A logic for CLDs

Maximilian Algehed
m.algehed@gmail.com

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1 Introduction

In this document we present a logic of causal loop diagrams.

2 Syntax of CLDL

$\langle V \rangle ::= a, b, ...$

$\langle \Gamma \rangle ::= V \oplus V \mid V \ominus V$

$\langle \Omega \rangle ::= V^+ \mid V^- \mid V^\perp$

V represents any observable variable, like $rainfall$, $waterLevel$, or $wages$. Γ represents causal relationships in the CLD as well as observations of the behaviour of variables.

3 Inference rules of CLDL

What follows are the inference rules of CLDL

$$\frac{\Gamma \vdash a \oplus b \qquad \Delta \vdash b \oplus c}{\Gamma, \Delta \vdash a \oplus c} \oplus i_\oplus \qquad \frac{\Gamma \vdash a \ominus b \qquad \Delta \vdash b \ominus c}{\Gamma, \Delta \vdash a \oplus c} \oplus i_\ominus$$

$$\frac{\Gamma \vdash a \oplus b \qquad \Delta \vdash b \ominus c}{\Gamma, \Delta \vdash a \ominus c} \ominus i_r \qquad \frac{\Gamma \vdash a \ominus b \qquad \Delta \vdash b \oplus c}{\Gamma, \Delta \vdash a \ominus c} \ominus i_l$$

$$\frac{\Gamma \vdash a^+ \qquad \Delta \vdash a^-}{\Gamma, \Delta \vdash a^\perp} -^\perp i \qquad \frac{\Gamma, \Delta \vdash A}{\Delta, \Gamma \vdash A} \mathit{Exchange}$$

$$\frac{\Gamma \vdash a^+ \qquad \Delta \vdash a \oplus b}{\Gamma, \Delta \vdash b^+} -^+ e_\oplus \qquad \frac{\Gamma \vdash a^+ \qquad \Delta \vdash a \ominus b}{\Gamma, \Delta \vdash b^-} -^+ e_\ominus$$

$$\frac{\Gamma \vdash a^- \qquad \Delta \vdash a \oplus b}{\Gamma, \Delta \vdash b^-} -^- e_\oplus \qquad \frac{\Gamma \vdash a^- \qquad \Delta \vdash a \ominus b}{\Gamma, \Delta \vdash b^+} -^- e_\ominus$$

$$\frac{\Gamma \vdash a^\perp \qquad \Delta \vdash a \oplus b}{\Gamma, \Delta \vdash b^\perp} -^\perp e_\oplus \qquad \frac{\Gamma \vdash a^\perp \qquad \Delta \vdash a \ominus b}{\Gamma, \Delta \vdash b^\perp} -^\perp e_\ominus$$

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash b^+} \mathit{Contraction} \qquad \frac{\Gamma \vdash B}{\Gamma, A \vdash B} \mathit{Weakening}$$

$$\frac{}{A \vdash A} \mathit{id}$$

4 Initial Results

Some initial theorems and definitions

No negative feedback. $a^+, a \ominus a \vdash a^\perp$
$a^-, a \ominus a \vdash a^\perp$

Proof. $\dfrac{\dfrac{a \ominus a \qquad a^+}{a^- \qquad\qquad a^+}}{a^\perp}$ Conversely for a^- □

5 Relationship to correctness of CLD inference algorithms

An algorithm for inference in CLDs is a function that takes a set Γ of assertions on the form $a \oplus b$ or $x \ominus y$ representing the CLD, and a set Ω of assertions on the form x^+ or x^-, representing observed changes in the values of the CLD, and returns a set κ of conclusions on the form a^+, a^-, or $a^\perp$. If an algorithm is correct it must follow that $\Gamma, \Omega \vdash \kappa$.