CNOEC PROJECT

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Team

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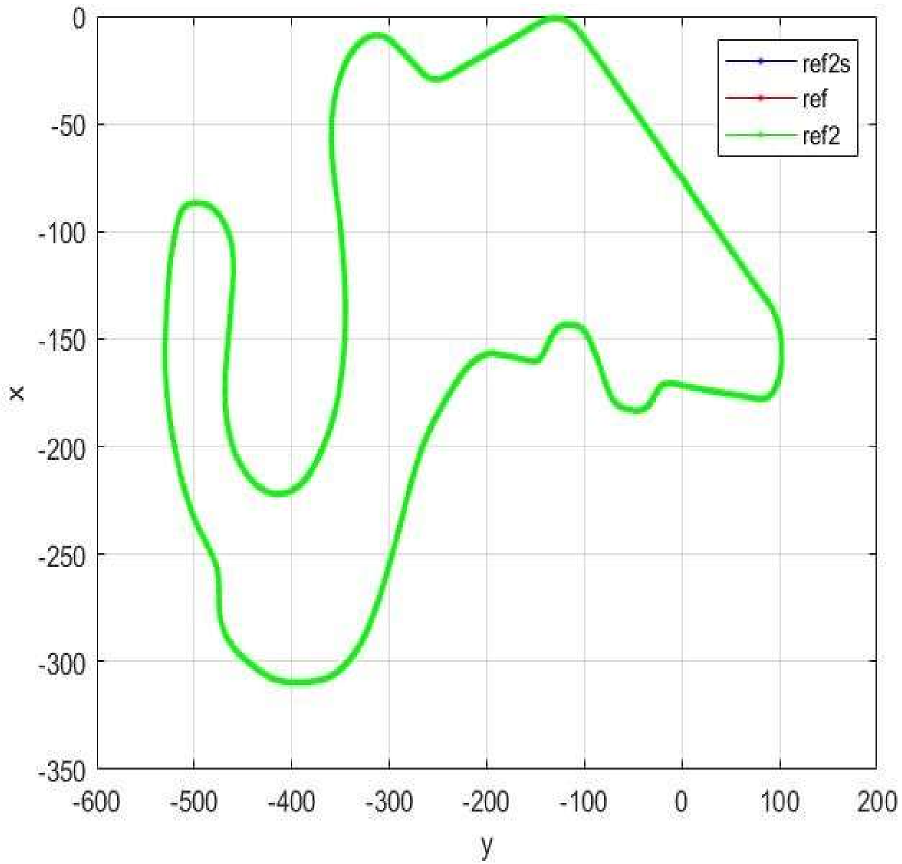
B.S.: Industrial Automation (UniBS)

M.Sc: Automation and Control engineering (PoliMi)

Project : qualitative goal

Goal: route optimization along a track using a drone.



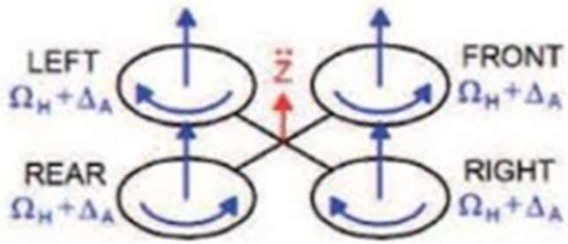
Project : reason why we chose the specific topic

Passions and interests about autonomous vechicles and Flight

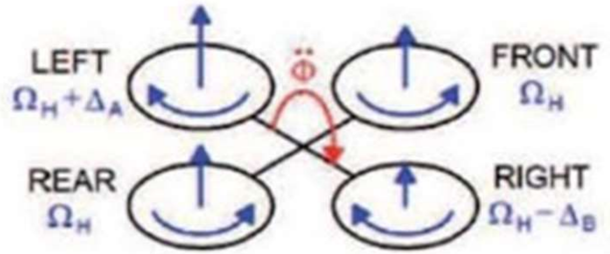
Flight physics

It has 6 dof , but it uses 4 actuators, and by varying the rotational speeds of them it can reach the desired maouvres

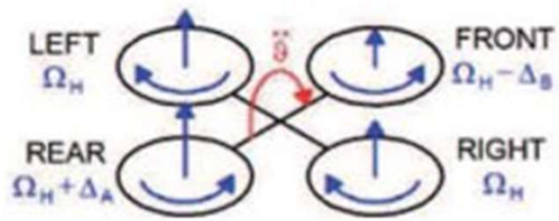
Traslation along z



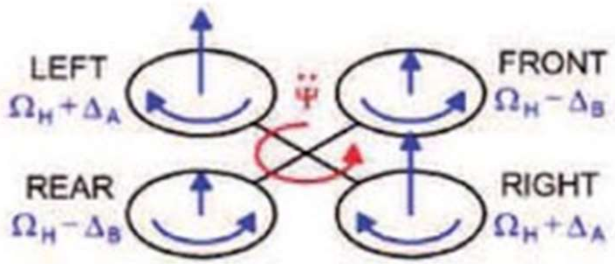
Roll with translation along x



Pitch with translation along y

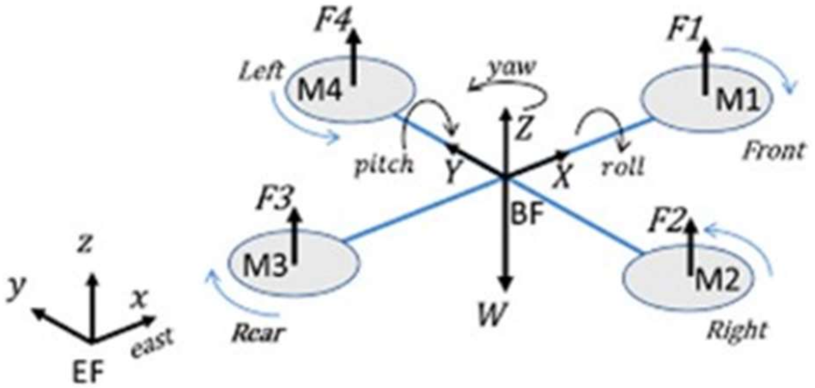


yaw

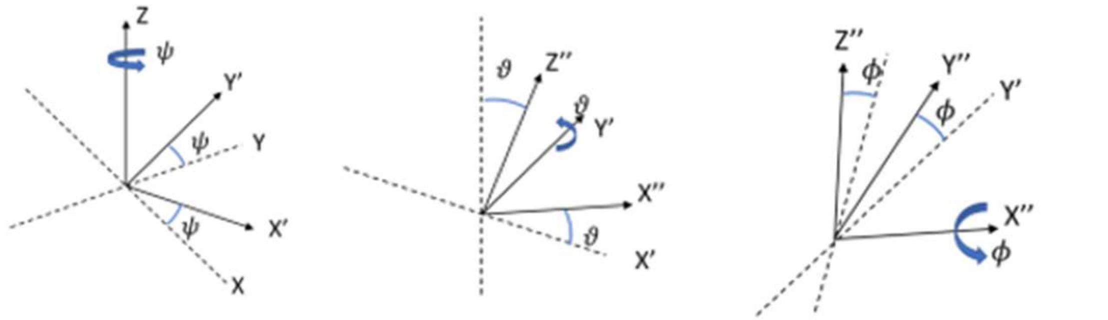


Kinematics of the system

Description of the quadrotor movement wrt a fixed reference frame called Earth, through three rotations along the roll,pitch , Yaw axes. And for the angular velocities I am going to consider the transfer matrix.



$R_Z(\psi)$ $\qquad$ $R_{Y'}(\vartheta)$ $\qquad$ $R_{X''}(\vartheta)$



Resulting rotation:

$$R_{BW} = \begin{bmatrix} c_\psi c_\vartheta & c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\psi s_\phi + c_\psi c_\phi s_\vartheta \\ c_\vartheta s_\psi & c_\phi c_\psi + s_\psi s_\vartheta s_\phi & c_\phi s_\vartheta s_\psi - c_\psi s_\phi \\ -s_\vartheta & c_\vartheta s_\phi & c_\vartheta c_\phi \end{bmatrix}$$

Kinematics relationship s:

$$\begin{cases} \dot{x}_W = c_\psi c_\vartheta \dot{x}_B + (c_\psi s_\theta s_\phi - s_\psi c_\phi)\dot{y}_B + (s_\psi s_\phi + c_\psi c_\phi s_\vartheta)\dot{z}_B \\ \dot{y}_W = c_\vartheta s_\psi \dot{x}_B + (c_\phi c_\psi + s_\psi s_\vartheta s_\phi)\dot{y}_B + (c_\phi s_\vartheta s_\psi - c_\psi s_\phi)\dot{z}_B \\ \dot{z}_W = -s_\vartheta \dot{x}_B + c_\vartheta s_\phi \dot{y}_B + c_\vartheta c_\phi \dot{z}_B \end{cases}$$

Matrice di trasferimento

$$T = \begin{bmatrix} s_\vartheta & 0 & 1 \\ -c_\vartheta s_\phi & c_\phi & 0 \\ c_\phi c_\vartheta & s_\phi & 0 \end{bmatrix}$$

$$\begin{cases} \dot{\psi} = \dfrac{-s_\phi}{c_\vartheta} q + r \\ \dot{\vartheta} = c_\phi q + s_\phi r \\ \dot{\phi} = p + s_\phi t_\theta q - c_\phi t_\vartheta r \end{cases}$$

Dynamics of the system

Newton-Euler quadcopter motion's equations , substituting le kinematic variables found in the slide before, and applying forces Torques generated bythe rotation of the propellers.

$$m\ddot{P} = \sum F_i \qquad m(\ddot{p} + \dot{o} \times \dot{p}) = R_{WB}\begin{bmatrix}0\\0\\-mg\end{bmatrix} + \begin{bmatrix}0\\0\\T\end{bmatrix} \qquad \begin{cases}\ddot{x}_B = g s_\vartheta + r\dot{y}_B - q\dot{z}_B \\ \ddot{y}_B = -g c_\vartheta s_\phi + p\dot{z}_B - r\dot{x}_B \\ \ddot{z}_B = -g c_\vartheta c_\phi + \dfrac{T}{m} + q\dot{x}_B - p\dot{y}_B\end{cases}$$

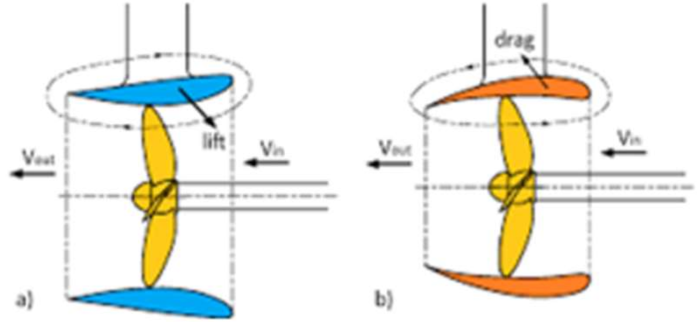
$$J\ddot{O} = \sum \tau_i \qquad J\ddot{o} = \begin{bmatrix}\tau_{roll}\\ \tau_{pitch}\\ \tau_{yaw}\end{bmatrix} - \dot{o} \times J\dot{o} \qquad \begin{cases}\dot{p} = \dfrac{\tau_{roll}}{J_x} - \dfrac{(J_z - J_y)}{J_x} rq \\ \dot{q} = \dfrac{\tau_{pitch}}{J_y} - \dfrac{(J_x - J_z)}{J_y} pr \\ \dot{r} = \dfrac{\tau_{yaw}}{J_z} - \dfrac{(J_y - J_x)}{J_z} pq\end{cases}$$

Aerodynamics of the propellers and definition of the inputs

Definition of forces and torques generated by a propeller in rotation around its axis, at constant speed in the air:

ThrustEffect:

$$T_i = C_T \rho A_{ri} r_i^2 \omega_i^2 = c_T\ \omega_i^2$$



A_{ri} : disk's area formed by the propeller rotating

r_i : propeller's radius

ω_i : propeller's rotatio speed

C_T : coefficient dependent from the propeller

ρ : air density

ϕ_i : angle between arm and x axis

DragEffect:

$$Q_i = c_Q \omega_i^2$$

$c_T = k$: coefficient(Thrust);

$d = l$: distance of the motor from the centre;

$c_Q = b$: coefficient(Drag);

$$T = \sum_{i=1}^{n} T_i$$

$$\tau_{roll} = \sum_{i=1}^{n} d_i \sin\phi_i\, |T_i|$$

$$\tau_{pitch} = \sum_{i=1}^{n} d_i \cos\phi_i\, |T_i|$$

$$\tau_{yaw} = \sum_{i=1}^{n} d_i \sigma_i |Q_i|$$

Matrix approximation for a quadcopter

$$\begin{bmatrix} T \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix} = \begin{bmatrix} c_T & c_T & c_T & c_T \\ 0 & dc_T & 0 & -dc_T \\ -dc_T & 0 & dc_T & 0 \\ -c_Q & c_Q & -c_Q & c_Q \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$

Non linear mathematical model

$$
\begin{cases}
\dot{x}_W = \cos\psi\cos\theta\dot{x}_B + (\cos\psi\sin\theta\sin\theta - \sin\psi\cos\phi)\dot{y}_B + (\sin\psi\sin\phi + \cos\psi\cos\phi\cos\theta)\dot{z}_B \\
\dot{y}_W = \cos\theta\sin\theta\dot{x}_B + (\cos\phi\cos\psi + \sin\psi\sin\phi\sin\theta)\dot{y}_B + (\sin\theta\cos\phi\sin\psi - \cos\psi\sin\phi)\dot{z}_B \\
\dot{z}_W = -\sin\theta\dot{x}_B - \cos\theta\sin\phi\dot{y}_B + \cos\theta\cos\phi\dot{z}_B \\
\dot{\psi} = \dfrac{-\sin\phi}{\cos\phi}q + r \\
\dot{\theta} = \cos\phi q + \sin\phi r \\
\dot{\phi} = p + \sin\phi\tan\theta q - \cos\phi\tan\theta r \\
\ddot{x}_B = g\sin\theta + r\dot{y}_B - q\dot{z}_B \\
\ddot{y}_B = -g\sin\phi\cos theta - r\dot{y}_B + p\dot{z}_B \\
\ddot{z}_B = -g\cos theta\cos phi + \dfrac{T}{m} + q\dot{x}_B - p\dot{y}_B \\
\dot{r} = \dfrac{\tau_\psi}{J_z} - \dfrac{J_y - J_x}{J_z}pq \\
\dot{q} = \dfrac{\tau_\theta}{J_y} - \dfrac{J_x - J_z}{J_y}pr \\
\dot{p} = \dfrac{\tau_\phi}{J_x} - \dfrac{J_z - J_y}{J_x}qr
\end{cases}
$$

The input space given by T and τ_ψ, τ_θ, τ_ϕ is decomposed into single rotor thrusts f=[f_1,f_2,f_3,f_4]
f_i is the thrust at rotor i=1,2,3,4.

$$
\begin{aligned}
T &= \sum f_i \\
\tau_\psi &= c_\tau(-f_1 + f_2 - f_3 + f_4) \\
\tau_\theta &= (-f_1 + f_3) \\
\tau_\phi &= l(f_2 - f_4)
\end{aligned}
$$

with the quadrotor's arm length l and the rotor's torque constant c_τ.

Linearization around Hovering position

$$
\begin{cases}
\dot{x}_W = \dot{x}_B \\
\dot{y}_W = \dot{y}_B \\
\dot{z}_W = \dot{z}_B \\
\dot{\psi} = r \\
\dot{\theta} = q \\
\dot{\phi} = p \\
\ddot{x}_B = g\theta \\
\ddot{y}_B = -g\phi \\
\ddot{z}_B = \dfrac{(f_1 + f_2 + f_3 + f_4)}{m} \\
\dot{r} = \dfrac{-f1 + f2 - f3 + f4}{J_z} \\
\dot{q} = \dfrac{-f1 + f3}{J_y} \\
\dot{p} = \dfrac{f2 - f4}{J_x}
\end{cases}
$$

Augmented dynamics

$$x = \begin{bmatrix} p & q & v & w & f & \boxed{\theta \quad v_\theta} \end{bmatrix}^T$$

$$u = \begin{bmatrix} \Delta f & \Delta v_\theta \end{bmatrix}^T$$

$$f_{k+1} = f_k + \Delta f_k \Delta t$$

$$\theta_{k+1} = \theta_k + \Delta\theta_k \Delta t$$

$$v_{\theta,k+1} = v_{\theta,k} + \Delta v_{\theta,k} \Delta t$$

$$A = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\frac{c_t}{c_q}}{J_z} & \frac{\frac{c_t}{c_q}}{J_z} & -\frac{\frac{c_t}{c_q}}{J_z} & \frac{\frac{c_t}{c_q}}{J_z} & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{l}{J_y} & 0 & -\frac{l}{J_y} & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{l}{J_x} & 0 & \frac{l}{J_x} & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & &
\end{bmatrix}
\quad
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

$$C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & &
\end{bmatrix}
\quad
D = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Additional elements for optimization

Circuit's Spline interpolation

Initial value of of θ for the θ_k interval

$$p_d(\theta_k) = \begin{cases} \rho_0(\theta_k)\text{with} & 0 \leq \theta_k \leq \theta_0 \\ \rho_1(\theta_k)\text{with} & \boxed{\theta_1} \leq \theta_k \leq \theta_2 \\ \dots & \\ \rho_{P-1}(\theta_k)\text{with} & \theta_{P-1} \leq \theta_k \leq \theta_P \end{cases}$$

$$\boxed{\theta_{k_{in}}(\theta_k)}$$

For 2-D point $p_d(\theta_k) = [x_d(\theta_k), y_d(\theta_k)]'$

$$x_d(\theta_k) = a(\theta_k)\left(\theta_k - \theta_{k_{in}}(\theta_k)\right)^3 + b(\theta_k)\left(\theta_k - \theta_{k_{in}}(\theta_k)\right)^2 + c(\theta_k)(\theta_k - \theta_{k_{in}}) + d(\theta_k)$$

$$y_d(\theta_k) = a(\theta_k)\left(\theta_k - \theta_{k_{in}}(\theta_k)\right)^3 + b(\theta_k)\left(\theta_k - \theta_{k_{in}}(\theta_k)\right)^2 + c(\theta_k)(\theta_k - \theta_{k_{in}}) + d(\theta_k)$$

Spline coefficients

$a(\theta_k), b(\theta_k), c(\theta_k), d(\theta_k)$

Optimization problems developed

- FHOCP single shooting with linear model and quad cost
- FHOCP multiple shooting with linear model and quad cost
- FHOCP single shooting with linear model and non linear cost 1° method
- FHOCP multiple shooting with linear model and non linear cost 1° method
- FHOCP single shooting with linear model and non linear cost 2° method
- FHOCP multiple shooting with linear model and non linear cost 2° method
- MPCC single shooting with linear model and non linear cost 1° method

Quadratic cost

The cost function choosen is a quadratic one in order to have a convex problem, in which is guaranteed to have a minimum in the optimization phase, and so a feasible solution. In this problem formulation we have the cost function composed by the first two terms which penalizes the difference between the coordinates desired and the actual ones, the cost on ω necessary for the stability of the solution since it forces the solver to keep low body rates whenever possible.

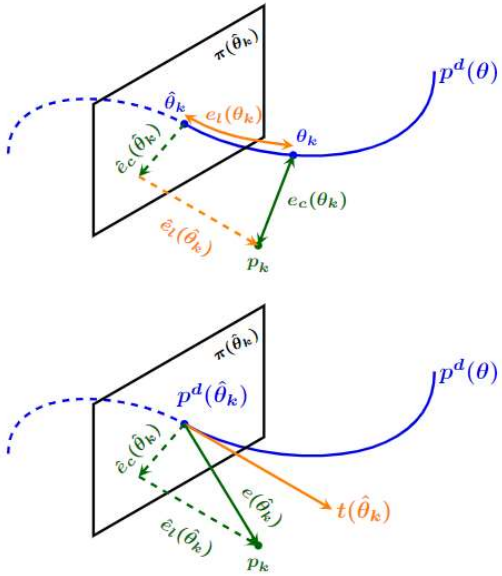
$$\|y_{ref} - y\|^2_{qy} + \|x_{ref} - x\|^2_{qx} + \|\omega_k\|^2_{q\omega} + \|\Delta v_{\theta_k}\|^2_{r\Delta v} + \|\Delta f_k\|^2_{r\Delta f} - \mu v_{\theta_k}$$

Non Linear cost

Let the position error at time k be $e(\theta_k) = \|p_k - p^d(\theta_k)\|$.Consider the tangent line of the desired path,$t(\theta_k) \in R^3$. $e(\theta_k)$ can be decomposed in a component that is projected onto $t(\theta_k)$,$e_l(\theta_k)$, and a component contained in $\pi(\theta_k)$,which is $e_c(\theta_k)$.

$$e_c(\theta_k) = \begin{bmatrix} 1 - t_x^2(\theta_k) & -t_x(\theta_k)t_y(\theta_k) & -t_x(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_y(\theta_k) & 1 - t_y^2(\theta_k) & -t_y(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_z(\theta_k) & -t_y(\theta_k)t_z(\theta_k) & 1 - t_z^2(\theta_k) \end{bmatrix} e(\theta_k) \tag{23}$$

$$\boldsymbol{e}(\theta_k) = \boldsymbol{p}_k - \boldsymbol{p}^d(\theta_k) \qquad e_l(\theta_k) = e(\theta_k) - e_c(\theta_k)$$



$$\boldsymbol{t}(\theta_k) = \frac{\partial \boldsymbol{p}^d(\theta_k)}{\partial \theta_k} = \begin{bmatrix} \frac{\partial x^d(\theta_k)}{\partial \theta_k} & \frac{\partial y^d(\theta_k)}{\partial \theta_k} & \frac{\partial z^d(\theta_k)}{\partial \theta_k} \end{bmatrix}^T$$

$$\|e_c(\theta_k)\|_{qc}^2 + \|e_l(\theta_k)\|_{ql}^2 + \|\omega_k\|_{q\omega}^2 + \|\Delta v_{\theta_k}\|_{r\Delta v}^2 + \|\Delta f_k\|_{r\Delta f}^2 - \mu v_{\theta_k}$$

Solution Algorithm

At each iteration of the SQP algorithm, a QP is solved to obtain search directions p_k in the space of primal variables (i.e. the decision variable x) and of the Lagrange multipliers λ_k, μ_k respectively for equality and inequality constraints).

$$
\begin{aligned}
&\min_{p_k} \nabla_x f(x^k)^T p^k + \frac{1}{2} p^{kT} H^k p^k \\
&\text{subj. to} \quad \nabla_x g(x^k)^T p^k + g(x^k) = 0 \\
&\qquad\qquad\quad \nabla_x h(x^k)^T p^k + h(x^k) \geq 0
\end{aligned}
\tag{27}
$$

- Lagrangian gradients $\nabla_x L(x^{k+1}, \lambda^{k+1}, \mu^{k+1})$, $\nabla_x L(x^k, \lambda^{k+1}, \mu^{k+1})$ then compute

$$
\begin{aligned}
y = \nabla_x L(x^{k+1}, \lambda^{k+1}, \mu^{k+1}) - \nabla_x L(x^k, \lambda^{k+1}, \mu^{k+1}) \\
s = x^{k+1} - x^k
\end{aligned}
$$

- Compute $H^{k+1} = H^k - \frac{H^k s s^T H^k}{s^T H^k s} + \frac{y y^T}{s^T y}$

After the QP has been solved,the update rule is performed:

$$
\begin{aligned}
x_{k+1} &= x_k + t_k p_k \\
\lambda_{k+1} &= \lambda_k + t_k \Delta\lambda_k \\
\mu_{k+1} &= \mu_k + t_k \Delta\mu_k
\end{aligned}
$$

FHOCP single shooting with linear model and quad cost

$$\min_{U} \sum_{i=0}^{N} \|y_{ref} - y\|^2_{qy} + \|x_{ref} - x\|^2_{qx} + \|\omega_k\|^2_{q\omega} + \|\Delta v_{\theta_k}\|^2_{r\Delta v} + \|\Delta f_k\|^2_{r\Delta f} - \mu v_{\theta_k}$$

$$z(i+1|t) = Az(i|t) + Bu(i|t), i = 0, ..., N-1$$

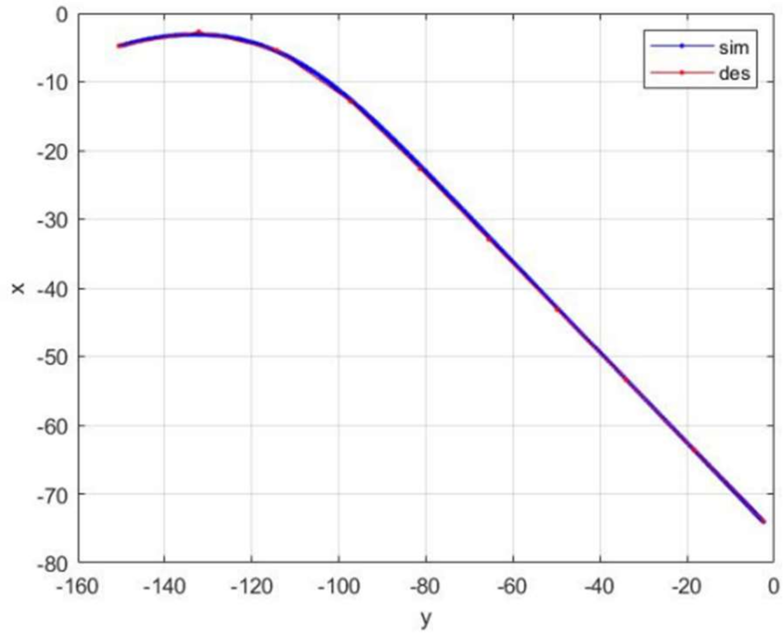
$$y(i|t) = Cz(i|t) + Du(i|t), i = 0, ..., N$$

subj. to

$$\Delta v_{\theta min} \leq \Delta v_{\theta} \leq \Delta v_{\theta max}$$

$$\Delta f_{min} \leq \Delta f \leq \Delta f_{max}$$

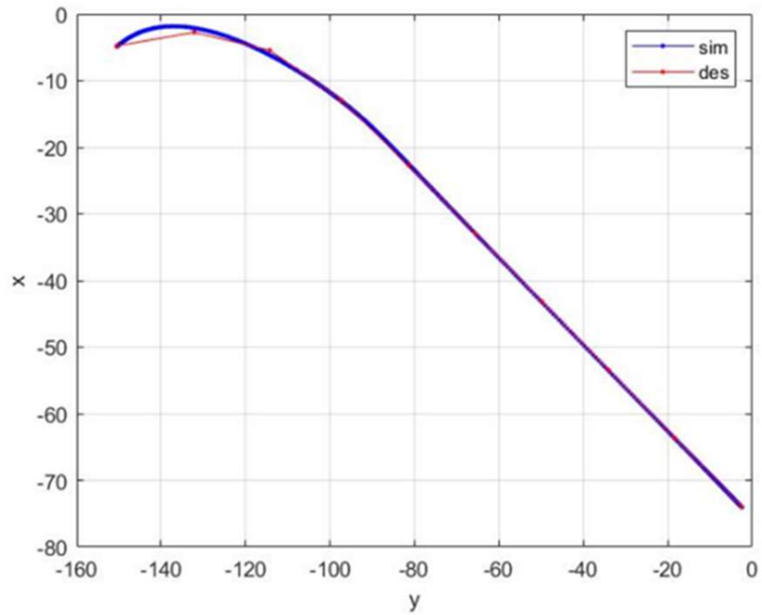
$$z(0|t) = z_0$$



FHOCP multiple shooting with linear model and quad cost

$$\min_{U,X} \sum_{i=0}^{N} \|y_{ref} - y\|_{qy}^2 + \|x_{ref} - x\|_{qx}^2 + \|\omega_k\|_{q\omega}^2 + \|\Delta v_{\theta_k}\|_{r\Delta v}^2 + \|\Delta f_k\|_{r\Delta f}^2 - \mu v_{\theta_k}$$

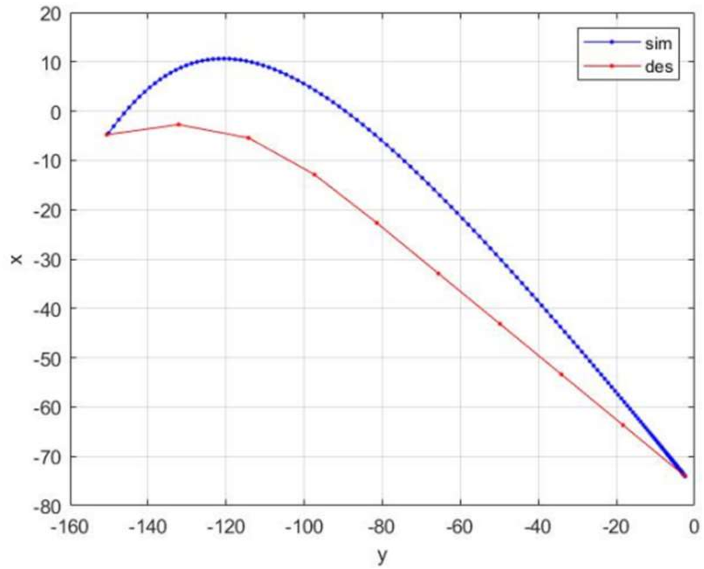
$$\text{subj. to} \quad \begin{aligned} & z(i+1|t) = Az(i|t) + Bu(i|t), i = 0, ..., N-1 \\ & y(i|t) = Cz(i|t) + Du(i|t), i = 0, ..., N \\ & \boxed{\begin{aligned} & \omega_{min} \leq \omega \leq \omega_{max} \\ & f_{min} \leq f \leq f_{max} \\ & 0 \leq v_\theta \leq v_{\theta max} \end{aligned}} \\ & \Delta v_{\theta min} \leq \Delta v_\theta \leq \Delta v_{\theta max} \\ & \Delta f_{min} \leq \Delta f \leq \Delta f_{max} \\ & z(0|t) = z_0 \end{aligned}$$



FHOCP single shooting with linear model and non linear cost 1° method

$$\boxed{\min_U \sum_{i=0}^{N} \|e_c(\theta_k)\|^2_{qc} + \|e_l(\theta_k)\|^2_{ql} + \|\omega_k\|^2_{q\omega} + \|\Delta v_{\theta_k}\|^2_{r\Delta v} + \|\Delta f_k\|^2_{r\Delta f} - \mu v_{\theta_k}}$$

$$\text{subj. to}\quad \begin{aligned} & z(i+1|t) = Az(i|t) + Bu(i|t), i = 0, ..., N-1 \\ & y(i|t) = Cz(i|t) + Du(i|t), i = 0, ..., N \\ & \Delta v_{\theta min} \le \Delta v_\theta \le \Delta v_{\theta max} \\ & \Delta f_{min} \le \Delta f \le \Delta f_{max} \\ & z(0|t) = z_0 \end{aligned}$$



$$e_c(\theta_k) = \begin{bmatrix} 1 - t_x^2(\theta_k) & -t_x(\theta_k)t_y(\theta_k) & -t_x(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_y(\theta_k) & 1 - t_y^2(\theta_k) & -t_y(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_z(\theta_k) & -t_y(\theta_k)t_z(\theta_k) & 1 - t_z^2(\theta_k) \end{bmatrix} e(\theta_k)$$

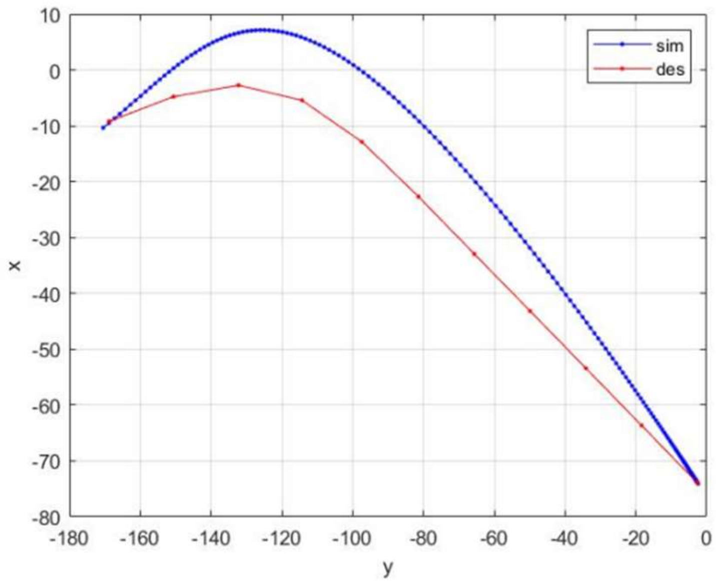
$$e_l(\theta_k) = e(\theta_k) - e_c(\theta_k)$$

$$t_y = \frac{(y_d(\theta_k) - y_k)}{\|(y_d(\theta_k) - y_k)\|_2} \qquad t_x = \frac{(x_d(\theta_k) - x_k)}{\|(x_d(\theta_k) - x_k)\|_2}$$

FHOCP multiple shooting with linear model and non linear cost 1° method

$$\min_U \sum_{i=0}^{N} \|e_c(\theta_k)\|^2_{qc} + \|e_l(\theta_k)\|^2_{ql} + \|\omega_k\|^2_{q\omega} + \|\Delta v_{\theta_k}\|^2_{r\Delta v} + \|\Delta f_k\|^2_{r\Delta f} - \mu v_{\theta_k}$$

$$\text{subj. to} \quad \begin{aligned} & z(i+1|t) = Az(i|t) + Bu(i|t), i = 0, ..., N-1 \\ & y(i|t) = Cz(i|t) + Du(i|t), i = 0, ..., N \\ & \boxed{\begin{aligned} & \omega_{min} \le \omega \le \omega_{max} \\ & f_{min} \le f \le f_{max} \\ & 0 \le v_\theta \le v_{\theta max} \end{aligned}} \\ & \Delta v_{\theta min} \le \Delta v_\theta \le \Delta v_{\theta max} \\ & \Delta f_{min} \le \Delta f \le \Delta f_{max} \\ & z(0|t) = z_0 \end{aligned}$$



$$e_c(\theta_k) = \begin{bmatrix} 1 - t_x^2(\theta_k) & -t_x(\theta_k)t_y(\theta_k) & -t_x(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_y(\theta_k) & 1 - t_y^2(\theta_k) & -t_y(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_z(\theta_k) & -t_y(\theta_k)t_z(\theta_k) & 1 - t_z^2(\theta_k) \end{bmatrix} e(\theta_k)$$

$$e_l(\theta_k) = e(\theta_k) - e_c(\theta_k)$$

$$t_x = \frac{(x_d(\theta_k) - x_k)}{\|(x_d(\theta_k) - x_k)\|_2} \qquad t_y = \frac{(y_d(\theta_k) - y_k)}{\|(y_d(\theta_k) - y_k)\|_2}$$

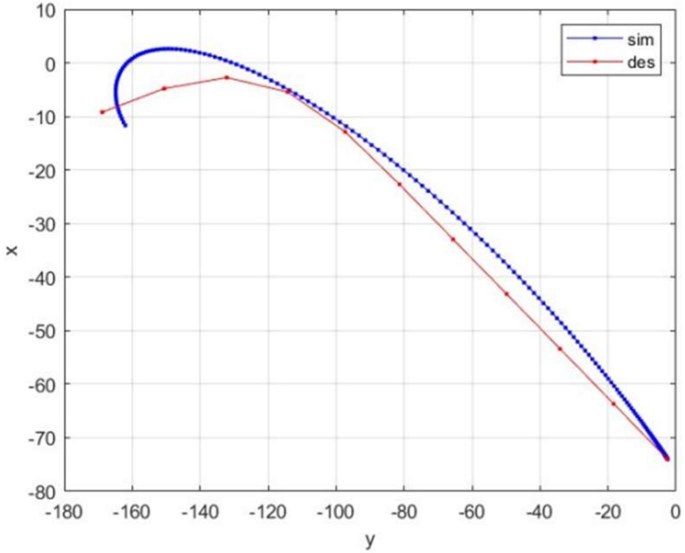
FHOCP single shooting with linear model and non linear cost 2° method

$$\min_{U} \sum_{i=0}^{N} \|e_c(\theta_k)\|^2_{qc} + \|e_l(\theta_k)\|^2_{ql} + \|\omega_k\|^2_{q\omega} + \|\Delta v_{\theta_k}\|^2_{r\Delta v} + \|\Delta f_k\|^2_{r\Delta f} - \mu v_{\theta_k}$$

$$\text{subj. to} \quad \begin{aligned} & z(i+1|t) = Az(i|t) + Bu(i|t), i = 0, ..., N-1 \\ & y(i|t) = Cz(i|t) + Du(i|t), i = 0, ..., N \\ & \Delta v_{\theta min} \leq \Delta v_\theta \leq \Delta v_{\theta max} \\ & \Delta f_{min} \leq \Delta f \leq \Delta f_{max} \\ & z(0|t) = z_0 \end{aligned}$$

$$e_c(\theta_k) = \begin{bmatrix} 1 - t_x^2(\theta_k) & -t_x(\theta_k)t_y(\theta_k) & -t_x(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_y(\theta_k) & 1 - t_y^2(\theta_k) & -t_y(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_z(\theta_k) & -t_y(\theta_k)t_z(\theta_k) & 1 - t_z^2(\theta_k) \end{bmatrix} e(\theta_k)$$

$$e_l(\theta_k) = e(\theta_k) - e_c(\theta_k)$$



$$t_x = 3a\big(x_d(\theta_k)\big)\left(\theta_k - \theta_{k_{in}}(\theta_k)\right)^2 + 2b\big(x_d(\theta_k)\big)\left(\theta_k - \theta_{k_{in}}(\theta_k)\right) + cx_d(\theta_k)$$

$$t_y = 3a\big(y_d(\theta_k)\big)\left(\theta_k - \theta_{k_{in}}(\theta_k)\right)^2 + 2b\big(y_d(\theta_k)\big)\left(\theta_k - \theta_{k_{in}}(\theta_k)\right) + cx_d(\theta_k)$$

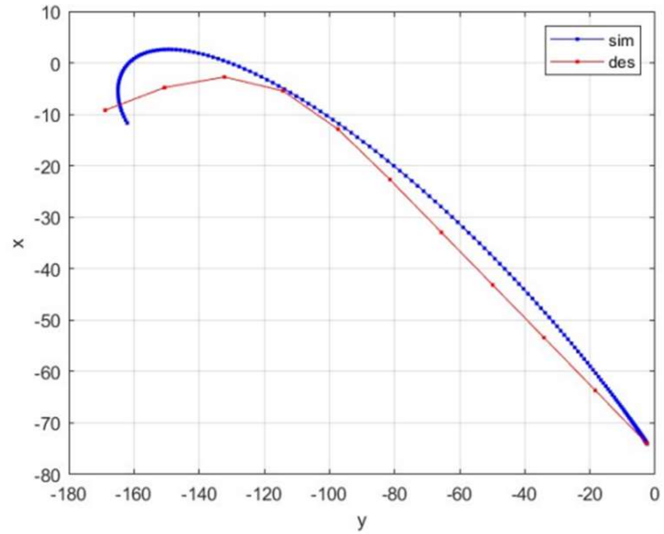
$$q_c(x_d(\theta_k)) = \sum_{j=0}^{M} \frac{e^{-\frac{1}{2}(x_d(\theta_k) - x_w^j(\theta_k)' \sum_x^- 1)(x_d(\theta_k) - x_w^j(\theta_k)}}{\sqrt{(2\pi)^3 |\sum_x|}}$$

$$q_c(y_d(\theta_k)) = \sum_{j=0}^{M} \frac{e^{-\frac{1}{2}(y_d(\theta_k) - y_w^j(\theta_k)' \sum_x^- 1)(y_d(\theta_k) - y_w^j(\theta_k))}}{\sqrt{(2\pi)^3 |\sum_y|}}$$

FHOCP multiple shooting with linear model and non linear cost 2° method

$$\min_U \sum_{i=0}^{N} \|e_c(\theta_k)\|^2_{qc} + \|e_l(\theta_k)\|^2_{ql} + \|\omega_k\|^2_{q\omega} + \|\Delta v_{\theta_k}\|^2_{r\Delta v} + \|\Delta f_k\|^2_{r\Delta f} - \mu v_{\theta_k}$$

$$\text{subj. to} \quad \begin{aligned} & z(i+1|t) = Az(i|t) + Bu(i|t), i = 0, ..., N-1 \\ & y(i|t) = Cz(i|t) + Du(i|t), i = 0, ..., N \\ & \omega_{min} \le \omega \le \omega_{max} \\ & f_{min} \le f \le f_{max} \\ & 0 \le v_\theta \le v_{\theta max} \\ & \Delta v_{\theta min} \le \Delta v_\theta \le \Delta v_{\theta max} \\ & \Delta f_{min} \le \Delta f \le \Delta f_{max} \\ & z(0|t) = z_0 \end{aligned}$$



$$e_c(\theta_k) = \begin{bmatrix} 1 - t_x^2(\theta_k) & -t_x(\theta_k)t_y(\theta_k) & -t_x(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_y(\theta_k) & 1 - t_y^2(\theta_k) & -t_y(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_z(\theta_k) & -t_y(\theta_k)t_z(\theta_k) & 1 - t_z^2(\theta_k) \end{bmatrix} e(\theta_k)$$

$$e_l(\theta_k) = e(\theta_k) - e_c(\theta_k)$$

$$t_x = 3a\big(x_d(\theta_k)\big)\left(\theta_k - \theta_{k\,in}(\theta_k)\right)^2 + 2b\big(x_d(\theta_k)\big)\left(\theta_k - \theta_{k\,in}(\theta_k)\right) + cx_d(\theta_k)$$

$$q_c(x_d(\theta_k)) = \sum_{j=0}^{M} \frac{e^{-\frac{1}{2}(x_d(\theta_k) - x_w^j(\theta_k)' \sum_x^- 1)(x_d(\theta_k) - x_w^j(\theta_k))}}{\sqrt{(2\pi)^3 |\sum_x|}}$$

$$t_y = 3a\big(y_d(\theta_k)\big)\left(\theta_k - \theta_{k\,in}(\theta_k)\right)^2 + 2b\big(y_d(\theta_k)\big)\left(\theta_k - \theta_{k\,in}(\theta_k)\right) + cx_d(\theta_k)$$

$$q_c(y_d(\theta_k)) = \sum_{j=0}^{M} \frac{e^{-\frac{1}{2}(y_d(\theta_k) - y_w^j(\theta_k)' \sum_x^- 1)(y_d(\theta_k) - y_w^j(\theta_k))}}{\sqrt{(2\pi)^3 |\sum_y|}}$$

MPCC single shooting with linear model and non linear cost 1° method

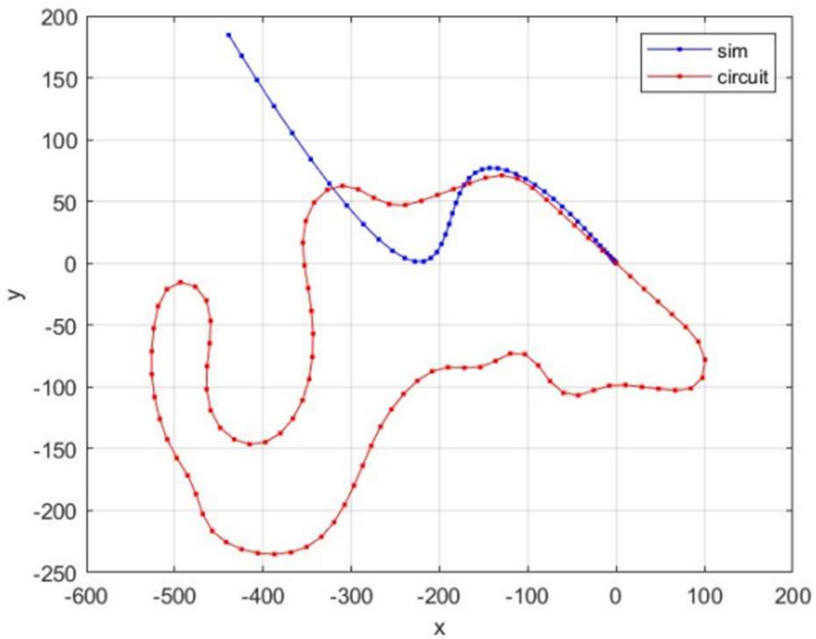
$$\min_{U} \sum_{i=0}^{N} \|e_c(\theta_k)\|^2_{qc} + \|e_l(\theta_k)\|^2_{ql} + \|\omega_k\|^2_{q\omega} + \|\Delta v_{\theta_k}\|^2_{r\Delta v} + \|\Delta f_k\|^2_{r\Delta f} - \mu v_{\theta_k}$$

$$\begin{aligned}
& z(i+1|t) = Az(i|t) + Bu(i|t), i = 0, ..., N-1 \\
& y(i|t) = Cz(i|t) + Du(i|t), i = 0, ..., N \\
\text{subj. to} \quad & \Delta v_{\theta min} \leq \Delta v_\theta \leq \Delta v_{\theta max} \\
& \Delta f_{min} \leq \Delta f \leq \Delta f_{max} \\
& z(0|t) = z_0
\end{aligned}$$

$$e_c(\theta_k) = \begin{bmatrix} 1 - t_x^2(\theta_k) & -t_x(\theta_k)t_y(\theta_k) & -t_x(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_y(\theta_k) & 1 - t_y^2(\theta_k) & -t_y(\theta_k)t_z(\theta_k) \\ -t_x(\theta_k)t_z(\theta_k) & -t_y(\theta_k)t_z(\theta_k) & 1 - t_z^2(\theta_k) \end{bmatrix} e(\theta_k)$$

$$e_l(\theta_k) = e(\theta_k) - e_c(\theta_k)$$

$$t_y = \frac{(y_d(\theta_k) - y_k)}{\|(y_d(\theta_k) - y_k)\|_2} \qquad t_x = \frac{(x_d(\theta_k) - x_k)}{\|(x_d(\theta_k) - x_k)\|_2}$$



Thanks for the attention