Assignment-1

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SECTION-A

F. Match the Following

1) Match the following: (2006-6M)

(A) $\int_0^{\frac{\pi}{2}} (\sin x)^{\cos x} \left(\cos x \cot x - \log (\sin x)^{\sin x}\right) dx$ (p) 1

(B) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ (q) 0

(C) Cosine of the angle of intersection of curves   (r) 6ln2
     $y = 3^{x-1} \log x$ and $y = x^x - 1$ is   (s) $\frac{4}{3}$

(D) Let $\frac{dy}{dx} = \frac{6}{x+y}$ where $y(0) = 0$ then value of y
     when $x + y = 6$ is

2) Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4x4 matrix given in the ORS. (2007-6M)

Column I

(A) $\int_{-1}^{1} \frac{dx}{1+x^2}$

(B) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(C) $\int_2^3 \frac{dx}{1-x^2}$

(D) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$

Column II

(p) $\frac{1}{2} \log \left(\frac{2}{3}\right)$

(q) $2 \log \left(\frac{2}{3}\right)$

(r) $\frac{\pi}{3}$

(s) $\frac{\pi}{2}$

3) (JEE Adv. 2014) **List-I**

P. The number of polynimials $f(x)$ with non-negative integer coeffecients of *degree* ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x) dx = 1$, is

Q. The number of points in the interval $\left[-\sqrt{13}, \sqrt{13}\right]$ at which $f(x) = \sin x^2 + \cos x^2$ attains its maximum value is

R. $\int_{-2}^2 \frac{3x^2}{(1+e^x)} dx$ equals

S. $\dfrac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_0^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}$

List-II

a) 8
b) 2
c) 4
d) 0

    P Q R S

(a) 3 2 4 1

(c) 3 2 1 4

    P Q R S

(b) 2 3 4 1

(a) 2 3 1 4

SECTION-B JEE MAIN/AIEEE

1) The area (*insq.units*) of the region $\{(x, y) :$ $y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is: [JEE M 2016]

   a) $\pi - \frac{4\sqrt{2}}{3}$
   b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$
   c) $\pi - \frac{4}{3}$
   d) $\pi - \frac{8}{3}$

2) The area (*insq.units*) of the region $\{(x, y) :$ $x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is: [JEE M 2017]

   a) $\frac{5}{2}$
   b) $\frac{59}{12}$
   c) $\frac{3}{2}$
   d) $\frac{7}{3}$

3) The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to: [JEE M 2017]

   a) -1
   b) -2
   c) 42
   d) 4

4) Let $g(x) = \cos x^2, f(x) = \sqrt{x}, \text{ and } \alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (*insq.units*) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta \text{ and } y = 0$, is:[JEE M 2018]

   a) $\frac{1}{2}\left(\sqrt{3} + 1\right)$
   b) $\frac{1}{2}\left(\sqrt{3} - \sqrt{2}\right)$
   c) $\frac{1}{2}\left(\sqrt{2} - 1\right)$
   d) $\frac{1}{2}\left(\sqrt{3} - 1\right)$

5) The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is: [JEE M 2018]

   a) $\frac{\pi}{2}$
   b) 4π
   c) $\frac{\pi}{4}$
   d) $\frac{\pi}{8}$

6) The value of $\int_0^{\pi} |\cos x|^3 \, dx$ is: [JEE M 2019-9 Jan(M)]

   a) 0
   b) $\frac{4}{3}$
   c) $\frac{2}{3}$
   d) $\frac{-2}{3}$

7) The area (*insq.units*) bounded by the parabola $y = x^2 - 1$, the tangent at the point $(2, 3)$ to it and the y-axis is: [JEE M 2019-9Jan(M)]

   a) $\frac{8}{3}$
   b) $\frac{32}{3}$
   c) $\frac{56}{3}$
   d) $\frac{14}{3}$

8) The value of $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x + \cos x} dx$ is: [JEE M 2019-9 April(M)]

   a) $\frac{\pi-2}{8}$
   b) $\frac{\pi-1}{4}$
   c) $\frac{\pi-2}{4}$
   d) $\frac{\pi-1}{2}$

9) The area (*insq.units*) of the region $A = \{(x, y) : x^2 \leq y \leq x + 2\}$ is: [JEE M 2019-9 April(M)]

   a) $\frac{10}{3}$
   b) $\frac{9}{2}$
   c) $\frac{31}{6}$
   d) $\frac{13}{6}$