1 SIGNIFICANT FIGURES AND SCIENTIFIC NOTATION

This chapter looks at significant figures and their importance in scientific disciplines. In Physics, Chemistry and Mathematics, one can only be as precise as the measurements they use in their calculations. Precision is generally limited by equipment. Being more precise than your least precise recordings can result in incorrect answers.

There are several rules that govern how many significant figures you can state your final answer to and what figures are significant. These rules are listed below with an example of each given.

1. If a number is not a zero, it is always significant
2. If at the end of an answer, after the decimal place, there is a zero, this is a significant figure
3. Zeroes between other non-zero digits are always significant
4. Zeroes that help position the non-zero digits in the right spot are not significant – generally scientific notation is ideal to provide answers under these circumstances
5. For multiplication and division, you round your answers to the same number of significant figures as the least precise measurement you used in calculating
6. For addition and subtraction, your final answer must be given to the same number of decimal places as the value with the smallest number of decimal places

Mastering these six rules will help you provide accurate answers to correct precision.

EXAMPLES

EXAMPLE 1
How many significant figures are in the number 67812.3628?

Using Rule 1, there are 9 significant figures.

EXAMPLE 2
How many significant figures are in the number 0.3450070?

Using Rule 2 and 3, there are 7 significant figures (the zero before the decimal place is a placeholder and hence is not significant), however the zeroes between 5 and 7 are significant, as is the final zero.

EXAMPLE 3
How many significant figures are in the number 0.00076?

Using Rule 4, there are two significant figures as all the other zeroes are simply holding the 7 and 6 in the correct place. One could also write this using scientific notation to be 7.6×10^{-4}*, however there will be more on scientific notation later in this chapter.*

EXAMPLE 4
Find the product of 35.8 and 90.23 to the correct number of significant figures

Product means to multiply the two numbers. Using Rule 5, we must identify the recording with the least number of significant figures. In this case, it is 35.8, which is to the precision of 3sf (significant figures). It is best to do the calculation in full, and then fix to avoid any possible rounding errors.

35.8 x 90.23 = 3230.234
.: 35.8x90.23 = 3230 (to 3sf, note that the last zero is not significant, hence this answer is better written in scientific notation, as shown later in this chapter)

EXAMPLES (CONTINUED)

EXAMPLE 5
Find the sum of 67.82, 89.921 and 54.234 to the correct number of significant figures

Sum means to add the three numbers together. Using rule 6, we must identify the number with the least number of decimal places, in this case, 2. It is best to do the calculation in full, and then fix to avoid any possible rounding errors.

67.82 + 89.921 + 54.234 = 211.975
.: 67.82 + 89.921 + 54.234 = 211.98 (to 2dp)

EXERCISE 1.1

Use of graphics calculator technology is appropriate for these questions

1. How many significant figures are the following numbers written to?
   a) 23
   b) 346
   c) 367.0
   d) 0.005
   e) 0.9180
   f) 0.1921
   g) 1300.0
   h) 1307
   i) 9019.24
   j) 876.42
   k) 9009
   l) 0.0000007609
   m) 23.000005
   n) 12.4309
   o) 12.000

2. Which has a greater number of significant figures?
   a) 67.004 or 0.0006574
   b) 0.009 or 0.90
   c) 8.0560 or 3.42761
   d) 0.0012 or 0.467
   e) 90.09 or 125.00
   f) 908.2 or 0.09

3. Explain how many significant figures the following measurements are to or whether there are multiple possible answers
   a) 300
   b) 450
   c) 130000
   d) 1340
   e) 560

4. Why may rounding results in calculations result in an incorrect final answer?

5. Calculate the product of these values to the correct number of significant figures
   a) 450.6 and 906.5
   b) 340.23 and 9
   c) 127.8 and 126.892
   d) 45.67 and 13.4
   e) 89.8128 and 912.44
   f) 12 and 13.6
   g) 0.0009 and 12.090
   h) 0.000710 and 12.9
   i) 0.00009 and 0.0405
   j) 0.010 and 0.001
   k) 12 and 439.1

6. Calculate the sum of these values to the correct number of significant figures
   a) 436.2 and 234.5
   b) 341.57 and 5
   c) 129.7 and 148.981 and 64.19
   d) 123.813 and 45.90
   e) 45.89 and 917.67
   f) 69.39 and 0.000260
   g) 0.00110 and 2.019
   h) 0.0008 and 0.0324
   i) 67 and 13.4
   j) 54.09 and 0.00192 and 1

7. Give an answer for the sum of 56.19 & 89.748 to the correct no. of significant figures

8. Give an answer for 78.324 divided by 78.10 to the correct number of decimal places

Scientific Notation is a way that scientists handle really big and really small numbers. It involves writing a number between 1 and 9.9999…. written to a power of ten. This can also be a great way to write numbers to a certain level of precision and significant figures. In order to write in scientific notation, there must be one figure before the decimal point and the appropriate number of significant figures. This is then written to a power of ten. If the number is smaller than 1 then the power of ten is negative. If the number is greater than 1, then it is either raised to 10^0 (1) or a positive power of ten.

The following examples show how to write four numbers in scientific notation form.

EXAMPLES

EXAMPLE 1
Write the number 42 in scientific notation

4.2×10^1

EXAMPLE 2
Write 219.75 in scientific notation to the precision of two significant figures

2.2×10^2

EXAMPLE 3
Write 0.00076 with appropriate scientific notation

7.6×10^{-4}

EXAMPLE 4
Write 0.00432 to the precision of two significant figures

4.3×10^{-3}

EXERCISE 1.2

1. Write the following numbers in scientific notation
   a) 421
   b) 397
   c) 1097
   d) 540.234
   e) 436.2
   f) 0.0012
   g) 0.01780
   h) 31 * 1000
   i) (31+89+90)*21+8
   j) 3.1415

2. Write the following to a precision of two significant figures in scientific notation form
   a) 0.00091721
   b) 140214
   c) 0.001240
   d) 0.73224
   e) 0.0001
   f) 21.742
   g) 2178913.31809
   h) 6.319

3. The speed of light, generally denoted by "c" in physics, is 300 million ms^{-1}. Write this in scientific notation form.

4. Derive the difference between the mass of a proton ($1.672621777 \times 10^{-27}$ kg) and the mass of an electron ($9.10938291 \times 10^{-31}$kg) and give your answer to three significant figures in scientific notation.

5. Use scientific notation to give an answer to the sum of 0.00123 and 0.323. Give your answer to an appropriate number of significant figures.

6. Use scientific notation to give an answer to the product of 1389.813 and 0.0006. Give your answer to an appropriate number of significant figures.

2 APPLICATION OF SIGNFICANT FIGURES IN CONTEXT

In experiments or calculations, you will need to give all measurements to a certain number of significant figures. This chapter focuses on a few contextual physics questions. For every question or statement, you should answer with an appropriate number of significant figures and units. Each question will rely on performing simple arithmetic with the formulae given however answers are only correct if they reflect the correct degree of precision from the original measurements. Clear and logical lines of reasoning are important as it allows other physicists to see how you derived your final answer.

EXAMPLES

EXAMPLE 1

The Universal Law of Gravitation calculates the force of gravity and is shown in the equation below. A physicist makes the conjecture that the force of gravity between the two masses is directly proportional to the product of the two masses.

$$F = G\frac{m_1 m_2}{r^2}$$

Test the possibility of this conjecture being correct by altering the values of m_1 and m_2 **numerically** while assuming G is constant non-zero real number and r is a constant value of 2.4m. First test with m_1 being 100.0kg and m_2 being 200.0kg, before doubling both these values.

*To test this conjecture, the values of m_1 and m_2 will be represented by 100kg and 200kg accordingly, giving a product of 20 000kg (2.000*10^4 kg). The product of 200kg and 400kg is 80 000kg (8.000*10^4 kg).*

*100.0*200.0 = 20 000 kg*
*.; 100.0*200.0 = 2.000*10^4 kg (4 sf)*

*200.0*400.0 = 80 000 kg*
*.; 200.0*400.0 = 8.000*10^4 kg (4 sf)*

$$F = G\frac{m_1 m_2}{r^2}$$

$$.: \ F = G\frac{2.000 * 10^4}{2.4^2}$$

$$.: \ F = G\frac{20000}{2.4^2}$$

$$.: \ F = 3472.22 * G$$

$.: \ F = ((3.5 * 10^3) * G)N$ *(to the precision of 2sf, where N is Newtons SI units for force)*

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = G\frac{8.000 * 10^4}{2.4^2}$$

EXAMPLE (CONTINUED)

$$F = G\frac{80000}{2.4^2} = 13888.88 * G$$

$F = ((1.4 * 10^4) * G)N$ *(to the precision of 2sf, where N is Newtons SI units for force)*

*As 4*3472.22 = 13888.88, the product of mass is quadrupled when both m_1 and m_2 are doubled. As G is constant, the force between the two objects is also quadrupled, hence the physicists claim of the force of gravity between two objects being directly proportional to the product of their mass is supported.*

EXERCISE 2.1

Use of graphics calculator technology is appropriate for these questions

1. The amount of kinetic energy an object has is directly proportional to the square of that objects velocity. This means that if a car doubles its velocity then the amount of kinetic energy is quadrupled.

    a) Represent this mathematically using symbols
    b) Explain why this is so based on the kinetic energy formula where $K = \frac{1}{2}mv^2$
    c) If a car has a velocity of 30.2kmh^{-1} and it doubles its velocity, what is its kinetic energy after the car reaches its new speed? (Hint: kmh^{-1} to ms^{-1} means to divide by 3.6)
    d) If an snail is travelling 0.0005kmh^{-1} and it doubles its velocity, what will its new velocity be?

2. Momentum is a measure of "mass in motion". Generally, the product of mass and velocity can give a good approximation of momentum: $p = mv$.
    a) Using this formula what is the momentum of a truck that has a mass of 5000kg and is travelling at 30ms^{-1}
    b) Another truck is travelling at the same speed with a mass of 6000.0kg. Calculate its momentum.
    c) Which value calculated has a higher degree of precision?

3. The exact formula for momentum is given by $p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$. Assume for all following questions that the speed of light, c, is given by $3.00\text{x}10^8\ \text{ms}^{-1}$.
    a) Repeat 2 (a) and 2(b) with the new formula. Is there a difference?
    b) Explain why, with reference to the formula and mathematics, there is no difference in objects travelling at 30ms^{-1} with the degree of precision of the measurements given.
    c) John works at a particle accelerator. One day he decides to accelerate a small object with a mass of 0.00001 grams to 900 000.0 ms^{-1}. Calculate momentum using both formulae and see if there is a difference.
    d) Photons are traditionally referred to as massless however travel at the speed of light. Does this mean that a photon has a large momentum as it travels at a high velocity? Explain.

4. A person sits in a tree approximately 5m above the ground. Calculate their gravitational potential energy if g = 9.8ms^{-2} and they have a mass of 60kg. Hint: E_{PG} = mgh

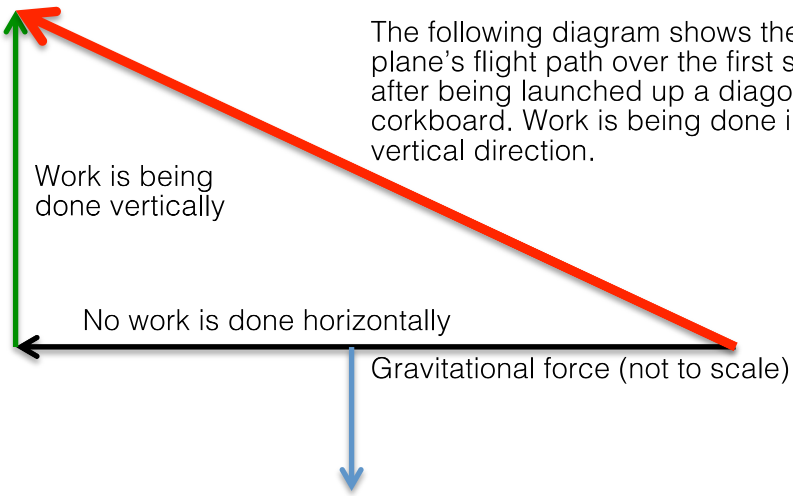
3 WORK DONE

The concept of work relies around there general rules. In order to have an understanding of what energy is, one must first understand this concept. Work is best defined as a force acting on an object causing it to move through a displacement. This means that work is done when you push a car and it begins to move, and if you are vacuuming a carpet, but no work would be done when you are holding a book above your head. The fact that no work is occurring when you hold that book comes down to the fact that there may be a force holding that book there however no mechanical work is being done on the book as the force does not noticeably move the book through a displacement (vector distance).

Another important thing to note though is that no work is done if a force is acting in a perpendicular nature to the force that moves an object through a displacement. This means that if another force is acting at 90 degrees, then no mechanical work is being done on the object. Take this example as moving a plate of sandwiches or moving a table (not by pushing it across the ground). In both circumstances, mechanical work is not being done.

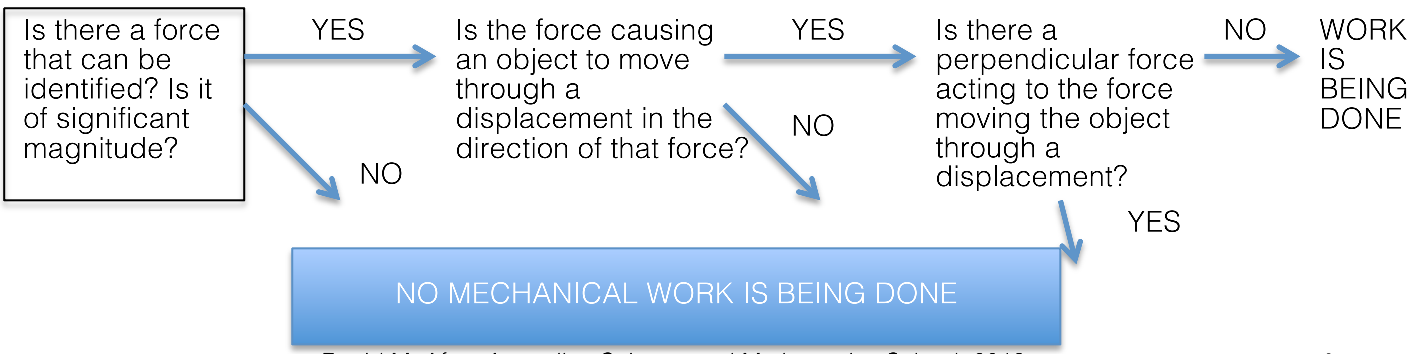
Perpendicular movement in a satellite object orbiting around the planet is also another example of when work is not done. Using vector diagrams one can easily see that as there will always be a force towards the center of the planet and the tangent to the circular path will be at 90 degrees to this force, no work will be done.

There are three rules that govern whether work is done, for work to be done, all three rules (conditions) must be satisfied. Sometimes it is best to draw a vector diagram; this involves drawing an arrow showing the direction of the force (generally the arrow would also be directly proportional to the magnitude of the force too) and a clearly labeled displacement. Remember to evaluate the horizontal vector component along with the vertical component. Below is an example involving a model plane.



The following diagram shows the model plane's flight path over the first second after being launched up a diagonal corkboard. Work is being done in the vertical direction.

Three Rules to check whether mechanical work is being done in the form of a flowchart



EXAMPLES

EXAMPLE 1
Is there work done when you push a car with a force of 7000N and it does not move?

No work is done, as the force does not move the object through a displacement.

EXAMPLE 2
Is there work being done when you walk up a flight of stairs?

Yes, work is being done, as you have vertical displacement, no perpendicular force and there is a force. Your gravitational potential energy also increases too as you are higher than your previous reference point.

EXAMPLE 3
A woman carries a briefcase across the yard and up a flight of stairs. In which phase of her journey to work is she doing work?

She is doing work when she carries the suitcase up a flight of stairs as the angle between the direction of the applied force and the direction of movement is not perpendicular, and the object is moving through a displacement, hence work is being done.

EXERCISE 3.1

1. Draw a vector diagram showing that a satellite in orbit around the Earth is doing no mechanical work being done on the satellite and write a small two sentence justification explaining this.
2. A box is moved a meter when a force is applied to push it. Is work being done?
3. A man carries a suitcase horizontally across a distance of 4.50m, is there work being done?
4. A man holds a barbell above his head. At this instance is there work being done? Explain
5. Tim plays with a wind up "Jack in the Box", when the clown springs up out of the box is there work being done?
6. Draw a vector diagram illustrating a box that is moved through a displacement of 7m along the floor by being tied to a string that is pulled at 42 degrees from the floor. Suggest why work is being done and the forces at work.
7. A large boulder drops from a cliff and accelerates downwards to the ground. Is work being done on the boulder?
8. A pencil is lifted over a displacement of 0.5m and an elephant is lifted over a displacement of 0.5m. Given the definition of work mentioned as a force acting to move an object over a displacement, which of the two scenarios results in the greater amount of mechanical work?
9. A helicopter is hovering above the ground. Is mechanical work being done on the helicopter?
10. When you are writing the answer to this question on a piece of paper, are you doing mechanical work?

There is a formula that allows one to derive the amount of work done in a situation. This formula is given as follows:

$$W = Fs\cos\theta$$

where W is work (measured in Joules)
F is the force causing the object to move (measured in Newtons)
s is the displacement over which the object moves (measured in meters)
θ is the angle between the applied force and the direction of the movement (degrees)

Remember that these answers are mathematical and as a result, it is important to take note of the precision. Note how many significant figures there are. Draw a diagram and list all the facts you know before you start.

Note that work done is measured in Joules, this is a measure based on James Prescott Joule, a famous physicist. All answers regarding work done should be in joules.

Another key point is that in some questions a mass is given. The SI units for mass is kilograms and mass is how much matter something is made up of. In order to convert this to weight, gravitational force, F_g, one must multiply by 9.8 (the gravitational acceleration on Earth in ms^{-2}). Weight is measured in Newtons and in some questions, is the force causing work to be done. See example 3 below for the process.

It is also important to know how to convert values into SI units with meters, centimeters, millimeters, and other prefixes which may sometimes apply for other measurements.

EXAMPLES

EXAMPLE 1
A box is pushed with a force of $5.00\text{x}10^{2}$ N to travel a constant velocity over 4.50m

$F = 500N$

$s = 4.50m$

$.: W = Fs\cos\theta$

$.: W = (500)(4.50)(1)$

$.: W = 2250J$

Note that there is no angle between applied force and movement force hence cos(0) is substituted in and a result of 1 is obtained

.: W = $2.25\text{x}10^3$J (3 sf) <- This is how much work has been done by the box

EXAMPLE 2
A box is pulled along the floor with a force of 500N to travel at a constant velocity over 3m. A rope that is at 48 degrees to the floor pulls the box. Derive work done.

$F = 500N(1sf)$

$s = 3m$

$\theta = 48^o$

$W = Fs\cos\theta$

$W = (500)(3)\cos(48)$

$.: W = 1003.69591J$

$.: W = 1000J(1sf)$

EXAMPLES (CONTINUED)

EXAMPLE 3

A box has a mass of 5.00kg, it is pushed through a vertical displacement of 3.00m with the minimum force that can allow it to travel at a constant velocity. Derive work done.

In this case, one must convert mass to weight. The minimum force needed to pull the box through a vertical displacement is the same as the weight. Hence, one simply finds the weight by multiplying mass by 9.8.

$F_g=mg$
$g=9.8ms^{-2}$

$\therefore F_g=(5.00)(9.8)$
$\therefore F_g= 49N$ *(2 sf)*

Hence the weight is 49N

$F = 49N$, $s = 3.00m$

$W = Fs \cos (theta)$
$\therefore W = (49)(3.00)\cos(0)$
$\therefore W = 147N$
$\therefore W = 1.5 \times 10^2 J$ *(2sf)*

EXERCISE 3.2

Use of graphics calculator technology is appropriate for these questions

1. Derive work done when a force of 600N is applied to an elevator and it travels over a displacement of 20.0m.
2. Is more work done when a weight trainer lifts a 1000N dumbbell to a height of 3.00m or when a 2000N dumbbell is lifted to a height of 1.50m?
3. Why is it that if the force of movement is perpendicular to the applied force that no work is done? Explain mathematically
4. A 2.1kg book is lifted over a displacement of 3.50m, what work is done?
5. A box is pulled along the ground by a pulley with a force of 740N over a displacement of 8.00m. If the pulley was at an angle of 52 degrees from the ground, derive work done on the box.
6. Complete this sentence: The work done in a situation is greatest when the angle between the direction of the applied and movement force is ___ degrees.
7. Show your response in question 6 mathematically to prove it
8. Which scenario has had more work done, a telephone lifted up to someone's ear or someone applying a force to a car stuck in mud without the car moving?
9. Draw a vector diagram of a box moving up a ramp with a frictional resistance of 500N and the ramp rises 12.0m for every 15.0m. If an object travels up this ramp to an elevation of 17.2m, and the object has a known mass of 1.36kg, derive the total work done in this situation.
10. Can a photon do work? Explain and justify your answer using the formula

For questions in exercise set 3.3, you need to rearrange a formula in order to derive the amount of work done. To do this, it is necessary to show the steps and derivation necessary to find the distance or force involved. Remember to follow the logic that if a term was in the denominator on one side, it will be in the numerator on the other side, and vice versa. This only applies to terms that are multiplied though, if a term is added or subtracted from another term then moving it to the other side of the equation will result in the inverse operation being done on the equation, either subtract or add.

Below shows the derivations of force, weight, angle and displacement based on the original formula.

DERIVATIONS

Force

$$W = Fs\cos\theta$$

$$\therefore \frac{W}{s\cos\theta} = F$$

Angle between applied and movement force

$$W = Fs\cos\theta$$

$$\therefore \frac{W}{Fs} = \cos\theta$$

Distance

$$W = Fs\cos\theta$$

$$\therefore \frac{W}{F} = s\cos\theta$$

$$\therefore \frac{W}{F\cos\theta} = s$$

Weight

$$W = Fs\cos\theta$$

$$W = (mg)\cos\theta$$

$$\therefore \frac{W}{\cos\theta} = mg$$

$$\therefore \frac{W}{\cos\theta g} = m$$

EXERCISE 3.3

Use of graphics calculator technology is appropriate for these questions

1. Next to each of the derivations in the box above, write an annotated step of sets explaining the process by which the value is made the subject of a formula using algebra.

2. Calculate the angle of applied force between the ground and a rope that pulls a box against 70N of frictional resistance across the floor at a minimum constant velocity over 3.95m

3. 3.51MJ of work is done on a large safety deposit box that is lifted from a mine. Assuming it travelled a vertical displacement of 500m, what is the weight of this safety deposit box?

4. A suitcase is loaded onto a plane. The applied force is 45.4 degrees from the direction of the movement force. If the suitcase has a mass of 5.65 kg and the amount of work done is 30.2J, over what distance did the suitcase travel?

5. Derive the force applied to a car when the work done is 4000.2J and it travels over a displacement of 0.03km.

6. A truck has 100MJ done over the course of x amount of seconds when it is lifted vertically upwards by a crane. If the truck has a mass of 3 metric tons, determine the number of seconds for which the truck was lifted.

7. The total amount of work done on a heavy physics textbook is 460.0J when it is lifted from the ground story of a building to the floor of the fifth floor (the sixth floor including the ground floor) of the building. If the book is lifted with a constant minimum velocity and each level on the building is 2.33m in height, derive the weight of the physics book.

8. Which has the greater weight, a child's colouring book which has been lifted from the ground floor to the floor of third floor of the building in question 7 under the same conditions as the textbook with a total amount of work done as 100.5J or a magazine which has been lifted from the floor of the sixth floor to the roof of the tenth floor of the same building with the same conditions, with a total amount of work being 124.6J?

Stopping distances and forces are important. In car crashes, manufacturers will often attempt to increase the stopping distance to reduce the impact force and hence make it safer for passengers.