Sue and Sally's Fish and Chip Shop

Sue and Sally are two entrepreneurs in the business world and want to set up their own franchise. Using concepts of differential calculus and motion, you are to help them succeed in running their own fish and chip shop business. This scaffolded investigation will cover aspects of optimisation, motion and properties of curves.

First off, before Sue and Sally enter the business world, they need to ensure that their pricing will be competitive. To do so, they have gone to a range of fish and chip shops across their local area in an attempt to analyse the pricing of a standard meal. From their travels, they found that shops closer to the ocean generally had a higher cost compared to those that were several kilometers inland, as fish and chips have a greater appeal for those who are at the beach. They also found that if they went further inland, the meals became more expensive. Sue attributes this to the cost of transport and getting fish from the ocean.

After calculating and number crunching, Sue found that the average price of fish and chips sold at stores could be graphed by $C = (1/5)x^2 - x + 4.50$, where x is the distance of the fish and chip shop from the ocean in kilometers and c is the average price of fish and chips.

1. Sketch the function given on a set of axes below – label any stationary points

2. What is the average cost of fish and chips at a shop next to the ocean?

3. Use technology to find the distance from the ocean where the cost of fish and chips is lowest

4. What is the cost of fish and chips at this distance from the ocean? Use algebra to prove this.

5. Find the derivative of the cost function

6. Algebraically find when the derivative equals zero

7. What is the significance of this value?

8. Construct a sign diagram and write a statement in context of the original problem

9. Find the second derivative of the cost function

10. What does the second derivative tell us about the rate of change in the pricing of fish and chips as one moves further inland?

11. What are the limitations imposed by the model that Sue found? What assumptions could be made? Comment on the average cost as distance from the ocean increases. Would it be necessary to impose a restriction or interval for which the model is an appropriate representation of the context?

Another important factor besides the cost is the popularity of the stores. Sue analysed how many people visited stores over the course of a few weeks and found that stores that were closer to the beach generally had more customers. Her observations could be graphed by the function $P = 1/x$, where P is popularity (measured by how many hundred customers purchase food from the kiosks in the course of one day) and x represents distance from the ocean in kilometers. She suggests that the model is valid for the interval $0.4<x<5$ km.

1. Graph the function on a set of axes below

2. What kind of model would the data fit? Exponential, logarithmic, surge… Why?

3. Sue and Sally live 2.4km from the ocean. If they built a fish and chip shop near their house, how many customers would they expect to have on a daily basis?

4. Why would it be appropriate to exclude all negative values from the model in context?

5. What is the limitation to the model? Is the model able to predict the popularity of fish and chip shops that are right next to the ocean (less than 200m away)?

6. What is happening as $x \to 0$?

7. Sketch popularity and cost on the same set of axes

8. Use sign diagrams to analyse what is happening to the cost and popularity as you move inland

9. If the cost of Sue's rent and fuel to get to the store can be graphed by $y=\frac{2}{3}x^2-3x+20$, where y is the cost in hundreds of dollars per week and x is the distance from the ocean and she wishes to sell her fish and chips for at least $3.00, find the location where her rent and fuel costs would be lowest under that condition.

10. What is the significance of the derivative of the equation in question nine? Find it and construct a sign diagram.

11. Construct an argument supported by the formulae over the first two sections as to where you think Sue may suggest to Sally for the location of their fish and chip shop and why

Sally is in charge of getting the building up and running and getting the supplies for the fish and chip shop. She has a strict budget and needs to make wise choices otherwise their business will be in debt. Upon deciding on a location for the fish and chip shop, she has begun to explore designs that will maximize the area of the kiosk while keeping the cost of the walls around the building to a minimum.

1. Sally has one possible fish and chip shop site with one existing wall to build off. She wants to explore the option of having the largest area possible with 40m of walls. Develop a diagram and an equation that will assist her in calculating the area of the shop.

2. Check your last answer and show that the area is given by $A = x * (40-2x)$

3. Using calculus and algebra, derive the maximum area possible

4. Verify your answer using technology

5. Sally decides to set an area of $200m^2$ and tries to find the smallest perimeter possible. If she is still considering the site with one existing wall of length, x, find an equation that would represent the perimeter if the other sides of the building are to be denoted by y and the diameter of the rectangular building is to be exactly 20m.

6. Using derivatives and algebra, approximate the perimeter of the building.

7. Verify your answer using technology

8. Sally decides to explore the possibility of having a circular shop. If she is to have an area of $200m^2$, find the radius of the shop would result.

9. Based on your observations, do you believe that the fish and chip shop should be circular or rectangular based on want Sally is aiming for (lowest cost, biggest area)? Why?

Sally needs to go to the market to get fresh fish for the "Grand Opening" of the store. Her car has unfortunately broken down so she needs to ride her bike to the market to pick up the supplies. If her displacement can be given by the function: $s(t) = -2x^2+19x$ where displacement is measured as 1 unit = 10m and x represents time in minutes, and she starts from the fish and chip shop, answer the following questions:

1. Why is it reasonable to assume that all values of $x < 0$ and all values of $y < 0$ do not make sense in context?

2. Find the velocity and acceleration functions

3. Is her speed ever increasing? Why, why not?

4. For which intervals is her velocity increasing?

5. When does she reach the market and start heading back to the fish and chip shop?

6. If she were to be delayed by one minute at a set of traffic lights, how long would her journey take? What would be her velocity at that time?

7. Draw a motion diagram for the s(t) function given in question 1

8. What is happening in her travel at t=2 minutes? Describe

9. Suggest any possible limitations to this model. What possible factors may need to be taken into consideration that are not?

Sally and Sue thank you for your help with setting up their new business! Thanks to your calculus skills and understanding of how to apply mathematics, there business is a great success.