FIZIKA

Understand, Apply, Achieve.

fizika.gramoly.org

ROUND I

Questions & Solutions

Organized by

GRAMOLY

In Collaboration with

Indian School of Physics

Physics Sir JEE

SPONSORS

Overleaf

PhysOlymp

Replit

AoPS Online

Art of Problem Solving

Math

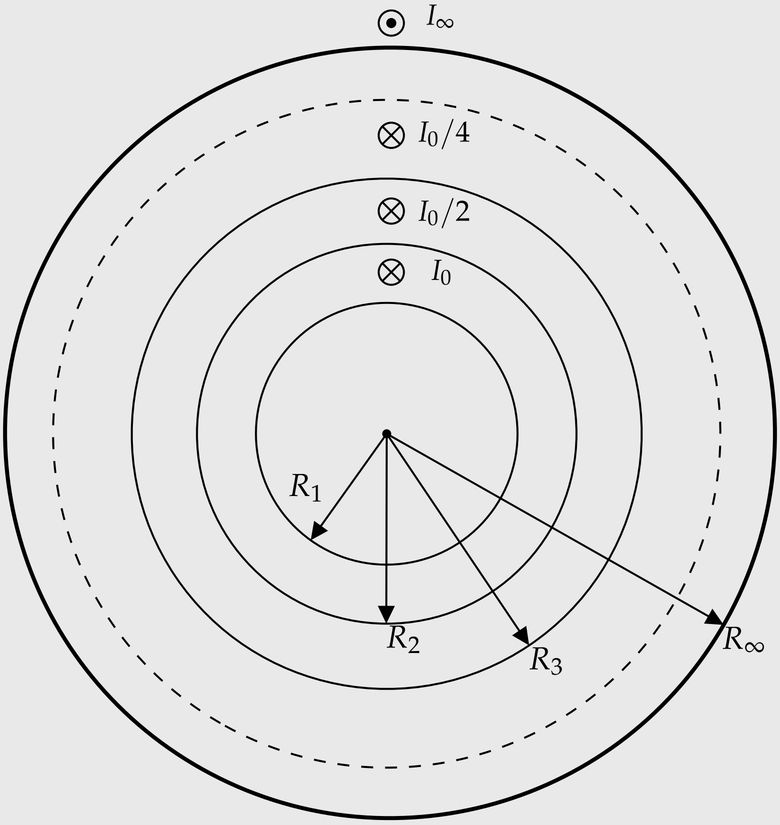
DISCUSSION

Discussion

@forum.gramoly.org

Problem 1

There is a system of infinite very long concentric cylindrical shells that have radius $R_1, R_2, R_3, R_4 .... R_\infty$ and having uniformly distributed current given as $I_0, \frac{I_0}{2}, \frac{I_0}{4}, \frac{I_0}{8}, ...$ and the outermost shell is having the current of I_∞. The current is going inwards in all the shells except the shell at infinity where its going outwards with respect to plane of the paper. Evaluate the ratio $\frac{I_\infty}{I_0}$ for which the outermost shell remains stress-free.



-Proposed by Nitin Sachan

Magnetic field due to a hollow cylinder will be zero at all interior points and for exterior points hollow cylinder will behave like a wire at the axis. Therefore for the outermost shell at infinity will experience pressure factor of 4 and due to its own current it will experience a factor of 8 in pressure expression. Therefore net pressure must be zero in order to make the outermost shell stress free.

$$\frac{\mu_0 I_0 I_\infty}{4\pi^2 R_\infty^2} + \frac{\mu_0 \left(\frac{I_0}{2}\right) I_\infty}{4\pi^2 R_\infty^2} + \frac{\mu_0 \left(\frac{I_0}{4}\right) I_\infty}{4\pi^2 R_\infty^2} + \cdots = \frac{\mu_0 I_\infty}{8\pi^2 R_\infty^2}$$

$$\mu_0 I_0 I_\infty \left(1 + \frac{1}{2} + \frac{1}{4} + \cdots + \infty\right) = \frac{\mu_0 I_\infty^2}{2}$$

This is a geometric progression with first term 1 and common ratio 1/2, hence we can write

$$I_0 \left(\frac{1}{1 - \frac{1}{2}}\right) = \frac{I_\infty}{2}$$

$$2I_0 = \frac{I_\infty}{2}$$

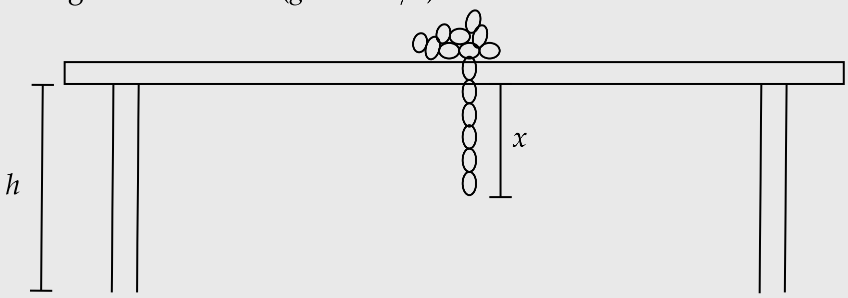
$$\boxed{\frac{I_\infty}{I_0} = 4}$$

Answer : 4

Solution 1

Problem 2

Given a peculiar chain of length 1 m, such that it's linear density varies as $\lambda(x) = x$, where x is distance from front edge A. It is kept on a table with a small hole such that the edge A starts falling through it and, the height of table is h. ($g = 10m/s$).



A) When $h > 1m > x$, then $v(x) = 2\sqrt{x}$.
B) When $h > x > 1m$, then $x(t) = 5t^2 + 2t + 1$.
C) When $1m > h$, $a(t) =$ is dependent of time.
D) When $1m > h$, $a(t)$ is independent of time.

-Proposed by Atharva Nilesh Mahajan

When $h > 1m > x$: We get

$$F = mg - v\frac{dm}{dt}$$

$$a = g - \frac{v}{m}\frac{dm}{dt}$$

$$v\frac{dv}{dx} = g - v\frac{\frac{dm}{dt}}{m} \tag{1}$$

Now, using $\lambda(x) = x$,

$$dm = xdx$$

Integrating:

$$m = \frac{x^2}{2}$$

Now substituting in (1), we get:

$$v\frac{dv}{dx} = g - \frac{v^2 x}{x^2/2} = 10 - \frac{2v^2}{x}$$

$$\implies v = 2\sqrt{x}$$

When $x = 1m$, velocity $= 2m/s$, and $a = g = 10m/s^2$

$$x - 1 = vt + \frac{1}{2}at^2$$

$$x - 1 = 2t + 5t^2$$

$$x = 5t^2 + 2t + 1$$

$$v = 10t + 2$$

When the string is not on the ground yet, $a(t) = 2m/s^2$
When all of the string falls on the ground, then $a(t) = 0m/s^2$
Answers: (A), (B), (C)

Solution 2

Problem 3

A Triangular $'L'$ story;
The self-inductance of an equilateral triangular loop of side l is L (as shown in figure-I). A rhombus of one internal angle 60° and side $'l'$ (as shown in figure-II) is placed very close to the triangle so that one of their sides align in the same plane (as shown in figure-III)

A)If the loop in figure-I were of side $'2l'$ instead, then its self-inductance would have been $'2L'$.

B) The self-inductance of the loop in figure-II is greater than $'2L'$.

C) The mutual inductance between the two loops in figure-III is $\frac{L}{3}$

D) The mutual inductance between the two loops in figure-III is $\frac{4L}{3}$

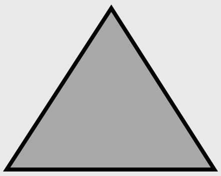


Figure I

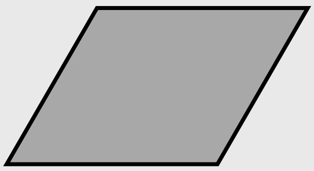


Figure II

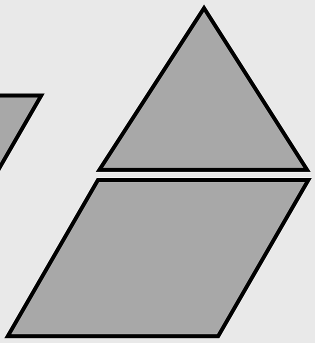
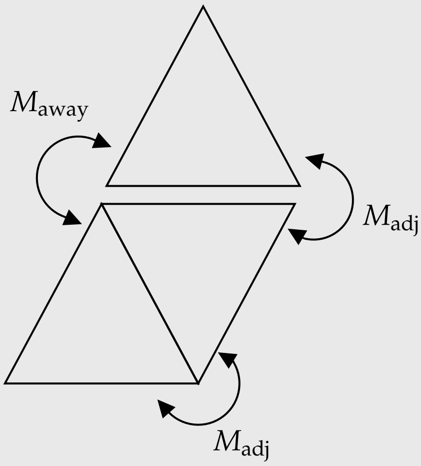


Figure III

-Proposed by Janardanudu Thallaparthi

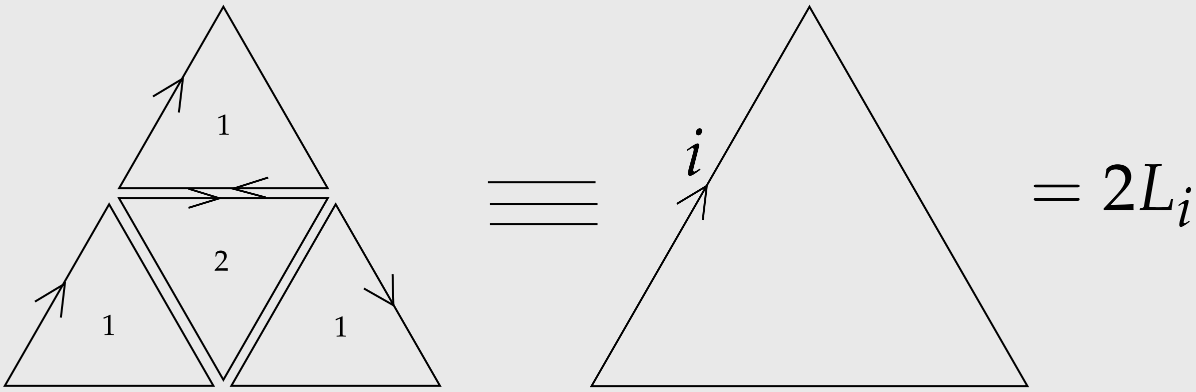
Divide the lower rhombus into two equilateral triangles. Let the mutual inductance of a pair of an equilateral triangle having a side (adjacent ones) be 'M_{adj}' and the ones away be 'M_{away}'. We need to solve for $M_{\text{adj}} + M_{\text{away}}$. Dimension analysis helps us realise a triangle of twice the side length has '$2L$' inductance. Taking this as a hint to solve the problem further, we build an extra triangle to the above figure to make it a '$2l$' side equilateral triangle (see the equivalence).



Each small traingle 'Labelled-1' haws one self flux term L_i, one self flux term L_i, one $M_{\text{adj}} \cdot i$ term, two $M_{\text{away}} \cdot i$ terms. Labelled-2 triangle has one L_i, three $M_{\text{adj}} \cdot i$ terms. So, counting properly,

$$4L_i + 6M_{\text{adj}} \cdot i + 6M_{\text{away}} \cdot i = 2L_i$$

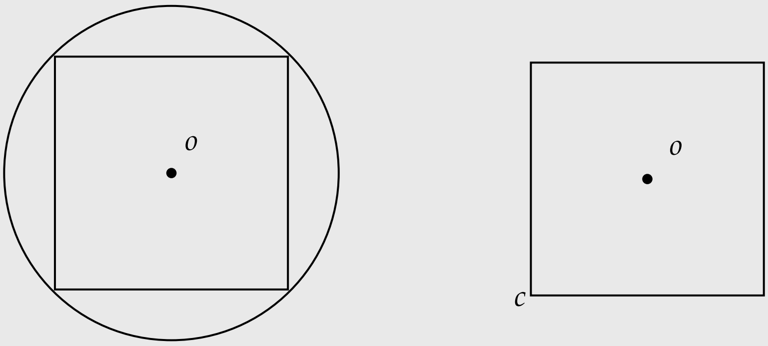
$$\Rightarrow |M_{\text{adj}} + M_{\text{away}}| = \frac{L}{3}$$



Answer : (A), (C)

Solution 3

Problem 4

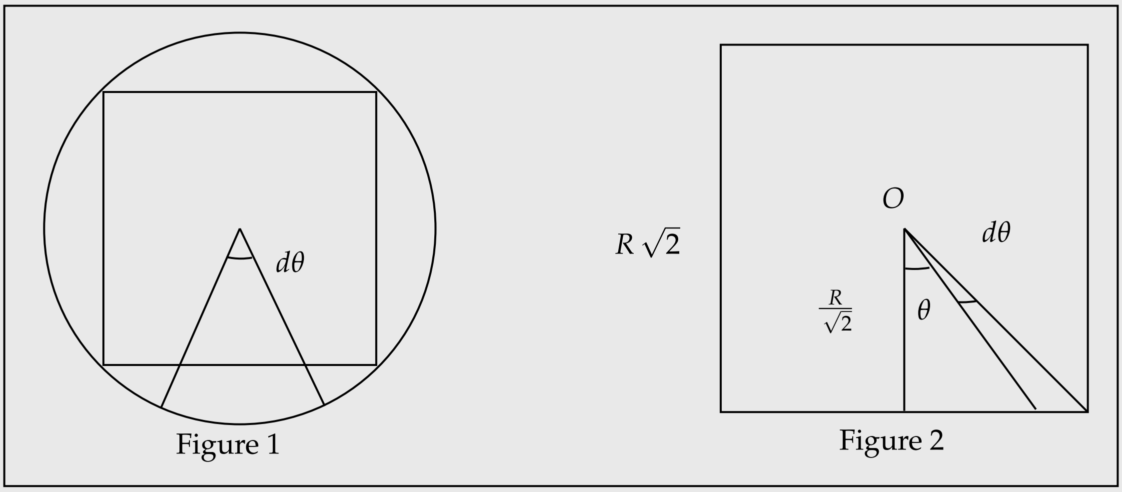


A uniformly charged non-conducting circular disc has its centre at potential V. Now a square plate is cut out of this disc keeping its surface charge density same. The new potential at center and corner are V_o and V_c respectively then

A) $V_o = 4V_c$
B) $V_o = 2V_c$
C) $V_o = 2V\ln\left(\sqrt{2}+1\right)$
D) $V_o = \frac{2\sqrt{2}}{\pi}V\ln\left(\sqrt{2}+1\right)$

-Proposed by Janardanudu Thallaparthi

Dimension analysis leads to $V = KR$.



Each $d\theta$ sectors contributes equally. So, $dV = \frac{d\theta}{2\pi}KR$. We use this result in Figure-2 for sectors of variable radii given by $\frac{R}{\sqrt{2}}\sec\theta$, assuming square to be made of "4" right isosceles plates. So,

$$V_o = 4\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{d\theta}{2\pi}K\left(\frac{R}{\sqrt{2}}\sec\theta\right)$$

$$\implies V_0 = \frac{2\sqrt{2}}{\pi}KR\int_0^{\frac{\pi}{4}}\sec\theta d\theta$$

$$\implies V_0 = \frac{2\sqrt{2}}{\pi}KR\ln\left(\sqrt{2}+1\right)$$

But $V = KR$, so we can write:

$$\boxed{\implies V_0 = \frac{2\sqrt{2}}{\pi} V \ln\left(\sqrt{2}+1\right)}$$

Answer: (D)

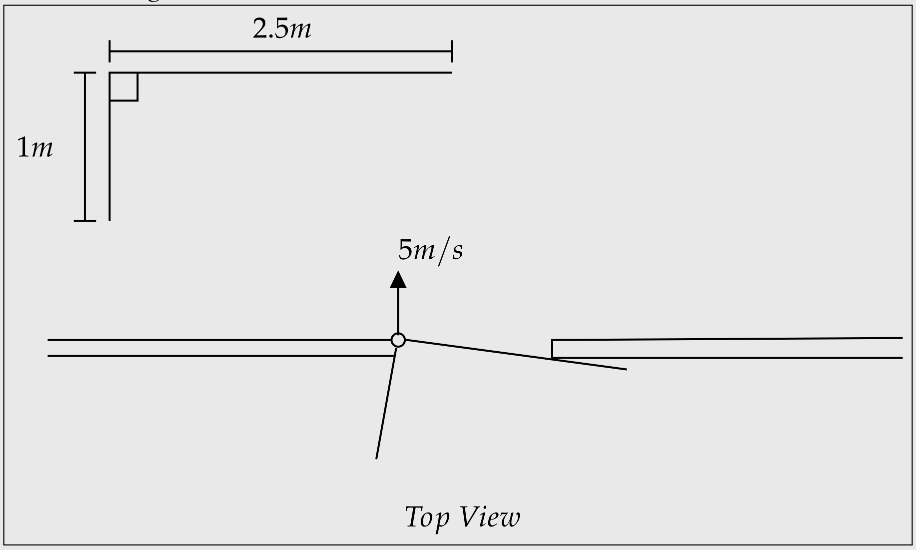
Solution 4

Problem 5

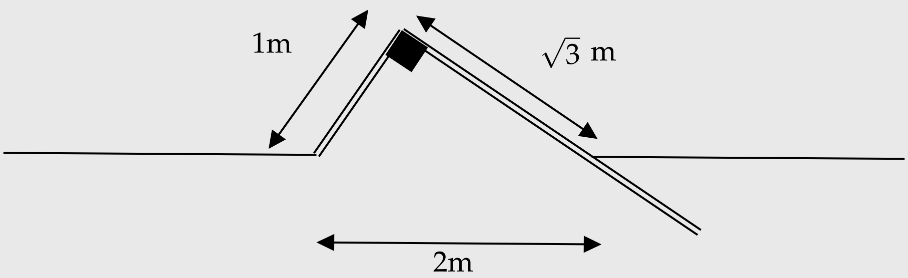
Consider a bead of mass $m = 1kg$ which is rigidly attached to 2 massless rods at right angles as shown. The lengths of the rods are 2.5 m and 1 m as shown. Let's say this arrangement is placed on a rough table with 2 rigid vertical barriers of $v = 5\ ms^{-1}$ perpendicular to the barrier, and it finally comes to rest at the position coordinate (a, b) assuming the initial position to be Origin.
Find the value of

$$4 \cdot |b\sqrt{3} - a|$$

If $\mu_{\text{table and bead}} = 0.5, \text{and} g = 10\ ms^{-2}$



-Proposed by Atharva Shivaram Mahajan



First of all, observe that the locus of the particle will be circular initially.
In this time, its speed will decrease at a constant rate of $\mu g \ \ m/s^2$

After this point, it will go in a straight line as the length of one rod is just $1m$

$$\begin{aligned} v^2 &= u^2 - 2as_1 \\ &= 5^2 - 2\cdot(0.5\cdot 10)\cdot\frac{\pi r}{3} \end{aligned} \tag{2}$$

$$\boxed{v = \sqrt{25 - 10\cdot\frac{\pi}{3}}} \tag{3}$$

After this, distance travelled will be

$$0 = v^2 - 2as_2$$

$$\implies s_2 = \sqrt{\frac{v^2}{2a}} = \sqrt{\frac{25 - 10\cdot\frac{\pi}{3}}{2\cdot(0.5\cdot 10)}} = \sqrt{\frac{25 - 10\frac{\pi}{3}}{10}}$$

$$\boxed{s_2 = \sqrt{\frac{5}{2} - \frac{\pi}{3}}} \tag{4}$$

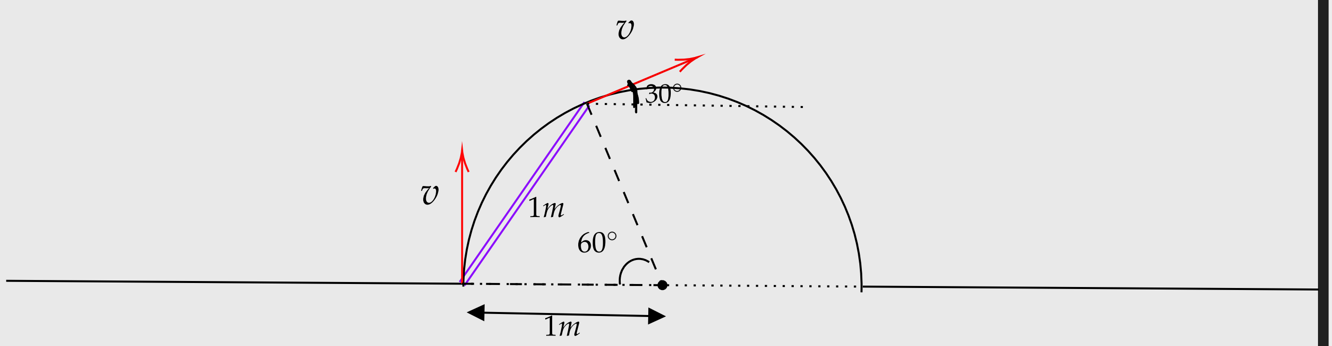
Final coordinate will be:

$$\begin{aligned} &(\cos(60) + s_2\cos(30),\ \sin(60) + s_2\sin(30)) \\ &= \left(\frac{1}{2} + \sqrt{\frac{5}{2} - \frac{\pi}{3}}\cdot\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} + \sqrt{\frac{5}{2} - \frac{\pi}{3}}\cdot\frac{1}{2}\right) \end{aligned}$$

So,

$$4|b\sqrt{3} - a| = 4\left|\frac{3}{2} + \sqrt{\frac{5}{2} - \frac{\pi}{3}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2} - \sqrt{\frac{5}{2} - \frac{\pi}{3}}\cdot\frac{\sqrt{3}}{2}\right|$$

$\boxed{\text{Ans : 4}}$



Solution 5

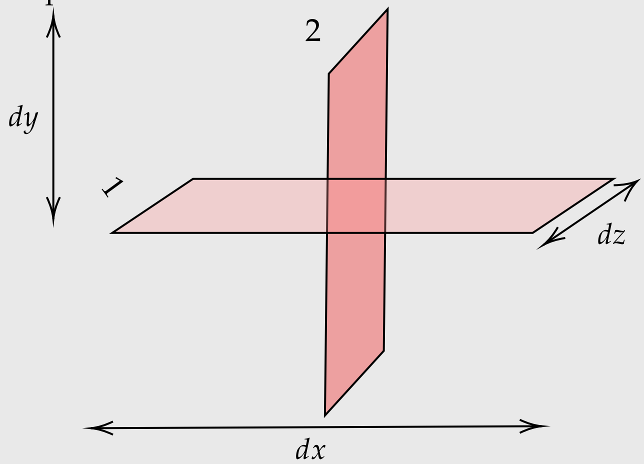
Problem 6

Consider a very low-resistivity fluid. The magnetic field is at time $t = 0$ sec is given by : $\vec{B} = B_0\ \vec{e_y}$.

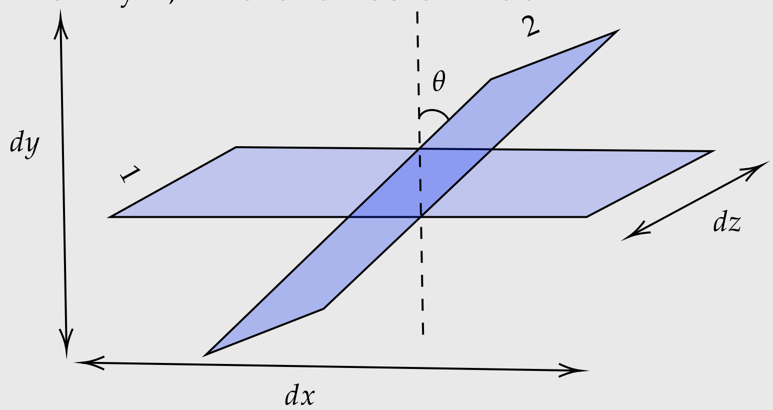
The velocity field is $\vec{V} = \frac{1}{1+(\frac{y}{m})^2}\ \vec{e_x}\ m/s$ always.
What is the magnitude of magnetic field at $(0,1)m$ at $t = 1$ second?
$A)B_0$
$B)B_0/2$
C) $\sqrt{3}B_0/2$
$D)$ $\sqrt{5}B_0/2$

-Proposed by Hemansh Shah

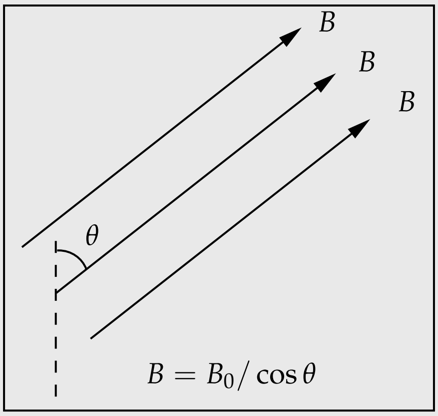
For a low resistivity fluid, consider a bunch of fluid elements forming a loop. As the fluid moves, the loop changes shape, expands/contacts, etc.
If the magnetic field through the loop changes, there is an electric field curl induced. But, that leads to the flow of current, which opposes the change in magnetic field. In the limit of very low resistivity, the magnetic field through the loop can not change. It stays a constant.
Now, let us consider two loops at $t = 0$ in the fluid as shown.



The loop 1 has flux $B_0 dxdz$ through it, Whereas loop 2 has flux 0 through it.
Now, let us look at where these loops will end up after a time t and how they will look.
The loops are moved ahead by vt, and oriented as shown below:



The flux through loop 2 is still 0. The flux through loop 1 is still $B_0 dxdz$.
Thus, the magnetic field looks as shown:



Solution 6

Problem 7

Consider the given figure, the blue lines are perfectly reflecting hyperboloid mirrors with coincident foci shown in blue. A beam of light (red) is directed in through an opening at the top left of the system. Two observers O_1 and O_2 shall receive the light. Find the ratio of the energy density of the multiply reflected beam seen at O_1 to that at O_2.

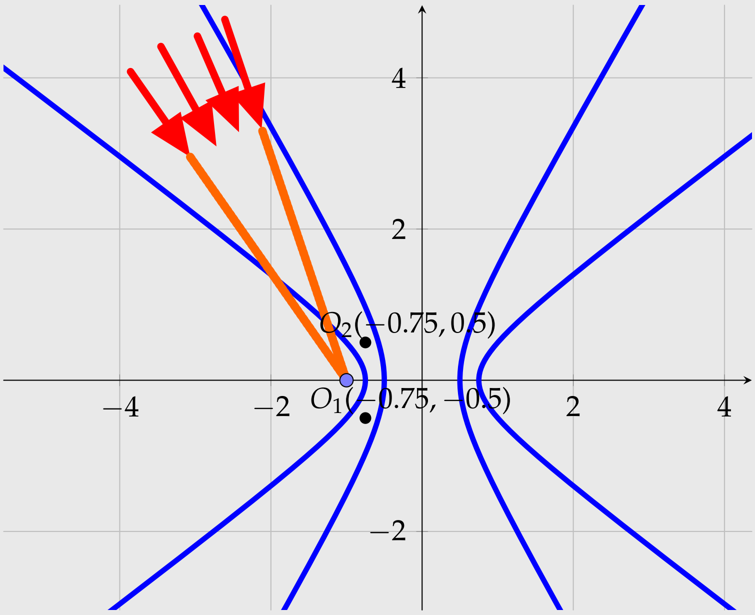
a) 0

b) 1/2

c) 1

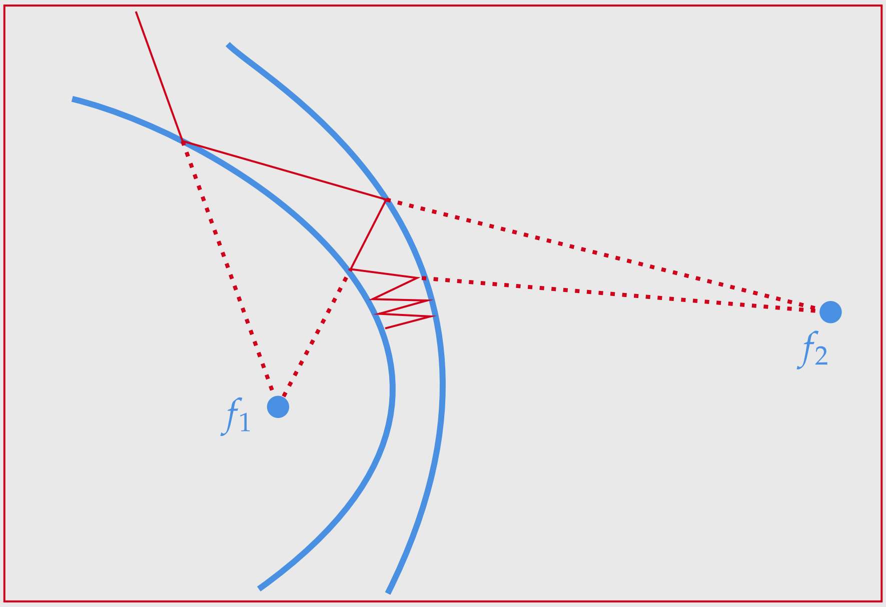
d) 2

Note: Neglect diffraction and other wave effects.



-Proposed by Hemansh Shah

Light rays going toward a focus of the hyperbola, upon reflection, change their paths so as to go through the other focus. Following the light path, we see that no rays can get to the point O_1.

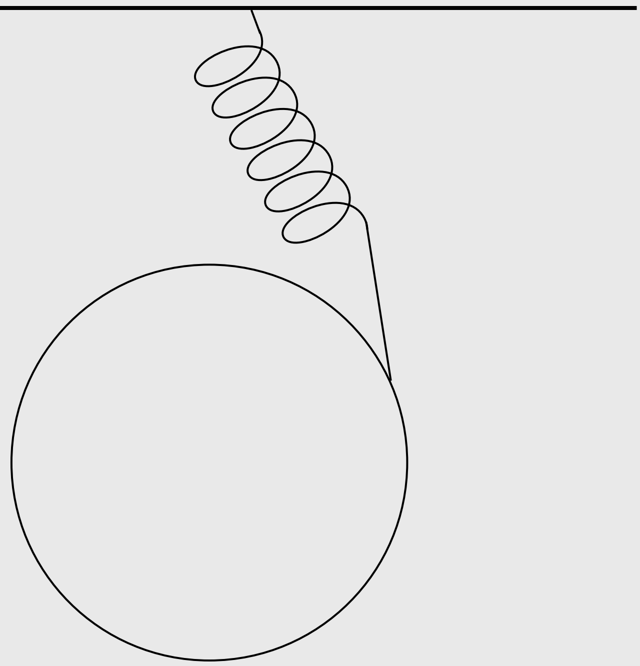


Hence, the answer is 0. **Answer: (a)**

Solution 7

Problem 8

A disc of mass M, and radius R is free to rotate about a fixed axis perpendicular to its surface and passing through its Centre of Mass. It is bound by a light spring, with a natural length of L. At its natural length, one end of the spring is connected to a fixed wall located at a distance of L from the disc, and the other end is connected to the disc. The disc is rotated so that the spring forms a tangent to it and is released from that position. Samar sticks a ring of mass $16M$ and radius R/N on the disc with the same fixed axis. He sticks this ring when the disc has maximum angular velocity and ensures he doesn't change its angular velocity. The spring snaps at a length $3\sqrt{L^2 + 2LR} - 2L$. You can assume that the spring exerts a force tangential to it at any given point. Friction is negligible everywhere. If $N \geq 1$, find the minimum N so the spring does not snap.



-Proposed by Yash Bughani

Let elongation intitially = X= $\sqrt{L^2+2LR}-L$
Let maximum elongation = Y = $3\sqrt{L^2+2LR}-3L$
Let stiffness = k

$$\frac{kX^2}{2} = \frac{I}{2}\omega_{max}^2$$

$$\boxed{\omega_{max} = \sqrt{\frac{2k}{M}}\left(\frac{X}{R}\right)}$$

After placing ring:

$$\frac{kY^2}{2} = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{16MR^2}{N^2}\right)\omega_{max}^2$$

$$kY^2 = MR^2\left(\frac{1}{2} + \frac{16}{N^2}\right)\left(\frac{2kX^2}{MR^2}\right)$$

$$Y^2 = \left(\frac{1}{2} + \frac{16}{N^2}\right)X^2$$

$$\frac{Y}{X} = \sqrt{1 + \frac{32}{N^2}}$$

Substituting Y and X:

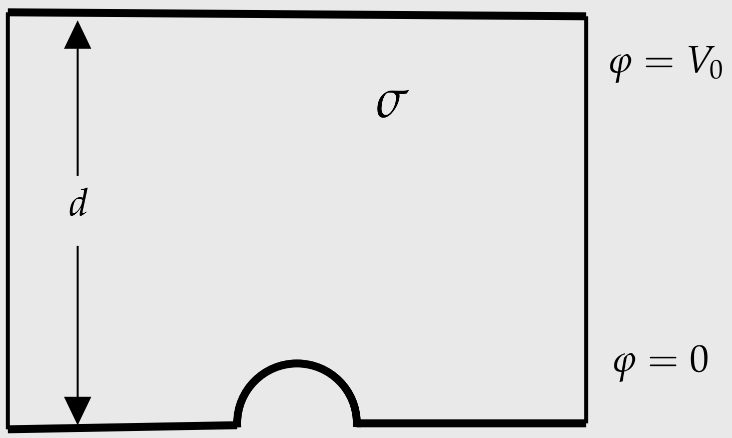
$$\frac{3\sqrt{L^2+2LR}-3L}{L^2+2LR-L} = \sqrt{1+\frac{32}{N^2}}$$

Hence $\boxed{N=2}$

Solution 8

Problem 9

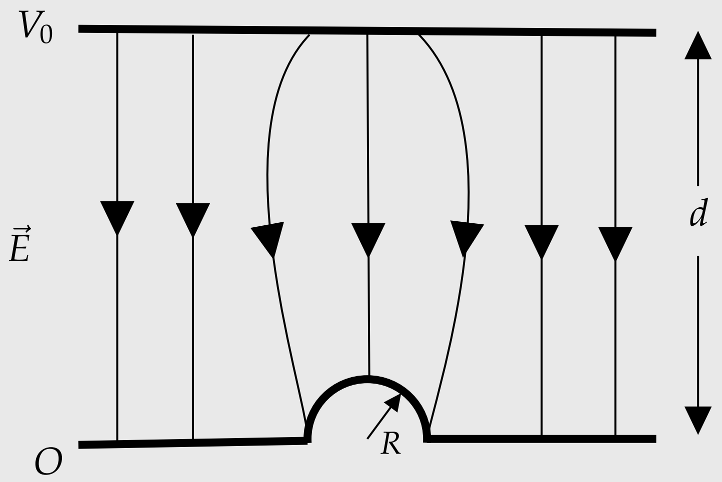
A potential difference of Vo initiates the flow of steady current from top to bottom of the conductor (conductivity of medium is σ). Atharva broke the conductor and a hemispherical bump is generated in the conductor. Now he initiates the flow of current, he found that current flowing into that hemispherical bump of radius R is $k\pi\sigma R^2\frac{Vo}{d}$, Assuming $d \gg R$ find the value of k?



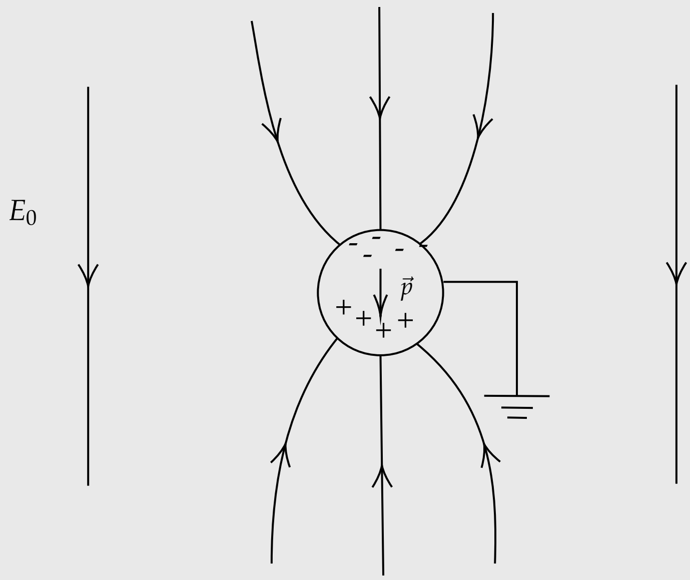
-Proposed by Harshit Gupta

Since $R << d$, $\vec{E}$ everywhere except near hemisphere will be:

$$\boxed{|\vec{E}| = \frac{V_0}{d}}$$

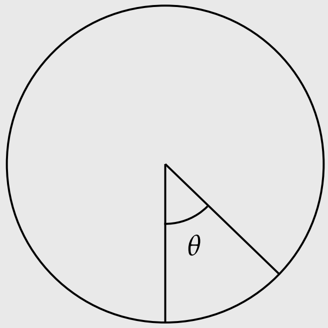


In uniform electric field grounded sphere gets polarised with dipole moment:

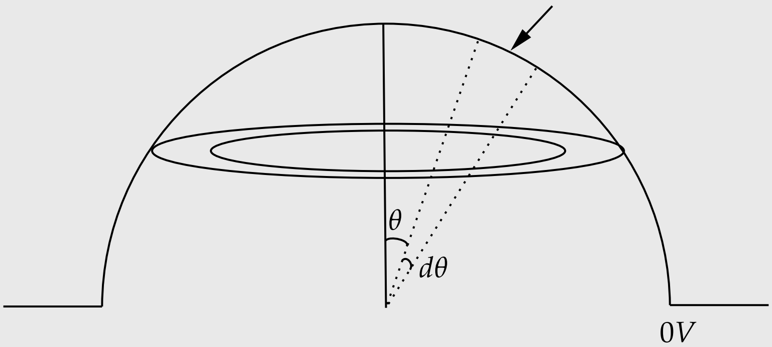


$$\boxed{|\vec{p}| = \frac{4}{3}\pi R^3 3\epsilon_0 \vec{E_0}}$$

So, surface charge density will be:



$$\boxed{\sigma = -3\epsilon_0 E_0 cos\theta}$$



$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0} = -3E_0 cos\theta\hat{r}}$$

Ohms law:

$$\vec{J} = \sigma\vec{E}$$

$$\vec{J} = -3E_0 \cos\theta\sigma\hat{r}$$
$$= -3\sigma\cos\theta\frac{V_0}{d}\hat{r}$$

So, total current flowing inside hemisphere will be

$$I = \int \vec{J} \cdot d\vec{A}$$

$$= \int_0^{\frac{\pi}{2}} 3\sigma\cos\theta\frac{V_0}{d} \cdot 2\pi R\sin\theta R d\theta$$

$$\boxed{\text{Hence, Answer=}\frac{3\pi R^2\sigma V_0}{d}}$$

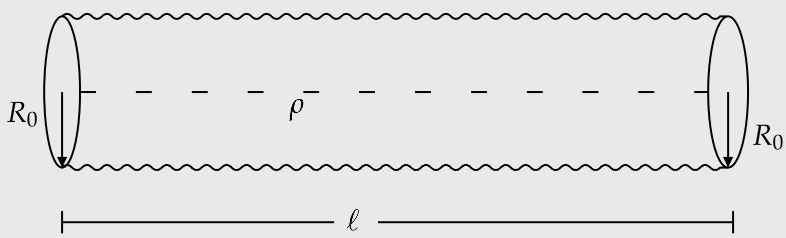
Answer: 3

Solution 9

Problem 10

Samar and Aayush live in a strange magical world. For doing magic, they need a perfect cylindrical wand. Samar and Aayush are sworn enemies and in a previous fight between them, Samar's wand broke so he made a new wand. He intended to make a perfect wand, but while using the charm "Rictumsempra", Aayush disturbed him and as a result, the radius of Samar's wand became:
$R(x) = R_0 + r\sin^2\frac{2\pi x}{a}$ and length of the wand is ℓ.
Somehow this wand reached Aditya in the Muggle world and Aditya wanted to calculate the resistance of the wand. He found the resistivity of the wand's material to be ρ. Determine the resistance of the wand that Aditya found. Assume that $r \ll R$ and $a \ll l$

a)The resistance of wand determined by Aditya will be $\frac{\rho l}{\pi R_o^2}$
b)The resistance of wand determined by Aditya will be $\frac{\rho l}{\pi R_o^2}\left(1 + \frac{r^2}{R_o^2}\right)$
c)The resistance of wand determined by Aditya will be $\frac{\rho l}{\pi R_o^2}\left(1 - \frac{r^2}{R_o^2}\right)$
d)The resistance of wand determined by Aditya will be $\frac{\rho l}{\pi R_o^2}\left(1 - \frac{r}{R_o}\right)$



-Proposed by Aditya

Let's analyse the magic wand at an arbitrary distance x from the left end, where $0 \leq x \leq l$
Infinitesimal resistance is

$$dR = \frac{\rho}{\pi\left(R_0 + r\sin^2\frac{2\pi x}{a}\right)^2}dl = \frac{\rho dx}{\pi R_0^2\left(1 + \frac{r}{R_0}\sin^2\frac{2\pi x}{a}\right)^2}$$

$$\approx \frac{\rho dx}{\pi R_0^2\left(1 + \frac{2r}{R_0}\sin^2\frac{2\pi x}{a}\right)}$$

Bionomial approximation gives

$$dR \approx \frac{\rho dx}{\pi R_0^2}\left(1 - \frac{2r}{R_0}\sin^2\frac{2\pi x}{a}\right) = \frac{\rho dx}{\pi R_0^2}\left[1 - \frac{r}{R_0}\left(1 - \cos\frac{4\pi x}{a}\right)\right] = \frac{\rho dx}{\pi R_0^2}\left(1 - \frac{r}{R_0} + \frac{r}{R_0}\cos\frac{4\pi x}{a}\right)$$

Now, we integrate dR over whole wand with limits from 0 to l

$$R = \int_0^l \frac{\rho dx}{\pi R_0^2}\left(1 - \frac{r}{R_0} + \frac{r}{R_0}\cos\frac{4\pi x}{a}\right)$$

$$= \frac{\rho dx}{\pi R_0^2}\left[\left(1 - \frac{r}{R_0}\right)x\right]_0^l + \left[\frac{ar}{4\pi R_0}\sin\frac{4\pi x}{a}\right]_0^l = \frac{\rho l}{\pi R_0^2}\left(1 - \frac{r}{R_0}\right)$$

Hence, the resistance is

$$\boxed{R = \frac{\rho l}{\pi R_0^2}\left(1 - \frac{r}{R_0}\right)}$$

Answer: (D)

Solution 10