

Solving Exponential Equations using logarithms

UNLESS OTHERWISE STATED, ASSUME LOG IS IN BASE 10

Name:

First off, we start with our equation in exponential form:

$$2^{x-6} = 3^{x-2}$$

You then log both sides and use the log law $\log a^n = n \log a$

$$(x-6) \log 2 = (x-2) \log 3$$

Expand out the brackets on the LHS and RHS

$$x \log 2 - 6 \log 2 = x \log 3 - 2 \log 3$$

Get all x terms on one side algebraically – remember to inverse operations when you switch sides

$$\begin{aligned} -6 \log 2 &= x \log 3 - x \log 2 - 2 \log 3 \\ 2 \log 3 - 6 \log 2 &= x \log 3 - x \log 2 \end{aligned}$$

Isolate the common term of x on the LHS

$$x (\log 2 - \log 3) = 2 \log 3 - 6 \log 2$$

Move the LHS non-x terms into the denominator of the RHS

$$\begin{aligned} X &= \frac{2 \log 3 - 6 \log 2}{(\log 2 - \log 3)} \end{aligned}$$

$$\begin{aligned} X &= \frac{\log (2^6) - \log (3^2)}{\log 2 - \log 3} \end{aligned}$$

Calculate using your Ti 84 graphics calculator

$$X = -3.30719$$

Substitute back into original equation to check

$$2^{2.5-6} = 3^{2.5-2}$$

The answers are incredibly close, but not the same. This is due to the fact that we rounded, as the real exponent deduced is irrational!

Solve the following exponential equations using logs. Show all necessary lines of mathematical communication and remember to assume base 10 in any given equation.

$$2^{x-4} = 3^{x+1}$$

$$4^{x+1} = 5^{x-7}$$

$$3^{x+4} = 2^{x+2}$$

$${}_{10}\log (2^{x-4})=5^{x+9}$$

$$20^{2-x}=10^{x+3.5}$$