

Exposure profile enhancement using cashflow information

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Abstract

We've introduced a method to interpolate present trade values, which serves to both reduce simulation costs and increase the granularity of the exposure profile. This technique is specifically designed to capture the inherent volatility of present values, while also accurately reflecting sudden trade jumps within the exposure profile. By doing so, we manage to significantly decrease computation time without compromising the accuracy of the results, as evidenced by maintaining a similar RMSE in the XVA calculations. This balance between efficiency and precision is crucial in ensuring the reliability of the model while optimizing performance.

Acknowledgments

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1 Introduction

From a bank’s perspective, XVA computations are notoriously expensive, largely because they rely on extensive Monte Carlo simulations that demand significant computational resources. The conventional approach to managing these costs involves sacrificing precision, which inevitably leads to a loss of crucial signal information. However, this trade-off between cost and accuracy is far from ideal.

Our goal is to develop a solution that reduces the overall computation time while preserving the integrity of the simulated signals. Rather than compromising on precision, we aim to maintain the accuracy of the most critical components of the calculation. By doing so, we seek to strike a balance where the essential details of the exposure profile are retained, ensuring that the results remain robust and reliable, even as we optimize the efficiency of the computational process. This approach not only addresses the cost concerns but also enhances the quality of the XVA computations, aligning with the bank’s need for both precision and performance.

2 XVA computation

2.1 Present values of trades

2.2 Exposure profile

2.3 Example : CVA

Portfolio trade present values are the cornerstone of any XVA computation. They provide the foundation for using a Monte Carlo approach to generate exposure profiles, which are essential for calculating various XVA metrics. Among these, Credit Valuation Adjustment (CVA) is one of the most prominent. CVA represents the expected loss due to counterparty default, accounting for the risk that the counterparty will fail to meet its obligations before the maturity of the contract. The formula for CVA is given by:

$$\text{CVA} = E^Q[L^*] = (1 - R) \int_0^T E^Q \left[\frac{B_0}{B_t} E(t) \mid \tau = t \right] d\text{PD}(0, t)$$

where:

- R is the recovery rate,
- $E^Q[\cdot]$ denotes the risk-neutral expectation,
- B_0 and B_t are the discount factors at times 0 and t , respectively,
- $E(t)$ is the exposure at time t ,
- τ is the time of default,
- $\text{PD}(0, t)$ is the cumulative probability of default from time 0 to time t ,
- T is the maturity of the contract.

As we can see, we need to integrate the expected exposure over a time schedule. This time schedule is continuous in an ideal world but when we simulate we first need to make this schedule discrete. The banks have a lot of counterparties for which this needs to be computed alongside with lots of other calculations of this kind but they have a limited time and also a limited computation capacity (RAM). The schedule has to have a low frequency which results in an integration bias when numerically integrating in the above formula.

in order to

- First step : Cashflow interpolation
- Second step : brownian bridge

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