# 08-04-2024 shift-2(1-15)

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1) If the image of the point (-4, 5) in the line $x + 2y = 2$ lies on the circle $(x + 4)^2 + (y - 3)^2 = r^2$, then $r$ is equal to:
   a) 1
   b) 2
   c) 75
   d) 3
2) Let $\overrightarrow{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$, $\overrightarrow{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ be three vectors. Let $\overrightarrow{r}$ be a unit vector along $\overrightarrow{b} + \overrightarrow{c}$. If $\overrightarrow{r} \cdot \overrightarrow{a} = 3$, then $3\lambda$ is equal to:
   a) 27
   b) 25
   c) 15
   d) 21
3) If $\alpha \neq a$, $\beta \neq b$, $\gamma \neq c$, and $\begin{vmatrix} \alpha & \beta & 1 \\ a & b & 1 \\ c & c & 0 \end{vmatrix} = 0$, then $\frac{a}{\alpha - a} + \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c}$ is equal to:
   a) 2
   b) 3
   c) 0
   d) 1
4) In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4th, 6th, and 8th terms is:
   a) 96
   b) 78
   c) 91
   d) 84
5) The number of ways five alphabets can be chosen from the alphabets of the word ”MATHEMATICS”, where the chosen alphabets are not necessarily distinct, is:
   a) 175
   b) 181
   c) 177
   d) 179
6) The sum of all possible values of $\theta \in [-\pi, 2\pi]$ for which $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is purely imaginary is equal to:
   a) $2\pi$
   b) $3\pi$
   c) $5\pi$
   d) $4\pi$
7) If the system of equations $x + 4y - z = \lambda$, $7x + 9y + \mu z = -3$, $5x + y + 2z = -1$ has infinitely many solutions, then $2\mu + 3\lambda$ is equal to:
   a) 2
   b) $-3$
   c) 3

d) $-2$

8) If the shortest distance between the lines $\frac{x-\lambda}{2} = \frac{y-4}{3} = \frac{z-3}{4}$ and $\frac{x-2}{4} = \frac{y-4}{6} = \frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of $\lambda$ is:
   a) $-\frac{13}{25}$
   b) $\frac{13}{25}$
   c) 1
   d) $-1$

9) If the value of $\frac{3\cos 36^\circ + 5\sin 18^\circ}{5\cos 36^\circ - 3\sin 18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$, where $a$, $b$, $c$ are natural numbers and $\gcd(a, c) = 1$, then $a + b + c$ is equal to:
   a) 50
   b) 40
   c) 52
   d) 54

10) Let $y = y(x)$ be the solution curve of the differential equation $\sec y \frac{dy}{dx} + 2x \sin y = x^3 \cos y$, $y(1) = 0$. Then $y(\sqrt{3})$ is equal to:
   a) $\frac{\pi}{3}$
   b) $\frac{\pi}{6}$
   c) $\frac{\pi}{4}$
   d) $\frac{\pi}{12}$

11) The area of the region in the first quadrant inside the circle $x^2 + y^2 = 8$ and outside the parabola $y^2 = 2x$ is equal to:
   a) $\frac{\pi}{2} - \frac{1}{3}$
   b) $\pi - \frac{2}{3}$
   c) $\frac{\pi}{2} - \frac{2}{3}$
   d) $\pi - \frac{1}{3}$

12) If the line segment joining the points $(5, 2)$ and $(2, a)$ subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of $a$ is:
   a) 6
   b) 8
   c) 2
   d) 4

13) Let $\overrightarrow{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{b} = 11\hat{i} - \hat{j} + \hat{k}$, and $\overrightarrow{c}$ be a vector such that $(\overrightarrow{a} + \overrightarrow{b}) \times \overrightarrow{c} = \overrightarrow{c} \times (2\overrightarrow{a} + 3\overrightarrow{b})$. If $2\overrightarrow{a} + 3\overrightarrow{b} \cdot \overrightarrow{c} = 1670$, then $2|\overrightarrow{c}|^2$ is equal to:
   a) 1627
   b) 1618
   c) 1600
   d) 1609

14) If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ has a local maximum at $x = \alpha$ and a local minimum at $x = \alpha^2$, then $\alpha$ and $\alpha^2$ are the roots of the equation:
   a) $x^2 - 6x + 8 = 0$
   b) $8x^2 + 6x - 8 = 0$
   c) $8x^2 - 6x + 1 = 0$
   d) $x^2 + 6x + 8 = 0$

15) There are three bags X, Y, and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins, and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random, and a coin drawn from it at random is found to be a one-rupee coin. Then the probability that it came from bag Y is:

a) $\frac{1}{3}$
b) $\frac{1}{2}$
c) $\frac{1}{4}$
d) $\frac{5}{12}$