Symmetric Numeric Transformation Systems:

A Novel Mathematical Discovery

Featuring the Tri-Set Numeric Interaction System & Number Mirror Symmetry

Abstract

This research introduces two innovative numeric transformation systems with unique properties:

- **Tri-Set Numeric Interaction System:** A framework organizing numbers into three sets with distinct combination rules
- **Number Mirror Symmetry:** A system rooted in modular arithmetic that reveals mirror relationships preserving digital roots

Both systems reveal intricate relationships between numeric sets and demonstrate unexpected computational behaviors and structural symmetries.

Introduction

Mathematical exploration often uncovers surprising patterns in numeric interactions.

- This research presents two independently discovered numeric transformation systems
- Both systems diverge from conventional computation methods
- They showcase the depth of number theory and modular arithmetic
- Potential applications include cryptography, algorithmic design, and mathematical education

Tri-Set Numeric Interaction System

Set Composition

Set 1	Set 2	Set 3
[1, 4, 7]	[2, 5, 8]	[3, 6, 9]

System 1 organizes single-digit numbers into three sets with distinct interaction rules when numbers are combined through addition.

Tri-Set Numeric Interaction: Rules

Intra-Set Interactions

When numbers within Set 1 or Set 2 are added, the result transforms to a number in the complementary set.

- Set 1: 1 + 4 = 5 (a Set 2 number)
- Set 2: 2 + 5 = 7 (a Set 1 number)

Inter-Set Interactions

When numbers from different sets (Set 1 and Set 2) are added, the result always belongs to Set 3.

- 1 + 2 = 3 (Set 3)
- 4 + 5 = 9 (Set 3)
- 7 + 8 = 15 → 1 + 5 = 6 (Set 3)

Tri-Set System: Set 3 Unique Behavior

Set 3 Interactions

- Adding two Set 3 numbers results in another Set 3 number:
  - 3 + 6 = 9 (Set 3)
  - 6 + 9 = 15 → 1 + 5 = 6 (Set 3)
- Adding a Set 3 number to a Set 1 number returns a Set 1 number:
  - 3 + 1 = 4 (Set 1)
  - 9 + 7 = 16 → 1 + 6 = 7 (Set 1)
- Adding a Set 3 number to a Set 2 number returns a Set 2 number:
  - 3 + 2 = 5 (Set 2)
  - 9 + 8 = 17 → 1 + 7 = 8 (Set 2)

Tri-Set System: Digit Reduction Rule

Multi-digit sums are reduced to a single digit by summing their digits (computing the digital root):

- This aligns with the number modulo 9, except multiples of 9 remain 9
- The digital root ensures all results stay within the single-digit framework

Examples

Operation	Result	After Reduction	Set Membership
1 + 4	5	5	Set 2
2 + 8	10	1 + 0 = 1	Set 1
1 + 2	3	3	Set 3
3 + 6	9	9	Set 3

Tri-Set System: Additional Operations

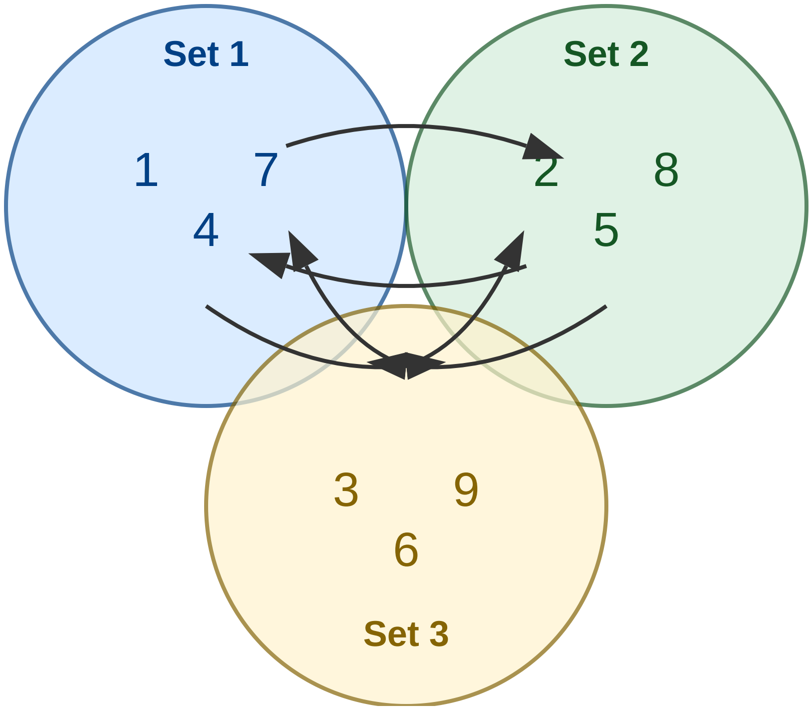
The patterns in System 1 extend beyond addition to other arithmetic operations:

Operation	Pattern Example	Result	Set Mapping
Subtraction	Set 1 - Set 1	7 - 4 = 3	→ Set 3
	Set 1 - Set 2	7 - 2 = 5	→ Set 2
	Set 3 - Set 1	6 - 1 = 5	→ Set 2
Multiplication	Set 1 × Set 1	4 × 7 = 28 → 2+8 = 10 → 1	→ Set 1
	Set 2 × Set 2	5 × 5 = 25 → 2+5 = 7	→ Set 1
	Set 3 × Set 3	3 × 6 = 18 → 1+8 = 9	→ Set 3
Division	Set 3 ÷ Set 3	9 ÷ 3 = 3	→ Set 3
	Set 1 ÷ Set 1	7 ÷ 7 = 1	→ Set 1

Mathematical Foundation: These patterns emerge because the sets align with modulo 3 arithmetic:

- Set 1 numbers all equal 1 mod 3 (remainder 1 when divided by 3)
- Set 2 numbers all equal 2 mod 3 (remainder 2 when divided by 3)
- Set 3 numbers all equal 0 mod 3 (divisible by 3)

Tri-Set System: Visual Representation



The diagram illustrates the transformation rules between the three sets.

Number Mirror Symmetry: Overview

Basic Pattern

System 2 defines a symmetric relationship where each single-digit number has a corresponding "mirror" number, with 9 acting as a pivotal element in the symmetry.

- Each single-digit number x (from 0 to 9) has a corresponding "mirror" number y
- This system uses modular arithmetic to establish the mirror relationship
- When either a number or its mirror is added to any starting number, the resulting sums have the same digital root

Number Mirror Symmetry: Pattern Characteristics

Mirror Definition

For each single-digit number x, its mirror y is defined such that:

$$y \equiv x \pmod{9}$$

This ensures that y and x produce the same effect on the digital root when added to any starting number.

Chosen Representatives

- $x = 0$, $y = 9$
- $x = 1$ to 8, $y = x - 9$ (yielding -8 to -1)
- $x = 9$, $y = 0$

These choices satisfy the condition $y \equiv x \pmod{9}$

Example: $9 \equiv 0 \pmod{9}$, $-8 \equiv 1 \pmod{9}$, etc.

Number Mirror Symmetry: Practical Examples

Key Function: Digital Root Preservation

Adding x or its mirror y to any starting number results in sums with the same digital root, due to their congruence modulo 9.

Example 1: Starting number 25

- Add x = 1:
  - 25 + 1 = 26
  - Digital root = 2 + 6 = **8**
- Add y = -8 (mirror of 1):
  - 25 + (-8) = 17
  - Digital root = 1 + 7 = **8**

Example 2: Starting number 25

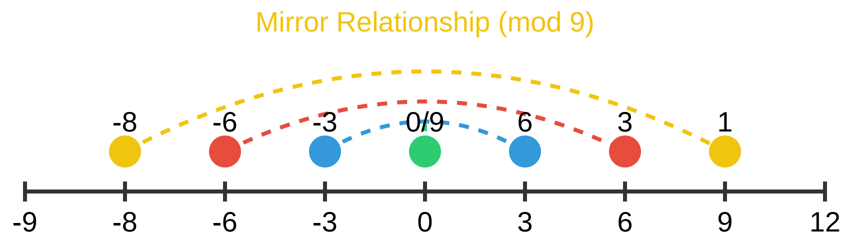
- Add x = 6:
  - 25 + 6 = 31
  - Digital root = 3 + 1 = **4**
- Add y = -3 (mirror of 6):
  - 25 + (-3) = 22
  - Digital root = 2 + 2 = **4**

In each case, the digital root remains consistent, illustrating the system's core symmetry.

Number Mirror Symmetry: Original Numbers and Mirrors

Original (x)	Mirror (y)	Modular Relationship
0	9	$9 \equiv 0 \pmod 9$
1	-8	$-8 \equiv 1 \pmod 9$
2	-7	$-7 \equiv 2 \pmod 9$
3	-6	$-6 \equiv 3 \pmod 9$
4	-5	$-5 \equiv 4 \pmod 9$
5	-4	$-4 \equiv 5 \pmod 9$
6	-3	$-3 \equiv 6 \pmod 9$
7	-2	$-2 \equiv 7 \pmod 9$
8	-1	$-1 \equiv 8 \pmod 9$
9	0	$0 \equiv 9 \pmod 9$

Number Mirror Symmetry: Visual Representation



Both members of each pair yield the same digital root
when added to any starting number

Addendum: Median Calculation in Number Mirror Symmetry

A "median" value for each original number x can be calculated using the formula:

$$z = (x + 5) \pmod 9$$

With the result mapped to 9 when the modulus is 0.

This creates ordered pairs (x, z) where x is the original number and z is its median value.

Calculation Examples

- For pair (0, 5): $0 + 5 = 5 \pmod 9$
- For pair (1, 6): $1 + 5 = 6 \pmod 9$
- For pair (4, 9): $4 + 5 = 9 \pmod 9$
- For pair (5, 1): $5 + 5 = 10 \equiv 1 \pmod 9$
- For pair (9, 5): $9 + 5 = 14 \equiv 5 \pmod 9$

Addendum: Median Values Table

Original (x)	Median (z)	Ordered Pair (x,z)	Calculation
0	5	(0,5)	0 + 5 = 5
1	6	(1,6)	1 + 5 = 6
2	7	(2,7)	2 + 5 = 7
3	8	(3,8)	3 + 5 = 8
4	9	(4,9)	4 + 5 = 9
5	1	(5,1)	5 + 5 = 10 ≡ 1 (mod 9)
6	2	(6,2)	6 + 5 = 11 ≡ 2 (mod 9)
7	3	(7,3)	7 + 5 = 12 ≡ 3 (mod 9)
8	4	(8,4)	8 + 5 = 13 ≡ 4 (mod 9)
9	5	(9,5)	9 + 5 = 14 ≡ 5 (mod 9)

Mathematical Significance

These systems reveal non-traditional numeric interaction rules with symmetries grounded in modular arithmetic:

- The **Tri-Set Numeric Interaction System** structure reflects modulo 3 properties
- The **Number Mirror Symmetry** system emphasizes digital root preservation via congruence modulo 9

Potential Applications

- Number theory (exploring cyclic patterns)
- Cryptographic modeling (modular-based algorithms)
- Algorithmic design (efficient computation methods)
- Mathematical education (illustrating modular arithmetic)

Conclusion

The numeric transformation systems introduced here highlight complex, symmetrical interactions that enrich our understanding of mathematical structures:

- The **Tri-Set Numeric Interaction System** provides a framework for understanding how numbers transfer between specific groups
- The **Number Mirror Symmetry** system demonstrates how different numbers can produce equivalent effects on digital roots
- Both systems provide a foundation for future research and practical applications
- Further exploration could reveal additional patterns and properties within these frameworks