$$\left.\begin{aligned}&\frac{d}{dt}\begin{pmatrix}U\\V\end{pmatrix}+\begin{pmatrix}0&-f\\f&0\end{pmatrix}\begin{pmatrix}U\\V\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\\&\begin{pmatrix}U(0)\\V(0)\end{pmatrix}=\begin{pmatrix}U_0\\V_0\end{pmatrix}\end{aligned}\right\}\quad\left.\begin{aligned}&\frac{d\bar{W}}{dt}+\tilde{F}\bar{W}=\bar{0}\\&\bar{W}(0)=\bar{W}_0\end{aligned}\right\}$$

$$\left.\begin{aligned}&\bar{W}=\exp(\tilde{A}t)\\&\tilde{A}+\tilde{F}=\tilde{0}\to\tilde{A}=-\tilde{F}\end{aligned}\right\}\quad \bar{W}=\exp(\underbrace{-\tilde{F}t}_{\tilde{M}})\bar{C}$$

$$\tilde{M}=-\tilde{F}t=\begin{pmatrix}0&ft\\-ft&0\end{pmatrix}\ \text{No simétrica.}\qquad \tilde{M}=\begin{pmatrix}i&-i\\-1&-1\end{pmatrix}\begin{pmatrix}ift&0\\0&-ift\end{pmatrix}\begin{pmatrix}i&-i\\-1&-1\end{pmatrix}^{-1}$$

$$\det(\tilde{M}-\lambda\tilde{I})=0\Rightarrow\det\begin{pmatrix}-\lambda&ft\\-ft&-\lambda\end{pmatrix}=0\Rightarrow\lambda^2+(ft)^2=0$$

$$\lambda^{\pm}=\pm ift\ \begin{cases}\lambda^+ & \begin{pmatrix}i\\-1\end{pmatrix}\\ \lambda^- & \begin{pmatrix}-i\\-1\end{pmatrix}\end{cases}$$

$$\left.\begin{aligned}&\frac{\partial U}{\partial t}-fV=0\\&\frac{\partial V}{\partial t}+fU=0\end{aligned}\right\}\quad\left.\begin{aligned}&\frac{\partial^2U}{\partial t^2}-f\frac{\partial V}{\partial t}=0\\&\frac{\partial V}{\partial t}=-fU\end{aligned}\right\}\quad\boxed{\frac{\partial^2U}{\partial t^2}+f^2U=0}\quad\begin{aligned}&U(0)=U_0\\&\left.\frac{\partial U}{\partial t}\right|_0=fV_0\end{aligned}\quad\begin{aligned}&U=\exp(\lambda t)\\&\lambda^2+f^2=0\to\lambda^{\pm}=\pm if\end{aligned}$$

$$\begin{aligned}&U(0)=U_0\\&V(0)=V_0\end{aligned}\quad\left.\begin{aligned}&\frac{\partial^2V}{\partial t^2}+f\frac{\partial U}{\partial t}=0\\&\frac{\partial U}{\partial t}=fV\end{aligned}\right\}\quad\boxed{\frac{\partial^2V}{\partial t^2}+f^2V=0}\quad\begin{aligned}&V(0)=V_0\\&\left.\frac{\partial V}{\partial t}\right|_0=-fU_0\end{aligned}\quad\begin{aligned}&V=\exp(\lambda t)\\&\lambda^2+f^2=0\to\lambda^{\pm}=\pm if\end{aligned}$$

$$\left.\begin{aligned}&U(t)=C_1\exp(ift)+C_2\exp(-ift)\\&V(t)=C_3\exp(ift)+C_4\exp(-ift)\end{aligned}\right\}\quad\boxed{t=0}\quad\left.\begin{aligned}&U_0=C_1+C_2\\&V_0=C_3+C_4\end{aligned}\right\}$$

$$\left.\begin{aligned}&\frac{\partial U}{\partial t}=C_1if\exp(ift)-C_2if\exp(-ift)\\&\frac{\partial V}{\partial t}=C_3if\exp(ift)-C_4if\exp(-ift)\end{aligned}\right\}\quad\boxed{t=0}\quad\left.\begin{aligned}&fV_0=if[C_1-C_2]\\&-fU_0=if[C_3-C_4]\end{aligned}\right\}$$

$$\begin{bmatrix}1&1&0&0\\0&0&1&1\\if&-if&0&0\\0&0&if&-if\end{bmatrix}\begin{bmatrix}C_1\\C_2\\C_3\\C_4\end{bmatrix}=\begin{bmatrix}U_0\\V_0\\fV_0\\-fU_0\end{bmatrix}\Rightarrow\begin{bmatrix}C_1\\C_2\\C_3\\C_4\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\{U_0-iV_0\}\\\frac{1}{2}\{U_0+iV_0\}\\\frac{1}{2}\{V_0+iU_0\}\\\frac{1}{2}\{V_0-iU_0\}\end{bmatrix}$$

$$\boxed{\begin{aligned}&U(t)=U_0\cos(ft)+V_0\sin(ft)\\&V(t)=V_0\cos(ft)-U_0\sin(ft)\end{aligned}}$$

$$\left.\begin{aligned} \frac{dx}{dt} &= u(t) \\ \frac{dy}{dt} &= v(t) \end{aligned}\right\} \Rightarrow \begin{aligned} \int_0^x dx' &= \int_0^t u(t')dt' \Rightarrow x = u_0 \int_0^t \cos(ft')dt' + v_0 \int_0^t \sin(ft')\,dt' \\ \int_0^y dy' &= \int_0^t v(t')dt' \Rightarrow y = v_0 \int_0^t \cos(ft')dt' - u_0 \int_0^t \sin(ft')dt' \end{aligned}$$

$$\left.\begin{aligned} \int_0^t \cos(ft')dt' &= \frac{1}{f}\sin(ft) \\ \int_0^t \sin(ft')dt' &= \frac{1}{f}(1-\cos(ft)) \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x &= \frac{u_0}{f}\sin(ft) - \frac{v_0}{f}\cos(ft) + \frac{v_0}{f} \\ y &= \frac{v_0}{f}\sin(ft) + \frac{u_0}{f}\cos(ft) - \frac{u_0}{f} \end{aligned}\right\}$$

Assume center: $\underline{\underline{x_* = \frac{v_0}{f}}}$, $\underline{\underline{y_* = -\frac{u_0}{f}}}$ then $\left.\begin{aligned} x - x_* &= \frac{u_0}{f}\sin(ft) - \frac{v_0}{f}\cos(ft) \\ y - y_* &= \frac{v_0}{f}\sin(ft) + \frac{u_0}{f}\cos(ft) \end{aligned}\right\}$

$$(x-x_*)^2 = \left(\frac{u_0}{f}\sin(ft) - \frac{v_0}{f}\cos(ft)\right)^2 = \frac{u_0^2}{f^2}\sin^2(ft) - 2\frac{u_0v_0}{f^2}\sin(ft)\cos(ft) + \frac{v_0^2}{f^2}\cos^2(ft)$$

$$+\ (y-y_*)^2 = \left(\frac{v_0}{f}\sin(ft) + \frac{u_0}{f}\cos(ft)\right)^2 = \frac{v_0^2}{f^2}\sin^2(ft) + 2\frac{u_0v_0}{f^2}\sin(ft)\cos(ft) + \frac{u_0^2}{f^2}\cos^2(ft)$$

$$(x-x_*)^2 + (y-y_*)^2 = \frac{u_0^2}{f^2}\left(\sin^2(ft) + \cos^2(ft)\right) + \frac{v_0^2}{f^2}\left(\sin^2(ft) + \cos^2(ft)\right) = \frac{u_0^2+v_0^2}{f^2} = R^2 \Rightarrow \underline{\underline{R = \frac{\sqrt{u_0^2+v_0^2}}{f}}}$$