A CROSS-LAYER DESIGN FOR MIMO RAYLEIGH FADING CHANNELS

Amine Maaref, *Student Member IEEE*, Sonia Aïssa, *Senior Member IEEE*
INRS-EMT
University of Quebec
Montreal, QC, Canada
{maaref, aissa}@inrs-emt.uquebec.ca

Abstract

This paper presents a cross-layer design for packet data transmission over a wireless channel employing transmit diversity. The design combines the use of adaptive modulation (AM) at the physical layer and truncated automatic repeat request protocol (T-ARQ) at the data link layer. The link adaptation process is intended to operate over a multiple-input multiple-output (MIMO) frequency-nonselective Rayleigh fading channel using transmit diversity through space-time block coding (STBC). Two metrics are used to evaluate the performance of the proposed MIMO cross-layer design, namely, the probability of outage and the achievable average spectral efficiency. Results show that this design achieves better performance than adaptive modulation used without ARQ retransmissions in the presence of STBC.

Keywords: Adaptive Modulation (AM); Multiple Input Multiple Output (MIMO); truncated Automatic Repeat reQuest (T-ARQ).

1. INTRODUCTION

Link adaptation through cross-layer design for packet data transmission over fading channels [1,2] has raised a lot of research interest, especially with the emergence of the high speed downlink packet access (HSDPA) [6]. The ultimate objective sought is to enhance the spectral efficiency of future wireless systems. In this paper, we combine the use of physical layer adaptive modulation and transmit diversity for MIMO packet data transmissions subject to a truncated automatic repeat request (T-ARQ) retransmission protocol at the data link layer. The aim is to minimize the probability of outage while maximizing the average spectral efficiency of the system under prescribed delay and error performance constraints.

The process of modifying transmission parameters, such as power and rate, to compensate for the variations in channel conditions is known as link adaptation. Multi-level adaptive modulation based on M-ary quadrature amplitude modulation (M-QAM) signal constellations has been widely investigated in the literature [4,5] and is a promising, yet simple technique for link adaptation at the physical layer since it allows a close adaptation of the constellation size to the variations of the fading channel, using higher or lower order modulation depending on the channel condition. An other link adaptation technique can be performed in the form of a data link layer T-ARQ retransmission protocol implemented at the packet level. With the truncated ARQ protocol, link layer acknowledgements are used for retransmission decisions of data packets, whereby an erroneously received packet is eventually retransmitted until a preset maximum number of retransmissions is reached.

Due to the fact that future wireless systems are being equipped with multiple antennas (some of them already are), link adaptation techniques have to be extended to encompass the features of the MIMO fading channel. In a MIMO system, the use of multiple antennas adheres to one of two distinct approaches that seek to increase either the diversity order or the information rate of the system. These two approaches are commonly denoted as MIMO diversity and spatial multiplexing, respectively. Herein, we consider the former approach as we extend the results in [2] to the case of a MIMO system using transmit diversity through space-time block coding (STBC).

The remainder of this paper is organized as follows. Section 2 introduces some physical and link layer aspects and presents useful approximations for the packet error rates of M-QAM modulations over AWGN channels. In Section 3, we describe the MIMO Fading channel model employed whereas in Section 4 we proceed to the performance analysis of our MIMO cross-layer design. Some numerical results are provided in Section 5 and concluding remarks are presented in the last section.

2. PHYSICAL AND LINK LAYER ASPECTS

2.1 Frame and Packet Formats

We consider a multiple-transmit multiple-receive antenna system as illustrated in Figure 1. A link layer truncated ARQ protocol controls packet retransmissions, initiating retransmission requests and sending them back to the transmitter

CCECE 2004 - CCGEI 2004, Niagara Falls, May/mai 2004

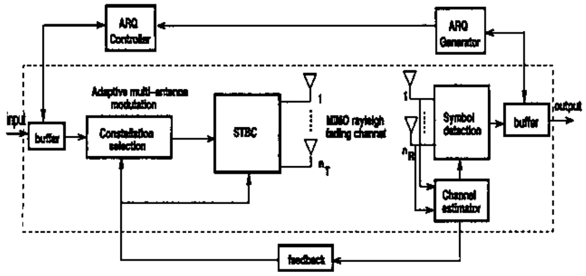

Fig. 1. System model.

whenever a packet is detected in error at the receiver. Frame transmission is considered at the physical layer [2]. A frame contains a fixed number of symbols, N_f, and a variable number of packets, N_b, from the data link layer. N_b is modulation-dependent, as packets are fractionated into a set of symbols belonging to a family of M-QAM signal constellations of size $\{M_n = 2^n\}_{\{n=1,\ldots,N\}}$, yielding the available set of data rates $\{n = log_2(M_n) : n = 1, \ldots, N\}$. Each N_p-bit packet comprises its own cyclic redundancy check (CRC) for ARQ error detection purposes as well as a payload content. After modulation at a given rate n, the N_p bits of each packet are mapped into N_p/n symbols to which N_c control symbols are added to form a frame at the physical layer. The frame length, therefore, satisfies the relationship: $N_f = N_c + N_b N_p/n$. Further details about the packet and frame structures can be found in [2,6]. Since only finite delays are tolerated by practical applications and buffers at both ends of the transmission link are of limited size, we truncate the number of retransmissions for the ARQ protocol to N_{arq} retransmissions. Accordingly, any target packet error rate PER_{link} that has to be achieved at the data link layer, is translated into an equivalent physical layer target PER, PER_{phy}, given by

$$PER_{phy} = PER_{link}^{1/N_{arq}}. \tag{1}$$

In the following, we seek to achieve this target PER through a combination of adaptive M-QAM and STBC transmit diversity.

2.2 M-QAM Packet Error Rate

The aim of this section is to present a useful approximation for the packet error rate PER_n when a M_n-QAM signal constellation is used over an AWGN channel, where $M_n = 2^n$ denotes the number of signal points in the constellation. For this purpose, we consider a single-input single-output (SISO) non-fading channel described by the relationship $y = x + v$, where y is the received sample, x is the transmitted symbol with energy E_s and v is the complex AWGN with variance $\sigma^2/2$ per dimension. The exact PER of coherent M_n-QAM with two dimensional Gray coding and equally likely symbols over an AWGN channel is well approximated by [2]

$$PER_n(\gamma) \approx \begin{cases} 1 & \text{if } \gamma < \gamma_{pn} \\ a_n \exp(-g_n\gamma) & \text{if } \gamma \geq \gamma_{pn} \end{cases} \tag{2}$$

where $\gamma = E_s/\sigma^2$ represents the received SNR, $\{a_n, g_n, \gamma_{pn}\}$ are constellation and packet-size dependent constants which are evaluated by least square fitting the PER expression of (2) to the exact PER expression given by [2, Eq. (23)]

$$\begin{aligned} PER_n(\gamma) &= 1 - \prod_{i=1}^{log_2 I} (1 - P_I^i(\gamma))^{N_p/n} \\ &\quad \times \prod_{j=1}^{log_2 J} (1 - P_J^j(\gamma))^{N_p/n}, \end{aligned} \tag{3}$$

In 3, $I = \lfloor n/2 \rfloor$ and $J = \lceil n/2 \rceil$ with $\lfloor . \rfloor$ and $\lceil . \rceil$ respectively denoting integer floor and integer ceil, whereas P_I^i and P_J^j are exact BERs corresponding to the i^{th} and j^{th} bit of two independent I-ary and J-ary PAMs, respectively, with $i \in \{1, \ldots, \log_2(I)\}$ and $j \in \{1, \ldots, \log_2(J)\}$, as given in [3].

3. MIMO FADING CHANNEL

We consider a discrete-time baseband channel model. A quasi-static independent flat Rayleigh fading MIMO diversity system, with $n_T > 1$ transmit and n_R receive antennas, is assumed. Flat fading is guaranteed by the fact that the delay spread of the channel is less than the symbol duration whereas the quasi-static assumption accounts for the fact that the channel characteristics remain constant at least for the period of transmission of an entire frame. Thus, the $\mathcal{K} \triangleq n_T n_R$ diversity system can be represented as the matrix $\mathbf{H} = [h_{i,j}]_{i,j=1}^{n_R,n_T}$, where $h_{i,j}$ is the channel coefficient between the j^{th} transmit and i^{th} receive antennas. Since we are concerned with Rayleigh fading, the channel coefficient $h_{i,j}$, $i = 1, \ldots, n_R$ and $j = 1, \ldots, n_T$, are modelled as i.i.d. complex circular Gaussian random variables, each with a $\mathcal{CN}(0, 1)$ distribution. Independent fading is assumed, which means that the $\mathcal{K}$ channel coefficients $h_{i,j}$ are independent. For this assumption to hold, sufficient spacing between antenna elements should be provided at both ends of the transmission link. As the channel matrix is unknown to the transmitter, adaptive modulation is performed using STBC [7] to achieve transmit diversity over the wireless link. STBC maps each N_f input QAM symbols into n_T orthogonal sequences of length T, where $T = N_f/R_c$ and R_c is the code rate of the space time block code used. Thus, the input-output relationship can be expressed as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{V}, \tag{4}$$

where the received signal $\mathbf{Y}$ is an $n_R \times T$ matrix, $\mathbf{X}$ is the $n_T \times T$ matrix of transmitted symbols and the receiver noise $\mathbf{V}$ is $n_R \times T$ with elements i.i.d. complex circular Gaussian random variables, each with a $\mathcal{CN}(0, \sigma^2)$ distribution. Finally, let P_T denote the total transmit power and define the average SNR per receive antenna as $\bar{\gamma} \triangleq (P_T/\sigma^2)$.

From [7], the effective induced channel by STBC is equivalent to 2 blocks of T sub-channels corresponding to the real and imaginary parts of the transmitted QAM symbols. Thus, the effective SNR at the receiver is $\gamma^{STBC} = (\|\mathbf{H}\|_F^4 \bar{\gamma}\sigma^2/2n_T)/(\|\mathbf{H}\|_F^2 \sigma^2/2)) = \bar{\gamma}\|\mathbf{H}\|_F^2/n_T$. $\|\mathbf{H}\|_F^2$ is the sum of $2\mathcal{K}$ independent χ^2 random variables (due to the

Rayleigh fading assumption), it is then χ^2 distributed, and hence, using a transformation of random variables, it can be shown that the received SNR, γ^{STBC}, is Gamma distributed with parameter $\mathcal{K}$ and mean $\mathcal{K}\bar{\gamma}/n_T$ according to the PDF, $p_{\gamma^{STBC}}(\gamma)$, given by

$$p_{\gamma^{STBC}}(\gamma) = \frac{1}{\Gamma(\mathcal{K})}\left(\frac{n_T}{\bar{\gamma}}\right)\left(n_T\frac{\gamma}{\bar{\gamma}}\right)^{\mathcal{K}-1}\exp\left(-n_T\frac{\gamma}{\bar{\gamma}}\right), \gamma > 0 \quad (5)$$

where $\Gamma(.)$ is the Gamma function [8].

4. CROSS-LAYER PERFORMANCE ANALYSIS

Adaptive modulation is performed on a frame-by-frame basis by splitting the range of the channel SNR into $N+1$ intervals, denoted $[\gamma_n, \gamma_{n+1})_{\{n=0,1,\ldots,N\}}$, where N is the number of M-QAM constellations available. Whenever, the SNR transmitted from the receiver to the transmitter falls within the interval $[\gamma_n, \gamma_{n+1})$, the constellation size M_n is selected, with $M_0 = 0$ referring to the case where no transmission is performed. Having set the target PER for the physical layer to PER_{phy}, the threshold levels $\{\gamma_n\}$ are set to the required SNR to achieve the target PER_{phy} over a non-fading AWGN channel [4], which leads to the following values

$$\gamma_n = \begin{cases} 0 & \text{if } n = 0 \\ K_n/g_n & \text{if } n = 1, \ldots, N \\ +\infty & \text{if } n = N+1 \end{cases} \quad (6)$$

where $K_n = -\log(PER_{phy}/a_n)$ is a power penalty factor incurred by choosing modulation level n.

Adaptive modulation operates at an average packet error rate, $\langle PER \rangle$, that is smaller than the target PER to be achieved at the physical layer PER_{phy}. This is due to the fact that our rate adaptive policy is conservative in the sense that we choose the constellation that guarantees an instantaneous PER no larger than PER_{phy}. To evaluate $\langle PER \rangle$, we first calculate the probability that constellation size $M_n = 2^n$ is chosen, which equals

$$\begin{aligned} a_n^{STBC} &= Prob\{\gamma_n \leq \gamma^{STBC} < \gamma_{n+1}\} \\ &= Q\left(\mathcal{K}, n_T\frac{\gamma_n}{\bar{\gamma}}\right) - Q\left(\mathcal{K}, n_T\frac{\gamma_{n+1}}{\bar{\gamma}}\right) \end{aligned} \quad (7)$$

where $Q(.,.)$ is the normalized incomplete gamma function defined as $Q(\alpha, x) \triangleq \Gamma(\alpha, x)/\Gamma(\alpha)$, whereas $\Gamma(.,.)$ is the incomplete Gamma function defined as $\Gamma(\alpha, x) \triangleq \int_x^{+\infty} t^{(\alpha-1)}\exp(-t)\,dt$ [8].

The average PER, $\langle PER \rangle$, has been defined in [2] based on the definition of the average bit error rate first adopted in [4]. It corresponds to the ratio of the average number of packets received in error over the total average number of transmitted packets. It is then given by

$$\langle PER \rangle = \frac{\sum_{n=1}^{N} R_n \langle PER \rangle_n}{\sum_{n=1}^{N} R_n a_n}, \quad (8)$$

where $\langle PER \rangle_n$ is the average PER corresponding to the choice of constellation size $M_n = 2^n$, R_n is the information rate of the combined AM and STBC in bits per channel use. Hence $R_n = R_c\frac{\log_2(M_n)/T_s}{W}$, T_s being the fixed symbol duration and W the signalling bandwidth. Assuming ideal Nyquist data pulses for each constellation, $W = 1/T_s$, yields $R_n = R_c\log_2(M_n)$.

In order to evaluate the average spectral efficiency $\langle SE \rangle$, we must account for the N_{arq}-truncated ARQ protocol implemented at the data link layer. Since each packet is transmitted up to $N_{arq}+1$ times, we can compute the average number of transmissions, needed for a packet to be either correctly received or discarded due to the truncated ARQ protocol, as a function of the maximum allowable number of retransmissions at the data link layer, N_{arq}. This average number, $\langle N \rangle_{N_{arq}}$, is given by (similar to [2, Eq. (10)] where a SISO channel is considered)

$$\begin{aligned} \langle N \rangle_{N_{arq}} &= 1 + \langle PER \rangle + \langle PER \rangle^2 + \ldots + \langle PER \rangle^{N_{arq}} \\ &= \frac{1 - \langle PER \rangle^{N_{arq}+1}}{1 - \langle PER \rangle}. \end{aligned} \quad (9)$$

As can be seen, when $N_{arq} = 0$ (no retransmission), $\langle N \rangle_{N_{arq}} = 1$, which corresponds to the case of adaptive modulation only and no ARQ.

The average spectral efficiency $\langle SE \rangle$ [2,5] is the sum of the information rates of each modulation level weighted by the probability a_n that the received SNR falls within the corresponding SNR region, as defined by (7), divided by the average number of transmissions per packet, $\langle N \rangle_{N_{arq}}$, it is then given by

$$\langle SE \rangle = \sum_{n=1}^{N} R_n a_n / \langle N \rangle_{N_{arq}}. \quad (10)$$

Since no data are transmitted when the received SNR falls below the threshold γ_1, the probability of outage, P_{out}, induced by the adaptive M-QAM modulation is given by

$$\begin{aligned} P_{out} &= Prob\{\gamma^{STBC} < \gamma_1\} \\ &= 1 - Q\left(\mathcal{K}, n_T\frac{\gamma_1}{\bar{\gamma}}\right). \end{aligned} \quad (11)$$

The probability of outage, $P_{out,N_{arq}}$, given up to N_{arq} allowable retransmissions at the data link layer, can then be expressed as follows

$$\begin{aligned} P_{out,N_{arq}} &= P_{out}\left[1 + \langle PER \rangle + \ldots + \langle PER \rangle^{N_{arq}}\right] \\ &= P_{out}\,\langle N \rangle_{N_{arq}}. \end{aligned} \quad (12)$$

Note that for the no-retransmission case ($N_{arq} = 0$), the probability of outage reduces to that of a fading channel subject to physical layer adaptive modulation, as reported in [4].

5. NUMERICAL RESULTS

Hereafter, we consider a full rate space-time block code with the simplest antenna configuration, i.e., $R_c = 1$, $n_T = 2$ and $n_R = 1$. Three values for the maximum allowable number of retransmissions N_{arq} are considered, namely $N_{arq} \in \{0, 1, 3\}$. Let $N_p = 1080$ be the number of bits per packet. The resulting set of parameters $\{a_n, g_n, \gamma_{pn}\}$ are summarized in Table I. Note that we found different values for the approx-

TABLE I

SET OF MODULATION-LEVEL AND PACKET-SIZE DEPENDENT CONSTANTS $\{a_n, g_n, \gamma_{pn}\}$ RESULTING FROM THE PER APPROXIMATION FOR $N_p = 1080$ BITS.

	BPSK	QPSK	8-QAM	16-QAM
a_n	107.95	109.06	93.44	85.01
g_n	1.0224	0.5117	0.1706	0.1025
γ_{pn}(dB)	6.7	9.7	14.7	16.7
	32-QAM	64-QAM	128-QAM	256-QAM
a_n	74.41	67.46	61.07	56.92
g_n	0.0394	0.0244	0.0097	0.0061
γ_{pn}(dB)	20.6	22.7	26.6	28.6

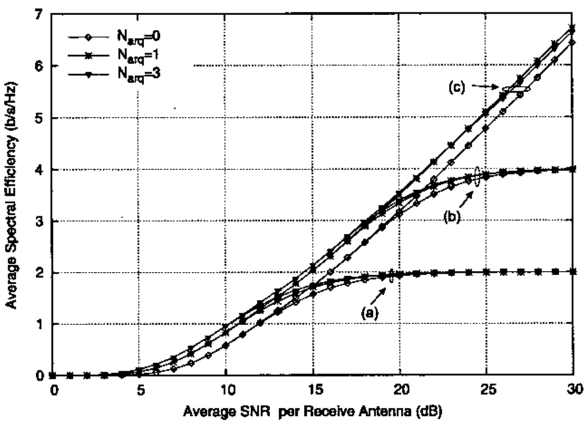

Fig. 2. Average spectral efficiency for different values of N_{arq} and different numbers of M-QAM constellations, namely, (a) N=2, (b) N=4 and (c) N=8.

imation parameters to those proposed in [2]. We believe that the proposed parameters yield a more accurate approximation for the exact PER expression given by (3).

Figure 2 shows the average spectral efficiency results as given by (3) for the three considered values of N_{arq} and for different numbers of M-QAM constellations. As can be observed from this figure, increasing the number of packet retransmissions yields an increased average spectral efficiency. This is especially true for $N_{arq} = 1$, that is with one retransmission allowed. This gain is more pronounced for low SNR in the case of a small number of M-QAM constellation (N=2) since for high SNR the curves corresponding to the different values of N_{arq} tend to converge towards the same value. Indeed, this is due to the fact that in high SNR the largest signal constellation available is transmitted, and hence using ARQ does not result in any advantage. Further increasing N_{arq} does not yield an increased gain in terms of spectral efficiency as is illustrated by the curve corresponding to $N_{arq} = 3$, which almost coincide with that of $N_{arq} = 1$. Figure 3 shows the outage probability given by (12) for the three values of the ARQ parameter. This figure clearly indicates how the use of ARQ diminishes the outage probability suffered by the system, and which arises because of the fact that adaptive modulation imposes that no packets are transmitted whenever the SNR falls below the switching threshold represented by γ_1.

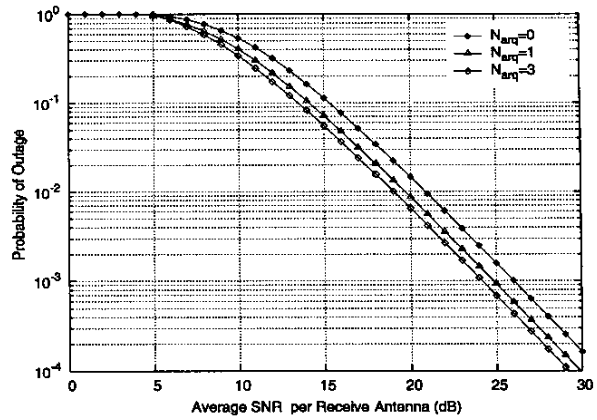

Fig. 3. Probability of outage P_{out} for different values of the ARQ parameter N_{arq}.

6. CONCLUSION

In this paper, we proposed a cross-layer design for MIMO diversity systems using STBC, extending the work in [2] for the SISO channel. Our proposed cross-layer design considers a frequency-flat independent Rayleigh fading channel with multiple transmit antennas and possibly multiple receive antennas, and combines the use of adaptive M-QAM at the physical layer with that of truncated ARQ at the data link layer. Results provided herein show how the proposed cross-layer design achieves higher performance than adaptive modulation used without ARQ retransmissions in the presence of STBC.

References

[1] S. Aïssa, A. Maaref and P. Mermelstein, "Uplink Packet Scheduling in the Presence of Interference Cancellation in Multi-Rate Wireless CDMA Networks," *Wiley Journal of Wireless Communications and Mobile Computing*, SI on Radio Resource Management, pp. 861-878, 2003.

[2] Q. Liu, S. Zhou and G.B. Giannakis, "Cross-Layer Combining of Adaptive Modulation and Coding with Truncated ARQ over Wireless Links," *IEEE Transactions on Wireless Communications*, to appear.

[3] K. Cho and D. Yoon, "On the General BER Expression of One- and Two-Dimensional Amplitude Modulations," *IEEE Transactions on Communications*, vol. 50, no. 6, pp. 1074–1080, July 2002.

[4] M.-S. Alouini and A.J. Goldsmith, "Adaptive Modulation over Nakagami Fading Channels," *Kluwer Journal on Wireless Communications*, vol. 13, no. 1-2, pp. 119–143, May 2000.

[5] S. Vishwanath and A.J. Goldsmith, "Variable-Rate Variable Power MQAM for Fading Channels," *IEEE Transactions on Communications*, vol. 45, no. 10, pp. 1218–1230, October 1997.

[6] 3GPP TR 25.848 V40.0, "Physical layer aspects of UTRA high speed downlink packet access," *release 4*, 2001.

[7] S. Sandhu and A. Paulraj, "Space Time Block Codes : A Capacity Perspective," *IEEE Communications Letters*, vol. 4, no. 12, pp. 384–386, December 2000.

[8] I.S. Gradshteyn and I.M. Ryzhik, "Table of Integrals, Series, and Products," *Academic Press, San Diego, California*, Sixth Edition, 2000.