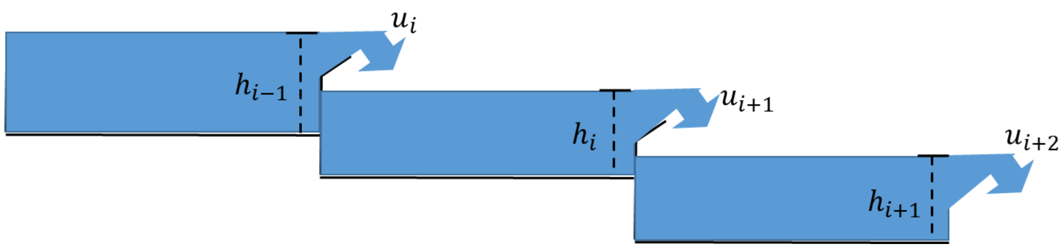
Irrigation channel

Consider the system consisting of a cascade of five pools of an irrigation network, discussed in the attached paper. The structure and the interconnection of the pools is illustrated in the following figure.



The continuous-time model of each pool, linearized around a nominal condition, assuming that no unexpected outflow is present, is

$$\alpha_i \dot{h}_i(t) = u_i(t - \tau_i) - u_{i+1}(t)$$

where the values of α_i and τ_i, for all $i = 1, \dots, 5$, are specified in the MATLAB file. Note that, since the model is linearized around a nominal condition, all the variables of the model above should be regarded as differences with respect to nominal values.

The corresponding discrete-time model (with sampling time $\tau_S = 4$ min) takes the following form

$$h_i(k+1) = h_i(k) + \frac{\tau_S}{\alpha_i}(u_i(k - k_i) - u_{i+1}(k))$$

Note that, in order to properly model the delay (in fact, the term $u_i(k - k_i)$ appears in the equation of $h_i(k+1)$), a number of auxiliary state variables (i.e., "delayed" versions of $u_i(k)$) must be defined, leading to a state-space model of order $k_i + 1$ for each pool. Overall, the resulting "centralized" discrete-time model is

$$x_{k+1} = Fx_k + Gu_k$$

where the matrices are specified in the corresponding MATLAB file. Assume that all the state variables are measurable.

Problem:

1. Decompose the state and input vectors into subvectors, consistently with the physical description of the system. Obtain the corresponding decomposed (discrete-time) model.
2. Compute the eigenvalues and the spectral radius of the (discrete-time) system. Is it open-loop asymptotically stable?
3. For different state-feedback control structures (i.e., centralized, decentralized, and different distributed schemes) perform the following actions
   a. Compute the discrete-time fixed modes
   b. Compute, if possible, the DISCRETE-TIME control gains using LMIs to achieve the desired performances. Apply, for better comparison, different criteria for computing the control laws.

c. Analyze the properties of the so-obtained closed-loop systems (e.g., stability, eigenvalues) and compute the closed-loop system trajectories of the pool water levels starting from a common random initial condition.