Formal semantics of GRACe

Sólrún Halla Einarsdóttir

Chalmers University of Technology

2/2/2017

Motivation/Background

- We would like to define formal semantics for our DSL
  - Formal reasoning
  - Robustness
  - Gain further insights
- We started by considering Causal Loop Diagrams, a specialisation of the GCMs our DSL describes.
- Work to extend and generalize these semantics is ongoing.

Causal Loop Diagrams (CLDs)

- A Causal Loop Diagram (CLD) is a directed graph used to display causal relationships between variables.
- Vertices represent variables, edges represent qualitative causal relationships, which can be positive or negative.
- CLDs have been modelled within GRACe.

Notation

- We denote a positive causal relationship between A and B by $A \xrightarrow{+} B$ and a negative one by $A \xrightarrow{-} B$.
- We denote the sign of the edge from A to B by s_{AB}, so $s_{AB} = +$ if $A \xrightarrow{+} B$ and $s_{AB} = -$ if $A \xrightarrow{-} B$.
- A vertex A has a sign s_A that denotes the total influence on A, so $s_A = +$ if there is an increase in A, $s_A = -$ if there is a decrease, $s_A = 0$ if there is no change and $s_A =?$ if we cannot determine the change.

Qualitative Probabilistic Networks (QPNs)

- A graph where vertices correspond to variables and edges to qualitative probabilistic influences.
- $s_{AB} = +$ means that greater values of A mean greater values of B are more likely, and $s_{AB} = -$ means that greater values of A mean smaller values of B are more likely.
- Possible to propagate an observed increase or decrease of one variable around the graph and find if other variables are likely to have increased or decreased.
- Broad enough to apply to many different systems and to be applicable to various real world situations.

QPN Issues

- QPNs were originally defined for acyclic graphs, theory relies on acyclicity, perhaps not the best fit to describe CLDs, in which cycles (feedback loops) are an important feature.
- Semantics of inference on QPNs is difficult to formalize since it relies heavily on not-so-simple probability theory.
- Can't be expanded to describe quantitative relationships unless we have information about the probability distributions and conditional probabilities involved.
- All inference in QPNs is probabilistic, may lead to results that are not as meaningful or concrete as we would like,

Difference equations

- Inspired by a system of tanks with water flowing from one to another.
- We consider the values of the graph's vertices to be functions of the same variable, such as a time variable t.
- We can describe the causal relationship from X to Y with

$$\frac{\partial Y}{\partial t} = G(X(t)),$$

  where G is monotonically increasing if $s_{XY} = +$ and monotonically decreasing if $s_{XY} = -$.
- If the vertex Y has parent vertices $X_1, \ldots, X_n$, we have

$$\frac{\partial Y}{\partial t} = \sum_{i=1} G_i(X_i(t)).$$

Difference equations

- More nuanced than classical CLDs, can have threshold values, rates of increase/decrease rather tahn just increase/decrease.
- Monotonically increasing mustn't necessarily be strictly increasing.
- Extend to quantitative reasoning by solving the appropriate differential equations.

Simplified qualitative difference equations

We consider a qualitative discrete time system where all values of vertex variables are either $+$, -, 0, or ?.

- Values have partial ordering $- < 0 < +$, but ? cannot be compared to the other values.
- The only strictly increasing function in this system is the identity function, the only strictly decreasing function is the negation function.
- For simplicity we consider the case where all initial values are set to zero.
- The value of variable X at time t, which we denote by X_t, then tells us whether there has been a net increase or decrease in X.

Comparison of approaches

- Same results when inferring on CLDs no matter which approach is used to describe the underlying semantics, which method is simpler to understand and reason about is a matter of opinion.
- Difficulties when working with the QPN approach described earlier, we are skeptical about its extensibility.
- Difference equation method was inspired by stock-and-flow-like systems, should extend well.