PHYSICS
Energy, Work and Forces

Contents

1 SIGNIFICANT FIGURES AND SCIENTIFIC NOTATION

This chapter looks at significant figures and their importance in scientific disciplines. In Physics, Chemistry and Mathematics, one can only be as precise as the measurements they use in their calculations. Precision is generally limited by equipment, by what is known as the resolution of an instrument (the smallest increment of measurement). Being more precise than your least precise recordings can result in incorrect answers.

There are several rules that govern how many significant figures you can state your final answer to and what figures are significant. These rules are listed below with an example of each given.

1. If a number is not a zero, it is always significant
2. If at the end of an answer, after the decimal place, there is a zero, this is a significant figure
3. Zeroes between other non-zero digits are always significant
4. Zeroes that help position the non-zero digits in the right spot are not significant – generally scientific notation is ideal to provide answers under these circumstances
5. For multiplication and division, you round your answers to the same number of significant figures as the least precise measurement you used in calculating
6. For addition and subtraction, your final answer must be given to the same number of decimal places as the value with the smallest number of decimal places

Mastering these six rules will help you provide accurate answers to correct precision.

EXAMPLES

EXAMPLE 1
How many significant figures are in the number 67812.3628?

Using Rule 1, there are 9 significant figures.

EXAMPLE 2
How many significant figures are in the number 0.3450070?

Using Rule 2 and 3, there are 7 significant figures (the zero before the decimal place is a placeholder and hence is not significant), however the zeroes between 5 and 7 are significant, as is the final zero.

EXAMPLE 3
How many significant figures are in the number 0.00076?

Using Rule 4, there are two significant figures as all the other zeroes are simply holding the 7 and 6 in the correct place. One could also write this using scientific notation to be 7.6×10^{-4}, however there will be more on scientific notation later in this chapter.

EXAMPLE 4
Find the product of 35.8 and 90.23 to the correct number of significant figures

Product means to multiply the two numbers. Using Rule 5, we must identify the recording with the least number of significant figures. In this case, it is 35.8, which is to the precision of 3sf (significant figures). It is best to do the calculation in full, and then fix to avoid any possible rounding errors.

$35.8 \times 90.23 = 3230.234$
.: $35.8 \times 90.23 = 3230$ (to 3sf, note that the last zero is not significant)

EXAMPLES (CONTINUED)

EXAMPLE 5

Find the sum of 67.82, 89.921 and 54.234 to the correct number of significant figures

Sum means to add the three numbers together. Using rule 6, we must identify the number with the least number of decimal places, in this case, 2. It is best to do the calculation in full, and then fix to avoid any possible rounding errors.

67.82 + 89.921 + 54.234 = 211.975
.: 67.82 + 89.921 + 54.234 = 211.98 (to 2dp)

EXERCISE 1.1 – SIGNIFICANT FIGURES

Use of graphics calculator technology is appropriate for these questions

1. How many significant figures are the following numbers written to?
   a) 23
   b) 346
   c) 367.0
   d) 0.005
   e) 0.9180
   f) 0.1921
   g) 1300.0
   h) 1307
   i) 9019.24
   j) 876.42
   k) 9009
   l) 0.0000007609
   m) 23.000005
   n) 12.4309
   o) 12.000

2. Which has a greater number of significant figures?
   a) 67.004 or 0.0006574
   b) 0.009 or 0.90
   c) 8.0560 or 3.42761
   d) 0.0012 or 0.467
   e) 90.09 or 125.00
   f) 908.2 or 0.09

3. Explain how many significant figures the following measurements are to or whether there are multiple possible answers
   a) 300
   b) 450
   c) 130000
   d) 1340
   e) 560

4. Why may rounding results in calculations result in an incorrect final answer?

5. Calculate the product of these values to the correct number of significant figures
   a) 450.6 and 906.5
   b) 340.23 and 9
   c) 127.8 and 126.892
   d) 45.67 and 13.4
   e) 89.8128 and 912.44
   f) 12 and 13.6
   g) 0.0009 and 12.090
   h) 0.000710 and 12.9
   i) 0.00009 and 0.0405
   j) 0.010 and 0.001
   k) 12 and 439.1

6. Calculate the sum of these values to the correct number of significant figures
   a) 436.2 and 234.5
   b) 341.57 and 5
   c) 129.7 and 148.981 and 64.19
   d) 123.813 and 45.90
   e) 45.89 and 917.67
   f) 69.39 and 0.000260
   g) 0.00110 and 2.019
   h) 0.0008 and 0.0324
   i) 67 and 13.4
   j) 54.09 and 0.00192 and 1

7. Give an answer for the sum of 56.19 & 89.748 to the correct no. of significant figures

8. Give an answer for 78.324 divided by 78.10 to the correct number of decimal places

Scientific Notation is a way that scientists handle really big and really small numbers. It involves writing a number between 1 and 9.9999…. written to a power of ten. This can also be a great way to write numbers to a certain level of precision and significant figures. In order to write in scientific notation, there must be one figure before the decimal point and the appropriate number of significant figures. This is then written to a power of ten. If the number is smaller than 1 then the power of ten is negative. If the number is greater than 1, then it is either raised to 10^0 (1) or a positive power of ten.

The following examples show how to write four numbers in scientific notation form.

EXAMPLES

EXAMPLE 1
Write the number 42 in scientific notation

4.2×10^1

EXAMPLE 2
Write 219.75 in scientific notation to the precision of two significant figures

2.2×10^2

EXAMPLE 3
Write 0.00076 with appropriate scientific notation

7.6×10^{-4}

EXAMPLE 4
Write 0.00432 to the precision of two significant figures

4.3×10^{-3}

EXERCISE 1.2 – SCIENTIFIC NOTATION

1. Write the following numbers in scientific notation
   a) 421
   b) 397
   c) 1097
   d) 540.234
   e) 436.2
   f) 0.0012
   g) 0.01780
   h) 31 * 1000
   i) (31+89+90)*21+8
   j) 3.1415
2. Write the following to a precision of two significant figures in scientific notation form
   a) 0.00091721
   b) 140214
   c) 0.001240
   d) 0.73224
   e) 0.0001
   f) 21.742
   g) 2178913.31809
   h) 6.319
3. The speed of light, generally denoted by "c" in physics, is 300 million ms^{-1}. Write this in scientific notation form.
4. Derive the difference between the mass of a proton ($1.672621777 \times 10^{-27}$ kg) and the mass of an electron ($9.10938291 \times 10^{-31}$kg) and give your answer to three significant figures in scientific notation.
5. Use scientific notation to give an answer to the sum of 0.00123 and 0.323. Give your answer to an appropriate number of significant figures.
6. Use scientific notation to give an answer to the product of 1389.813 and 0.0006. Give your answer to an appropriate number of significant figures.

2 APPLICATION OF SIGNFICANT FIGURES IN CONTEXT

In experiments or calculations, you will need to give all measurements to a certain number of significant figures. This chapter focuses on a few contextual physics questions. For every question or statement, you should answer with an appropriate number of significant figures and units. Each question will rely on performing simple arithmetic with the formulae given however answers are only correct if they reflect the correct degree of precision from the original measurements. Clear and logical lines of reasoning are important as it allows other physicists to see how you derived your final answer. Make sure that you draw a picture or diagram to understand the context.

For some of these questions, but not many, you may need to know the SI units and basic conversions, these will be covered over later chapters however will be introduced in this chapter as a core skill. You will need to use them with later energy and work questions over the proceeding chapters, hence it is important to note and memorize this table.

Look in the left column for the SI unit, and convert using the right side of this table.

Common measurements and SI units		Conversion prefixes	
Speed	ms^{-1} (metres per second)	Giga (G)	10^9 (there are a billion SI units)
Velocity	ms^{-1} (metres per second)	Mega (M)	10^6 (there are a million SI units)
Energy	J (Joules)	Kilo (k)	10^3 (there are a thousand SI units)
Work	J (Joules)	Hecto (h)	10^2 (there are a hundred SI units)
Acceleration	ms^{-2} (metres per second squared)	Deca	10^1 (there are ten SI units)
Time	s (seconds)	Deci	10^{-1} (a tenth of an SI unit)
Weight	N (Newtons)	Centi (c)	10^{-2} (a hundredth of an SI unit)
Mass	Kg (Kilograms)	Milli (m)	10^{-3} (a thousandth of an SI unit)
Force	N (Newtons)	Micro (μ)	10^{-6} (a millionth of an SI unit)
Volume	L (litre)	Nano (n)	10^{-9} (a billionth of an SI unit)
Angles	degrees	Pico (p)	10^{-12} (a trillionth of an SI unit)

Another key term that is used in physics and mathematics is direct and inverse proportionality, denoted by the symbol: $\propto$

If a value is directly proportional to another value, then it means that when you change one of the values, another value will change by that same amount. For example, Force is directly proportional to mass, so hence when you double the mass, you double the force (as long as it is a fixed acceleration).

Go ahead, try this concept using the formula: $F = ma$, where acceleration is a fixed value of $4ms^{-2}$ and the mass is 12kg. Double the mass and see what happens to the force. This is a good example of direct proportionality. One can represent this mathematically by: $F \propto m$. Remember to learn direct proportionality, this will help you make sense of physics.

Inverse proportionality is where if you increase one value, another value is decreased by the same amount. Think of an example, perhaps take Newton's Second Law and rearrange to find a value in the denominator. Again, assume one value is fixed when surveying this.

EXERCISE 2.1 – CONVERTING UNITS

1. Convert these values into SI units

a) 20 gigametres
b) 3.4 millilitres
c) 36 MJ
d) 90.8 GJ
e) 23.4mm
f) 0.197dm
g) 1.807nm
h) 1318.3ng
i) 9019.24pm
j) 876.42 mins
k) 9009 days
l) 0.0000007609 Gm
m) 23.000005 fs
n) 12.4309GL
o) 12.000 ML

EXAMPLES

EXAMPLE 1

The Universal Law of Gravitation calculates the force of gravity and is shown in the equation below. A physicist makes the conjecture that the force of gravity between the two masses is directly proportional to the product of the two masses.

$$F = G\frac{m_1 m_2}{r^2}$$

Test the possibility of this conjecture being correct by altering the values of m_1 and m_2 **numerically** while assuming G is constant non-zero real number and r is a constant value of 2.4m. First test with m_1 being 100.0kg and m_2 being 200.0kg, before doubling both these values.

*To test this conjecture, the values of m_1 and m_2 will be represented by 100kg and 200kg accordingly, giving a product of 20 000kg (2.000*10^4 kg). The product of 200kg and 400kg is 80 000kg (8.000*10^4 kg).*

*100.0*200.0 = 20 000 kg*
*.; 100.0*200.0 = 2.000*10^4 kg (4 sf)*

*200.0*400.0 = 80 000 kg*
*.; 200.0*400.0 = 8.000*10^4 kg (4 sf)*

$$F = G\frac{m_1 m_2}{r^2}$$

$$.:\ F = G\frac{2.000 * 10^4}{2.4^2}$$

$$.:\ F = G\frac{20000}{2.4^2}$$

$$.:\ F = 3472.22 * G$$

$.:\ F = ((3.5 * 10^3) * G)N$ *(to the precision of 2sf, where N is Newtons SI units for force)*

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = G\frac{8.000 * 10^4}{2.4^2}$$

EXAMPLE (CONTINUED)

$$F = G\frac{80000}{2.4^2} = 13888.88 * G$$

$F = ((1.4 * 10^4) * G)N$ *(to the precision of 2sf, where N is Newtons SI units for force)*

*As 4*3472.22 = 13888.88, the product of mass is quadrupled when both m_1 and m_2 are doubled. As G is constant, the force between the two objects is also quadrupled, hence the physicists claim of the force of gravity between two objects being directly proportional to the product of their mass is supported.*

EXERCISE 2.2 – SOLVING PHYSICS PROBLEMS

Use of graphics calculator technology is appropriate for these questions

1. The amount of kinetic energy an object has is directly proportional to the square of that objects velocity. This means that if a car doubles its velocity then the amount of kinetic energy is quadrupled.
   a) Represent this mathematically using symbols
   b) Explain why this is so based on the kinetic energy formula where $K = \frac{1}{2}mv^2$
   c) If a car has a velocity of 30.2kmh^{-1} and it doubles its velocity, what is its kinetic energy after the car reaches its new speed? (Hint: kmh^{-1} to ms^{-1}, divide by 3.6)
   d) If a snail is travelling 0.0005kmh^{-1} and it doubles its velocity, what will its new velocity be?

2. Momentum is a measure of "mass in motion". Generally, the product of mass and velocity can give a good approximation of momentum: $p = mv$.
   a) Using this formula what is the momentum of a truck that has a mass of 5000kg and is travelling at 30ms^{-1}
   b) Another truck is travelling at the same speed with a mass of 6000.0kg. Calculate its momentum.
   c) Which value calculated has a higher degree of precision?

3. The exact formula for momentum is given by $p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$. Assume for all following questions that the speed of light, c, is given by 3.00x10^8 ms^{-1}.
   a) Repeat 2 (a) and 2(b) with the new formula. Is there a difference?
   b) Explain why, with reference to the formula and mathematics, there is no difference in objects travelling at 30ms^{-1} with the degree of precision of the measurements given.
   c) John works at a particle accelerator. One day he decides to accelerate a small object with a mass of 0.00001 grams to 900 000.0 ms^{-1}. Calculate momentum using both formulae and see if there is a difference.
   d) Photons are traditionally referred to as massless however travel at the speed of light. Does this mean that a photon has a large momentum as it travels at a high velocity? Explain.

4. A person sits in a tree approximately 5m above the ground. Calculate their gravitational potential energy if g = 9.8ms^{-2} and they have a mass of 60kg. Hint: E_{PG} = mgh

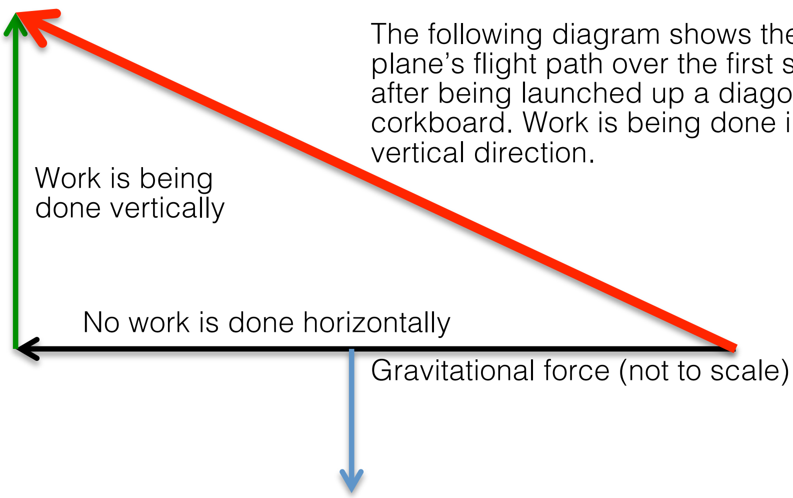
3 WORK DONE

The concept of work revolves around three general rules, which are outlined in the flow chart. In order to have an understanding of what energy is, one must first understand this concept. Work is best defined as a force acting on an object causing it to move through a displacement. This means that work is done when you push a car and it begins to move, and if you are vacuuming a carpet, but no work would be done when you are holding a book above your head. The fact that no work is occurring when you hold that book comes down to the fact that there may be a force holding that book there however no mechanical work is being done on the book as the force does not noticeably move the book through a displacement (vector distance).

Another important thing to note though is that no work is done if a force is acting in a perpendicular nature to the force that moves an object through a displacement. This means that if another force is acting at 90 degrees, then no mechanical work is being done on the object. Take this example as moving a plate of sandwiches or moving a table (not by pushing it across the ground). In both circumstances, mechanical work is not being done.

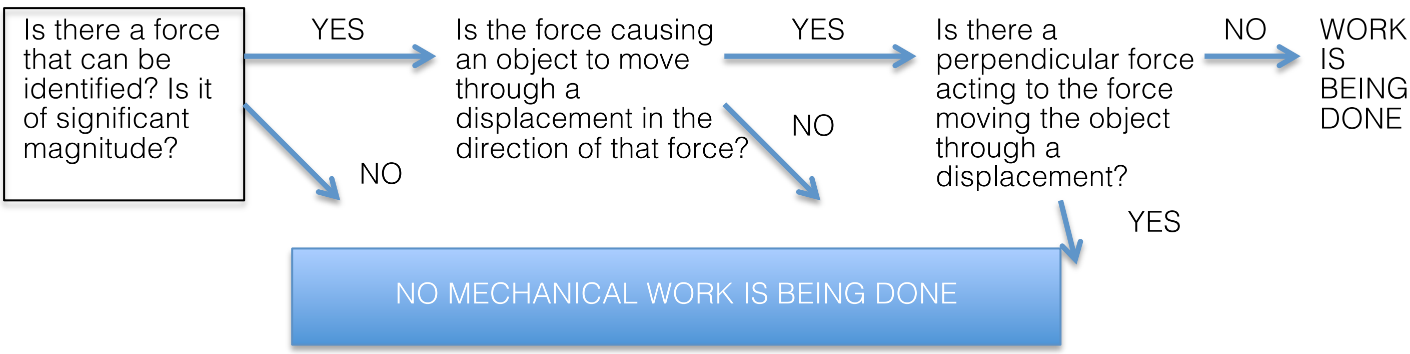
Perpendicular movement in a satellite object orbiting around the planet is also another example of when work is not done. Using vector diagrams one can easily see that as there will always be a force towards the center of the planet and the tangent to the circular path will be at 90 degrees to this force, no work will be done.

Sometimes it is best to draw a vector diagram; this involves drawing an arrow showing the direction of the force (generally the arrow would also be directly proportional to the magnitude of the force too) and a clearly labeled displacement. Remember to evaluate the horizontal vector component along with the vertical component. Below is an example involving a model plane.



The following diagram shows the model plane's flight path over the first second after being launched up a diagonal corkboard. Work is being done in the vertical direction.

Three Rules to check whether mechanical work is being done in the form of a flowchart



EXAMPLES

EXAMPLE 1

Is there work done when you push a car with a force of 7000N and it does not move?

No work is done, as the force does not move the object through a displacement.

EXAMPLE 2

Is there work being done when you walk up a flight of stairs?

Yes, work is being done, as you have vertical displacement, no perpendicular force and there is a force. Your gravitational potential energy also increases too as you are higher than your previous reference point.

EXAMPLE 3

A woman carries a briefcase across the yard and up a flight of stairs. In which phase of her journey to work is she doing work?

She is doing work when she carries the suitcase up a flight of stairs as the angle between the direction of the applied force and the direction of movement is not perpendicular, and the object is moving through a displacement, hence work is being done.

EXERCISE 3.1 – CONCEPTUAL WORK QUESTIONS

1. Draw a vector diagram showing that a satellite in orbit around the Earth is doing no mechanical work and write a small two sentence justification explaining this. (Remember that vectors show the force explained)
2. A box is moved a metres when a force is applied to push it. Is work being done?
3. A man carries a suitcase horizontally across a distance of 4.50m, is there work being done?
4. A man holds a barbell above his head. At this instance is there work being done? Explain
5. Tim plays with a wind up "Jack in the Box", when the clown springs up out of the box is there work being done?
6. Draw a vector diagram illustrating a box that is moved through a displacement of 7m along the floor by being tied to a string that is pulled at 42 degrees from the floor. Suggest why work is being done and the forces at work.
7. A large boulder drops from a cliff and accelerates downwards to the ground. Is work being done on the boulder?
8. A pencil is lifted over a displacement of 0.5m and an elephant is lifted over a displacement of 0.5m. Given the definition of work mentioned as a force acting to move an object over a displacement, which of the two scenarios results in the greater amount of mechanical work?
9. A helicopter is hovering above the ground. Is mechanical work being done on the helicopter?
10. When you are writing the answer to this question on a piece of paper, are you doing mechanical work?

There is a formula that allows one to derive the amount of work done in a situation. This formula is given as follows:

$$W = Fs\cos\theta$$

where W is work (measured in Joules)
F is the force causing the object to move (measured in Newtons)
s is the displacement over which the object moves (measured in metress)
θ is the angle between the applied force and the direction of the movement (degrees)

Remember that these answers are mathematical and as a result, it is important to take note of the precision. Note how many significant figures there are. Draw a diagram and list all the facts you know before you start.

Note that work done is measured in Joules, this is a measure based on James Prescott Joule, a famous physicist. All answers regarding work done should be in joules.

Another key point is that in some questions a mass is given. The SI units for mass is kilograms and mass is how much matter something is made up of. In order to convert this to weight, gravitational force, F_g, one must multiply by 9.8 (the gravitational acceleration on Earth in ms^{-2}). Weight is measured in Newtons and in some questions, is the force causing work to be done. See example 3 below for the process.

It is also important to know how to convert values into SI units with metress, centimetress, millimetress, and other prefixes which may sometimes apply for other measurements.

EXAMPLES

EXAMPLE 1
A box is pushed with a force of 5.00×10^2 N to travel a constant velocity over 4.50m

$$F = 500N$$
$$s = 4.50m$$
$$.: W = Fs\cos\theta$$
$$.: W = (500)(4.50)(1)$$
$$.: W = 2250J$$

Note that there is no angle between applied force and movement force hence cos(0) is substituted in and a result of 1 is obtained

.: W = 2.25×10^3J (3 sf) <- This is how much work has been done by the box

EXAMPLE 2
A box is pulled along the floor with a force of 500N to travel at a constant velocity over 3m. A rope that is at 48 degrees to the floor pulls the box. Derive work done.

$$F = 500N(1sf)$$
$$s = 3m$$
$$\theta = 48^o$$
$$W = Fs\cos\theta$$
$$W = (500)(3)\cos(48)$$
$$.: W = 1003.69591J$$
$$.: W = 1000J(1sf)$$

EXAMPLES (CONTINUED)

EXAMPLE 3

A box has a mass of 5.00kg, it is pushed through a vertical displacement of 3.00m with the minimum force that can allow it to travel at a constant velocity. Derive work done.

In this case, one must convert mass to weight. The minimum force needed to pull the box through a vertical displacement is the same as the weight. Hence, one simply finds the weight by multiplying mass by 9.8.

$F_g = mg$
$g = 9.8ms^{-2}$

$\therefore F_g = (5.00)(9.8)$
$\therefore F_g = 49N$ (2 sf)

Hence the weight is 49N

$F = 49N$, $s = 3.00m$

$W = Fs \cos(\text{theta})$
$\therefore W = (49)(3.00)\cos(0)$
$\therefore W = 147N$
$\therefore W = 1.5 \times 10^2 J$ (2sf)

EXERCISE 3.2 – WORK CALCULATIONS

Use of graphics calculator technology is appropriate for these questions

1. Derive work done when a force of 600N is applied to an elevator and it travels over a displacement of 20.0m.
2. Is more work done when a weight trainer lifts a 1000N dumbbell to a height of 3.00m or when a 2000N dumbbell is lifted to a height of 1.50m?
3. Why is it that if the force of movement is perpendicular to the applied force that no work is done? Explain mathematically
4. A 2.1kg book is lifted over a displacement of 3.50m, what work is done?
5. A box is pulled along the ground by a pulley with a force of 740N over a displacement of 8.00m. If the pulley was at an angle of 52 degrees from the ground, derive work done on the box.
6. Complete this sentence: The work done in a situation is greatest when the angle between the direction of the applied and movement force is ___ degrees.
7. Show your response in question 6 mathematically to prove it
8. Which scenario has had more work done, a telephone lifted up to someone's ear or someone applying a force to a car stuck in mud without the car moving?
9. Draw a vector diagram of a box moving up a ramp with a frictional resistance of 500N and the ramp rises 12.0m for every 15.0m. If an object travels up this ramp to an elevation of 17.2m, and the object has a known mass of 1.36kg, derive the total work done in this situation.
10. Can a photon do work? Explain and justify your answer using the formula

For questions in exercise set 3.3, you need to rearrange a formula in order to derive the amount of work done. To do this, it is necessary to show the steps and derivation necessary to find the distance or force involved. Remember to follow the logic that if a term was in the denominator on one side, it will be in the numerator on the other side, and vice versa. This only applies to terms that are multiplied though, if a term is added or subtracted from another term then moving it to the other side of the equation will result in the inverse operation being done on the equation, either subtract or add.

Below shows the derivations of force, weight, angle and displacement based on the original formula.

DERIVATIONS

Force

$$W = Fs\cos\theta$$

$$\therefore \frac{W}{s\cos\theta} = F$$

Angle between applied and movement force

$$W = Fs\cos\theta$$

$$\therefore \frac{W}{Fs} = \cos\theta$$

Distance

$$W = Fs\cos\theta$$

$$\therefore \frac{W}{F} = s\cos\theta$$

$$\therefore \frac{W}{F\cos\theta} = s$$

Weight

$$W = Fs\cos\theta$$

$$W = (mg)\cos\theta$$

$$\therefore \frac{W}{\cos\theta} = mg$$

$$\therefore \frac{W}{\cos\theta g} = m$$

EXERCISE 3.3 – WORK DERIVATIONS

Use of graphics calculator technology is appropriate for these questions

1. Next to each of the derivations in the box above, write an annotated step of sets explaining the process by which the value is made the subject of a formula using algebra.

2. Calculate the angle of applied force between the ground and a rope that pulls a box against 70N of frictional resistance across the floor at a minimum constant velocity over 3.95m

3. 3.51MJ of work is done on a large safety deposit box that is lifted from a mine. Assuming it travelled a vertical displacement of 500m, what is the weight of this safety deposit box?

4. A suitcase is loaded onto a plane. The applied force is 45.4 degrees from the direction of the movement force. If the suitcase has a mass of 5.65 kg and the amount of work done is 30.2J, over what distance did the suitcase travel?

5. Derive the force applied to a car when the work done is 4000.2J and it travels over a displacement of 0.03km.

6. A truck has 100MJ done over the course of x amount of seconds when it is lifted vertically upwards by a crane. If the truck has a mass of 3 metric tons, determine the number of seconds for which the truck was lifted.

7. The total amount of work done on a heavy physics textbook is 460.0J when it is lifted from the ground story of a building to the floor of the fifth floor (the sixth floor including the ground floor) of the building. If the book is lifted with a constant minimum velocity and each level on the building is 2.33m in height, derive the weight of the physics book.

8. Which has the greater weight, a child's colouring book which has been lifted from the ground floor to the floor of third floor of the building in question 7 under the same conditions as the textbook with a total amount of work done as 100.5J or a magazine which has been lifted from the floor of the sixth floor to the roof of the tenth floor of the same building with the same conditions, with a total amount of work being 124.6J?

Stopping distances and forces are important. In car crashes, manufacturers will often attempt to increase the stopping distance to reduce the impact force and hence make it safer for passengers. Discuss how a car manufacturer could perhaps achieve this goal.

The idea and concepts behind force, including the two dimensions, vectors, resultant vectors and magnitude will be discussed in a later chapter, however a brief introduction is necessary to understand the following section on work. Read through the following to get a basic introduction to some common types of forces that can influence the amount of work done.

The normal force is a force that acts perpendicular to the horizontal plane of contact. You will see this in the example in the moment. Think again of what this would mean on a ramp as these will be the examples of work that will be demonstrated over the next section of this chapter.

Friction is a force caused by two surfaces sliding over each other. This force is the result of tiny microscopic bumps on the surface of objects that appear smooth. These microscopic bumps act against an objects movement.

This worked example is the same as what the questions given in the next exercise set will be like. When attempting to find work done, consider the forces acting against the object and their magnitude. Remember that force is a vector quantity, meaning that size and direction matter; hence one cannot simply derive the sum of all forces as they are acting in different directions. Play close attention to the annotated example and the justification it gives.

EXAMPLES

EXAMPLE 1

A slope rises 1.00 m vertically for every 14.0 m of its inclined length. A truck of known mass 3.00×10^3 kg travels a distance of 40.0 m up the slope at a uniform speed. Find the work done if the frictional resistance is 80.0 N. Consider work done on the engine.

It asks for the work done on the truck's engine, this means that one must consider what forces are acting against the truck going up the ramp. In this case, there are four forces:

- *The force of gravity (F_g) which is found by the sum of mass and gravitational acceleration, acting downwards (NOTE that force is a vector quantity, hence one must specify a direction)*
- *The force of friction (F_f) is 80N diagonally downward*
- *The force of the ramp (F_r) pushing the tires up the slope (a force exerted by the engine)*
- *The normal force (F_n) and acts in a perpendicular nature to the slope*

EXAMPLES (CONTINUED)

let θg, θf, θr, θn be the angles between the various force vectors and the displacement vector "s" (s = 40 metress in the direction of travel)

Total work = $((F_g)\cos(\theta g) + (F_f)\cos(\theta f) + (F_r)\cos(\theta r) + (F_n)\cos(\theta n)) \times s$

In context, one can interpret that $\theta f = 180°$, as the force of friction is acting 180 degrees from the movement of the truck. $\theta r = 0°$, as the force is acting in the same direction as the direction of movement. $\theta n = 90°$, as the normal force always acts in a perpendicular nature to the ramp.

.: Total work = $((F_g)\cos(\theta g) + (-F_f) + F_r + 0)\times s$

One can rewrite this as such by considering F_f as being in the negative direction due to the fact that friction acts in the opposing direction and force is a vector quantity where direction matters.

F_g angle is also interpolated by drawing a diagram, the force of gravity points directly downward and is equal to the slope – rise over run (see diagram), hence the angle F_g = -1/14.

Total work = $((F_g)(-1/14) - F_f + F_r)\times s$

This can be set to zero as total work would be the change in kinetic energy and in this case, the truck travels up the slope ramp at uniform speed.

$0 = ((F_g)(-1/14) - F_f + F_r)\times s$

From this one can derive the total amount of work done by the engine by rearranging the equation to make F_r x s the subject. This is again due to the fact that one needs to know the force exerted by the engine over the displacement.

$F_r \times s = (F_g/14 + F_f)\times s$

And then values can simply be substituted in accordingly

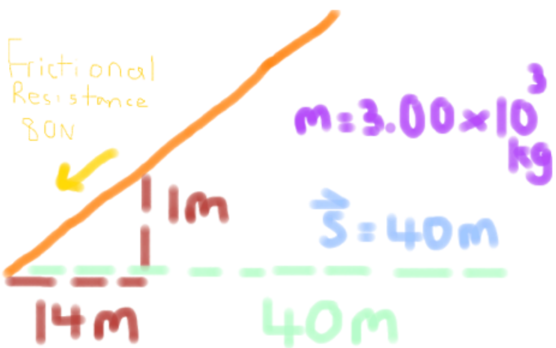
$[(3.00\times10^3\ \text{kg})(9.80\ \text{metres/s}^2)/14 + 80\text{N}] \times 40$ metres

Hence a value is obtained in Joules (SI units for work)

= 87200 J (3 sf)

And it is then corrected to three significant figures as g was assumed to be 9.80ms^{-2}, to 3sf

Hence, the total amount of work done on the engine is $8.72\text{x}10^4$ J (3sf)



This diagram offers a rough idea of the situation. It is not a proper force vector diagram.

Notice how the key facts from the questions are labeled on the diagram.

EXAMPLES (CONTINUED)

EXAMPLE 2

Consider work done against gravity and friction for the same question.

Work done is the same as a change in energy, to calculate the amount of work done against gravity, one can simply consider the scenario in terms of Gravitational Potential energy which is the product of mass, gravitational acceleration and height, along with work done against friction, which is the product of frictional resistance and displacement. One can then simply substitute this into the work done formula to compute the work done.

Please note that some of these energy ideas will become more apparent in the next chapter when we explore energy, which is changes when work is done.

$E_{PG} = mgh$

Where m is mass, E_{pg} *is gravitational potential energy, g is gravitational acceleration* ($9.80ms^{-2}$) *and height (in metres) is above the reference level, which in this case is 2.5871m, as shown below*

Work occurs for the vertical displacement, hence the elevation after 40m must be derived.

40/14 = 2.8571m,

hence, the ramp rises 2.8571m over the course of 40m, providing a displacement, s, value for the work done formula.

From these calculations,

Work = mgh = $(3.00*10^3)(9.80)(2.857) = 76060.74 = 7.61*10^4$ *J*
To compute the work done against friction = $(80)(40) = 3200 = 3.2*10^3$ *J*
Total work done against gravity and friction = (76060.74+3200) =79260.74 J = $7.93x10^3$ *(3 sf)*

EXERCISE 3.4 – WORK DONE AGAINST FORCES

Use of graphics calculator technology is appropriate for these questions

1. Draw a vector diagram for the first example showing all the forces involved that are acting on the truck
2. Calculate the work done against gravity on the truck by rearranging the equation in the first example
3. A slope rises 1.80 m vertically for every 11.58 m of its inclined length. A truck of known mass 3.00×10^4 kg travels a distance of 50.2 m up the slope at a uniform speed. Find the work done if the frictional resistance is 60.9N. Consider work done on the engine.
4. Use the gravitational potential energy formula where $E_{PG} = mgh$ to derive the work done against gravity by the truck in question 3 going up the same ramp in that question
5. Prove the relationship between work done and gravitational potential energy (made in the first sentence of the second example) with a contextual situation (Hint: consider raising a book through a vertical displacement)

EXERCISE 3.5 – WORK DONE WITH VARIABLE FORCE

Use of graphics calculator technology is appropriate for these questions

1. Draw a vector diagram of the following situation and solve for the work done. Suggest identify the forces involved in the scenario. A box is dragged across a floor by a 100N force directed 60.2° above the horizontal. The object is pulled 8.0m total.

2. Draw a vector diagram of the following situation and solve for the work done. A horizontal force F pulls a 10.1 kg carton across the floor at constant speed. The coefficient of sliding friction between the carton and the floor is 0.30, with the carton moving a displacement of 5.3m.

Variable force measures work done as the area under the curve on the graph. This following work done question set is an extension for those who understand the concept and process of integration with calculus. An example is given below assuming knowledge of integration. Note that this form of work calculation means that the particle is accelerating however a constant force is applied to the object.

EXAMPLES

EXAMPLE 1

A force F = 2x + 5 acts on a particle. Find the work done by the force during the displacement of the particle from x =0.0m to x = 2.0m. Given that the force is in Newtons.

Work done W $= \int F(x)dx$

Thus W $= \int F(x)dx \cos 0^{o}$

$= \int F(x)dx$

$= \int (2x + 5)dx$

Note that f(0) = 0, hence f(2) will represent the energy (based on area under the curve)

$f(2)= 2x^2/2 + 5x \mid$

$= 2^2 + 5 \times 2$

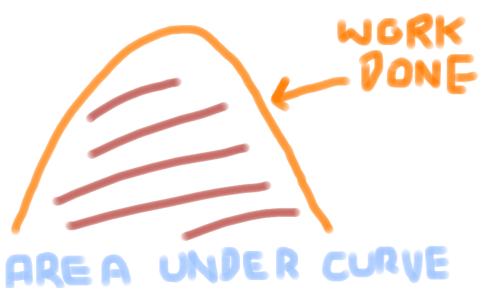
$= 14$ *J* (2 sf)

Hence the work done over the first two metres by a particle with force 2x+5 acting on it is 14J (2sf)

3. A force (4x + 8)N acts on a particle. Find the work done by the force during the displacement of the particle from x =0.0m to x = 3.0m.

4. A force (4.5x + 8.1)N acts on a particle. Find the work done by the force during the displacement of the particle from x =0.0m to x = 3.8m.

5. A force ($4x^2$ + 13)N acts on a particle. Find the work done by the force during the displacement of the particle from x =0.0m to x = 12.7m.



4 GRAVITATIONAL POTENTIAL ENERGY

By now, you probably know that work is simply a transfer or change of energy. The conservation of energy states that “energy can not be created nor destroyed, it can only transfer to other form(s) of energy”. One of these forms of energy is gravitational potential energy and this is generally evaluate relative to a reference point.

Say I were to stand on a chair, as a responsible writer – I don’t recommend that anyone does this as it may be unsafe, however in principle, I have increased my gravitational potential energy, commonly denoted as E_{PG} (read it backwards through the subscript and it makes sense).

Now hang on, you are probably curious about that word “potential” in there. The word potential simply means that the energy is stored. Think about Chemical Potential Energy, this energy is necessarily being used in excess quantities but it is still there. The same goes for gravitational potential energy, when you raise an object up, it gains Gravitational Potential Energy from Kinetic Energy (the energy of you moving the object). This build up of energy, when the object is released will then change to kinetic energy.

When the object has the most gravitational energy, generally when it is being held still before being dropped, it has stored energy. Think about how much work is being done when an object drops from a height based on the understanding that work is simply a force moving an object through a displacement. Now consider the statement at the top of this chapter.

$$E_{GP} = mgh$$

Gravitational Potential Energy is simple to calculate, you just need three values: height (above a reference point), mass and gravitational acceleration (which if surveyed at or near Earth’s surface is $9.80ms^{-2}$). Mass is in SI units of kilograms. One can assume that if no reference point is given that the reference level is the surface of the Earth.

Sometimes a question may ask by how much has the gravitational potential energy increased. It is important that when making a final statement to respond to the original question, that you word that sentence according to the original context. Remember that the SI unit for energy is Joules, just like work. Newton metres can also be used as another way of expressing the unit however as $1J=1Nm^{-1}$

Gravitational Potential Energy is just one of the forms of energy discussed in this textbook. The other forms over the next few chapters include:

- Kinetic Energy and Rotational Kinetic Energy (Chapter 5)
- Elastic Potential Energy (Chapter 6)
- Electrical Energy and Electrical Potential Energy (Chapter 7)
- Nuclear Energy, Fission and Fusion (Chapter 8)
- The Law of Conservation of Energy (Chapter 9)

EXAMPLES

EXAMPLE 1

A small pencil with a weight of 0.092N was raised from an elevation of 2.00m above sea level to 5.00m above sea level. By how much has the pencil’s Gravitational Potential Energy increased relative to the reference point of….

EXAMPLES (CONTINUED)

(i) two metres above sea level

$F_g = mg = 0.092N$, $g=9.8ms^{-2}$, $h=3m$ (as 5-2=3)

As the weight is 0.092N, one can calculate mass by dividing by 9.80 as 1kg = 9.8N

.: $F_g/g = m$.: $0.092/9.80=m$.: $m= 0.009387755102kg$

$E_{PG}=mgh$, .: $E_{PG} = (0.00939)(9.80)(3.00) = 0.276066J = 2.76x10^{-1}J$ (3sf)

Hence the pencil's GPE has increased by $2.76x10^{-1}J$

(ii) sea level

$E_{PG}=mgh$, .: $E_{PG} = (0.00939)(9.80)(5.00) = 0.46011 = 4.60x10^{-1}J$ (3sf)
Hence the pencil's GPE has increased by $4.60x10^{-1}J$ (3sf)

EXERCISE 4.1 – GPE CALCULATIONS

Use of graphics calculator technology is appropriate for these questions

1. An object has a mass of 40kg and travels a displacement of 50m horizontally right. By how much has the gravitational potential energy increased relative to the ground?

2. The same object (in question 1) is moved by a force to travel 5m vertically upwards from the ground. By how much has the gravitational potential energy increased relative to the ground?

3. An aeroplane of a mass of 18000kg is elevated to a height of 500m. By how much has the gravitational potential energy of the aeroplane increased relative to the ground?

4. A small boat is pushed up an inclined plane with a 1m rise over 10m horizontally. If the mass of the boat is 1000kg, determine how much the gravitational potential energy has increased by from the bottom of the ramp to the top.

5. Consider the work done formula. Write the work done formula for a pencil (like that in the example was lifted in the example) that is lifted through a vertical displacement in terms of the variables mass, gravitational acceleration and height. What do you notice?

6. A small beachball is pushed upwards from a height of 2m above the sand to a peak of 3.5m while it is going over the net before going back down to the Earth and landing on the surface. With the sand as a reference level, answer the following questions:
   a) Draw a diagram of the situation with all relevant facts
   b) What is the initial gravitational potential energy?
   c) What is the gravitational potential energy difference between the initial position and when the ball was going over the net?
   d) What is the final gravitational potential energy?
   e) What is the difference in gravitational potential energy from the initial hit to when the ball landed on the sand?

7. An ant walks vertically up a drainpipe to a height of 4cm above the ground. If the mass of the ant is presumed to be 1.4 grams, determine the gravitational potential energy increase.

8. While taking a photograph on the seventh floor of a building, a tourist accidentally drops their sandwich. Assuming that the height of each floor is 2300mm, that he dropped the sandwich from 1.3m above the floor of the seventh level, that the first level is the ground floor and the weight of the sandwich is 200 grams, what is the initial gravitational potential energy relative to the ground and the final gravitational potential energy?

9. The tourist in question 8 happens to notice that he dropped his sandwich and quickly manages to make his way to the balcony of the third floor (which is inline with the floor of this level). If he happens to catch the sandwich here at 1.3m above the floor of this level, what would the gravitational potential energy of the sandwich be relative to the ground?

10. What is the gravitational potential energy of an elephant sitting on a skyscraper if the elephant has a weight of 49000N and the height of the skyscraper is 190000000000nm?

11. A man has a mass of 650N, determine how much his gravitational potential energy increases by if he were to climb a flight of stairs 4.2m high.

12. Energy cannot be destroyed nor created (a key focus of Chapter 9), why is it that gravitational potential energy decreases as an object descends towards a reference level?

13. Determine the amount of gravitational potential energy a 5.00kg bowling ball has relative to the ground if it is 12mm from the ground's surface

14. If it was lifted from the ground, how much work has the bowling ball in question 13 done?

15. A pencil has been lifted to a height of 170.6m. If the work done is 15.883J, determine the mass of the pencil in kilograms.

16. Determine the gravitational potential energy of the pencil (from question 15)

17. A pair of glasses is lifted to the top of a desk, which is 300.5cm tall. By how much has the gravitational potential energy increased if the glasses has a mass of 19150 mg and the person lifted it from his eye level, which when sitting is 1.2m above the ground?

18. A feather and a bag of marbles are both at the same elevation. Which object would be expected to have a greater gravitational potential energy relative to the same reference point?

Another way of calculating gravitational potential energy is given by:

$$E_{pg} = \frac{-GMm}{r}$$

where G is the gravitation constant (given by , M is the mass of the attracting body, m is the mass of the object and r is the distance between the two centers.

The following example outlines how this formula can be used to calculate gravitational potential energy. This secondary formula is generally used for objects that are not near the Earth's surface.

EXAMPLES

EXAMPLE 1

A small planetary body of 120300kg attracts a small rock that is 5.00kg that is 40.0m above the surface. Assuming the gravitational constant is 6.67384×10^{-11} m^3 kg^{-1} s^{-2}, derive the gravitational potential energy per kilogram and hence determine the attraction for the 5.00kg mass.

$$E_{pg} = \frac{-GMm}{r}$$

$G = 6.67384 \times 10^{-11}$ m^3 kg^{-1} s^{-2}

M = 120300kg

m = 5.00kg

r = 40m

All the values are listed above, we can adapt this formula down to what is printed below as we want to determine the energy per kilogram then derive the overall energy for the rock by simply multiplying by 5. You can check that you get the same result by substituting the values into the same equation and evaluating.

$$E_{pg} = \frac{GM}{r}$$

$$.: E_{pg} = \frac{(6.67384 * 10^{-11})(120300)}{40}$$

$$.: E_{pg} = 2.01 * 10^{-7} J / kg$$

.: The gravitational potential energy is $2.01 * 10^{-7} J / kg$ *, hence for a 5.00kg rock, the gravitational potential energy is* $5(2.01 * 10^{-7}) J / kg$ *, which is* 1.00×10^{-5}*J. (3sf)*

EXERCISE 4.2 – GPE WITH MASS AND ATTRACTION

Use of graphics calculator technology is appropriate for these questions

1. Find the gravitational Potential energy of the Earth due to the sun if mass of earth is 5.98×10^{24} Kg and mass of sun is 1.99×10^{30} Kg and earth is 150 million Km away from sun?
2. A small planetary body of 150006100kg attracts a small man that is 65.00kg that is 20.5m above the surface. Assuming the gravitational constant is 6.67384×10^{-11} m^3 kg^{-1} s^{-2}, derive the gravitational potential energy.
3. As the mass of the object that attracts the object with GPE, does the overall GPE increase or decrease?
4. As the distance between the two objects of attraction increase, does the GPE increase or decrease?
5. What can be stated in terms of proportionality from the formula shown?
6. A small spherical object has a mass of 30kg attracts a small fly that is 2g that is and 1cm away. Assuming the gravitational constant is 6.67384×10^{-11} m^3 kg^{-1} s^{-2}, derive the gravitational potential energy.

With the following diagrams, an unknown value is written on the diagram. The objective is to determine the unknown value through rearranging and manipulating formulae through derivations. An example is given in the box below.

A common process to remember when it comes to rearranging equations is that if a term is in the denominator on one side, it will be in the numerator on the other side (if this term is multiplied by another term or is by itself, elsewise the entire expression must be moved).

EXAMPLES

EXAMPLE 1

Find the unknown value in the diagram

EXAMPLES (CONTINUED)

Height is the value that must be derived

E_{PG} = mgh

$$\therefore \frac{E_{PG}}{mg} = h$$

$$\therefore \frac{210}{(5.00)(9.8)} = h$$

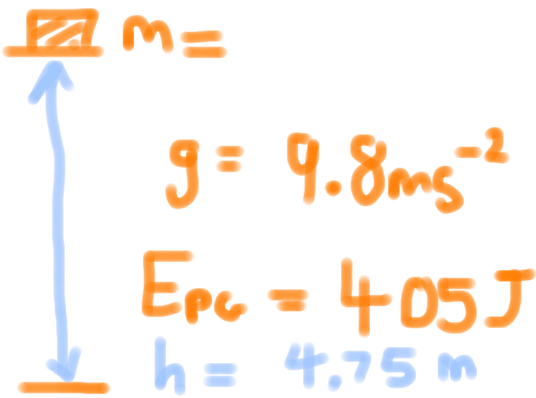
$$h = 4.30m(2sf)$$

Hence the height of the box in the diagram is 4.30m (2sf)

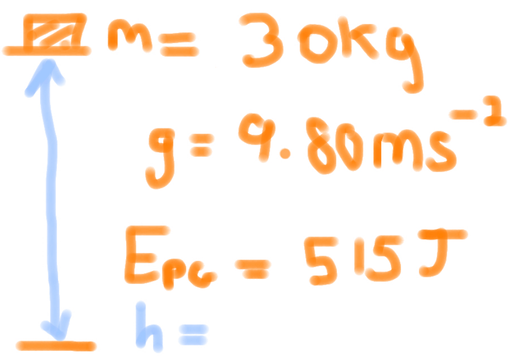
EXERCISE 4.3 – FINDING UNKNOWN VALUES

Use of graphics calculator technology is appropriate for these questions

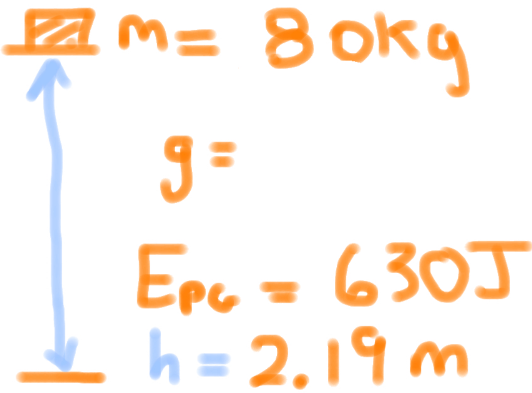
1.



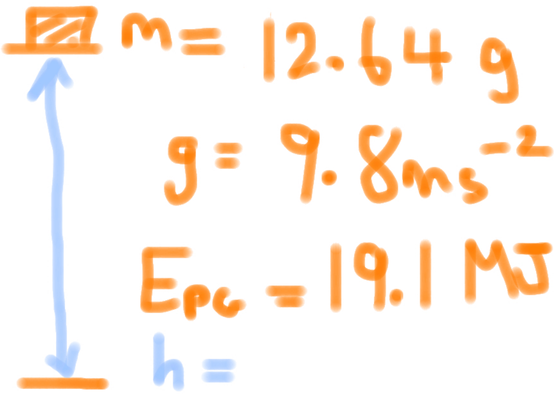
2.



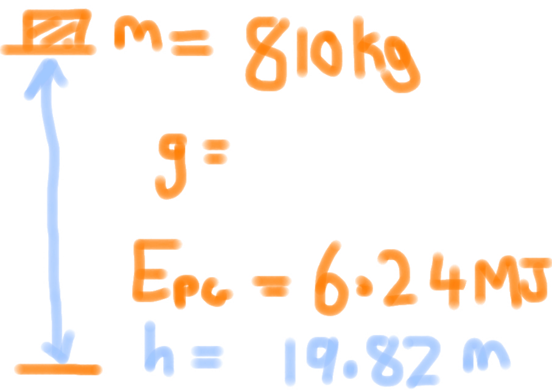
3.


m= 80kg
g=
Epg = 630J
h = 2.19 m

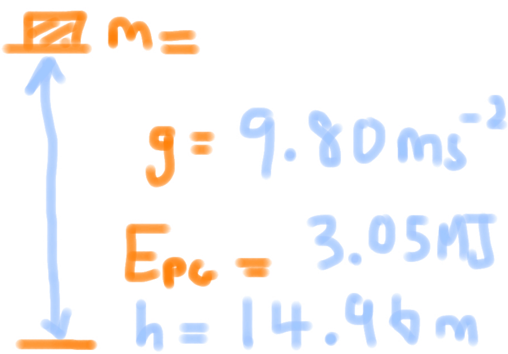
4.


m= 12.64 g
g= 9.8ms-2
Epg = 19.1 MJ
h =

5.


m= 810kg
g=
Epg = 6.24MJ
h = 19.82 m

6.


m=
g= 9.80ms-2
Epg = 3.05MJ
h = 14.96m

EXAMPLES

EXAMPLE 1

Determine the gravitational acceleration the planet Meigon, if an object that is 1.1m above the surface of the planet with a mass of 7.02kg has 13.2KJ of gravitational potential energy.

E_{PG} =mgh

One must find, g, gravitational acceleration. One can only assume g to be 9.80ms^{-2} at or near Earth's surface. NOTE: That gravitational acceleration (g) is measured in ms^{-2} (meters per second per second), just like normal acceleration.

.: 13.2 = (7.02)(g)(1.1)

$$.: \frac{13.2}{7.02 * 1.1} = g$$

.: g = 1.7ms^{-2} (2 sf)

Hence the gravitational acceleration of the planet Meigon is 1.7ms^{-2} (2sf), which is comparable to that of the moon.

EXERCISE 4.4 – GPE DERIVATIONS

Use of graphics calculator technology is appropriate for these questions

1. A ball is stationary on a platform above the surface of the Earth. If the gravitational potential energy is 305J and the ball has a mass of 2.3kg, derive how high the platform is above the surface of the Earth?
2. A small object is suspended 102m above the Earth. If the gravitational potential energy is 90.4J, what is the objects mass?
3. An umbrella is dropped from the top of a three level building. Before it drops it has a gravitational potential energy of 28.4J. If the mass of the umbrella is 400.5 grams, determine how high the third level of the building is from the ground.
4. If the umbrella was dropped from twice the height is the gravitational potential energy twice as much?
5. Determine the gravitational acceleration the planet Doogon, if an object that is 1.2m above the surface of the planet with a mass of 4.30kg has 16.2KJ of gravitational potential energy.
6. Gravitational acceleration (g) on the moon is one sixth that of Earth. Determine how high a hammer is dropped from if it possesses 1.83KJ of gravitational potential energy and if the hammer weighs 205.2g.
7. A circus performer is walking across a tightrope. His known to be 58.3kg. The gravitational potential energy is halved when the tightrope is lowered by 2.5m. If the height of the tent is 8m and the height of the man walking the tightrope is 1.75m (hence the tightrope can not be at an elevation of more than 6.25m), determine the elevation of the tightrope and the amount of gravitational potential energy. **EXTENSION:** How much work has the circus performer done?

5 KINETIC ENERGY AND ROTATIONAL KINETIC ENERGY

Kinetic energy is the energy related to movement. When an object has both mass and velocity (speed), then the object has some kinetic energy. This is similar to momentum, which is the impetus gained by a moving object.

There are many cases when kinetic energy, typically denoted by “K”, can be observed within the real world, like when a boulder falls from the top of the cliff and accelerates towards the ground. Cars, planes, trains and other vehicles all have kinetic energy when they are moving.

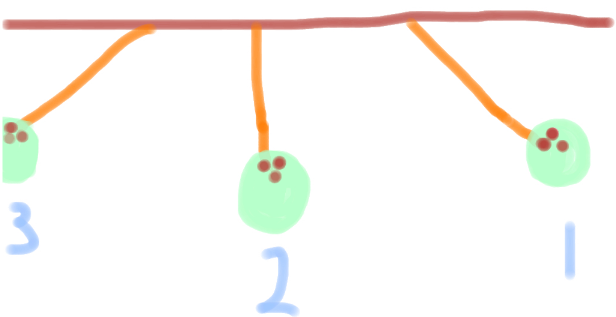
Calculating kinetic energy can be done using the following formula:

$$K = \frac{1}{2}mv^2$$

where m is mass (kg) and v is speed (ms^{-1})

Based on this formula, one can also deduce that if you were to double the speed of an object, then you would be quadrupling the amount of kinetic energy that the object possesses and hence mathematically this can be shown by: $K \propto v^2$

In Chapter 10, you will learn how potential energy is converted into kinetic energy and other common energy transformations. There must be some kinetic energy in any object that moves.



A good example of an object with kinetic energy is a pendulum. When it rises up to the top of its swing, it looses its speed and temporarily is stationary. This is to do with the energy changing from kinetic energy to gravitational potential energy. As it continues its swing downwards, the pendulum gains kinetic energy as its speed increase (the pendulum has a fixed mass). While this example is good at describing the conservation of energy, it does show a good example of kinetic energy.

In this chapter, you will also learn about rotational kinetic energy where an object that is spinning around a central axis possesses a different amount of energy to the standard formula. This will be discussed towards the end of the chapter and mainly has applications with objects that are spinning such as baseballs or cricket balls or even a spinning top, etc.

EXAMPLES

EXAMPLE 1

From the diagram in the beginning of the chapter, suggest where the bowling ball pendulum possesses the greatest amount of Kinetic Energy

The bowling ball possesses the greatest amount of kinetic energy in position 2, as it has the fastest velocity at this point (as this is the bottom of the swing)

EXAMPLES (CONTINUED)

EXAMPLE 2

Does a car parked in a garage have any kinetic energy?

No, because the car is stationary and hence the object does not possess this form of energy.

EXAMPLE 3

When a train slows down to stop at a station, what is happening to the amount of kinetic energy the train has?

The amount of kinetic energy decreases as it pulls into the station as the speed of the train decreases

EXERCISE 5.1 – CONCEPTUAL KINETIC ENERGY QUESTIONS

1. A paper plane glides through the air. Does this object have kinetic energy?
2. A koala is sitting in a tree and eating eucalyptus leaves. Does the koala have kinetic energy?
3. A monorail is transporting tourists along a scenic route of a busy city. If it is moving between stations does it have any kinetic energy?
4. The same monorail in question 3 is stopped at a station and passengers are getting on the train. At this moment does the train have any kinetic energy?
5. Which object possesses more kinetic energy: a truck travelling down a busy highway or a small car travelling at the same speed down the highway?
6. An object halves its speed; by what degree is its kinetic energy decreased?
7. Does a rollercoaster possess kinetic energy when it is at the bottom of the first hill?
8. Does a skateboarder riding down a ramp possess kinetic energy?
9. Explain why a pendulum possesses kinetic energy
10. Can a photon have kinetic energy?
11. Can an object have kinetic energy and no momentum?

EXAMPLES

EXAMPLE 1

Calculate the kinetic energy possessed by a bullet train with a mass of 14000 Mg as it travels between towns at a rate of 254.6kmh^{-1}.

EXAMPLES (CONTINUED)

$$K = \frac{1}{2}mv^2$$

velocity (speed) will need to be converted into ms^{-1}. To do so there is a simple conversion technique in physics that can be used. Divide the kmh^{-1} value by 3.6.

$$v = \frac{254.6}{3.6}$$

$$.:\ v = 84.87ms^{-1}$$

Mass must be converted to SI units (kilograms). In order to do this, one would multiply by 1000.

$$.:\ m = 14000 * 1000 = 14000000kg$$

Substitute values into the equation accordingly now that they are in SI units.

$$K = \frac{1}{2}mv^2$$

$$.:\ K = \frac{1}{2}(14000000)(84.87)^2$$

$$.:\ K = 50420418300J$$

$$.:\ K = 5.0 * 10^9 J$$

Hence the kinetic energy of the bullet train is $5.0*10^9J$

EXERCISE 5.2 – KINETIC ENERGY CALCULATIONS

Use of graphics calculator technology is appropriate for these questions

1. A car is travelling $20ms^{-1}$ down a busy road, if the mass of the vehicle was approximately 2000kg, what is kinetic energy of the vehicle?
2. When a car doubles its speed, by what factor is its kinetic energy increased and how does this explain why people generally don't leave a save distance between vehicles at such a speed?
3. A small toy car rolls horizontally across a ramp with a velocity of $1.9ms^{-1}$ with a mass of 21.2 grams. Derive the kinetic energy.
4. What is the kinetic energy that a shopping trolley has when it is pushed along the road with a constant velocity of $3ms^{-1}$ with a mass of 20.5kg?
5. What is the kinetic energy of a small bowling ball when rolled across a horizontal ramp with a constant velocity of $6kmh^{-1}$ and a mass of 6.2kg?
6. Derive the kinetic energy of a pencil rolling across a table at $0.2ms^{-1}$ with an 8g mass.

EXAMPLES

EXAMPLE 1

In a conserved system, no energy is lost to the surroundings. Based on this, the kinetic energy at the zero reference level from an object falling downward must be equal to the energy at the starting point where the energy is in the form of gravitational potential energy. If an object started falling from 4.0m above sea level (assume the reference level for gravitational potential energy is sea level) off the jetty, with a known mass of 20.0kg, derive the final velocity as it fits the water.

$$K = \frac{1}{2}mv^2$$
$$G_{PE} = mgh$$
$$\frac{1}{2}mv^2 = mgh$$
$$mv^2 = 2mgh$$
$$v^2 = \frac{2mgh}{m}$$
$$v^2 = 2gh$$
$$v = \sqrt{2gh}$$

In this process, we set KE to GPE as the two must be equal in the system based on the conservation of energy. Mass on the LHS has been moved to the RHS after multiplying both sides by 2. The masses cancel out from the numerator and denominator and the square root of both sides is taken.

The formula v = sqrt(2gh) can be used in any similar questions to that above, note that for any questions in the derivations section rely on the conservation of energy ignoring the affects of friction and air resistance (for now).

$$v = \sqrt{2gh}$$
$$.:v = \sqrt{2(9.80)(4.0)}$$
$$.:v = \sqrt{2(39.2)}$$
$$.:v = \sqrt{78.4}$$
$$.:v = 8.85ms^{-1}(3sf)$$

Hence the velocity when the object hits the water is 8.85 m/s (3sf)

Again, for any questions that use “g” in this section, assume 9.80ms^{-2} to the precision of 3sf. This is the value of gravitational acceleration. In London and other parts of the world it is closer to 9.81ms^{-2}.

- **INTERESTING QUESTION TO CONSIDER**
  Why doesn’t the final velocity depend on mass?