Generative Ratio Matching Networks

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Goal

A *stable* learning algorithm for *implicit* deep generative models with *high* dimensional data

- MMD networks are stable but perform poorly on high dimensional data
- Adversarial methods with zero-sum games (GANs, MMD-GANs, etc) can scale up to high dimensional (image) data but are not stable in general

Key Ideas

1. Learn a low-dimensional sub-space projection in which the density ratio between the data and the generator is close to the density ratio in the original space
2. Train the generator via the MMD loss in this space

Learning Low Dimensional Sub-space Projection $f_\theta(x)$

We'd like to learn a parameterized transformation $f_\theta(x)$ by minimising the squared difference between the pair of density ratios:

$$
\begin{aligned}
D(\theta) &= \int q_x(x) \left(\frac{p_x(x)}{q_x(x)} - \frac{\bar{p}(f_\theta(x))}{\bar{q}(f_\theta(x))} \right)^2 dx \\
&= C - \left(\int \bar{q}(f_\theta(x)) \left(\frac{\bar{p}(f_\theta(x))}{\bar{q}(f_\theta(x))} \right)^2 df_\theta(x) - 1 \right) \\
&= C - \mathrm{PD}(\bar{q}, \bar{p})
\end{aligned}
$$

We can *minimise* the squared difference by *maximizing* Pearson Divergence in the low dimensional space ❤️

Pearson Divergence Maximisation

We carry out a Monte Carlo approximation,

$$\mathrm{PD}(\bar{q}, \bar{p}) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\bar{p}(f_\theta(x_i))}{\bar{q}(f_\theta(x_i))} \right)^2 - 1$$

where $x_i^q \sim q_x$.

We use a MMD based density ratio estimator (Sugiyama et al., 2012) under the fixed-design setup: $\hat{r}_q = \mathbf{K}_{q,q}^{-1} \mathbf{K}_{q,p} \mathbf{1}$.

- $\mathbf{K}_{q,q}$ and $\mathbf{K}_{q,p}$ are Gram matrices defined by $[\mathbf{K}_{q,q}]_{i,j} = k(f_\theta(x_i^q), f_\theta(x_j^q))$ and $[\mathbf{K}_{q,p}]_{i,j} = k(f_\theta(x_i^q), f_\theta(x_j^p))$.

Density Ratio Estimation via (Infinite) Moment Matching

Maximum mean discrepancy

$$\mathrm{MMD}_{\mathcal{F}}(p, q) = \sup_{f \in \mathcal{F}} \left(\mathbb{E}_p[f(x)] - \mathbb{E}_q[f(x)] \right)$$

Gretton et al. (2012) show that it is sufficient to choose $\mathcal{F}$ to be a unit ball in an reproducing kernel Hilbert space $\mathcal{R}$ with a characteristic kernel k.

Using this definition of MMD, the density ratio estimator $r(x)$ can be derived as the solution to

$$\min_{r \in \mathcal{R}} \left\| \int k(x; .) p(x) dx - \int k(x; .) r(x) q(x) dx \right\|_{\mathcal{R}}^2 .$$

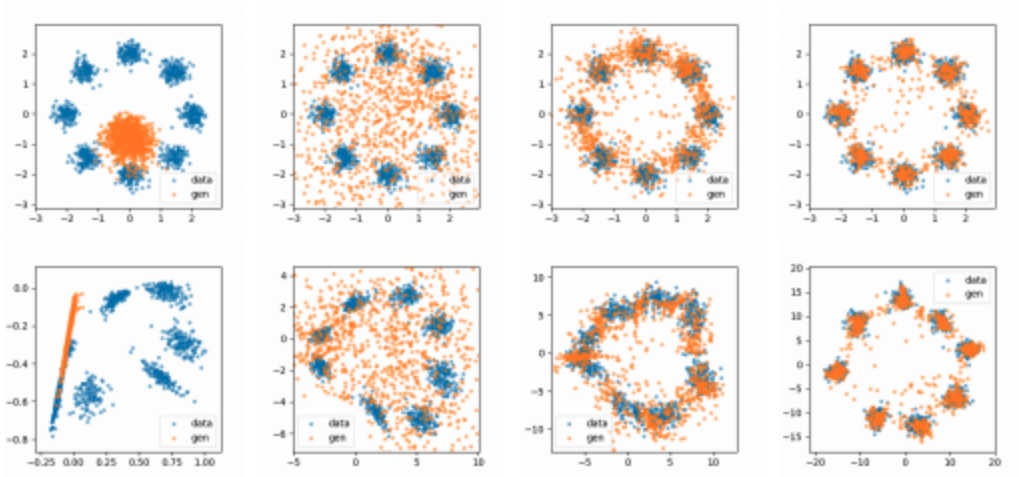
Generator Training

- The generator $G\gamma$ is trained by minimizing the empirical estimator of MMD,

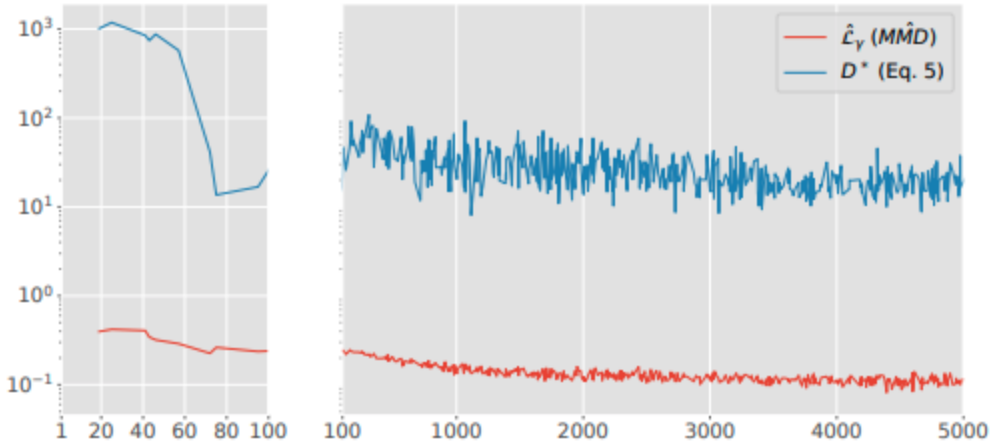
$$\min_{\gamma} \left[\frac{1}{N^2} \sum_{i=1}^{N} \sum_{i'=1}^{N} k(f_\theta(x_i), f_\theta(x_{i'})) - \frac{2}{NM} \sum_{i=1}^{N} \sum_{j=1}^{M} k(f_\theta(x_i), f_\theta(G_\gamma(z_j))) + \frac{1}{M^2} \sum_{j=1}^{M} \sum_{j'=1}^{M} k(f_\theta(G_\gamma(z_j)), f_\theta(G_\gamma(z_{j'}))) \right]$$

with respect to it's parameters γ.

The Ring dataset: Illustration of the Method and Stability

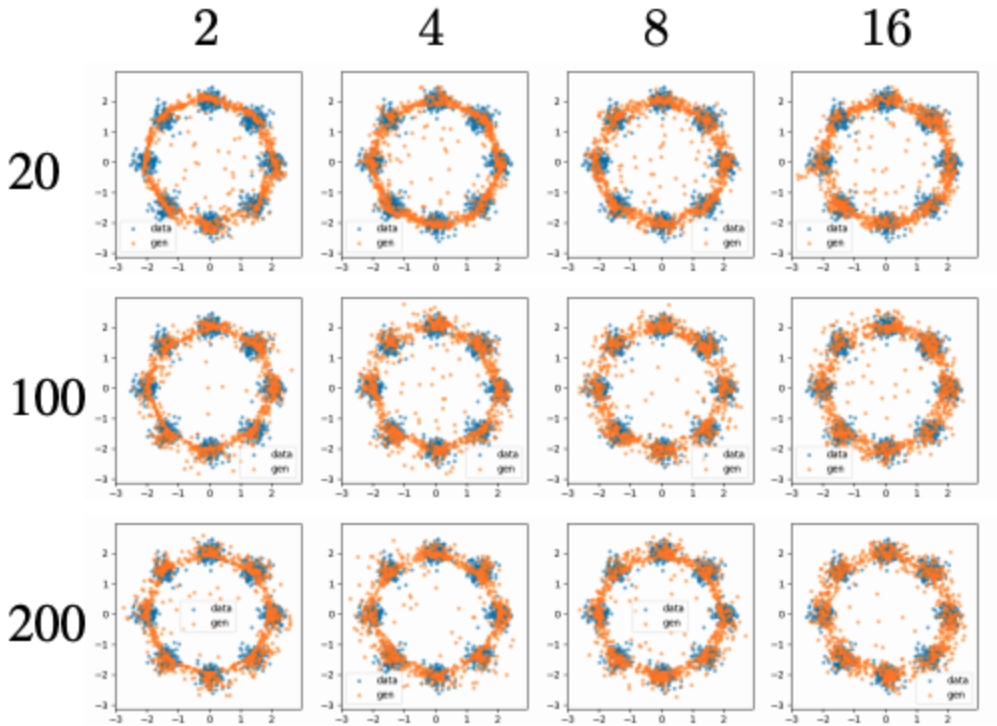


(a) Data and samples in the original (top) and projected space (bottom) during training; four plots are at iteration 10, 100, 1000 and 10, 000 respectively. Notice how the projected space separates $\bar{p}$ and $\bar{q}$.

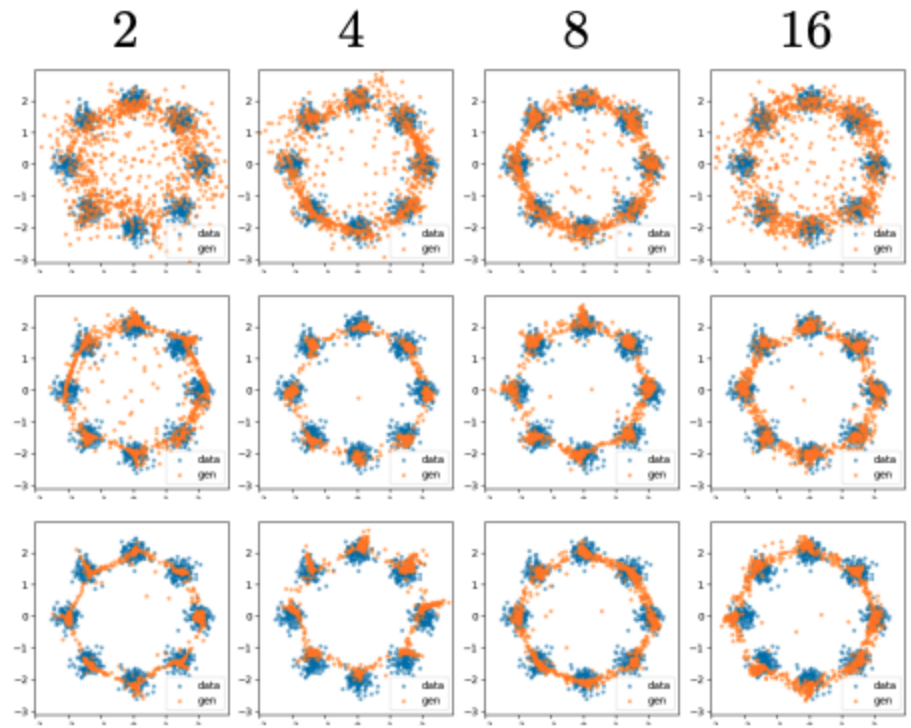


(b) Trace of $\hat{\mathcal{L}}_\gamma$ and D^* (equation (5)) during training. The left plot is for iteration 1 to 100 and the right plot is for 100 to 5,000, with the same y-axes in the log scale.

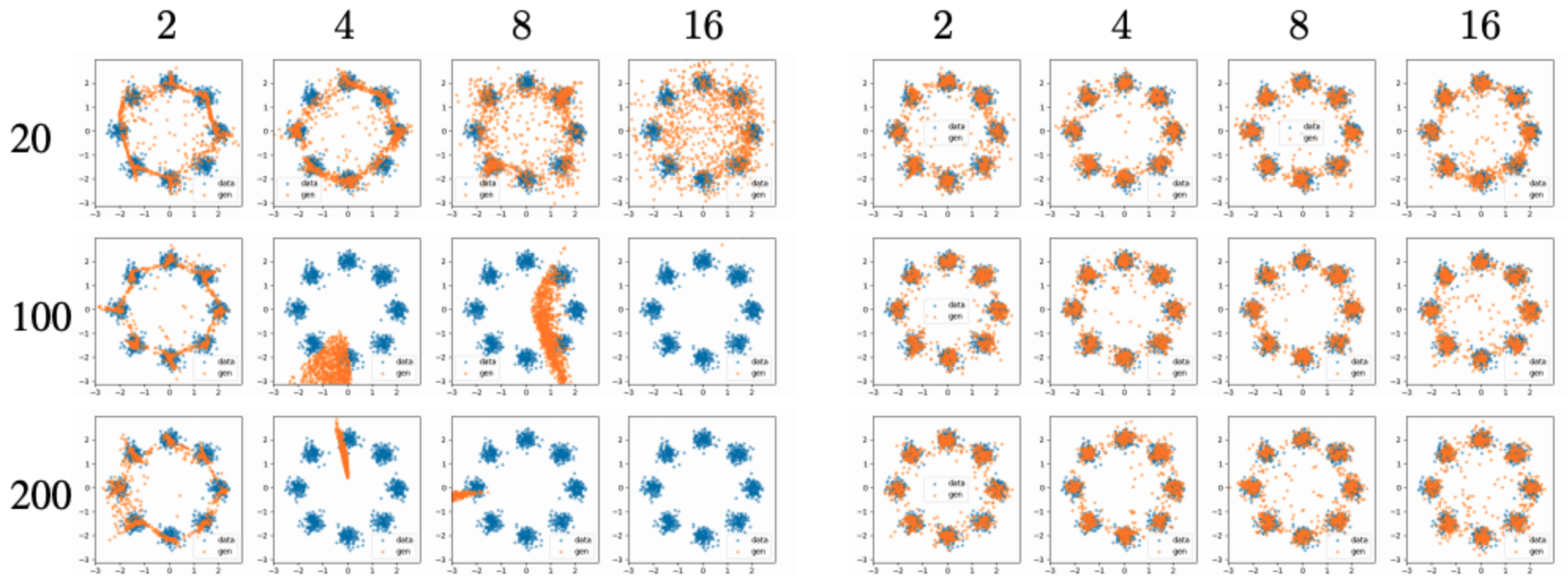
Figure 1: Training results with projected dimension fixed to 2.



(a) MMD-nets



(b) MMD-GANs



(a) GAN

(b) GRAM-net

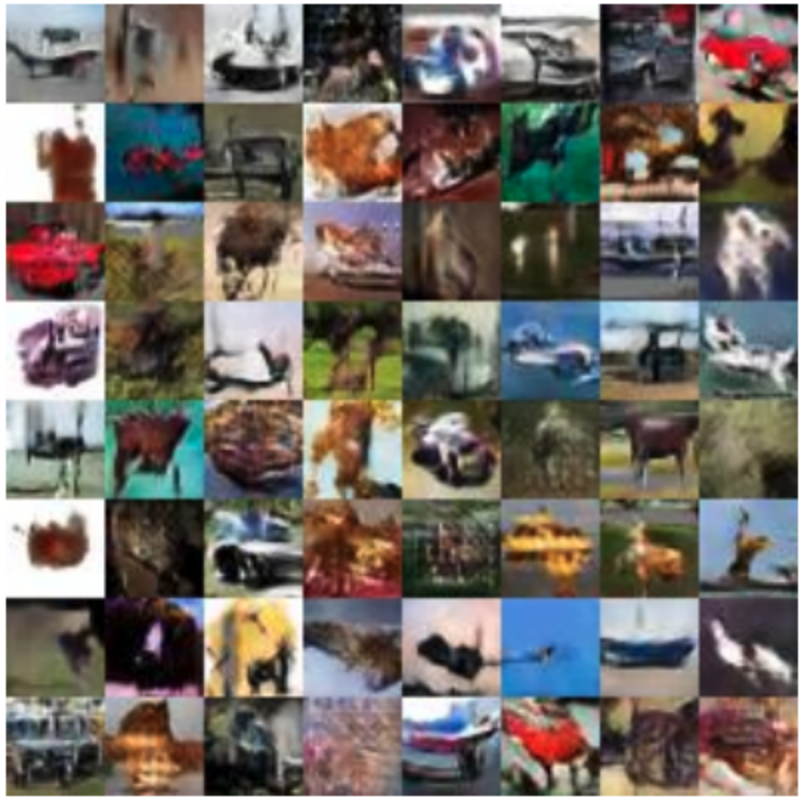
Figure 2: Training after 2,000 epochs by varying noise dimension h and the hidden layer size of critic model. For each model, each row is a different layer size in $[20, 100, 200]$ and each column is a different h in $[2, 4, 8, 16]$. Half of the GAN training diverges while all GRAM training converges.

Quantitative Results: Sample Quality

Table 1: Sample quality (measured by FID; lower is better) of GRAM-nets compared to GANs.

Arch.	Dataset	MMD-GAN	GAN	GRAM-net
DCGAN	Cifar10	40.00 ± 0.56	26.82 ± 0.49	$\mathbf{24.85 \pm 0.94}$
Weaker	Cifar10	210.85 ± 8.92	31.64 ± 2.10	$\mathbf{24.82 \pm 0.62}$
DCGAN	CelebA	41.105 ± 1.42	30.97 ± 5.32	$\mathbf{27.04 \pm 4.24}$

Qualitative Results: Random Samples



(a) CIFAR10

(b) CelebA