CONFIDENCE INTERVALS: THE FOOD CRITIC

A food critic who writes for the newspaper is claimed to be too harsh on businesses. The critic claims that the average rating he gives a restaurant is 3.2 stars (out of 5). The manager of a local restaurant collects his reviews every weekend for the course of 1 year (52 samples) and finds that the mean rating he gave was 2.6 stars. The standard deviation is known to be 0.4 stars. Assume the reviews are normally distributed.

1. Draw the distribution that the food critic claimed with z scores and a scaled x axis (draw three standard deviations to the left and right of the mean)

(2)

2. Construct a 95% confidence interval using the sample taken by the restaurant owner

(2)

3. What does this confidence interval mean in context?

(2)

4. Is the food critic correct when he claims that the average rating he gives is 3.2 stars?

(1)

5. The food critic claims that a 0.99 confidence interval should be taken. If a confidence interval is in the form of $\bar{x} - a(\frac{\sigma}{\sqrt{n}}) < \mu < \bar{x} + a(\frac{\sigma}{\sqrt{n}})$, and at the 0.95 confidence level, 'a' is given by 1.96, derive the value of 'a' at the 0.99 level.

(3)

6. Derive the number of samples that should be taken at the 0.95 confidence level to be certain that the population mean differs from the sample mean by 0.1 stars using the formula $a\frac{\sigma}{\sqrt{n}} = x$

(2)

7. Derive the number of samples that should be taken at the 0.99 confidence level to be certain that the population mean differs from the sample mean by 0.1 stars using the formula $a\frac{\sigma}{\sqrt{n}} = x$

(2)

8. What can be concluded about the 0.99 and 0.95 confidence level and the number of samples necessary to have a certain accuracy?

(1)

9. The width of a confidence interval can be given by $w = \frac{2 * 1.96\sigma}{\sqrt{n}}$, show the derivation that makes the value of n the subject.

(3)

10. Determine the number of samples needed at the 0.95 confidence interval to be within 0.2 stars of the mean using the width formula.

(2)

11. Mathematically show how the confidence interval width decreases as the value of n increases.

(2)

12. An additional year of samples was taken. From the new set of 52 samples it was found that the new sample mean was 3.1. Is there evidence to support that the food critic has become less harsh with his reviewing?

(3)

13. Is there a possibility that he hasn't changed? What is the probability that he has not changed?

(5)

14. What are some of the limitations, generalisations and assumptions made in this investigation? Is there a possibility that the sample is inappropriate?

(4)