $\alpha$. Then the number of points in $\mathbb{R}$ where $f$ is not differentiable is ______________.

12) If $\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}}[(nk+1) + (nk+2) + \ldots + (nk+n)] = 33 \lim_{n\to\infty} \frac{1}{n^{k+1}}[1^k + 2^k + 3^k + \ldots + n^k]$, then the integral value of $k$ is equal to ______________.

13) Let the equation of two diameters of a circle $x^2+y^2-2x+2fy+1 = 0$ be $2px-y = 1$ and $2x+py = 4p$. Then the slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2 - y^2 = 3$ passing through the center of the circle is equal to ______________.

14) The sum of diameters of the circles that touch (i) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\frac{8}{5}, \frac{6}{5}\right)$ and (ii) the y-axis, is equal to ______________.

15) The line of shortest distance between the lines $\frac{x-2}{0} = \frac{y-1}{1} = \frac{z}{1}$ and $\frac{x-3}{2} = \frac{y-5}{2} = \frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane $P : ax - y - z = 0$ $(a > 0)$. If the image of the point $(1, 1, -5)$ in the plane $P$ is $(\alpha, \beta, \gamma)$, then $\alpha + \beta - \gamma$ is equal to ______________.